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Validation of EDQNM for subgrid and supergrid Models

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1. Introduction

The advantage of two-point closure models over one-point closure models is that they retain one of the most important turbulence characteristics, the energy transfer mechanism among eddies of different sizes. These models require closure hypotheses on the higher order terms. Among the different models derived by the direct-interaction approximation, DIA, of Kraichnan the simplest one is the eddy-damped quasi-normal Markovianized, EDQNM, theory of Orszag (1970). It requires a very simple numerical scheme to solve the so-called triadic integral. This closure is based on the quasi-normal approximation in the equation for the third-order cumulants where fourth-order terms are replaced by products of second order terms. The approximation gives an unrealistic increase of the third order cumulant and leads to negative energies. The introduction of an eddy damping term eliminates that unphysical effect.

2. Isotropic Turbulence

When a particular transformation (Crocco & Orlandi, 1985) for the wave numbers is introduced, the EDQNM expression of the energy transfer term is

$$T(k) = \frac{1}{k}\int_0^1 d\beta \int_{1-\beta}^{1+\beta} d\gamma [k^3\Sigma(k,\beta k,\gamma k) + (\frac{k}{\beta})^3\Sigma(k,k/\beta,\gamma k/\beta)] \tag{1}$$

where the integration is to be performed only in a triangle in the (β,γ) plane. $\Sigma(k,p,q)$ is given by

$$\Sigma(k,p,q) = k^3 pq B(k,p,q) D(k,p,q)\Big[\frac{E(p)}{p^2} - \frac{E(k)}{k^2}\Big]\frac{E(q)}{q^2} \tag{2}$$

where $B(k,p,q)$ is a geometrical factor and $D(k,p,q)$ is the relaxation frequency that results from the Markovianization of a certain integral:

$$D(k,p,q) = \frac{1 - e^{-[\eta(k)+\eta(p)+\eta(q)]t}}{\eta(k)+\eta(p)+\eta(q)}$$

The damping function $\eta(k)$ completes the EDQNM closure and the expression generally adopted is

$$\eta(k) = \nu k^2 + \lambda\Big(\int_0^k p^2 E(p) dp\Big)^{\frac{1}{2}}$$

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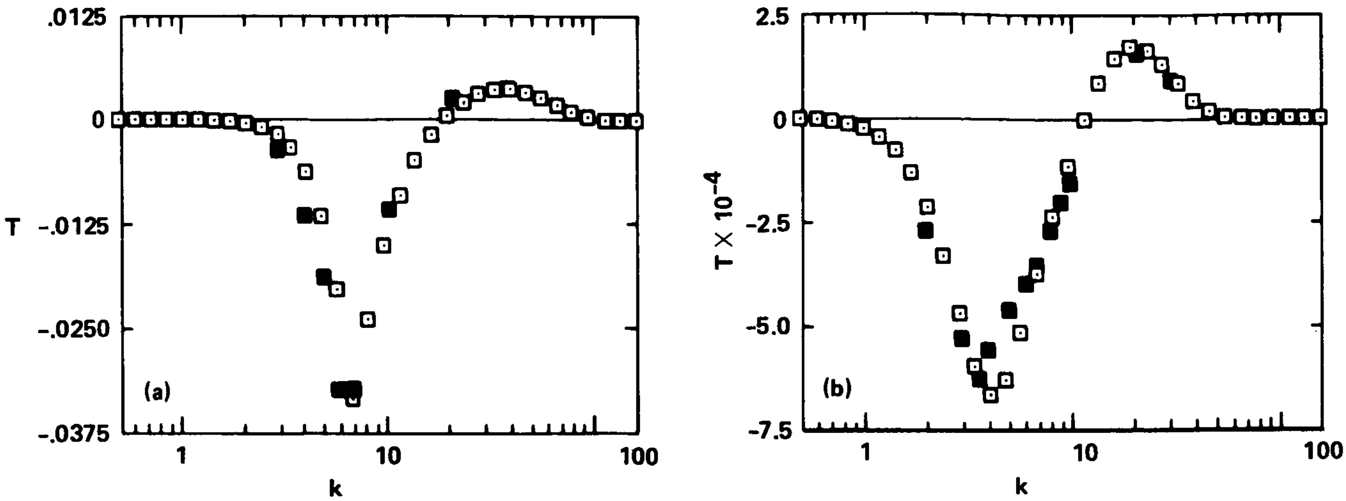

FIGURE 1. Energy transfer distribution: (a) $t = 0.54$, (b) $t = 4.34$, □ EDQNM , ■ DS.

Comparisons between EDQNM and direct simulation, DS, are possible only at very low Reynolds numbers. In the past these comparisons were limited to the evolution in time of globally averaged quantities such as turbulent energy , dissipation and skewness . In the first part of the CTR summer program a detailed comparison of the energy transfer between scales in a DS and the transfer given by EDQNM has been made. Only the accurate prediction of the energy transfer distribution is sufficient to demonstrate that the EDQNM closure describes correctly the complex mechanism of energy transfer among different scales. An even better check of the closure would be obtained by comparison of the detailed interaction distribution $\Sigma(k,p,q)$ in the (β,γ) plane. This has been done by Wray, but the evaluation of $\Sigma(k,p,q)$ within a DS requires $O(N^2)$ operations and the calculation was thus limited to a simulation of only $N = 32^3$ nodes. The resulting Σ distribution was very sparse and it was difficult to infer a continuous distribution from a single realization. Once the transfer $T(k)$ is determined, the energy spectrum evolution is obtained by solving

$$\frac{\partial E(k)}{\partial t} + 2\nu k^2 E(k) = T(k) \quad . \tag{3}$$

Starting from the initial spectrum used in the DS, (3) together with (1) and (2) have been solved. figure 1 shows the energy transfer term at two different times and indicates very good agreement with the DS results.

The ability of the EDQNM closure to produce accurate transfer spectra encourages its use as a closure model (subgrid or supergrid) in a DS. A simulation at high Reynolds numbers, with an inertial range extending for at least one decade, might then be obtained by introducing both a subgrid and a supergrid model to account for the transfer between the computed scales and the unresolved scales both at low and high wave numbers. The energy spectrum is then subdivided into three regions

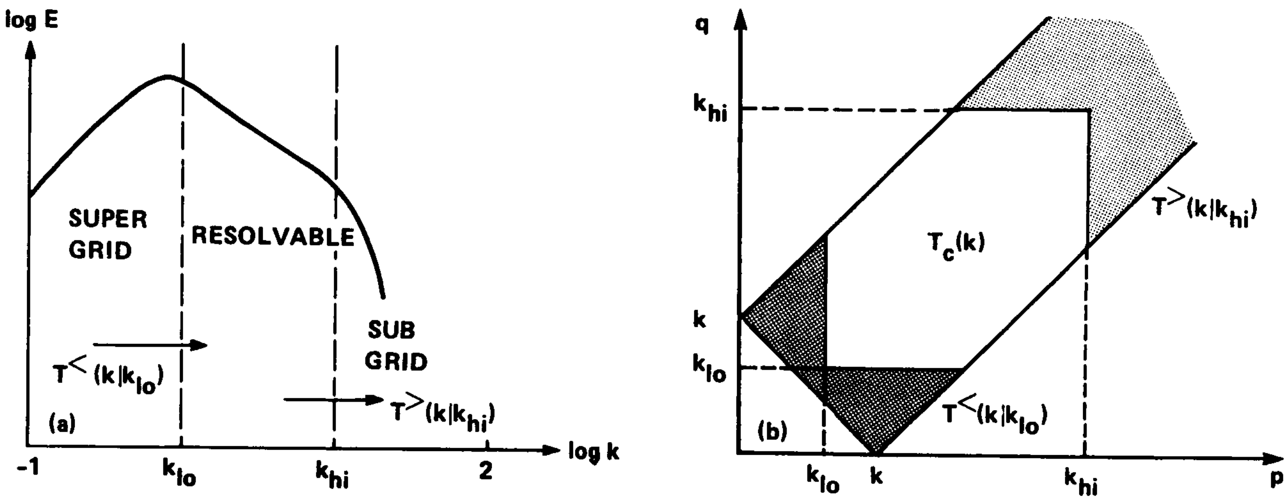

FIGURE 2. (a) Partition of the energy spectrum into resolved and unresolved scales. (b) Partition of the domain of integration for $\Sigma(k,p,q)$.

as figure 2a shows. In the DS range $k_{lo} < k < k_{hi}$ the energy transfer can be decomposed into two parts,

$$T(k) = T_c(k) + T_{sgs}(k|k_{lo}, k_{hi}),$$

where $T_c(k)$ is the transfer due to interactions between wave numbers within the DS, $T_{sgs}(k|k_{lo}, k_{hi})$ is the transfer due to interactions involving wave numbers $p, q < k_{lo}$ (supergrid $T^<(k|k_{lo})$), and interactions involving wave numbers $p, q > k_{hi}$ (subgrid $T^>(k|k_{hi})$). The transfers due to interactions outside the DS are evaluated by (1). figure 2b shows the contribution of supergrid and subgrid ranges to the integral (1). When k is very close to k_{hi}, interactions between sub- and supergrid scales occur, and these can be easily calculated.

The integral (1) is calculated by discretizing the triangle into quadrilaterals of different sizes whose areas are related to the number of points per octave used to represent the energy spectrum. Thus, at each point inside the domain in the (β, γ) plane, $\Sigma(k,p,q)$ represents the interaction of wave numbers p and q. When $T^<(k|k_{lo})$ (supergrid) is calculated, $\Sigma(k,p,q)$ is set to 0 unless p or q is less than k_{lo}. In a similar way when $T^>(k|k_{hi})$ is calculated, $\Sigma(k,p,q)$ is set to 0 unless p or q is greater than k_{hi}.

To compare the transfers due to unresolved scales obtained by the EDQNM integral with those obtained by DS, the flowfield of a DS with a 128^3 resolution has been considered and the transfers across several cutoff wave numbers k_c, have been calculated. Figures 3 and 4 show that the distributions of $T(k)$, $T^<(k|k_c)$, and $T^>(k|k_c)$ obtained from DS are in very good agreement with those calculated by EDQNM at both small and large times. As a result of this very good agreement it is hoped that a DS can be used to simulate decaying turbulence at high Re. At

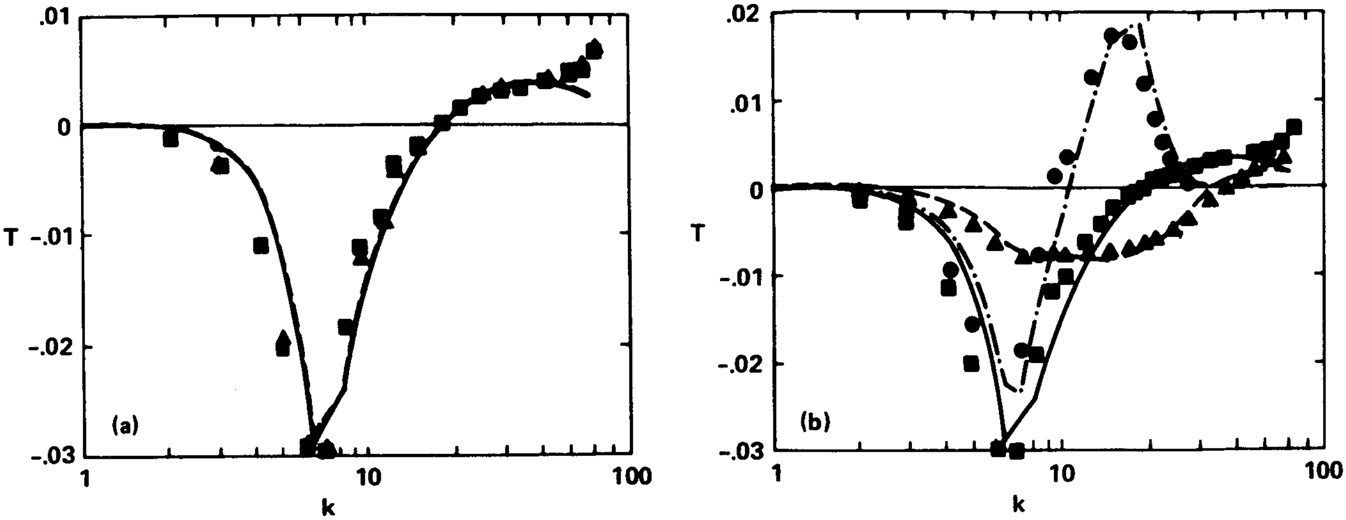

FIGURE 3. Energy transfer distribution at $t = 0.54$ (a) $k_c = 4$, (b) $k_c = 16$: EDQNM —— $T(k)$, —·— $T^{>}(k|k_c)$, ---- $T^{<}(k|k_c)$; DS □ $T(k)$, ○ $T^{>}(k|k_c)$, △ $T^{<}(k|k_c)$.

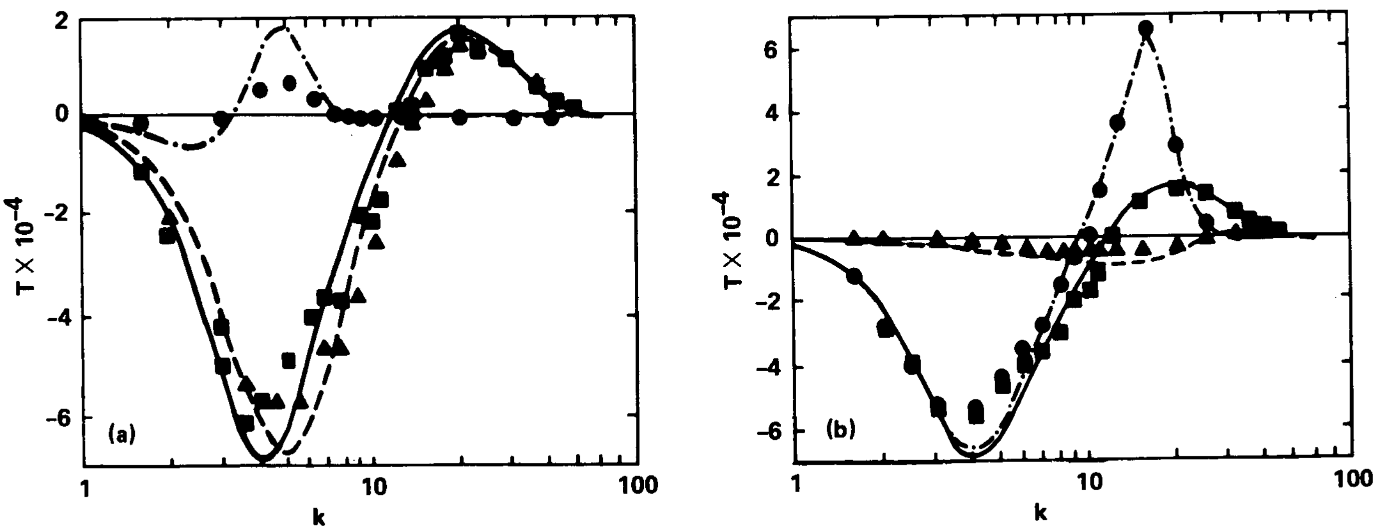

FIGURE 4. Energy transfer distribution at $t = 4.34$ (a) $k_c = 4$, (b) $k_c = 16$: EDQNM —— $T(k)$, —·— $T^{>}(k|k_c)$, ---- $T^{<}(k|k_c)$; DS □ $T(k)$, ○ $T^{>}(k|k_c)$, △ $T^{<}(k|k_c)$.

each time step the EDQNM calculation with $k_{min} << k_{lo}$ and $k_{max} >> k_{hi}$ gives the transfers $T^{<}(k|k_{lo})$ and $T^{>}(k|k_{hi})$ necessary to drive the simulation at the large scales and remove energy at the small ones.

Table 1.
Comparison between RDT and DS

case RR128: $S = 28.28, \nu = 0.01$

	$St=0$	$St=2$		$St=4$		$St=6$		$St=8$		$St=10$	
		RDT	DS	RDT	DS	RDT	DS	RDT	DS	RDT	DS
q^2	20.89	15.42	15.12	18.74	20.42	24.77	25.70	33.34	31.11	44.42	34.76
$\overline{u_2'^2}/q^2$	.373	.428	.477	.502	.628	.534	.724	.544	.779	.525	.819
$\overline{u_3'^2}/q^2$	.329	.242	.195	.181	.068	.166	.027	.170	.012	.191	.007
$\overline{u_1'^2}/q^2$	.308	.329	.328	.316	.304	.299	.249	.286	.207	.284	.174
$\overline{u_2'u_3'}/\sqrt{\overline{u_2'^2}\,\overline{u_3'^2}}$	.0	.535	.596	.579	.740	.558	.797	.528	.812	.498	.820

Case S64NJ: $S = 10.00, \nu = 0.02$

	$St=0$	$St=2$		$St=4$		$St=6$	
		RDT	DS	RDT	DS	RDT	DS
q^2	20.89	4.29	5.19	4.10	5.02	4.81	5.16
$\overline{u_2'^2}/q^2$	.373	.452	.461	.535	.598	.601	.699
$\overline{u_3'^2}/q^2$	.329	.259	.165	.166	.047	.125	.018
$\overline{u_1'^2}/q^2$	.308	.302	.374	.299	.355	.273	.283
$\overline{u_2'u_3'}/\sqrt{\overline{u_2'^2}\,\overline{u_3'^2}}$	.0	.539	.843	.607	.645	.602	.471

3. Anisotropic Turbulence (rapid distortion case)

In the case of anisotropic turbulence the expression for the energy transfer term is much more difficult to derive and it is not clear whether the eddy damping term should have a tensorial or a scalar form. The time evolution of the correlations consists in part of interactions between turbulence and mean fields and in part of higher correlations. If interactions among the turbulent fields (slow terms) are neglected, a calculation based on rapid-distortion theory, RDT, is possible. It is convenient to work in a reference frame in which, for turbulence with some symmetry, all the second order correlations can be derived from three quantities N_1, N_2, N_3. In the simple case of a shear flows with $S = \partial U_2/\partial x_3$ the equations are (Craya 1958):

$$\begin{aligned}
\frac{\partial N_1}{\partial t} + 2\nu k^2 N_1 - 2S\frac{k_2k_3}{k^2}N_1 - Sk_2\frac{\partial N_1}{\partial k_3} &= \Omega_1(k,\theta,\varphi),\\
\frac{\partial N_2}{\partial t} + 2\nu k^2 N_2 - 2S\frac{k_1}{k}N_3 - Sk_2\frac{\partial N_2}{\partial k_3} &= \Omega_2(k,\theta,\varphi),\\
\frac{\partial N_3}{\partial t} + 2\nu k^2 N_3 - S\frac{k_2k_3}{k^2}N_3 - S\frac{k_1}{k}N_1 - Sk_2\frac{\partial N_3}{\partial k_3} &= \Omega_3(k,\theta,\varphi),
\end{aligned} \tag{4}$$

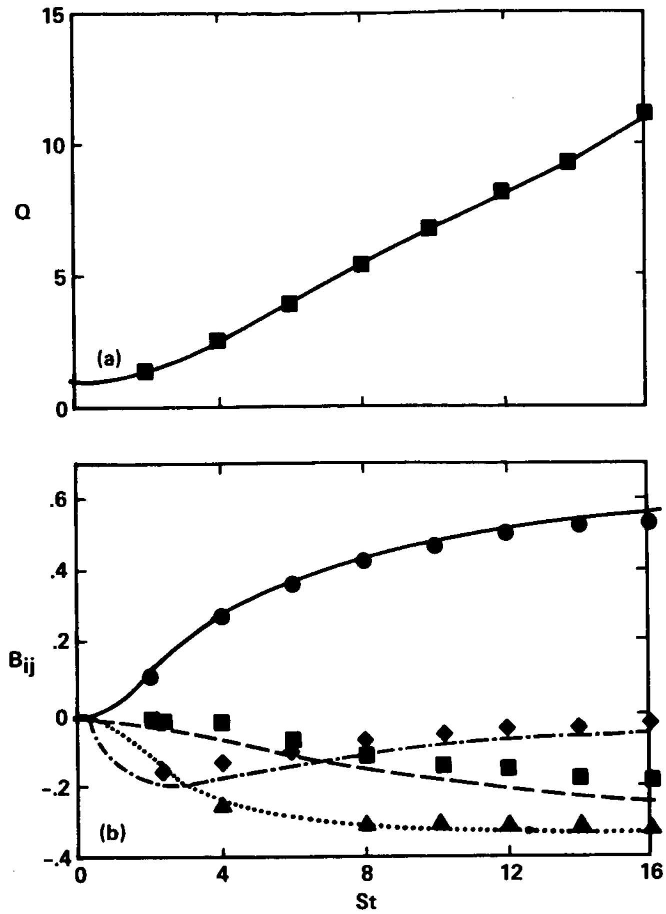

FIGURE 5. Evolution of (a) turbulent kinetic energy, (b) Reynolds-stress anisotropy tensor. The present results are: —— B_{22}, ---- B_{11}, ······ B_{33}, —·— B_{23}. The solid symbols are the corresponding DS results.

where $\Omega_l(k, \theta, \varphi)$ are the non-linear terms that are discarded in RDT. Introducing the modified wave number system $(k_1, k_2, k_3 + Stk_2)$ the system of equations (4) can be easily solved.

Lee et al (1987) performed a DS by following the isotropic decay of an initial spectrum until it reaches a value for the skewness of about -0.5. This field was then used as the initial condition for a highly sheared simulation. Flow field structures similar to those found near the walls of a channel are obtained even when the non-linear terms are neglected. We calculated this case by using EDQNM for the isotropic decay and RDT for the high shear evolution. figure 5 shows good agreement between the present results and those obtained by DS. A further comparison between the RDT calculations and cases S64NJ and RR128 of Rogers et al (1986) has been completed. Table 1 shows that due to the predominant effect of the viscosity on the shear, the RDT calculation predicts the behavior of the total energy also at the later times, St, but it does not predict well the time evolution of each component. This is consistent with the analysis done by Brasseur during the CTR summer school. He analyzed the effects of the slow pressure terms and found that these withdraw energy from $\overline{u_2'^2}$, transfer it to $\overline{u_3'^2}$ and from there to $\overline{u_1'^2}$ .Our calculation shows a faster decay of the $\overline{u_3'^2}$ than the one obtained by DS. On the contrary $\overline{u_1'^2}$ agrees with the DS because the amount of energy it is receiving from $\overline{u_2'^2}$ is comparable to the amount of energy it is transfering to $\overline{u_3'^2}$.

4. Conclusions

From these preliminary calculations we conclude that the two-point EDQNM closure accurately describes the behavior of second order moments. This closure can be applied as subgrid and supergrid models for Large Eddy Simulations at higher Reynolds numbers. In the case of homogeneous anisotropic turbulence, when the non-linear terms are introduced the calculation becomes quite onerous but is still considerably less expensive than the calculation of a DS. The major merit of two-point closure models is that they can be easily applied to flows at Reynolds numbers that are unreachable by a DS. Work is in progress to derive expressions for the non-linear terms that give good global conservation properties.

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