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Modeling scalar flux and the energy and dissipation equations

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Closure models derived from the Two-Scale-Direct-Interaction Approximation have been compared with data from direct simulations of turbulence. In the working session, we have restricted our attention to 1) anisotropic scalar diffusion models, 2) models for the energy dissipation equation, and 3) models for energy diffusion.

1. Anisotropic eddy-diffusivity model for turblent scalar flux

The scalar flux is represented by a gradient diffusion model

$$\overline{u_i'\theta'} = -D_{ij}\frac{\partial\overline{\theta}}{\partial x_j}$$

with a diffusivity tensor D_{ij} that depends on the mean strain and vorticity tensors (Yoshizawa, 1985).

$$D_{ij} = C_K\frac{k_\theta^2\epsilon}{\epsilon_\theta^2}\delta_{ij} - \frac{k_\theta^3\epsilon}{\epsilon_\theta^3}[C_{KA}(\frac{\partial\overline{u}_i}{\partial x_j} + \frac{\partial\overline{u}_j}{\partial x_i}) + C'_{KA}(\frac{\partial\overline{u}_i}{\partial x_j} - \frac{\partial\overline{u}_j}{\partial x_i})].$$

The accuracy of the model is shown in table 1 by comparison at several times t of the actual fluxes with the modeled fluxes in a direct numerical simulation of homogeneous turbulence in uniform shear S having a uniform scalar gradient. The scalar diffusivities D_{22}, D_{12}, and D_{21} are represented well by the model but D_{33} and D_{11} are not. The performance of the model might be improved by the inclusion of unsteady terms in C_K suggested by the TSDIA analysis.

$$C_K \longrightarrow C_K + a\frac{1}{\epsilon_\theta}\frac{\partial k_\theta}{\partial t} + b\frac{k_\theta}{\epsilon_\theta^2}\frac{\partial\epsilon_\theta}{\partial t} + c\frac{k_\theta}{\epsilon_\theta\epsilon}\frac{\partial\epsilon}{\partial t}$$

Table 1. Evaluation of the Scalar Diffusion Model from case C128U of Rogers, Moin, and Reynolds (1986) ($C_K = .187$, $C_{KA} = .132$, $C'_{KA} = .06$)

St	D_{22}		$-D_{12}/D_{22}$		$-D_{21}/D_{22}$		D_{33}/D_{22}		D_{11}/D_{22}	
	model	data	model	data	model	data	model	data	model	data
8	.068	.090	1.97	2.38	1.32	1.20	.737	1.98	.936	5.58
10	.102	.108	2.74	2.63	1.27	1.23	.695	1.98	.826	6.52
12	.150	.146	2.54	2.56	1.24	1.23	.663	1.82	.760	6.60
14	.205	.194	2.60	2.45	1.17	1.23	.649	1.77	.717	6.44
16	.250	.265	2.51	2.21	1.19	1.15	.634	1.66	.746	5.77

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2. A model for the dissipation of kinetic energy

Here we contrast the familiar $k - \epsilon$ model (model 1),

$$\frac{D\epsilon}{Dt} = C_{\epsilon 1}\frac{\epsilon}{k}P - C_{\epsilon 2}\frac{\epsilon^2}{k} + \frac{\partial}{\partial x_i}(C_{\epsilon\epsilon}\frac{k^2}{\epsilon}\frac{\partial \epsilon}{\partial x_i}),$$

$$C_{\epsilon 1} \simeq 1.4,\ C_{\epsilon 2} \simeq 1.9,\ C_{\epsilon\epsilon} \simeq 0.07,$$

with the model derived via the TSDIA approach (model 2; Yoshizawa, 1987) ,

$$\frac{D\epsilon}{Dt} = C_{\epsilon 1}\frac{\epsilon}{k}P - C_{\epsilon 2}\frac{\epsilon^2}{k} + C'_{\epsilon 1}k(\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \overline{u}_j}{\partial x_i})^2 + \bullet\bullet\bullet \ ,$$

$$C_{\epsilon 1} = C_{\epsilon 2} \simeq 1.70.$$

Note that for both models, the eddy-viscosity approximation implies that

$$C_{\epsilon 1}\frac{\epsilon}{k}P = C_{\epsilon 1}C_\nu k(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i})^2 \ ,$$

where $C_\nu \simeq 0.09$. For homogeneous turbulence in uniform shear S, model 1 reduces to

$$\frac{\partial \epsilon}{\partial t} = C_{\epsilon 1}\frac{\epsilon}{k}P - C_{\epsilon 2}\frac{\epsilon^2}{k} \ ,$$

and model 2 reduces to

$$\frac{\partial \epsilon}{\partial t} = C_{\epsilon 1}\frac{\epsilon}{k}P - C_{\epsilon 2}\frac{\epsilon^2}{k} + 2C'_{\epsilon 1}kS^2 \ .$$

These two models were tested against the homogenous shear turbulence fields of Rogers et al (1986) for $8 \leq St \leq 14$. The resulting "constants" were found to be $.97 \leq C_{\epsilon 1} \leq 1.2$ for model 1, and $1.7 \leq C_{\epsilon 1} \leq 1.9$, $-.025 \leq C'_{\epsilon 1} \leq -.018$ for model 2. Note that the negative value of $C'_{\epsilon 1}$ implies that the effect of rotation, given by the third term of the model, acts to reduce the dissipation rate. The simulation data also support the relationship $C_{\epsilon 1} = C_{\epsilon 2} \simeq 1.7$ suggested by TSDIA.

3. A model for the diffusion of kinetic energy

The diffusion term

$$D_k = \frac{\partial}{\partial x_i}(\frac{1}{2}\overline{u'_i u'_j u'_j} + \overline{p' u'_i})$$

in the equation for kinetic energy is usually modeled as (model 1)

$$D_k = -\frac{\partial}{\partial x_j}(C_K\frac{k^2}{\epsilon}\frac{\partial k}{\partial x_j}),$$

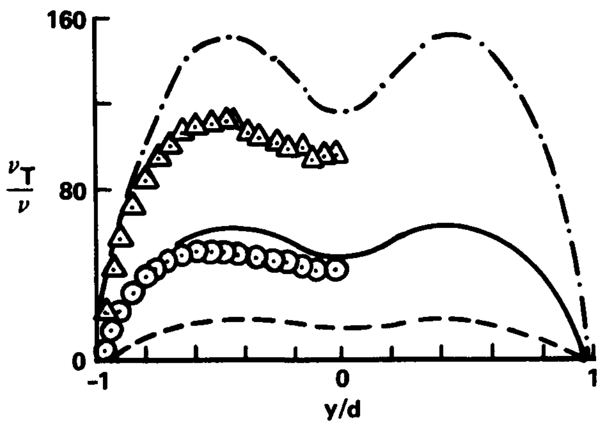

FIGURE 1. The eddy viscosity distribution in turbulent channel flow. Experimental data of Hussain & Reynolds: $\triangle$ $R = 32300$, $\circ$ $R = 13800$. Model: —·— $R = 32454$, —— $R = 12581$, ---- $R = 3666$.

whereas the TSDIA analysis (Yoshizawa, 1982) indicates the presence of a cross-diffusion term

$$D_k = -\frac{\partial}{\partial x_j}(C_{KK}\frac{k^2}{\epsilon}\frac{\partial k}{\partial x_j}) + \frac{\partial}{\partial x_j}(C_{K\epsilon}\frac{k^3}{\epsilon^2}\frac{\partial \epsilon}{\partial x_j}).$$

These models have been compared with the turbulent channel flow data of Kim et al (1987) (hereafter KMM) for $100 < y^+ < 180$. The Reynolds number is 3300, based on channel half-height and centerline velocity, and the centerline is at $y^+ = 180$.

When the data was fit with a single term of the model, the constants were estimated to be $.11 \leq C_{KK} \leq .12$ when $C_{K\epsilon}$ was set to zero, and $.06 \leq C_{K\epsilon} \leq .08$ when C_{KK} was set to zero. At high Reynolds number, the eddy viscosity distribution ($\nu_T = \overline{u'v'}/S$) has maxima off the centerline of the channel. The cross-gradient term in model 2, when incorporated into a $k - \epsilon$ model, can produce the off-axis maxima whereas model 1 cannot. However, at the low Reynolds number of the simulation, the eddy viscosity did not exhibit the off-centerline maxima strongly enough to allow the two constants to be found simultaneously from the data alone.

If the constants are taken as $C_{KK} = .08$ and $C_{K\epsilon} = .03$, the locations of the maxima and their values are reproduced. The data of KMM indicate a maximum $\nu_T/\nu = 16$ at $y/d = \pm.5$ while the model gives a maximum of 18 at $y/d = \pm.47$. A comparison of the eddy-viscosity distribution of model 2 and experimental data at higher Reynolds numbers is shown in figure 1.

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