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SUMMARY

Input-output characterization of fiber composites is studied theoretically by tracing SH waves in the media. A fiberglass epoxy composite is modeled as a homogeneous transversely isotropic continuum plate.

The reflection of a SH wave at a stress-free plane boundary in a semi-infinite transversely isotropic medium is considered first. It is found that an incident SH wave reflects only a similar SH wave back into the medium. It is also established that the angle of reflection of the reflected wave is equal to the angle of incidence of the incident wave.

The phase velocity of the SH waves and the delay time of the SH waves in reaching the receiving transducer are computed as functions of a reflection index, defined as the number of reflections of the SH waves from the bottom face of the continuum plate. The directivity function corresponding to the shear stress associated with the SH waves in the continuum plate is also derived as a function of the reflection index.

A theoretical output voltage from the receiving transducer is calculated for a tone burst (a periodic input voltage of finite duration). The output voltage is shown for tone bursts of duration 60 microseconds and center frequencies 0.75, 1.00 and 1.25 MHz. The study enhances the quantitative and qualitative understanding of the nondestructive evaluation (NDE) of fiber composites which can be modeled as transversely isotropic media.

INTRODUCTION

Certain fiber composites are often modeled as equivalent homogeneous solids [1]. Continuum models of this sort are useful in analyzing wave propagation in fiber composites when the wave lengths under consideration are long compared with the mean fiber diameter.

The input-output characterization of a homogeneous transversely isotropic elastic plate is investigated by tracing SH waves. Following the work in [2], the reflection of an SH wave at a stress-free plane boundary in a semi-infinite transversely isotropic medium is considered first. It is reestablished that an incident SH wave reflects only a similar SH wave. It is also reestablished that the angle of reflection is equal to the angle of incidence whenever the plane boundary where the reflection occurs is parallel to the isotropic plane of the transversely isotropic medium.

The plane of isotropy of the equivalent transversely isotropic contiuum plate lies in the midplane of the plate and is parallel to each face of the plate [1]. The SH waves experience multiple reflections at each face of the plate. The phase velocity of the SH waves and the delay time for the SH waves arriving at the receiving transducer are computed as functions of a reflection index, which is defined as the number of reflections from the bottom face of the continuum plate. The directivity function corresponding to the shear stresses associated with the SH waves in the continuum plate is derived also in terms of the reflection index.

Finally, a theoretical ouput voltage from the receiving transducer is calculated for a tone burst (a periodic input voltage of finite duration). The theoretical output voltage is expressed as an infinite sum over the reflection index.

REFLECTION OF INCIDENT SH WAVE AT STRESS-FREE PLANE BOUNDARY IN SEMI-INFINITE TRANSVERSELY ISOTROPIC MEDIUM WITH PLANE OF ISOTROPY PARALLEL TO PLANE BOUNDARY

1. REFLECTED SH WAVE

A plane progressive SH wave may be represented as

$$(u, v, w) = A(P_x, P_y, P_z)\exp\{i\omega(S_x x + S_y y + S_z z - t)\} \tag{1}$$

for example, see [3], where u, v and w are the displacement components of a point in the medium along the x, y and z axes, respectively; A is the amplitude of particle displacement; P_x, P_y and P_z are the components of the unit vector of particle displacement along the x, y and z axes, respectively; $i = \sqrt{-1}$; ω denotes radian frequency; S_x, S_y and S_z are the components of the slowness vector, which points in the same direction as the normal to the wave front and has magnitude equal to the reciprocal of the magnitude of the phase velocity, see [4], along the x, y and z axes, respectively; and t denotes time.

A plane progressive SH wave is incident on the plane boundary of a semi-infinite linearly elastic transversely isotropic continuum with plane of isotropy parallel to the plane boundary. Define a cartesian coordinate system (x, y, z) as follows: the plane boundary of the medium contains the x and y axes, and the z axis is the zonal axis of the medium (see Fig. 1). The generalized Hooke's Law, when written relative to the (x, y, z) coordinate system is [4]

$$\tau_{xx} = C_{11}u{,}x + C_{12}v{,}y + C_{13}w{,}z$$
$$\tau_{yy} = C_{12}u{,}x + C_{11}v{,}y + C_{13}w{,}z$$
$$\tau_{zz} = C_{13}u{,}x + C_{13}v{,}y + C_{33}w{,}z$$
$$\tau_{xz} = C_{44}(u{,}z + w{,}x)$$
$$\tau_{yz} = C_{44}(v{,}z + w{,}y)$$
$$\tau_{xy} = C_{66}(u{,}y + v{,}x) \qquad (2)$$

where for $i = j$ τ_{ij} is a normal stress and for $i \neq j$ τ_{ij} is a shear stress; "," denotes partial differentiation with respect to the succeeding variable; C_{11}, C_{12}, C_{13}, C_{33}, and C_{44} are the five independent elastic constants for a linearly elastic transversely isotropic medium; and C_{66} is equal to $1/2(C_{11}$-$C_{12})$.

The stresses associated with a plane progressive SH wave can then be evaluated by substituting the expression for the displacement components at a point in the medium, given by eqn. (1), into eqn. (2), as

$$\tau_{xx} = i\omega A\,(C_{11}S_xP_x + C_{12}S_yP_y + C_{13}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{yy} = i\omega A\,(C_{12}S_xP_x + C_{11}S_yP_y + C_{13}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{zz} = i\omega A\,(C_{13}S_xP_x + C_{13}S_yP_y + C_{33}S_zP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{xz} = i\omega A\,(C_{44}S_zP_x + C_{44}S_xP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{yz} = i\omega A\,(C_{44}S_zP_y + C_{44}S_yP_z)\exp\{i\omega(S_xx + S_yy + S_zz - t)\}$$
$$\tau_{yx} = i\omega A\,(C_{66}S_yP_x + C_{66}S_xP_y)\exp\{i\omega(S_xx + S_yy + S_zz - t)\} \qquad (3)$$

The stress boundary conditions on a stress-free plane boundary require [3]

$$\tau_{xz}^{(I)} + \tau_{xz}^{(R)} = 0$$
$$\tau_{yz}^{(I)} + \tau_{yz}^{(R)} = 0$$
$$\tau_{zz}^{(I)} + \tau_{zz}^{(R)} = 0 \tag{4}$$

where the $\tau_{ij}^{(I)}$ represent the stresses associated with the incident wave and the $\tau_{ij}^{(R)}$ represent the stresses associated with the reflected waves. The stresses determined by eqn. (3) satisfy the boundary conditions, eqn. (4), only when the frequency, ω, of any reflected wave is equal to the frequency of the incident wave [3] and

$$S_x^{(I)} = S_x^{(R)}$$
$$S_y^{(I)} = S_y^{(R)} \tag{5}$$

where $S_x^{(I)}$ and $S_y^{(I)}$ are the x and y components of the slowness vector of the incident wave, respectively, and $S_x^{(R)}$ and $S_y^{(R)}$ are the x and y components of the slowness vector of any reflected wave, respectively [2]. Eqn. (5) indicates that the incident and the reflected waves lie in the same plane, called the plane of incidence. Without loss of generality, the plane of incidence can be assumed to coincide with the y-z plane; the analysis is thus simplified. Then, the x components of the slowness vectors of the incident and reflected waves can be allowed to vanish, thereby satisfying the relation

$$S_x^{(I)} = S_x^{(R)} = 0 \tag{6}$$

An SH wave with slowness surface in a plane containing the zonal axis of a transversely isotropic medium possesses a transverse displacement [5]. Thus, for an SH wave traveling in the y-z plane in the (x, y, z) coordinate system, the unit vector of particle displacement is given by

$$(P_x, P_y, P_z) = (1, 0, 0) \tag{7}$$

A wave with P_x, P_y and P_z given by eqn. (7) and x component of slowness vector given by eqn. (6) has stresses determined from eqn. (3) satisfying

$$\tau_{xx} = \tau_{yy} = \tau_{zz} = \tau_{yz} = 0$$
$$\tau_{xy} \neq 0$$
$$\tau_{xz} \neq 0 \tag{8}$$

A P wave with slowness surface in a plane containing the zonal axis of a transversely isotropic medium possesses a vanishing transverse displacement [5]. Therefore, a P wave traveling in the plane x=0 in the (x, y, z) coordinate system has a unit vector of particle displacement

$$(P_x, P_y, P_z) = (0, P_y^{(P)}, P_z^{(P)}) \tag{9}$$

where $P_y^{(P)}$ and $P_z^{(P)}$ are, respectively, the directionally dependent y and z components of the unit vector of particle displacement of a P wave.

A wave P_x, P_y and P_z satisfying eqn. (9) and S_x determined by eqn. (6) has stresses given by eqn. (3) satisfying

$$\tau_{xy} = \tau_{xz} = 0$$
$$\tau_{yz};\ \tau_{xx};\ \tau_{yy};\ \tau_{zz} \neq 0 \tag{10}$$

An SV wave with slowness surface in a plane containing the zonal axis of a transversely isotropic medium also has a vanishing transverse displacement [5]. The unit vector of particle displacement of an SV wave traveling in the y-z plane in the (x, y, z) coordinate system is then

$$(P_x, P_y, P_z) = (0, P_y^{(SV)}, P_z^{(SV)}) \tag{11}$$

where $P_y^{(SV)}$ and $P_z^{(SV)}$ are, respectively, the directionally dependent y and z components of the unit vector of particle displacement of an SV wave.

A wave with unit vector of particle displacement determined by eqn. (11) and x component of slowness vector given eqn. (6) has stresses evaluated from eqn. (3) satisfying

$$\tau_{xy} = \tau_{xz} = 0$$
$$\tau_{yz};\ \tau_{xx};\ \tau_{yy};\ \tau_{zz} \neq 0 \tag{12}$$

An SH wave is incident on the plane boundary of a semi-infinite transversely isotropic medium. The stresses at the point of incidence associated with the incident SH wave are constrained by eqn. (8) and satisfy

$$\tau_{yz}^{(I)} = \tau_{zz}^{(I)} = 0$$
$$\tau_{xz}^{(I)} \neq 0 \tag{13}$$

where the superscript $^{(I)}$ is used to denote properties associated with the incident wave.

The stress boundary conditions for any wave reflected by an incident SH wave can be expressed by using eqn. (4) and eqn. (8) as

$$\tau_{yz}^{(R)} = \tau_{zz}^{(R)} = 0$$
$$\tau_{xz}^{(R)} = -\tau_{xz}^{(I)} \neq 0 \tag{14}$$

where the superscript $^{(R)}$ is used to denote properties associated with the reflected wave. Eqn. (10) indicates that the τ_{yz} and τ_{zz} components of the stress tensor associated with a P wave are not zero and eqn. (12) indicates that the τ_{yz} and τ_{zz} components of the stress tensor associated with an SV wave are also nonzero. Therefore, no P wave or SV wave will be reflected back into the medium by an incident SH wave, because a reflected P wave or SV wave would have associated stresses that would violate the stress boundary conditions. Eqn. (8) indicates that a reflected SH wave can satisfy the boundary conditions. Therefore, an incident SH wave will cause only an SH wave to be reflected back into the medium, as shown in [2].

2. SLOWNESS SURFACE OF SH WAVE

The equations of motion relative to the (x, y, z) coordinate system are [4]

$$\tau_{xx},x + \tau_{xy},y + \tau_{xz},z = \rho u,tt$$
$$\tau_{xy},x + \tau_{yy},y + \tau_{yz},z = \rho v,tt$$
$$\tau_{xz},x + \tau_{yz},y + \tau_{zz},z = \rho w,tt \tag{15}$$

where ρ is the density of the medium and "$,tt$" denotes the second derivative of the preceeding variable with respect to time.

When the components of stress are calculated according to eqn. (3) and the displacement components are calculated according to eqn. (1), the equations of motion can be written as [2]

$$(C_{11}S_x^2 + C_{66}S_y^2 + C_{44}S_z^2 - \rho)P_x + (C_{12} + C_{66})S_xS_yP_y + (C_{13} + C_{44})S_xS_zP_z = 0$$
$$(C_{12} + C_{66})S_xS_yP_x + (C_{66}S_x^2 + C_{11}S_y^2 + C_{44}S_z^2 - \rho)P_y + (C_{13} + C_{44})S_yS_zP_z = 0$$
$$(C_{13} + C_{44})S_xS_zP_z + (C_{13} + C_{44})S_yS_zP_y + (C_{44}S_x^2 + C_{44}S_y^2 + C_{33}S_z^2 - \rho)P_z = 0 \tag{16}$$

Eqn. (16) can be rewritten in matrix form as

$$[B]\,(P_x,P_y,P_z)^T = [0] \tag{17}$$

where $[B]$ is a 3x3 matrix with entries

$$b_{11} = (C_{11}S_x^2 + C_{66}S_y^2 + C_{44}S_z^2 - \rho)$$

$$b_{22} = (C_{66}S_x^2 + C_{11}S_y^2 + C_{44}S_z^2 - \rho)$$

$$b_{33} = (C_{44}S_x^2 + C_{44}S_y^2 + C_{33}S_x^2 - \rho)$$

$$b_{12} = b_{21} = (C_{12} + C_{66})S_xS_y$$

$$b_{13} = b_{31} = (C_{13} + C_{44})S_xS_z$$

$$b_{23} = b_{32} = (C_{13} + C_{44})S_yS_z \qquad (18)$$

(P_x, P_y, P_z) is the unit vector of particle displacement and [0] is the 3x1 zero matrix. The matrix $[B]$ as defined by eqn. (18) is a symmetric matrix; therefore, there exist three real eigenvalues. One eigenvalue corresponds to an SH wave, another eigenvalue corresponds to a P wave, and the third eigenvalue corresponds to an SV wave [2].

The plane wave solution is found by setting the determinant of matrix $[B]$, the matrix of coefficients of P_x, P_y and P_z, equal to zero [6]. Expanding the determinant of $[B]$ and solving for the three roots yields equations for the three slowness surfaces. The slowness surface for an SH wave is [7]

$$C_{66}(S_x^2 + S_y^2) + C_{44}S_z^2 = \rho \qquad (19)$$

The values of the material constants C_{66}, C_{44} and ρ are given for a representative fiberglass epoxy composite in [4] as C_{66}=3.243x10^9 N/m^2, C_{44}=4.422x10^9 N/m^2 and ρ =1850 kg/m^3. The slowness surface of an SH wave traveling in the y-z plane in the fiberglass epoxy composite is obtained by substituting the numerical values of the constants into eqn. (19) and setting the x component of the slowness vector equal to zero. The first quadrant of the slowness surface thus obtained is shown in Fig. 2.

3. ANGLE OF REFLECTION OF REFLECTED SH WAVE

According to eqn. (5) the y component, $S_y^{(I)}$, of the slowness vector of the incident SH wave is equal to the y component, $S_y^{(R)}$, of the slowness vector of the reflected SH wave. The z component, $S_z^{(R)}$, of the slowness vector of the reflected SH wave is found from the equation for the slowness surface of an SH wave, eqn. (19), by setting $S_y^{(R)}$ equal to $S_y^{(I)}$ and $S_x^{(R)}$ equal to zero, according to eqn. (6). Then, $S_z^{(R)}$ is found to satisfy

$$S_z^{(R)} = \pm S_z^{(I)} \tag{20}$$

where $S_z^{(I)}$ is the z component of the slowness vector of the incident SH wave. The slowness vector of the incident wave points out of the medium, and the slowness vector of the reflected wave points into the medium; therefore, the relationship between $S_z^{(R)}$ and $S_z^{(I)}$ must be [2]

$$S_z^{(R)} = -S_z^{(I)} \tag{21}$$

The angle of incidence is defined as the angle between the slowness vector of the incident wave and the normal to the boundary at the point of incidence. The angle of incidence $\theta^{(I)}$ of the incident SH wave is then

$$\theta^{(I)} = \arctan\left(\frac{S_y^{(I)}}{-S_z^{(I)}}\right) \tag{22}$$

where the point of incidence is taken to be on the plane boundary of the semi-infinite body (see Fig. 1).

The angle of reflection is defined, in a manner similar to the angle of incidence, as the angle between the slowness vector of the reflected wave and the normal to the boundary at the point of reflection. The angle of reflection $\theta^{(R)}$ of the reflected SH wave is then

$$\theta^{(R)} = \arctan\left(\frac{S_y^{(R)}}{S_z^{(R)}}\right) \tag{23}$$

Evaluating $\theta^{(R)}$ using eqn. (5) and eqn. (21) and comparing the resulting expression to eqn. (22) establishes

$$\theta^{(R)} = \theta^{(I)} \tag{24}$$

Thus, the angle of reflection of the reflected SH wave is equal to the angle of incidence of the incident SH wave [2].

INPUT-OUTPUT CHARACTERIZATION OF FIBER COMPOSITE

1. PHASE VELOCITY AND DELAY TIME

Certain fiber composites may be modeled as a linearly elastic transversely isotropic homogeneous continua [1]. A fiber composite modeled as such a solid in the form of an infinite plate where the plane of isotropy lies in the midplane of the plate is to be studied. A cartesian coordinate system (x, y, z) is chosen such that the x-y plane coincides with the plane of isotropy and the plate is bounded above by the plane $z = h/2$ and bounded below by the plane $z = - h/2$.

A transmitting transducer and a receiving transducer are assumed to be coupled to the top face of the plate in the y-z plane and separated by a distance L. The input electrical voltage of the transmitting transducer is a known function of time, $V_i(t)$. The output electrical voltage of the receiving transducer is an unknown function of time, $V_o(t)$. The transmitting transducer converts the input electrical voltage into a stress that travels through the plate as stress waves. The receiving transducer converts the stress associated with the stress waves into an output voltage. In the following analysis, only those stress disturbances at the receiving transducer associated with SH waves generated by the transmitting transducer will be considered.

The SH waves produced by the transmitting transducer experience multiple reflections at each face of the plate before reaching the receiving transducer. Since the isotropic plane of the plate lies in the midplane of the plate and is parallel to the top and bottom faces of the plate, each reflection may be treated as the reflection of a plane progressive SH wave on the plane boundary of a semi-infinite linearly elastic tranversely isotropic continuum with its isotropic plane parallel to the boundary [2]. Therefore, at each reflection the incident SH wave produces only a reflected SH wave and the angle of reflection of the reflected SH wave is equal to the angle of incidence of the incident SH wave.

The transmitting transducer and the receiving transducer are coupled to the same face of the plate, therefore, any SH wave traveling from the transmitting transducer to the receiving transducer must experience an odd number of reflections. Let the total number of reflections experienced by an arbitrary SH wave traveling from the transmitting transducer to the receiving transducer be equal to $2n$-1, where n is called the reflection index and is a positive integer equal to the number of reflections from the bottom face of the plate. Assume the angle of incidence of the SH wave at the receiving transducer is θ_n. The angle of reflection is equal to the angle of incidence at each reflection experienced by the SH wave, so the angle of incidence at each reflection is θ_n (see Fig. 3). The distance r_n traveled by the SH wave between each reflection is calculated from Fig. 3 as

$$r_n = \frac{h}{\cos\theta_n} \tag{25}$$

The distance b_n traveled in the y direction by the SH wave between each reflection is

$$b_n = r_n \sin\theta_n \tag{26}$$

Using eqn. (25) to evaluate r_n in eqn. (26), the distance b_n can be expressed as

$$b_n = h \tan\theta_n \tag{27}$$

The SH wave travels a distance b_n in the y direction between each reflection, between the transmitting transducer and the initial reflection, and between the final reflection and the receiving transducer; so the total distance B_n traveled in the y direction by the SH wave is

$$B_n = 2nh \tan\theta_n \tag{28}$$

The transmitting transducer and the receiving transducer are separated by a distance L in the y direction, therefore; the total distance traveled by the SH wave must be L. Eqn. (28) can then be expressed as

$$L = 2nh \tan\theta_n \tag{29}$$

The angle of incidence at the receiving transducer must therefore satisfy

$$\theta_n = \arctan\left(\frac{L}{2nh}\right) \tag{30}$$

where n is a positive integer.

Eqn. (30) allows all the trigonometric functions of θ_n to be calculated (see Fig. 4). The tangent of θ_n is given by

$$\tan\theta_n = \frac{L}{2nh} \tag{31}$$

The sine of θ_n is given by

$$\sin\theta_n = \frac{L}{\sqrt{L^2 + 4n^2h^2}} \tag{32}$$

and the cosine of θ_n is given by

$$\cos\theta_n = \frac{2nh}{\sqrt{L^2+4n^2h^2}} \tag{33}$$

The distance r_n traveled by the SH wave between each reflection can be evaluated using eqn. (25) and eqn. (33) as

$$r_n = \frac{\sqrt{L^2+4n^2h^2}}{2n} \tag{34}$$

The SH wave travels a distance r_n between each reflection, between the transmitting transducer and the first reflection, and between the last reflection and the receiving transducer; so the total distance R_n traveled by the SH wave between the transmitting transducer and the receiving transducer is

$$R_n = \sqrt{L^2+4n^2h^2} \tag{35}$$

The phase velocity $C_{SH}(\theta)$ of an SH wave traveling in a transversely isotropic continuum is [4]

$$C_{SH}(\theta) = \{C_{66}\sin^2\theta + C_{44}\cos^2\theta\}^{1/2} \tag{36}$$

Using eqn. (32) and eqn. (33), the phase velocity can be expressed in terms of n as

$$C_{SH}(\theta_n) = \left\{C_{66}\left[\frac{L^2}{L^2+4n^2h^2}\right] + C_{44}\left[\frac{4n^2h^2}{L^2+4n^2h^2}\right]\right\}^{1/2} \tag{37}$$

The time delay t_n is defined as the time taken for the SH wave to reach the receiving transducer after being produced by the transmitting transducer. The time delay is thus

$$t_n = \frac{R_n}{C_{SH}(\theta_n)} \tag{38}$$

Eqn. (38) can be evaluated using eqn. (35) and eqn. (37) as

$$t_n = \{L^2 + 4n^2h^2\}\{C_{66}L^2 + C_{44}4n^2h^2\}^{-1/2} \tag{39}$$

The time delay for a typical transducer geometry is shown in Table 1. The transducers are assumed to be coupled to a fiberglass epoxy composite plate having continuum model with material constants C_{44}=4.422x10^9 N/m^2 and C_{66}=3.234x10^9 N/m^2 and separated by a distance L=10 cm. The plate is assumed to be of thickness h=5 cm. The time delay is shown as a function of the reflection index for values of n from 1 to 9.

2. ASSUMPTIONS ON TRANSDUCERS

The transmitting transducer and the receiving transducer are assumed to be transverse transducers that transform an electrical voltage into a uniform shear stress or a uniform shear stress into an electrical voltage, respectively. The following approach parallels that of [8].

If an input voltage V_i of amplitude V and frequency ω is applied according to

$$V_i = V \exp\{-i\omega t\} \tag{40}$$

where $i = \sqrt{-1}$ and t denotes time, the shear stress τ_{xz} that is introduced into the medium at the transducer-medium interface by the transmitting transducer is

$$\tau_{xz}(t) = F_1(\omega) V \exp\{-i(\omega t + \phi_1)\} \tag{41}$$

where $F_1(\omega)$ is the frequency dependent transduction ratio for the transmitting transducer in transforming a voltage into a stress and ϕ_1 is a phase angle. In eqn. (43) and eqn. (44) the harmonic character of the signals is expressed in complex notation, but only the real parts of these and subsequent equations should be considered. The amplitude T of the applied stress is then

$$T = F_1(\omega)V \tag{42}$$

Similarly, if a stress wave producing a shear stress component τ_{zz}' of amplitude T' and frequency ω that impinges on the receiving transducer is defined as

$$\tau_{zz}'(t) = T' \exp\{-i\omega t\} \tag{43}$$

then the output voltage V_o from the receiving transducer is

$$V_o(t) = F_2(\omega)T' \exp\{-i(\omega t + \phi_2)\} \tag{44}$$

where $F_2(\omega)$ is the frequency dependent transduction ratio for the receiving transducer in transforming a shear stress to a voltage, and ϕ_2 is a phase angle. Thus, the amplitude V' of the output electrical voltage is

$$V' = F_2(\omega)T' \tag{45}$$

The characteristics of $F_1(\omega)$ and $F_2(\omega)$ are unknown except that the product $F_1(\omega)F_2(\omega)$ is dimensionless.

3. DIRECTIVITY FUNCTION

The directivity function of the shear stress, τ_{zz}, due to an SH wave traveling in a transversely isotropic medium is evaluated from the far-field asymptotic solutions of the displacement components produced by a harmonic point load imbedded in an infinite body in [2]. The directivity function $D^{(SH)}$ is given by [2]

$$D^{(SH)} = \frac{C_{44}\omega S_z^*}{4\pi(C_{66}/\rho)(C_{44}/\rho)^{1/2}}\left[\left(\frac{C_{66}}{\rho}S_y^*\right)^2 + \left(\frac{C_{44}}{\rho}S_z^*\right)^2\right]^{1/2} \tag{46}$$

where S_y^* and S_z^* are the y and z components, respectively, of a point on the slowness surface of an SH wave where the normal to the slowness surface is parallel to the line connecting the transmitting transducer and the point at which the directivity function is to be evaluated.

The shear stress, radiated by the transmitting transducer, that is incident at the receiving transducer with angle of incidence θ_n, is assumed to be equivalent to the shear stress at a point M_n associated with an SH wave traveling in a semi-infinite transversely isotropic medium (see Fig. 5). Except for the reflection coefficients at each face of the plate the shear stress can be computed as if there were no bottom boundary to the plate [2].

The slope, m, of the line connecting the transmitting transducer to the point M_n is found from Fig. 5 to be

$$m = \frac{\Delta y}{\Delta z} = -\frac{2nh}{L} \tag{47}$$

Eqn. (47) establishes that the line connecting the transmitting transducer to the point M_n lies in the y-z plane. Therefore, a point on the slowness surface, ($S_{x_n}^*$, $S_{y_n}^*$, $S_{z_n}^*$), where the normal is parallel to the line connecting the transmitting transducer to the point M_n may be found by setting $S_{x_n}^*$ equal to zero.

The normal to the slowness surface of an SH wave is found at any point in the y-z plane by setting the x component of the slowness vector, S_x, equal to zero and differentiating the equation for the slowness surface, eqn. (19), with respect to S_z, the z component of the slowness vector. Solving the subsequent equation for the slope, $-dS_y/dS_z$, of the normal yields

$$\frac{dS_y}{dS_z}=\frac{C_{44}S_z}{C_{66}S_y} \tag{48}$$

Combining eqn. (47) and eqn. (48) establishes the constraint on $S_{y_n}^*$ and $S_{z_n}^*$, the point on the slowness surface where the normal is parallel to the line connecting the transmitting transducer to the point M_n, as

$$\frac{C_{44}S_{z_n}^*}{C_{66}S_{y_n}^*}=-\frac{2nh}{L} \tag{49}$$

The point ($S_{x_n}^*$, $S_{y_n}^*$, $S_{z_n}^*$) must lie on the slowness surface of an SH wave and have slope satisfying eqn. (49). Therefore, $S_{y_n}^*$ and $S_{z_n}^*$ can be found by setting $S_{x_n}^*$ equal to zero and then solving eqn. (19) and eqn. (49) simultaneously. Then the value of $S_{y_n}^*$ is

$$S_{y_n}^*=\left\{\frac{\rho}{C_{66}\left[1+\left(\frac{C_{66}}{C_{44}}\right)\left(\frac{L^2}{4n^2h^2}\right)\right]}\right\}^{1/2} \tag{50}$$

and the value of $S_{z_n}^*$ is

$$S_{z_n}^*=-\frac{2nh}{L}\left\{\frac{C_{66}\rho}{C_{44}\left[C_{44}+C_{66}\left(\frac{L^2}{4n^2h^2}\right)\right]}\right\}^{1/2} \tag{51}$$

Substituting eqn. (50) and eqn. (51) into eqn. (46) produces an expression for $D_n^{(SH)}$, the value of the directivity function associated with an angle of incidence θ_n at the receiving transducer. The expression for $D_n^{(SH)}$ as a function of n is

$$D_n^{(SH)} = -\frac{nh}{L}\frac{\omega}{2\pi}\left[\frac{\rho^{3/2}}{C_{44}+C_{66}\left(\frac{4n^2h^2}{L^2}\right)}\right]\left[C_{44}\left(1+\frac{4n^2h^2}{L^2}\right)\right]^{1/2} \tag{52}$$

The directivity function for an SH wave traveling in a transversely isotropic plate is calculated as a function of the angle of incidence at the receiving transducer by treating eqn. (30) and eqn. (52) as a pair of parametric equations with parameter n equal to the reflection index. The directivity function $D^{(SH)}$ is shown for an SH wave traveling in a fiberglass epoxy composite plate as a function of the angle of incidence at the receiving transducer in Figs. 6, 7 and 8. The transducer geometry is assumed to be L=10 cm and h=5 cm. The composite plate has with material properties C_{44}=4.422x10^9 N/m^2 and C_{66}=3.234x10^9 N/m^2 [4]. The directivity function is shown for frequencies equal to 0.75, 1.00 and 1.25 MHz.

4. STRESS FIELD RADIATED BY TRANSMITTING TRANSDUCER

If there were no bottom boundary to the plate and the SH waves were propagating in an infinite half-space, the SH wave incident at the receiving transducer with angle of incidence θ_n would travel to a point M_n, in a time t_n, traveling a distance R_n (see Fig. 6). The amplitude of the hypothetical shear stress at the point M_n is $T_n^{(M)}$ and is defined as [2]

$$T_n^{(M)} = T\frac{D_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \tag{53}$$

where T is the magnitude of the point load applied by the transmitting transducer in the z direction, $D_n^{(SH)}$, the value of the directivity function for an SH wave associated with the point M_n is given by eqn. (52) and α is the SH wave attenuation constant of the medium. The SH wave, however, does not propagate in an infinite half-space, but instead experiences $2n$-1 reflections. The amplitude T_{o_n} of the shear stress at the receiving transducer is thus obtained by modifying eqn. (53) as

$$T_{o_n} = \frac{TD_n^{(SH)}(Q^{(SH-SH)})^{2n-1}}{R_n}\exp\{-\alpha R_n\} \tag{54}$$

where $Q^{(SH\text{-}SH)}$ is the reflection coefficient for SH waves and is equal to -1 [2].

The amplitude V_{o_n} of the output voltage from the receiving transducer is obtained from eqn. (45) and eqn. (54) and is

$$V_{o_n} = \frac{-F_2(\omega)TD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \tag{55}$$

Substituting eqn. (42) into eqn. (55) yields

$$V_{o_n} = \frac{-F_1(\omega)F_2(\omega)VD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \tag{56}$$

where V is the amplitude of the input voltage. Introducing a possible electrical signal amplification factor, K, eqn. (56) can be written as

$$V_{o_n} = \frac{-KF_1(\omega)F_2(\omega)VD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \tag{57}$$

5. OUTPUT VOLTAGE DUE TO TONE BURST

Assume the input voltage V_i to the transmitting transducer is a periodic function of time, of center frequency ω and duration t_i. Mathematically the input voltage can be expressed as the sum of two periodic functions of frequency ω that are 180° out of phase, one beginning at time t=0 and one beginning at time t=t_i. The input voltage is then given by

$$V_i = V\exp\{i\omega t\}\,\mathrm{U}(t) - V\exp\{i\omega t\}\,\mathrm{U}(t - t_i) \tag{58}$$

where V is the amplitude of the input voltage, $i = \sqrt{-1}$, and U(x) is the unit step function defined as

$$\mathrm{U}(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad x \geq 0 \end{cases} \tag{59}$$

The periodic nature of the signal in eqn. (58) and subsequent equations is expressed in complex notation, but only the real part of the signal is to be considered.

The voltage output V_o at the receiving transducer is found by considering the two terms on the right-hand side of eqn. (61) independently and superposing the voltage output associated with each input signal. Consider the input voltage V_i' given by the first term on the right-hand side of eqn. (58)

$$V_i' = V\exp\{i\omega t\}\,\mathrm{U}(t) \tag{60}$$

The stress waves produced by a transmitting transducer with input voltage V_i' will have a periodic character with respect to time. Therefore, the amplitude of the output voltage V_{o_n}'

associated with the SH wave experiencing $2n$-1 reflections will also be periodic in time and the output voltage V_o' at the receiving transducer must be found by the superposition of the contributions of each reflection path.

An SH wave experiencing $2n$-1 reflections between the transmitting transducer and the receiving transducer will be in phase at the receiving transducer with an SH wave traveling in a semi-infinite body to a point M_n (see Fig. 5). The τ_{xz} stress associated with a plane progressive SH wave is given by eqn. (3) and can be calculated at point M_n as

$$\tau_{xzM} = T_{M_n} \exp\left\{ i\left[\omega\left(S_{y_n} L - S_{z_n} 2nh - t \right)\right]\right\} \tag{61}$$

where T_{M_n} is the amplitude of the stress and S_{y_n} and S_{z_n} are the y and z components of the slowness vector of the SH wave, respectively. The shear stress τ_{xz_o} at the receiving transducer is then

$$\tau_{xz_o} = T_{o_n} \exp\left\{ i\left[\omega\left(S_{y_n} L - S_{z_n} 2nh - t \right)\right]\right\} \tag{62}$$

where T_{o_n} is the amplitude of the stress and is given by eqn. (54). The output voltage V_{o_n}' at the receiving transducer is found by substituting eqn. (62) into eqn. (44) to be

$$V_{o_n}'(t) = F_2(\omega) T_{o_n} \exp\left\{ i\left[\omega\left(S_{y_n} L - S_{z_n} 2nh - t \right) + \phi \right]\right\} \tag{63}$$

where ϕ is a phase angle. The output voltage is then calculated using eqn. (42) and eqn. (54) and including a possible electrical signal amplification factor K as

$$V_{o_n}' = \frac{-KF_1(\omega)F_2(\omega)VD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\}\exp\left\{i\left[\omega\left(S_{y_n}L - S_{z_n}2nh - t\right) + \phi\right]\right\} \quad (64)$$

where V is the amplitude of the input voltage and the $D_n^{(SH)}$ is given by eqn. (52). The total output voltage V_o' is found by summing eqn. (64) over all n and retarding each contribution by the delay time t_n to be

$$V_o' = \sum_{n=1}^{\infty} \frac{-KF_1(\omega)F_2(\omega)VD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_n}L - S_{z_n}2nh - t\right) + \phi\right]\right\} \mathrm{U}(t - t_n) \quad (65)$$

where t_n is given by eqn. (38).

The output voltage V_o'' associated with the input voltage characterized by the second term on the right-hand side of eqn. (58) is found similarly to be

$$V_o'' = \sum_{n=1}^{\infty} \frac{KF_1(\omega)F_2(\omega)VD_n^{(SH)}}{R_n}\exp\{-\alpha R_n\} \times$$

$$\exp\left\{i\left[\omega\left(S_{y_n}L - S_{z_n}2nh - t\right) + \phi\right]\right\} \mathrm{U}(t - (t_i - t_n)) \quad (66)$$

where t_i is the duration of the input signal.

In order to evaluate the infinite sum in eqn. (65), S_{y_n} and S_{z_n} must be found as functions of n. The components of the slowness vector $(0, S_{y_n}, S_{z_n})$ associated with the SH wave experiencing $2n$-1 reflections between the transmitting transducer and the receiving transducer are constrained by eqn. (22) to satisfy

$$\theta_n = \arctan\left(\frac{S_{y_n}}{-S_{z_n}}\right) \tag{67}$$

Substituting eqn. (30) into eqn. (67) yields

$$\frac{L}{2nh} = \frac{S_{y_n}}{-S_{z_n}} \tag{68}$$

Solving eqn. (11) and eqn. (68) simultaneously determines S_{y_n} and S_{z_n} as

$$S_{y_n} = \frac{L}{2nh}\left\{\frac{\rho}{C_{66}\left(\frac{L^2}{4n^2h^2}\right)+C_{44}}\right\}^{1/2}$$

$$S_{z_n} = -\left\{\frac{\rho}{C_{66}\left(\frac{L^2}{4n^2h^2}\right)+C_{44}}\right\}^{1/2} \tag{69}$$

The output voltage V_o associated with the input voltage V_i determined by eqn. (58) is found by adding eqn. (65) and eqn. (66). Substituting in the values of the constants given by eqns. (35), (39) and (69) allows V_o to be written as

$$V_o(t) = -KF_1(\omega)F_2(\omega)V \sum_{n=1}^{\infty}\left\{\frac{D_n^{(SH)}}{\sqrt{L^2+4n^2h^2}}\exp\{-\alpha\sqrt{L^2+4n^2h^2}\}\times\right.$$

$$\exp\left\{i\left\{\omega\left[\frac{L^2}{2nh}\left(\frac{\rho}{C_{66}\left(\frac{L^2}{4n^2h^2}\right)+C_{44}}\right)^{1/2}+2nh\left(\frac{\rho}{C_{66}\left(\frac{L^2}{4n^2h^2}\right)+C_{44}}\right)^{1/2}-t\right]+\phi\right\}\right\}\times$$

$$\left\{U\left(t-\{L^2+4n^2h^2\}\{C_{66}L^2+C_{44}4n^2h^2\}^{-1/2}\right)-\right.$$

$$\left.\left.U\left(t-\left[\{L^2+4n^2h^2\}\{C_{66}L^2+C_{44}4n^2h^2\}^{-1/2}+t_i\right]\right)\right\}\right\} \qquad (70)$$

where $D_n^{(SH)}$ is a function of n according to eqn. (52).

The output voltage predicted by eqn. (70) is shown as a function of time in Figs. 9, 10 and 11. The transducers are assumed to be coupled to a representative fiberglass epoxy composite plate. The composite plate can be modeled as a transversely isotropic continuum plate with material properties ρ=1850 kg/m^3, C_{44}=4.422x10^9 N/m^2 and C_{66}=3.234x10^9 N/m^2 and thickness h=5 cm and transducer separation L=10 cm [4]. The continuum plate is assumed to have SH wave attenuation constant α equal to zero. The normalized output voltage V_o/KF_1F_2V is shown for center frequencies equal to 0.75, 1.00 and 1.25 MHz. and a signal duration of 60μs; the phase angle ϕ is set equal to zero.

CONCLUSION AND DISCUSSION

The input-output characterization of certain fiberglass epoxy composite plates can be studied using a transversely isotropic continuum plate model when the wavelengths under consideration are long compared to the mean fiber diameter. For wavelengths close to or less than the mean fiber diameter, a continuum plate model is no longer appropriate and the inhomogeneities of the composite must be considered.

In the analysis of a transversely isotropic continuum plate, the angle of reflection of a reflected SH wave is equal to the angle of incidence of the incident SH wave and only an SH wave is reflected by the incident SH wave for reflections at the top or bottom face of the plate. This is due to the fact that the isotropic plane of the plate is parallel to the two faces of the plate. When this geometry is present, the plate can be modeled as a semi-infinite transversely isotropic medium, except for the reflection coefficients encountered at each reflection which must be factored in separately. Treating the continuum plate as a semi-infinite body reduces the complexity of tracing stress waves through the medium. If this special geometry does not exist, the use of this simplified analysis is inappropriate and the plate must be analyzed in a more complicated manner.

It is possible to compute the relationship between the input voltage to the transmitting transducer and the output voltage from the receiving transducer in terms of the material constants, the geometry of the transducer arrangement and a single variable, the reflection index n. Using the fact that the angle of incidence of an incident SH wave is equal to the angle of reflection of the reflected SH wave, the angle θ with which the final wave of any series of waves from the transmitting transducer to the receiving transducer is incident at the receiving transducer can be computed as a function of the reflection index. The nonzero components of the slowness vector of an SH wave traveling in a plane perpendicular to the isotropic plane of the plate, the phase velocity of the SH wave, the time delay in reaching the receiving transducer, the directivity function of the shear stress produced by the transmitting transducer and the output voltage from the receiving transducer can then be computed as functions of the reflection index and the geometric and physical properties of the plate.

The output voltage from the receiving transducer, as predicted by eqn. (70), is shown in Figs. 9, 10 and 11 as functions of time for input voltages of 60 μs duration and center frequencies of 0.75, 1.00 and 1.25 MHz., respectively. All three output voltages are observed to exhibit similar behavior, being characterized by simple harmonic functions experiencing discontinuities in amplitude. Checking the times at which these changes in amplitude occur against Table 1, which shows the time delays for reflection indices from 1 to 9, reveals that the amplitude changes occur at the time delays for these values of the reflection index. Thus the discontinuities in the output voltages correspond with the arrival times of different wave paths. Since the time delay is independent of the frequency of the wave, these discontinuities occur at the same time in each output voltage regardless of the frequency of the associated input voltage.

This study enhances the theoretical understanding of the nondestructive evaluation (NDE) of transversely isotropic media such as certain fiber composites.

REFERENCES

[1] J.D. Achenbach, "A Theory of Elasticity with Microstructures for Directionally Reinforced Composites," OSIM - Courses and Lectures, No. 167 (International Center for Mechanical Sciences), Springer-Verlag, N.Y., 1965.

[2] J.H. Williams, Jr. and P. Liao, "Acousto-Ultrasonic Input-Output of Unidirectional Fiber Composite Plate by SH Waves," NASA Contractor Report 4087, August 1987.

[3] E.G. Henneke II, "Reflection-Refraction of a Stress Wave at a Plane Boundary Between Anisotropic Media," Journal of Acoustical Society of America, Vol. 51 Part 2, April 1972, pp. 210-217.

[4] E.R.C. Marques and J.H. Williams, Jr. "Stress Waves in Transversely Isotropic Media, The Homogeneous Problem," NASA Contractor Report 3977, May 1986.

[5] M.J.P. Musgrave, "On the Propogation of Elastic Waves in Acolotropic Media, "Proceedings of the Royal Society of London, Series A, Vol. 226, 1954, pp. 339-355.

[6] R.D. Kriz and H.M. Ledbetter, "Elastic Representation Surfaces of Unidirectional Graphite/Epoxy Composites," National Bureau of Standards, 2nd USA/Japan Conference in Composite Materials, NASA Langley Research Center, Hampton, Virginia, 1984.

[7] V.T. Buchwald, "Elastic Waves in Anisotropic Media," Proceedings of the Royal Society, Series A, Vol. 253, 1959, pp. 563-580.

[8] J.H. Williams, Jr., H. Karagulle and S.S. Lee, "Ultrasonic Input-Output for Transmitting and Receiving Longitudinal Transducers Coupled to Same Face of Isotropic Elastic Plate," Materials Evaluation, Vol. 40, May 1982, pp. 655-662.

Table 1 Time delay in reaching receiving transducer for SH wave in composite plate for reflection indices n from 1 to 9.

reflection index n	time delay t_i (microseconds)
1	2.28
2	3.46
3	4.82
4	6.25
5	7.71
6	9.18
7	10.6
8	12.2
9	13.6

$$L = 10 \text{ cm}$$

$$h = 5 \text{ cm}$$

$$C_{44} = 4.422 \times 10^9 \text{ N/m}^2$$

$$C_{66} = 3.230 \times 10^9 \text{ N/m2}$$

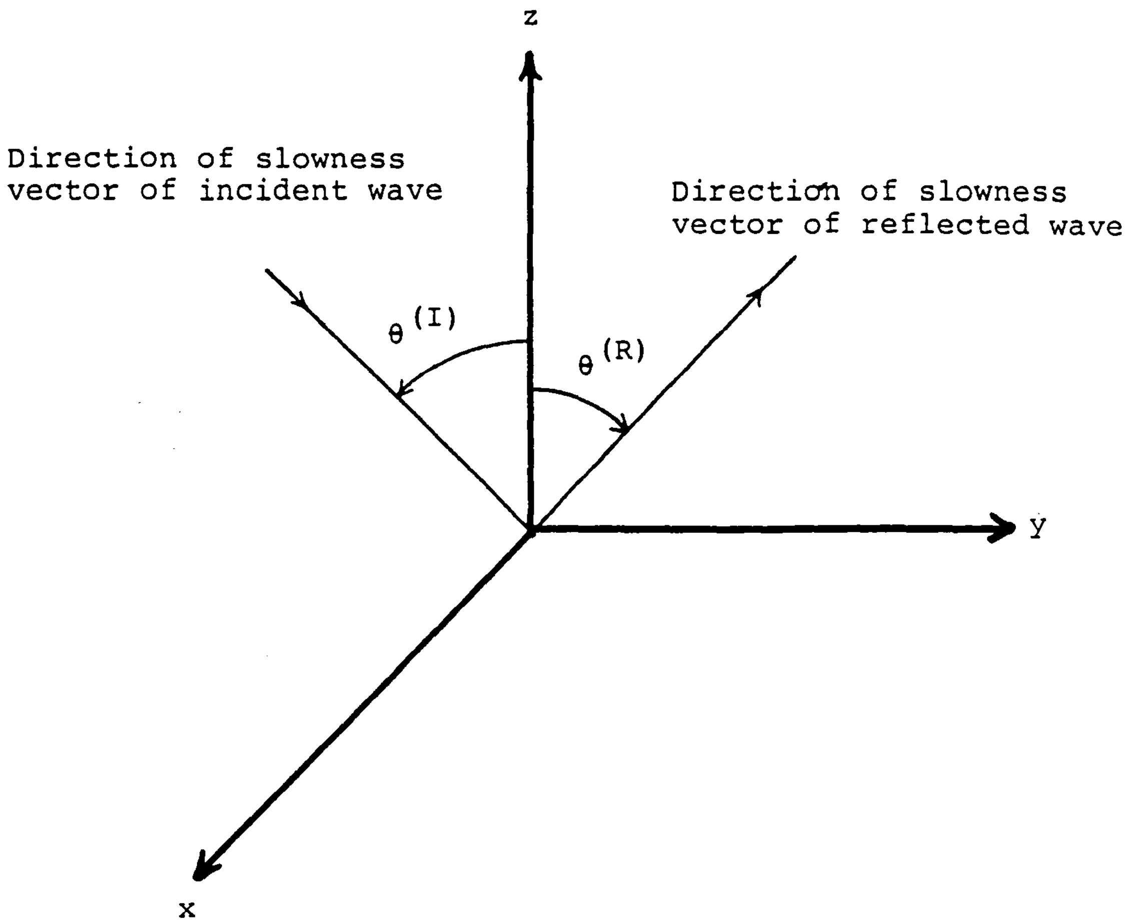

Fig. 1 Angle of incidence $\theta^{(I)}$ of incident SH wave and angle of reflection $\theta^{(R)}$ of reflected SH wave shown in y-z plane of (x,y,z) coordinate system.

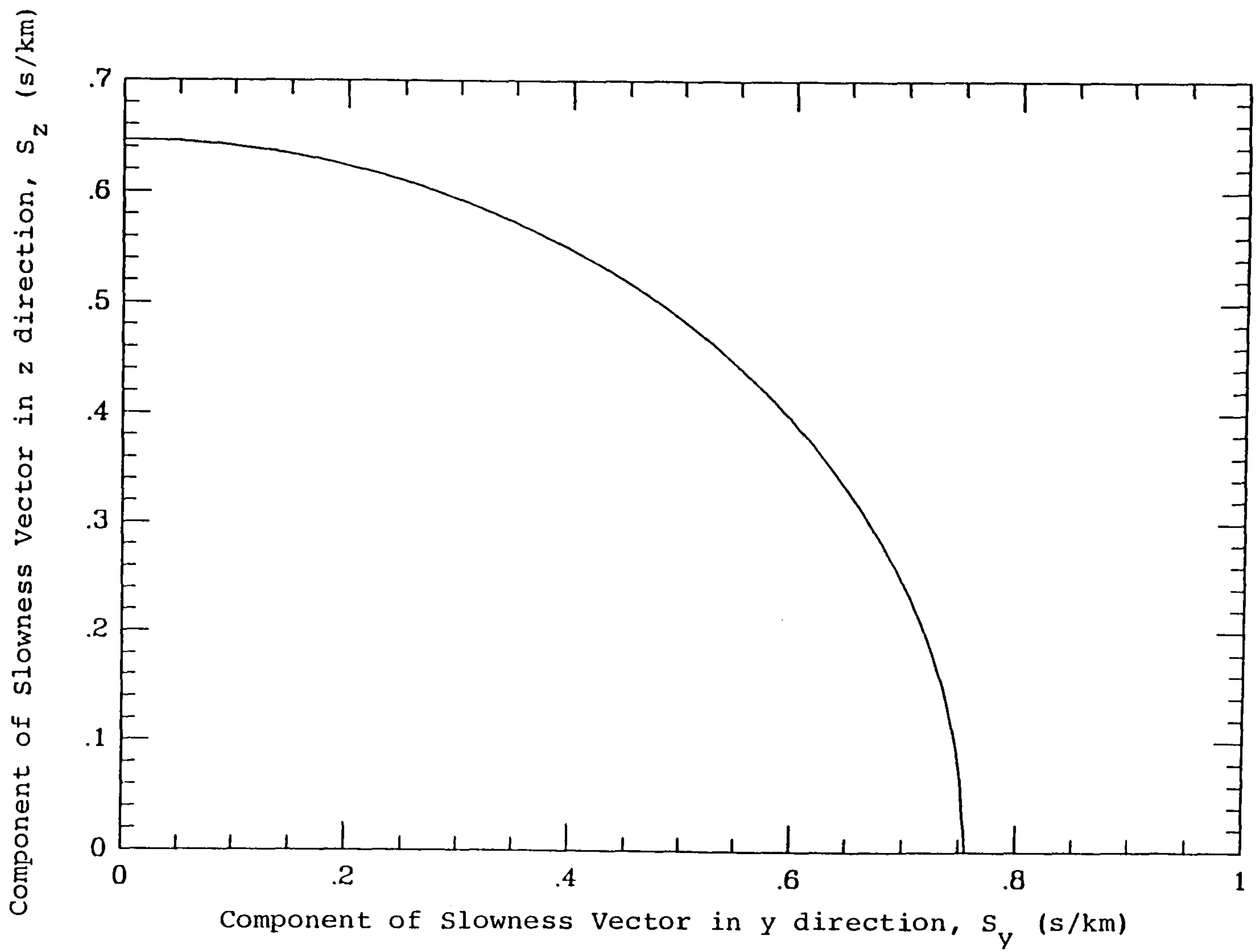

Fig. 2 First quadrant of slowness surface of SH wave in fiberglass epoxy composite in y-z plane.

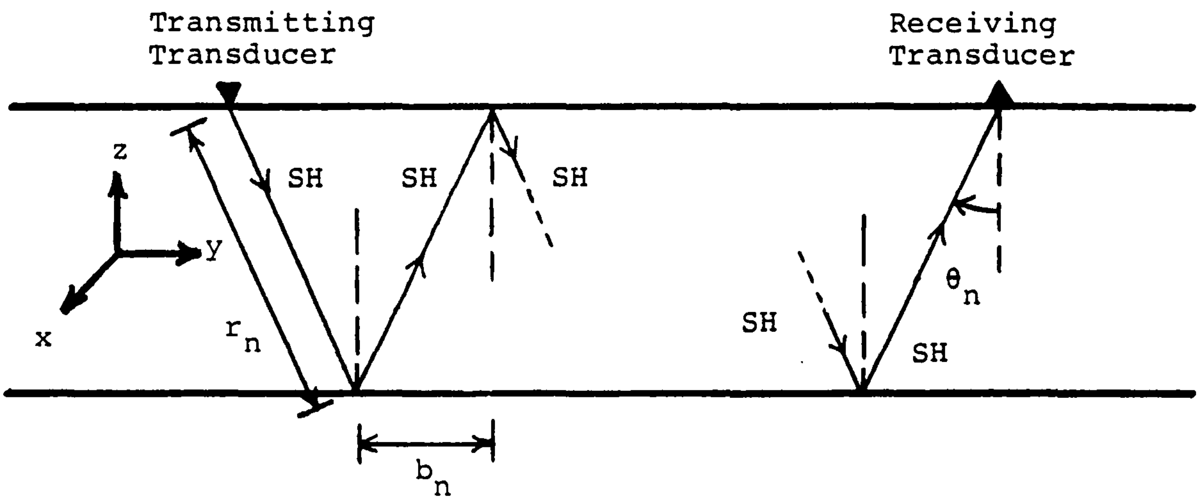

Fig. 3 SH wave experiencing 2n-1 reflections between transmitting transducer and receiving transducer.

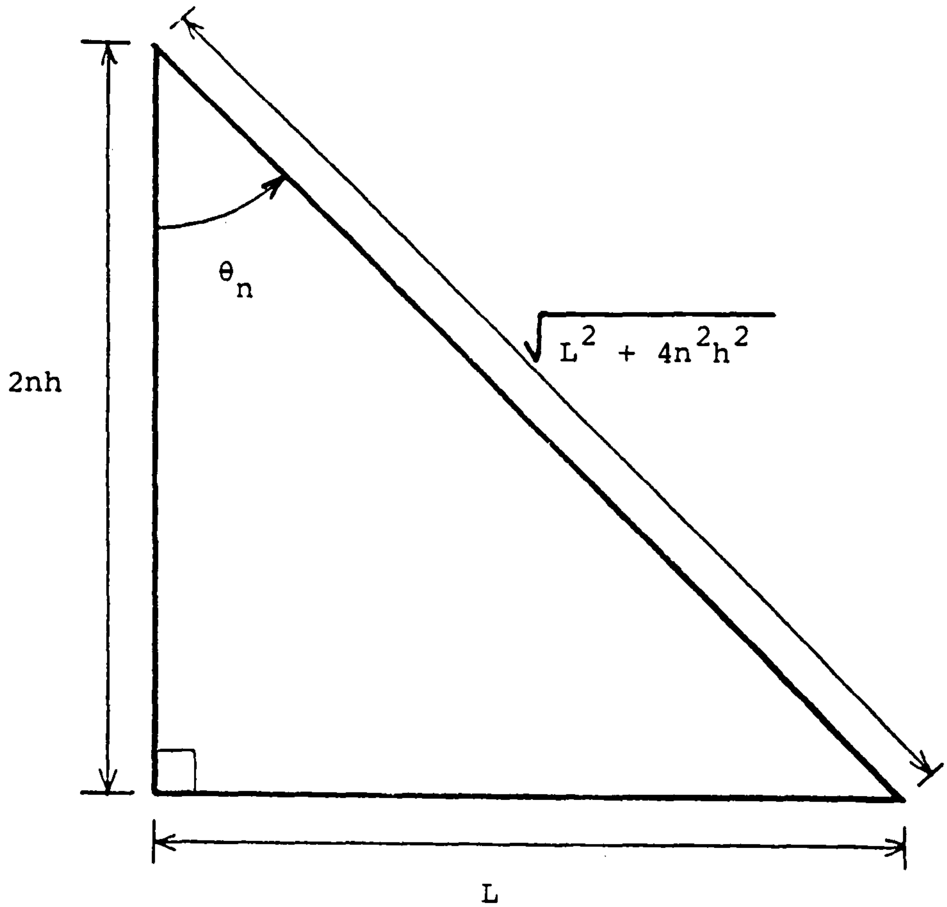

Fig. 4 Right triangle from which trigonometric functions of θ_n are derived.

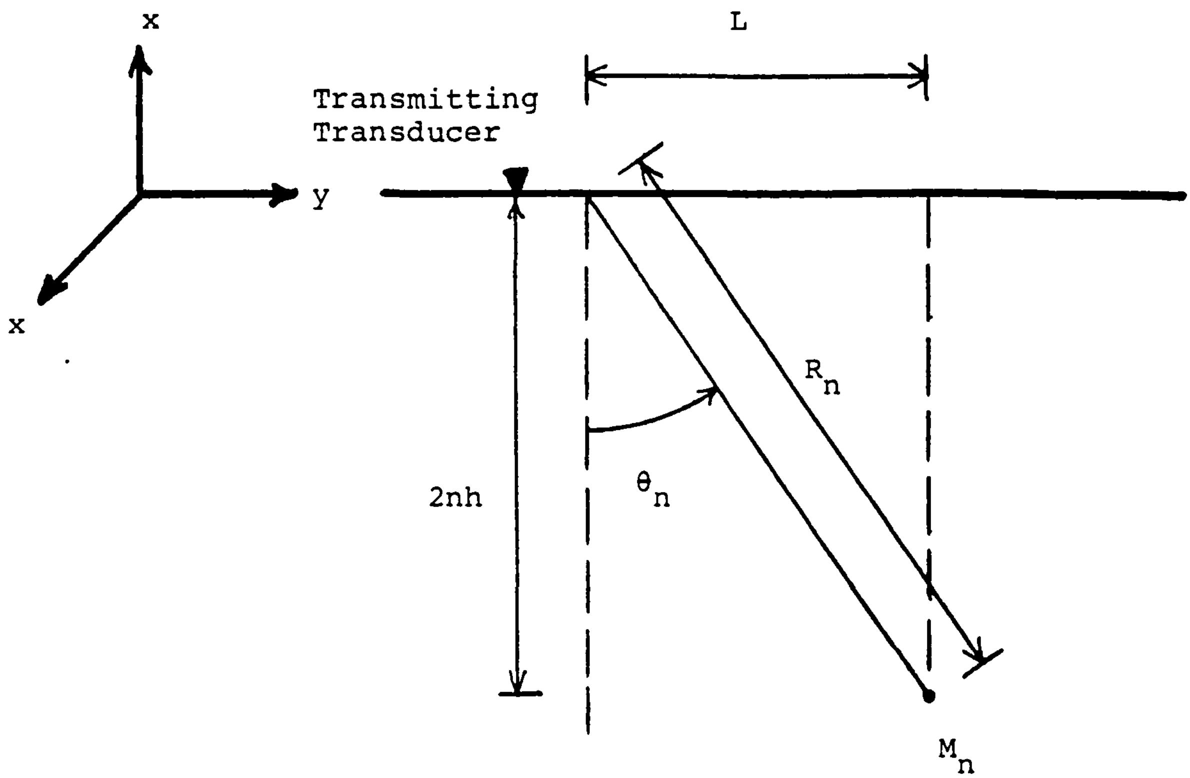

Fig. 5 SH wave traveling distance R_n to hypothetical point M_n in semi-infinite body.

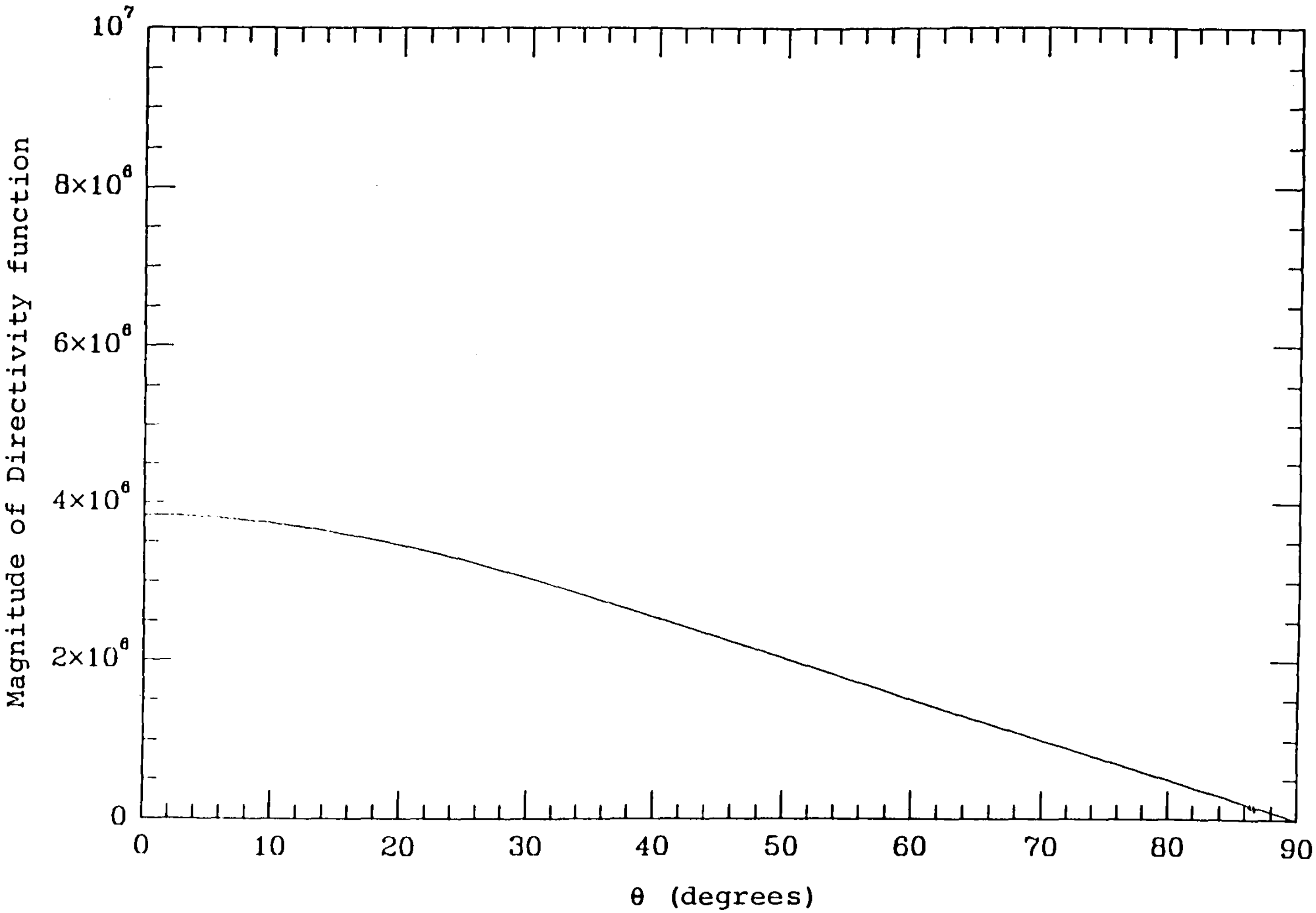

Fig. 6 Directivity function $D^{(SH)}$ of shear stress as function of angle from the z axis for frequency 0.75 MHz.

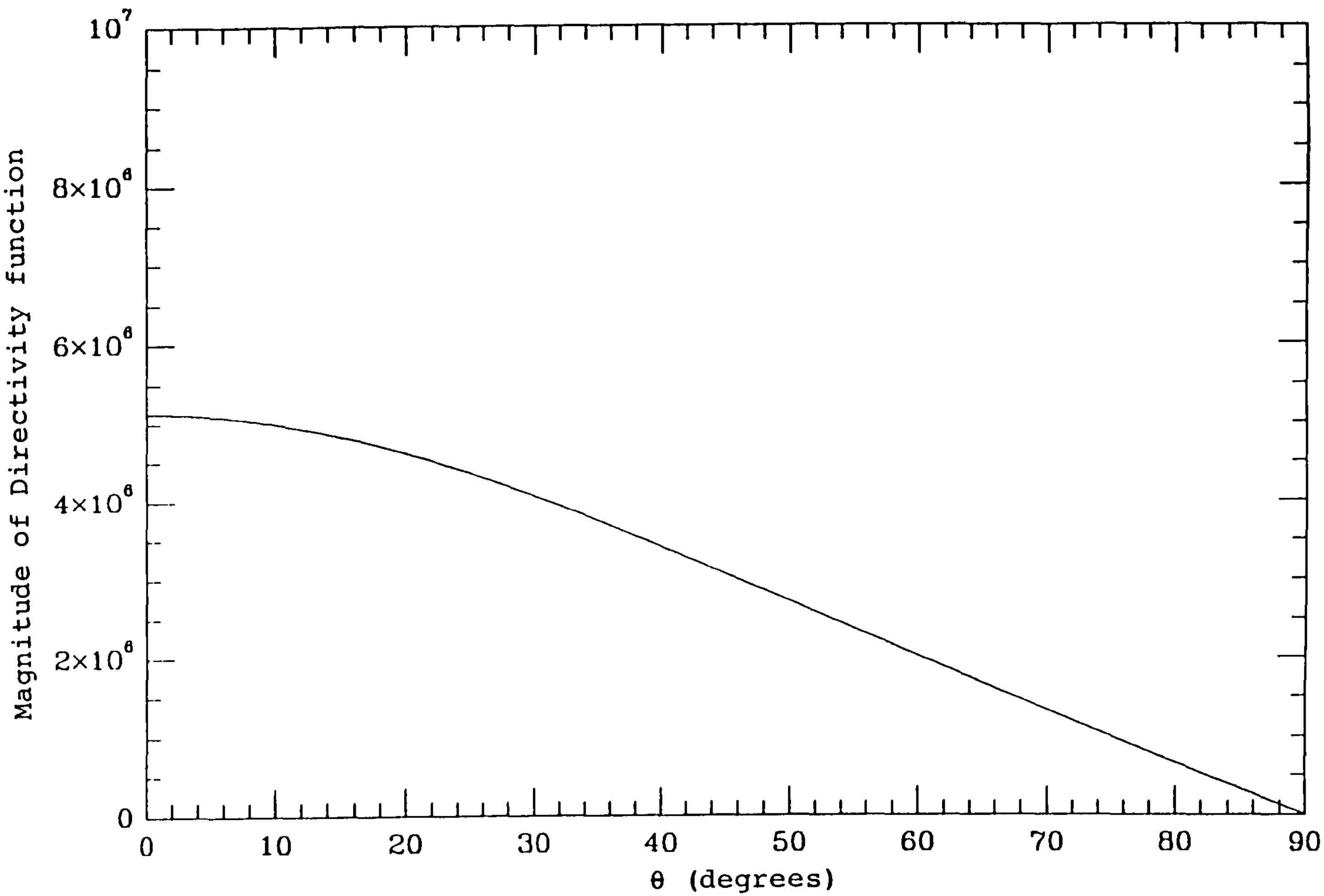

Fig. 7 Directivity function $D^{(SH)}$ of shear stress as function of angle from z axis for frequency 1.00 MHz.

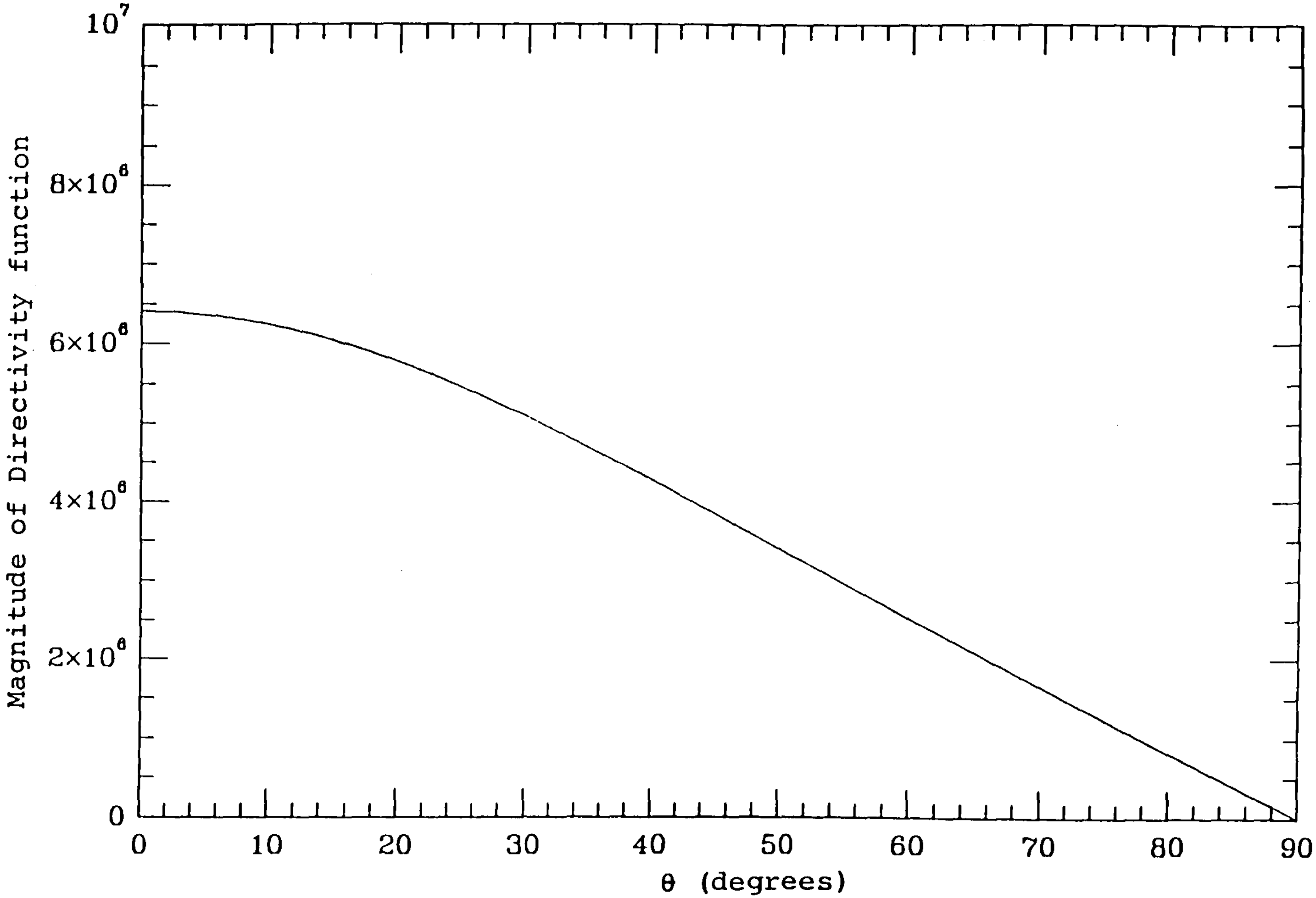

Fig. 8 Directivity function $D^{(SH)}$ of shear stress as a function of angle from z axis for frequency 1.25 MHz.

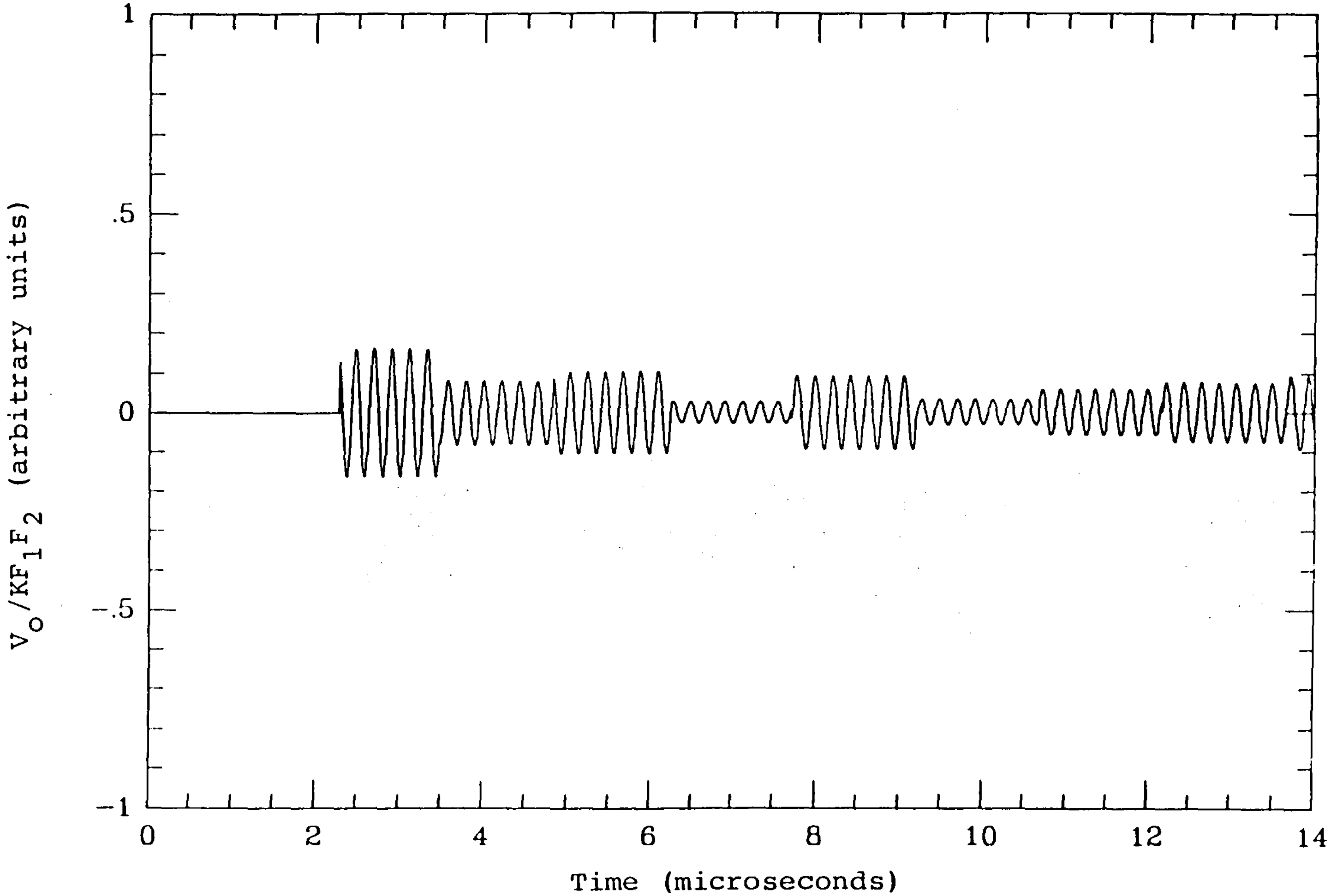

Fig. 9 Normalized output voltage as function of time for frequency 0.75 MHz.

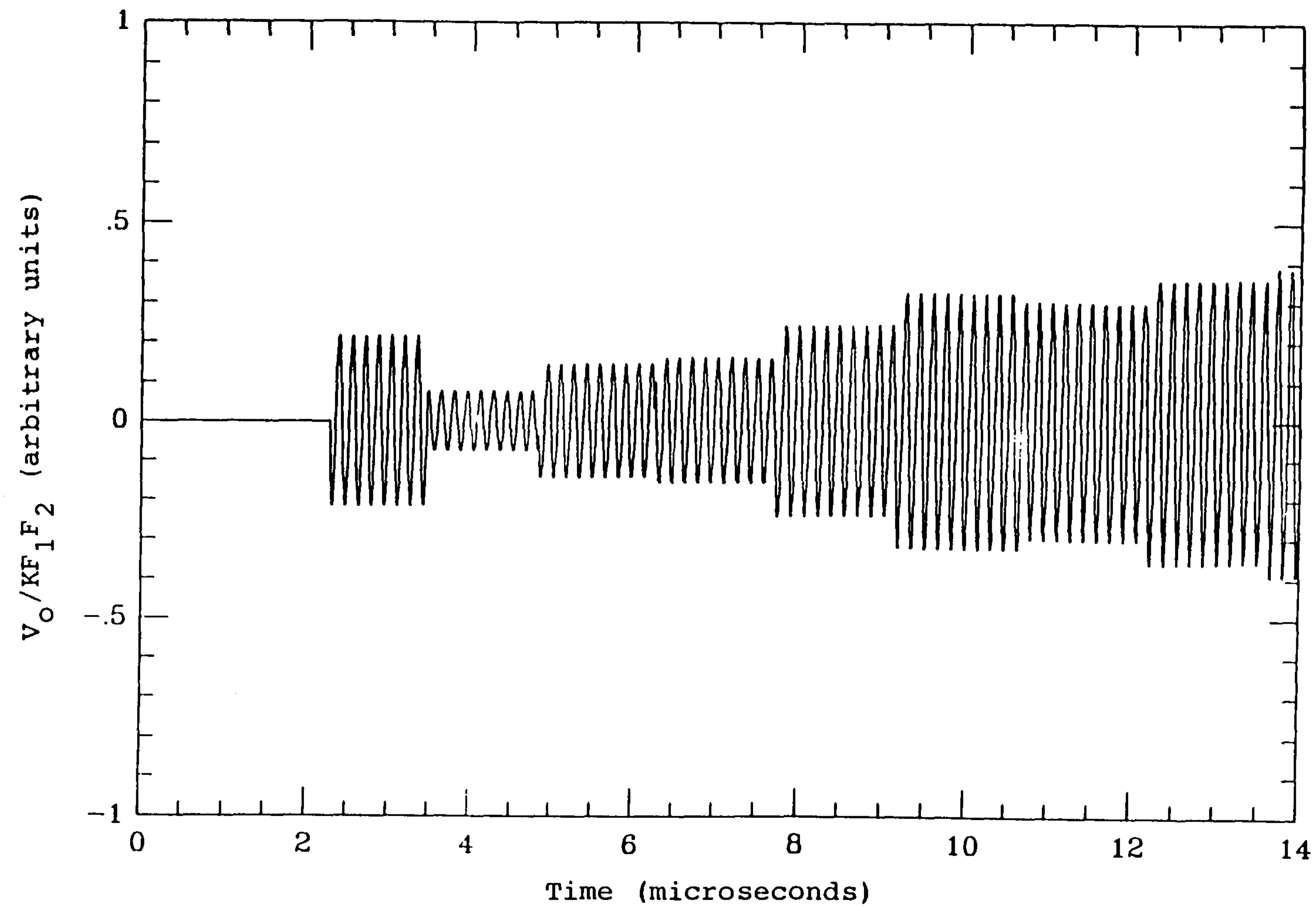

Fig. 10 Normalized output voltage as function of time for frequency 1.00 MHz.

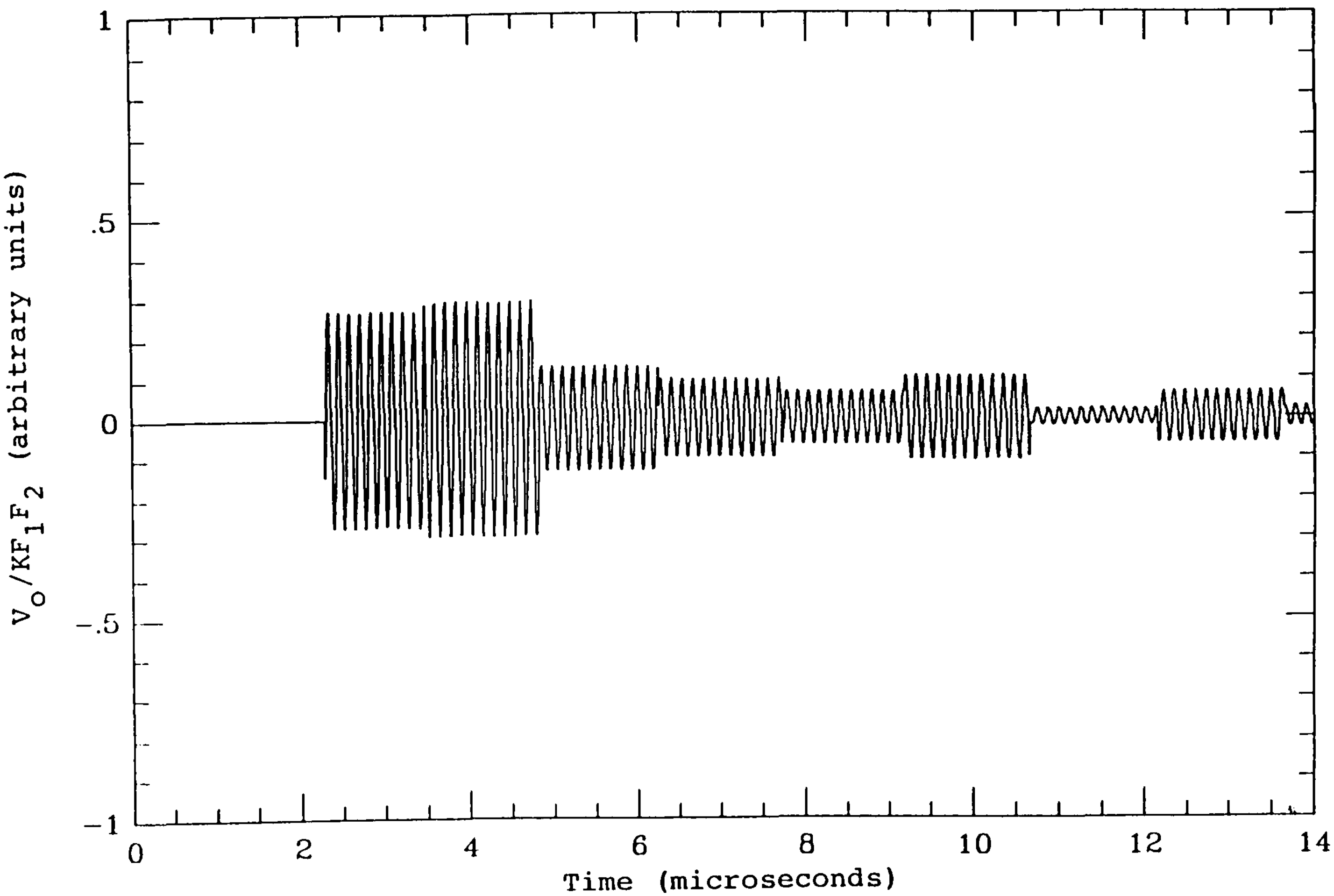

Fig. 11 Normalized output voltage as function of time for frequency 1.25 MHz.

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16. Abstract

Input-output characterization of fiber composites is studied theoretically by tracing SH waves in the media. A fiberglass epoxy composite is modeled as a homogeneous transversely isotropic continuum plate. The reflection of a SH wave at a stress-free plane boundary in a semi-infinite transversely isotropic medium is considered first. It is found that an incident SH wave reflects only a similar SH wave back into the medium. It is also established that the angle of reflection of the reflected wave is equal to the angle of incidence of the incident wave. The phase velocity of the SH waves and the delay time of the SH waves in reaching the receiving transducer are computed as functions of a reflection index, defined as the number of reflections of the SH waves from the bottom face of the continuum plate. The directivity function corresponding to the shear stress associated with the SH waves in the continuum plate is also derived as a function of the reflection index. A theoretical output voltage from the receiving transducer is calculated for a tone burst (a periodic input voltage of finite duration). The output voltage is shown for tone bursts of duration 60 μs and center frequencies 0.75, 1.00, and 1.25 MHz. The study enhances the quantitative and qualitative understanding of the nondestructive evaluation (NDE) of fiber composites which can be modeled as transversely isotropic media.

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