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BASE REACTION OPTIMIZATION OF MANIPULATORS WITH REDUNDANT KINEMATICS

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Abstract

Manipulators used in space applications are operated under microgravity conditions. Base reactions of space manipulator are directly exerted on the supporting space structure. It is desirable to make these reactions as small as possible in order to reduce their influence on the dynamics of the supporting space structure. Furthermore, in delicate experiments conducted in space, the test specimen would have to be moved carefully without subjecting it to excessive accelerations and jerks. It follows that minimization of base reactions and limitation of end-effector accelerations and jerks are important objectives for space manipulators.

In this presentation, a trajectory generation method for space manipulators is introduced. The approach developed employs a manipulator with redundant kinematics. The method is implemented in two steps. First, the end-effector trajectory is developed to satisfy motion requirements. Next, the joint trajectories are developed to minimize base reactions. This presentation describes the analytical development of the method, and presents an example to illustrate the method.

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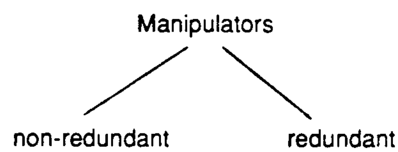
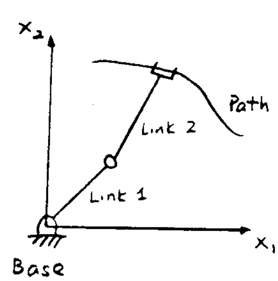
Types of Manipulators

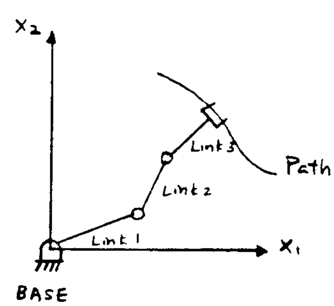
Manipulators can be categorized according to their degrees of freedom into two groups: nonredundant and redundant. Nonredundant manipulators have the minimum number of degrees of freedom required to follow a general trajectory. If a manipulator has more than the minimum number of degrees of freedom required to perform a task, then it is called a redundant manipulator.



2 DOF Manipulator

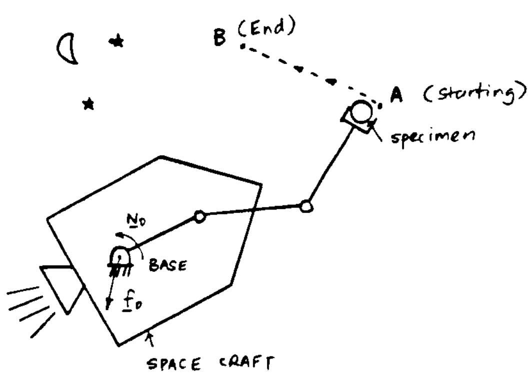


3 DOF Manipulator
(1 degree of redundancy)



Space Manipulator of NASA

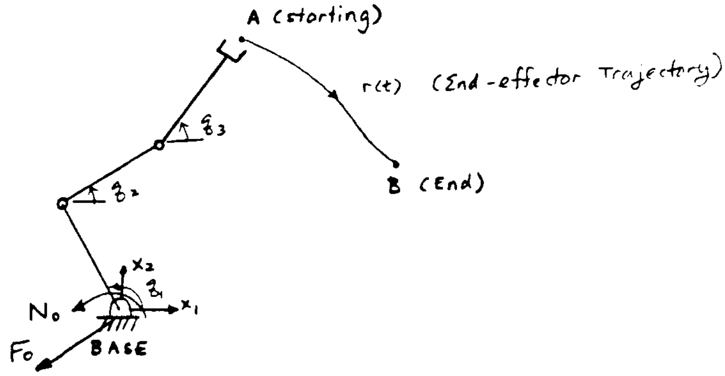
Manipulators used in space are operated under microgravity conditions. Base reactions of a space manipulator are directly exerted on the supporting space structure. It is desirable to make the reactions as small as possible in order to reduce their influence on the dynamics of the space structures. Besides the requirement of small base reactions, delicate experiments conducted in space require that test specimens be moved carefully without subjecting them to excessive accelerations and jerks.



- desire to move specimens without excessive acceleration
- require base reactions ($\underline{f}_0$, $\underline{N}_0$) to be as small as possible

Problem Statement

The trajectory problem for redundant manipulators which we are going to address has two requirements: (1) to move a redundant manipulator according to task specifications; (2) to minimize base reactions ($\mathbf{N}_0$ and $\mathbf{F}_0$) transmitted by manipulators to the base during motion.



A Planar Redundant Manipulator

- plan end-effector trajectory to satisfy acceleration constraint

- determine joint trajectories that MINIMIZE BASE REACTIONS

Proposed Approach

The trajecotry planning problem can be approached by splitting the problem into two parts. This enables us to deal with the end-effector trajectory and joint trajectories separately. The first part generates the end-effector trajectory that satisfies task specifications. The second part obtains joint space solution that minimizes base reactions.

Part 1

Determination of end-effector trajectory

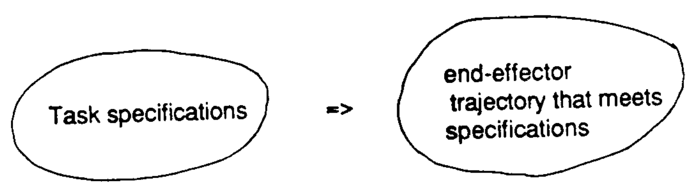
Part 2

Finding joint trajectories that minimize base reactions

Part 1: End-effector Trajectory Generation

The first part of the approach deals with the generation of end-effector trajectory, **r**(t) to satisfy certain task specifications. The task specifications of interest are total distance of the straight-line path (D_T), maximum acceleration of end-effector trajectory (a_{max}), and total time of task (T).

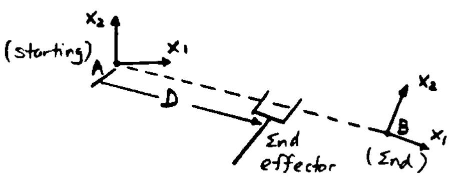
IDEA:



Task Specs.	End-effector
1. D_T : total distance of the straight path	r(t)
2. a_{max} : max. acceleration of the path	
3. T : time to accomplish the task	

Point-to-point Motion

A point-to-point motion is considered as the motion of an end effector moving from a specified initial position to a specified final position. A simple way to execute this motion is to move the origin of a coordinate frame fixed to the end effector along a straight-line path that connects the two points.



- accomplish by a straight line from A to B

$$v(t) = \frac{dD}{dt}$$ (speed along the straight path)

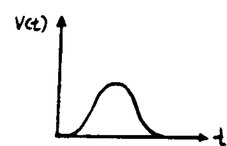
End-effector Trajectory Description

One of the requirements in planning end-effector trajectory is to have zero velocity at initial and final positions. Cycloid curve which satisfies this requirement can be used to describe the linear speed of the end-effector trajectory. Furthermore, it has smooth kinematics properties and can be defined by using only three constants (a, b, c).

Linear Speed:

Cycloid: $v(p) = b(1 - \cos p)$

$t(p) = a(p - c \sin p)$



P is the parameter.

$p = 0$ (starting position)

$p = 2\pi$ (end position)

Determination of a, b , and c

The three constants a, b, and c can be determined by forcing cycloid function to satisfy three motion constraints. By solving the following three equations, we can obtain the valus for the constants.

Total Time of Task : $T = 2 \pi a$

Total Distance : $D_T = 2 \pi a b (1 + 0.5 c)$

Max. Acceleration:

$$|a_{max}| = b / a (1 - c^2)^{-0.5}$$

From above 3 eqns. => a, b , c

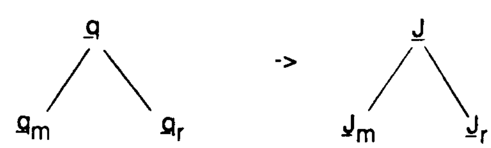
Part 2: Determination of Joint Trajectories

The basic idea of proposed approach is to pose the inverse kinematics problem as an optimization problem with a cost function that measures the base reactions. The approach begins by partitioning the joint variable vector, **q** into 2 portions. Then the Jacobian matrix is partitioned into a nonsingular square Jacobian matrix and a submatrix. Using these partitioned matrices, we are able to represent the motion of all the joints in terms of an optimization parameter matrix. The unique joint space solution can be determined by finding the optimal parameter matrix for the optimization problem.

Recall

$$\dot{\underline{r}} = \underline{J}\,\dot{\underline{q}} \qquad (1)$$

Step 1 : Partition $\underline{q}$ & $\underline{J}$



$(|\underline{J}_m| \neq 0)$

Step 2: Express $\dot{\underline{q}}_m$ & $\ddot{\underline{q}}_m$ *in terms of* $\dot{\underline{q}}_r$ *and* $\ddot{\underline{q}}_r$

$$\dot{\underline{q}}_m = g_1(\dot{\underline{q}}_r) \qquad (2)$$

$$\ddot{\underline{q}}_m = g_2(\dot{\underline{q}}_r, \ddot{\underline{q}}_r) \qquad (3)$$

Step 3 : Setup a cost function, $J_c(\dot{\underline{q}}_r, \ddot{\underline{q}}_r)$

Step 4: minimize J_c

$$\min(J_c) \Rightarrow \quad \dot{\underline{q}}_r, \ddot{\underline{q}}_r \quad \underset{(2,3)}{\Rightarrow} \dot{\underline{q}}_m, \ddot{\underline{q}}_m$$

Part 2 Determination of Joint Trajectories (cont'd)

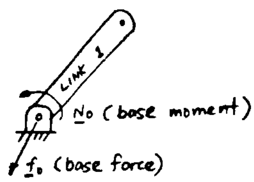
For the purpose of optimization,

$$\dot{\underline{q}}_r = \underline{C}\ \underline{f}(t)$$

where $\underline{f}(t) = [1, t, t^2, \ldots, t^k]^T$

$\underline{C}$ = constant coefficient matrix

For this problem:



To minimize base reactions,

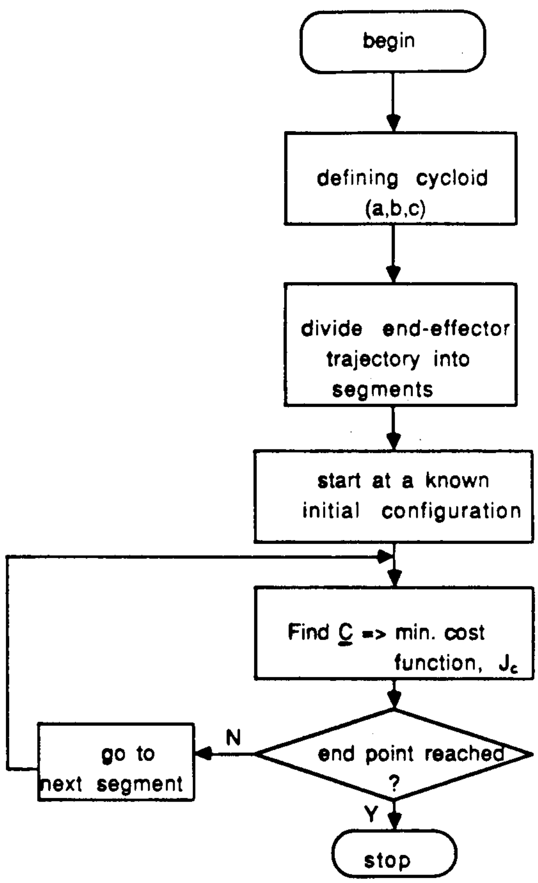
Local Cost Function: $J_C = \underline{B}^T\ \underline{Q}\ \underline{B}$

where $\underline{B} = \begin{bmatrix} \underline{f}_0 \\ \underline{N}_0 \end{bmatrix}$

$\underline{Q}$: positive definite weighting matrix

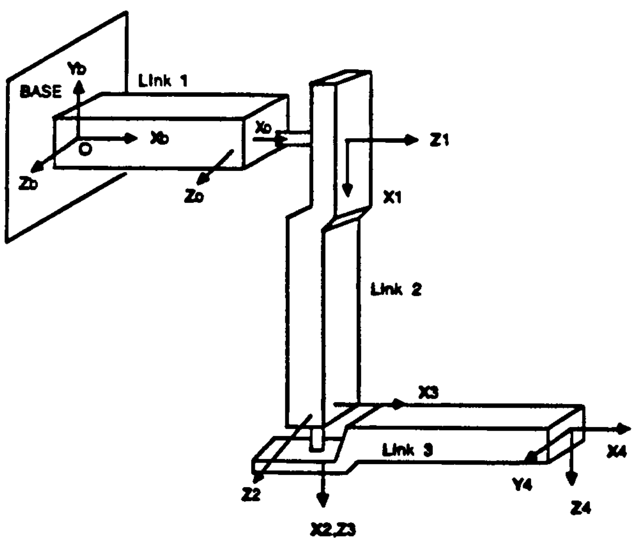
Algorithm of the Proposed Approach

An algorithm of the proposed approach and a computer program written in Pascal have been developed to implement this methodology. The flowchart below illustrates the basic algorithm.



Illustrative Example

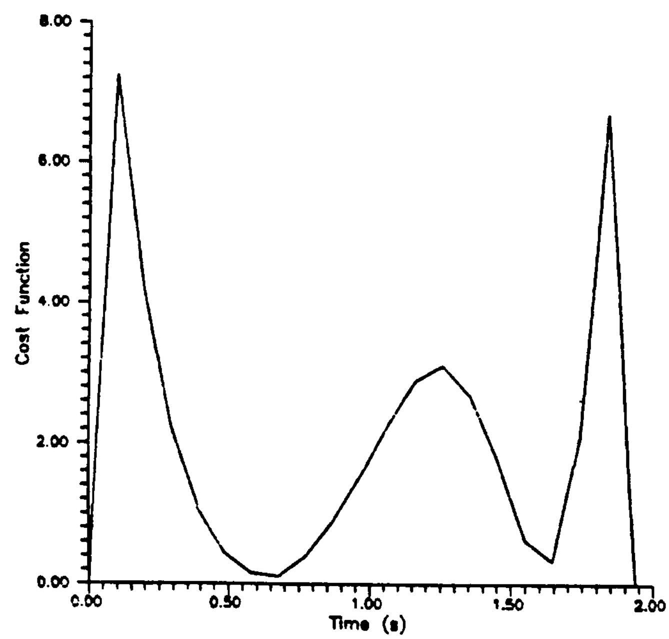
A 4 degrees of freedom spatial manipulator as shown below is studied. It has three links with lengths of l_1, l_2, and l_3 respectively. For point-to-point spatial motion, three degrees of freedom are required. Therefore, one degree of redundancy is available. The reference frame $X_bY_bZ_b$ is fixed at the base. Link 1 is mounted to the supporting structure and the other two links are each driven by a differential drive mechanism which has two outputs that rotate about orthogonal axes. For the purpose of kinematic and dynamic analyses, this mechanism can be considered as two intersecting revolute joints.



A 4 DOF Traction-Drive Manipulator

Results of Example : Optimal Cost Function

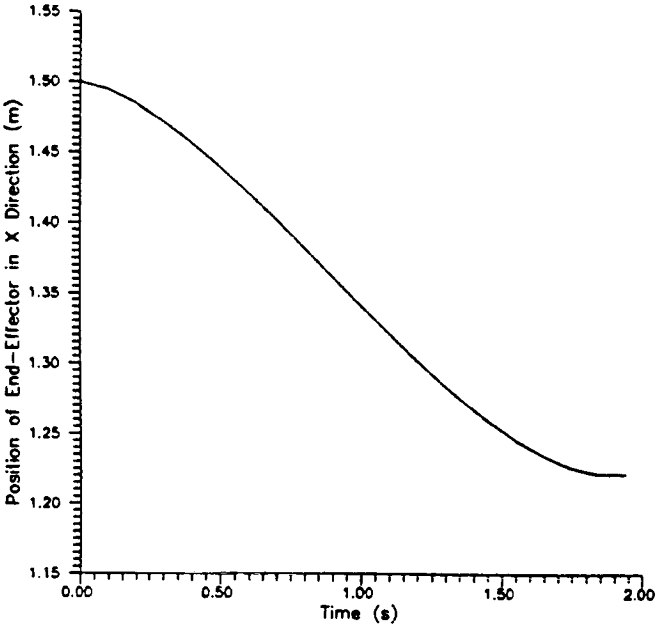
Using the algorithm developed in this research, the time history of cost function given by J = $\mathbf{R}^T\mathbf{Q}\,\mathbf{R}$ (**Q** = identity matrix) is shown in Figure 2.



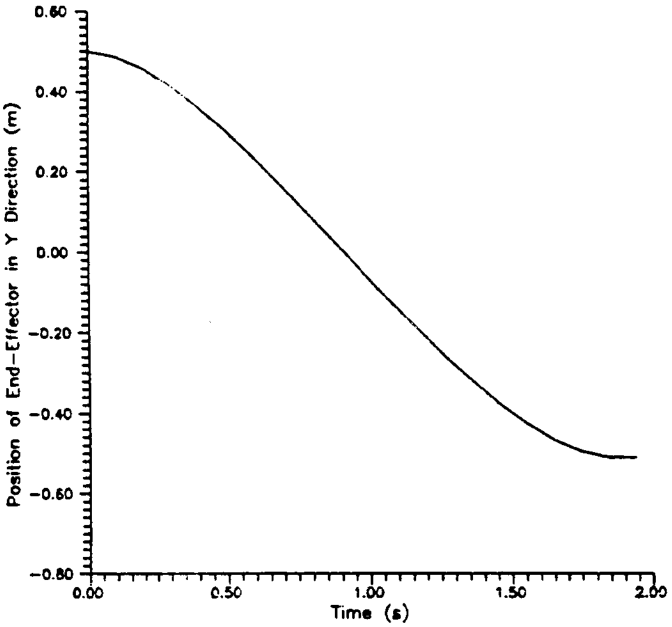
Optimal Base Reactions Cost Function

End-effector Trajectory

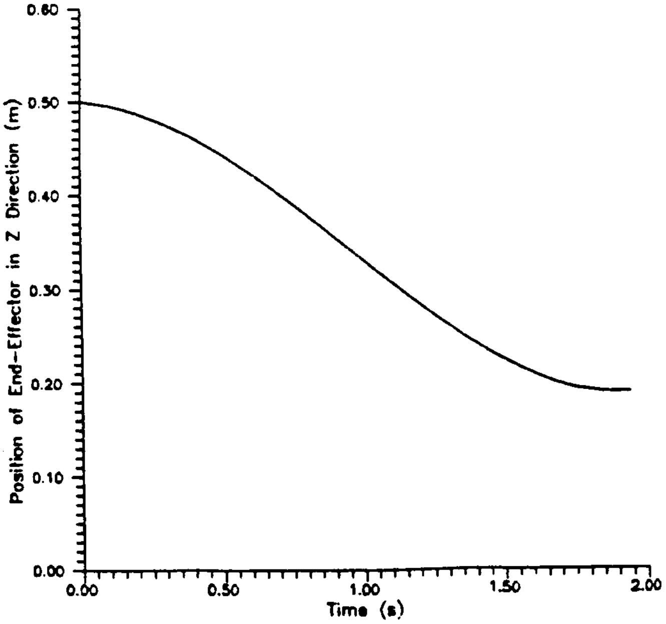
The end-effector trajectory defined by cycloid curve is shown in the figures below.



End-Effector Trajectory in X_b Direction



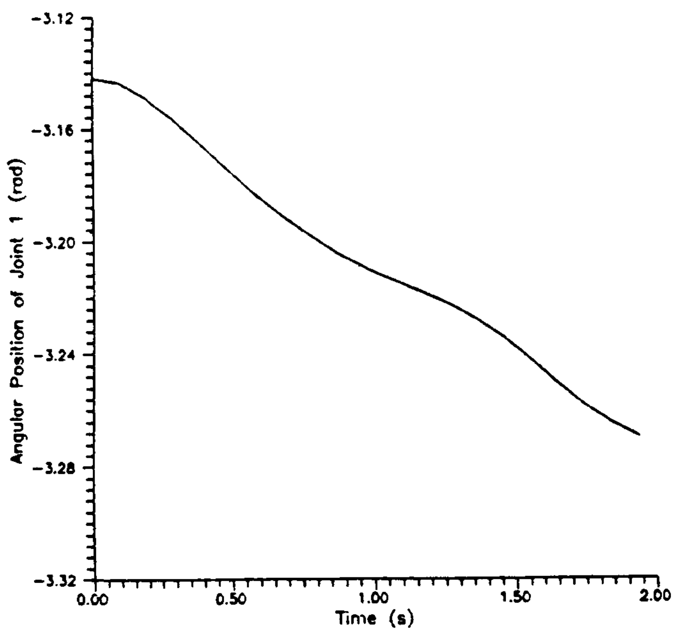
End-Efector Trajectory in Y_b Direction



End-Effector Trajectory in Z_b Direction

Optimal Joint Trajectory

A typical joint trajectory that minimizes base reactions is shown in the figure. This figure shows that using the proposed approach, we can obtain smooth joint trajectory.



Optimal Trajectory of joint 1

Conclusions

In this presentation we have shown how kinematic redundancy can be employed in planning joint trajectories to minimize base reactions exerted by the manipulator on the supporting space structure. The results of the example show that small base reactions are exerted on the space structure. The major advantage of this approach is no special restrictions are imposed on the cost function. The disadvantage is that it is computationally intensive because an optimization routine is required to find the optimal joint trajectories.

(+) We have obtained a workable approach for this problem.

It can handle cost functions which include dynamics relatively easily.

(-) The approach is computationally intensive.