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Relations Between Two-Point Correlations and Pressure Strain Terms

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We study the structure of the two-point spatial correlations (velocity-velocity, velocity-scalar and scalar-scalar) with a view to improve turbulence closure models. The linear model for the two-point correlations proposed by Naot et al. provides a method of including the information about the turbulence structure in the turbulence models. We test the assumptions and adequacy of this model against the homogeneous shear flow simulation data base. The model performs poorly in some details and we suggest how it may be improved. We also test the models for rapid pressure-strain terms in a variety of flows including axisymmetric expansion and contraction flows, homogeneous shear flow, channel flow and boundary layer.

Introduction

Two-point correlations are often considered to include much information on the structure of turbulence, and on the modeling of various terms in the equations governing turbulent quantities such as the Reynolds stresses or the turbulent heat fluxes. In particular, the rapid pressure-velocity gradient terms in the Reynolds stress equations or pressure-temperature term in the heat flux equations, as well as the viscous decay terms of these equations may be exactly calculated if the corresponding two-point correlations are known with sufficient accuracy. It was therefore the purpose of the present project to study the two-point correlations, and to improve turbulence closure models for pressure terms by a better understanding of the two-point correlations.

The report is organized in three sections. The first section contains some general observations on two-point correlations; Section 2 contains the assessment of linear two-point correlation models, which is followed by our study of linear pressure-strain models in Section 3. Finally, some conclusions from the present work are presented.

1. Two-point correlations

Two-point correlations (velocity-velocity, velocity-scalar, scalar-scalar) were examined for homogeneous shear and channel flows. The primary focus was on the homogeneous shear case due to its simplicity. The numerical data base generated by Rogers et al. (1986) provided the "raw data." The C128 simulation series was studied in most detail. The simplest case is that of scalar-scalar correlation in a

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homogeneous shear flow. The flow is in the x-direction, with a uniform velocity gradient in the y-direction. Uniform scalar gradients were applied in the x-, y-, and z-directions, further referred to as scalars A, B, and C, respectively. Details of the simulations may be found in Rogers et al. (1986). Contour plots of the scalar-scalar two-point correlations in the x-y plane at nondimensional time $St = 12$ are shown in Figs. 1a, b, and c for the three corresponding temperature gradients. All cases show an inclination of about 15°-20° with respect to the x-axis. This inclination appears to represent the influence of dU/dy (mean shear). At earlier nondimensional times the correlations show a steeper angle. This behavior is similar to that observed by Rogers et al. (1986) for vorticity correlations. Correlation contours in the y-z plane (not shown) are nearly elliptical, with the direction of the scalar gradient defining the major axis. The influence of mean shear is also clearly evident in the contours of the velocity-scalar correlations $\overline{uB}$ and $\overline{wC}$ when plotted in the x-y plane (not shown). The influence of the mean scalar gradient on the velocity-scalar correlations is better seen in the y-z plane. In Fig. 2 the correlations $\overline{vB}$ and $\overline{wC}$ are plotted in the y-z plane at nondimensional time $St = 12$. The correlations decrease more slowly in the direction of the applied scalar gradient, a feature also noted in the scalar-scalar correlations.

The contours of the velocity-velocity two-point correlations $\overline{uu}, \overline{vv}$ and $\overline{ww}$ in plane x-y are shown in Fig. 3. While the $\overline{uu}$ and $\overline{ww}$ show an inclination of about 22° the $\overline{vv}$ does not show this orientation. This suggests a strong influence of the mean velocity gradient dU/dy on the velocity-velocity two point correlations. It may be noted that the derivation of the linear pressure-strain models (as presented by Naot *et al.*) assumes no direct dependence of the two point correlations on the mean velocity gradients.

Finally, the channel flow results of the two-point correlations with separation vector in the y-direction show asymmetries not possible in the homogeneous shear case. The influence of the wall appears to be quite persistent and leads to asymmetries in the correlations. It may be noted that some of these features are well described by a model proposed by Hunt (details may be found in his report in the present volume).

The principal conclusion from this part of the study is that the two-point correlations (velocity-velocity, velocity-scalar, and scalar-scalar) show strong dependence not only on their single-point analogs but also on the mean velocity gradients and mean scalar gradient. Thus models which inadequately represent these dependencies may not be very successful.

2. Linear two-point correlation model

The two-point correlations may be represented by a model in which they are linearly related to the corresponding single-point correlations by arbitrary functions of r, the magnitude of the separation vector only. One such function is required for scalar-scalar correlation, two for scalar-velocity correlations, and six for velocity-velocity correlations (but three of these are related to the other three by continuity relations. Such models have been proposed by Naot et al. (1973) for the velocity-velocity correlations and by Miklavic and Wolfshtein (1987) for the velocity-scalar and scalar-scalar correlations. We tried to fit the data for the homogeneous shear case to this model. Typical results for scalar-velocity correlation functions are shown in Fig. 4 for functions G and R.

The different curves correspond to different combinations of profiles of the two-point correlations (from simulations) used to obtain the model function. Considering function G, which contains the isotropic part of the correlation, most (but not all) profiles appear to give similar results for small separations, but not for large separations. The function R representing the non-isotropic part has small values for the small separations, and becomes important only for larger separations, and there it shows unacceptable scatter. Three velocity-velocity model functions (not shown) show a very similar behavior.

We did not have sufficient time to test the validity of the linear two-point correlation model in other flow fields. However, examination of the governing equations, as well as results on the rapid pressure-strain term (to be described in Section 3) suggest that the two-point correlation model may perform reasonably well for axially-symmetrical turbulence (in particular for compression).

The linear models tested here can be considerably improved by accounting for the dependence of the two-point correlations on the mean velocity gradients, and in addition on the mean scalar gradient for correlations involving the scalar fields. Our study suggests that both the irrotational and rotational components of the mean deformation rate should be included in such extensions. Detailed exploration of these possibilities was not conducted.

3. Linear pressure-strain model

We considered here the model of Naot, Shavit and Wolfshtein (1973, hereafter referred to as NSW) or that of Launder, Reece and Rodi (1975, hereafter referred to as LRR) (the two models are identical, although the derivation is quite different). In both models the rapid pressure-strain terms are related to the Reynolds stresses and velocity gradients by a single coefficient ϕ (for NSW) or C_2 (for LRR). The test here was to calculate the value of ϕ corresponding to different Reynolds stress components from the simulation data base for the rapid term in various flows.

In Figs. 5a and b, the calculated value of the NSW coefficient ϕ is plotted against total strain for the axisymmetric expansion and contraction flows, respectively. The data base used was from Lee et al. (1985) and the simulation details may be found there. The scatter is acceptable, but (at least in the compression) the value of ϕ changes with the strain (which corresponds to time in this case). This situation is typical for all the compression cases studied, but not to all expansion cases.

In the case of shear flows it was impossible to get a single value of ϕ (from different Reynolds stress components, indicated as the subscript on ϕ in the figures). The behavior of the homogeneous shear flow is very similar to the plane channel and boundary layer shown in Figs. 6a and b, respectively. The ϕ values obtained from different stress components differ a lot amongst each other but do not change much with the distance from the wall (in the log- layer). These results make the linear model unsuitable for shear flows. However, if we do not require tensorial symmetries in the model, it is possible to use different values of ϕ for different components. In this case it may be possible to use such a linear model.

We tried to seek a correlation of the total pressure terms with the three model constants suggested by NSW. The coefficients β, γ, Λ were computed for the homogeneous shear case. This model appears to be a logical choice, as most coefficients do not change rapidly as a function of the nondimensional time St.

An even better result is obtained if we consider the combined total pressure velocity and viscous decay terms.

Conclusions

We now summarize our conclusions from the present work. Linear two-point correlation models appear to be imperfect even for simple turbulent flows. For shear flows it is necessary to relate the two-point correlations not only to the Reynolds stresses, but also to *all* mean velocity gradients and mean scalar gradients. Even so, it may be necessary to use nonlinear modeling to account for asymmetries.

The current linear models for the pressure-strain terms (in the models considered) can work only if different values of the coefficient ϕ are used for each direction, but then the evolution of ϕ and its spatial variation is a serious problem. Considerable improvement may be obtained if we consider the total pressure-strain terms. The models work even better when the total pressure-strain terms are combined with the viscous decay terms.

With everything said, we should bear in mind that all these conclusions are based on low Reynolds number turbulence. It is desirable to confirm these conclusions by comparison with Large Eddy Simulations at higher Reynolds numbers.

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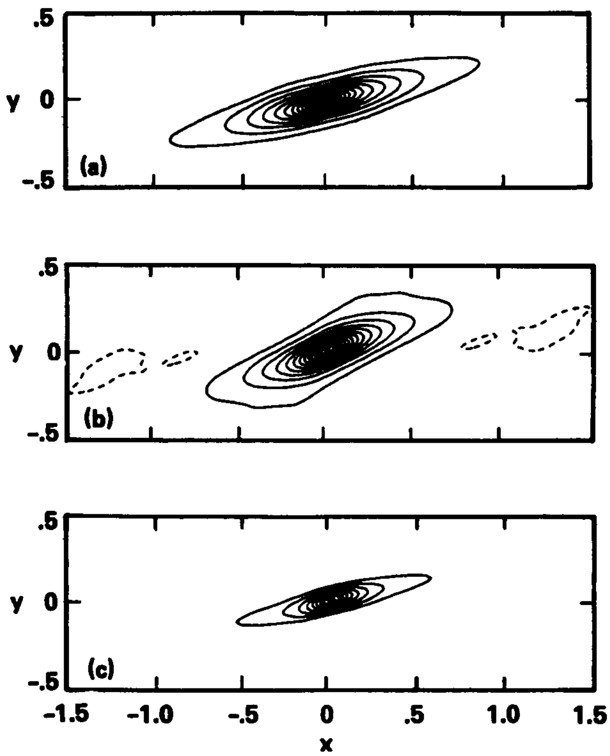

FIGURE 1. Iso-correlation contours of scalar-scalar two-point correlation in x-y plane at $St = 12$. a) Scalar gradient in x-direction (Scalar-A); b) Scalar gradient in y-direction (Scalar-B); c) Scalar gradient in z-direction (Scalar-C).

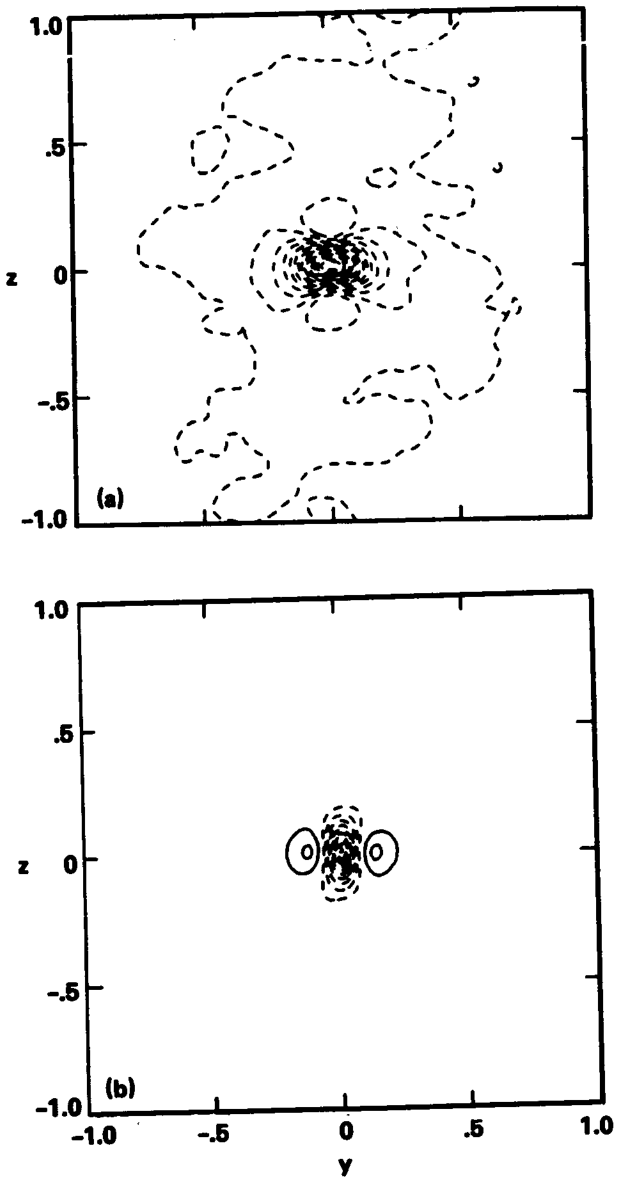

FIGURE 2. Iso-correlation contours of velocity-scalar two-point correlation in y-z plane at $St = 12$. a) $\overline{vB}$ correlation; b) $\overline{wC}$ correlation.

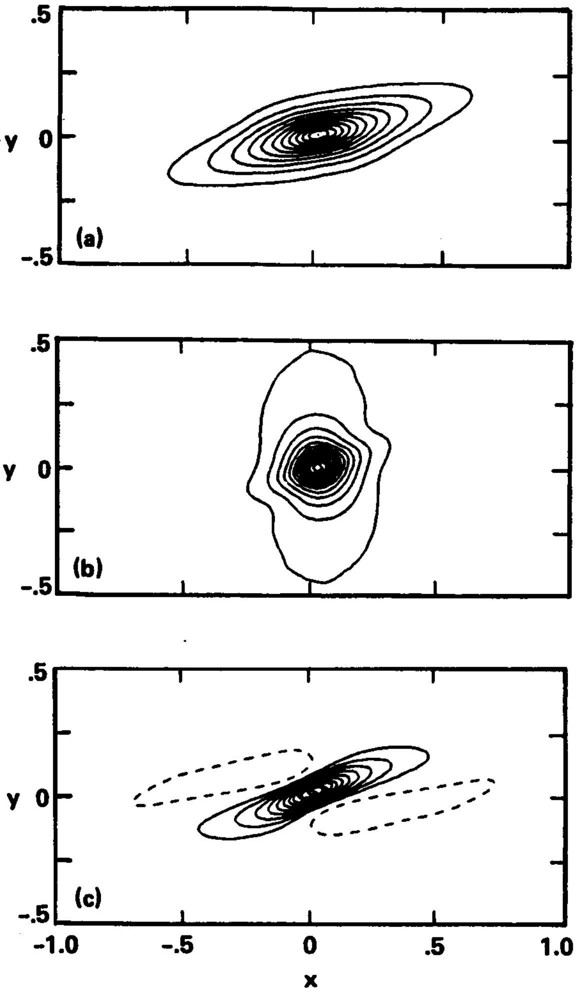

FIGURE 3. Iso-correlation contours of velocity-velocity two-point correlation in x-y plane at $St = 12$. a) $\overline{uu}$ correlation; b) $\overline{vv}$ correlation; c) $\overline{ww}$ correlation.

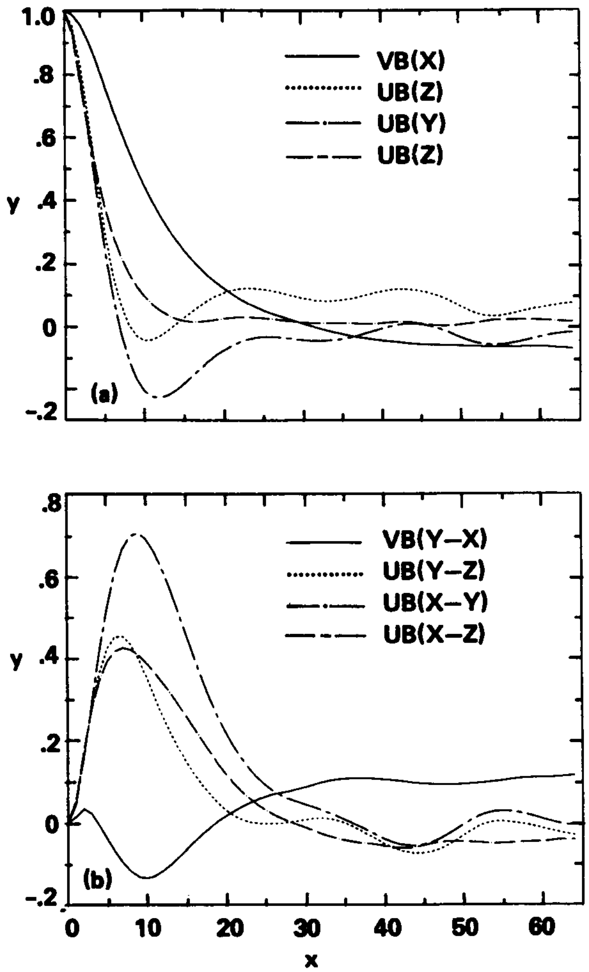

FIGURE 4. Two-point scalar-velocity correlation model functions for uniform shear flow at $St = 12$. a) Different estimates of the model function G; b) Different estimates of the model function R.

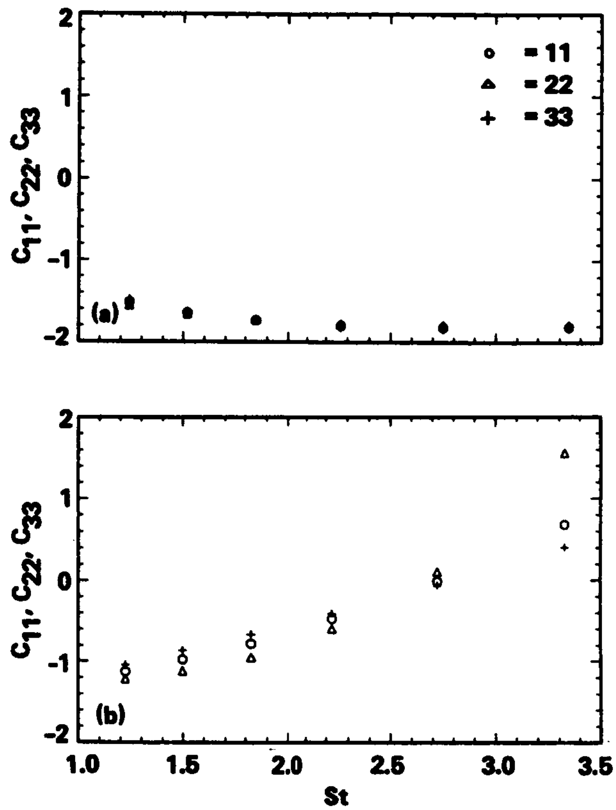

FIGURE 5. Values of the NSW coefficient ϕ in velocity-velocity two-point correlation model for axisymmetric turbulence. a) Expansion flow; b) Contraction flow.

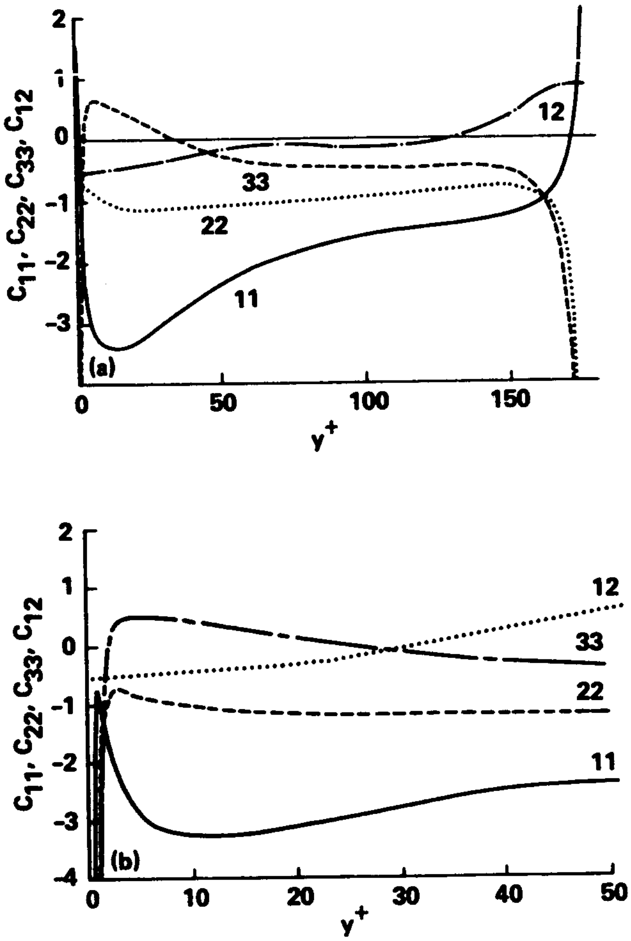

FIGURE 6. Values of the NSW coefficient ϕ in velocity-velocity two-point correlation model. a) Channel flow; b) Boundary layer.