

A BENCHMARK FOR GALACTIC COSMIC
RAY TRANSPORT CODES

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COSMIC RAY TRANSPORT BENCHMARK

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A nontrivial analytic benchmark solution for galactic cosmic ray transport is presented for use in transport code validation. Computational accuracy for a previously-developed cosmic ray transport code is established to within one percent by comparison with this exact benchmark. Hence, solution accuracy for the transport problem is mainly limited by inaccuracies in the input spectra, input interaction databases, and the use of a straight ahead/velocity-conserving approximation.

INTRODUCTION

With the advent of the Space Station era, and future manned lunar bases and Mars missions under consideration, considerable attention must be given to developing methods for shielding against the high-energy heavy ion (HZE) component of galactic cosmic rays, since their ranges are generally comparable to their mean free paths for nuclear interaction in tissue and low density shield materials. Because these HZE particles include nearly all nuclear species, and possess a broad spectrum of energies, detailed laboratory measurements for all possible ion-shielding combinations are not practical; therefore, accurate calculational methods describing the interactions and transport of these energetic ions in bulk matter are needed to properly evaluate the shielding effectiveness of spacecraft structures and the self-shielding factors of the astronauts themselves.

Whenever galactic cosmic rays traverse bulk matter, their radiation fields change composition through interactions with the target materials encountered. Aside from continuously losing energy through collisions with orbital electrons, the incident ions and target nuclei also undergo nuclear fragmentation (breakup) reactions. These fragmentations result in the production of secondary and subsequent generation reaction products which alter the isotopic composition of the transported radiation field. Studies of these radiation fields are presently hampered by the lack of an adequate nuclear fragmentation cross section data base over the broad spectrum of energies and fragmenting species involved. Because the experimental fragmentation data base is sparse and no adequate quantitative fundamental theory exists, a semiempirical formulation was developed for use in cosmic ray studies (1). Unfortunately, it lacks charge and mass conservation for secondary fragments (2). Recently, an alternative semiempirical fragmentation

model, which is based upon more fundamental physical considerations and which conserves fragment mass and charge, has been formulated (3). Its predicted cross sections agree with available experimental fragmentation data to the extent that these data agree among themselves. This fragmentation model has been successfully incorporated into a recently-developed galactic cosmic ray (GCR) transport code (4,5) for use in space radiation shielding applications. In the GCR code, methods previously developed for nucleon transport (6) were extended to HZE transport by a combination of analytic and numerical tools. The GCR ion transport problem was transformed to an integral along the characteristic curve of that particular ion. Fragment velocity conservation then enables the perturbation series (6) to be replaced by a simple numerical procedure. The resulting method reduces the difficulty associated with low-energy discretization, eliminates any restrictions to a particular functional form for the stopping power, and is computationally simple and nondemanding upon computer resources. Details of the solution method and a comparison with an alternative code are published elsewhere (4,5).

In the present report we address the question of GCR transport code validation. Ideally, validation should be accomplished using detailed transport data obtained from carefully planned and controlled experiments; unfortunately, there exists a paucity of such data. Although useful for comparison purposes, the atmospheric propagation measurements used previously (5) are clearly not definitive since they consist of integral fluences of as many as ten different nuclear species combined into a single datum. Although limited quantities of HZE dosimetry measurements from manned space missions (e.g. Skylab) are also available (7), numerous assumptions concerning the relationships between dosimeter locations and spacecraft shield thicknesses

and geometry must be made in order to estimate astronaut doses using GCR codes. Since many of these assumptions may involve inherently large uncertainties (a factor of two or greater), it becomes difficult to attribute sources of any comparison differences to particular assumptions or approximations which may have been used in the analyses. In the absence of definitive GCR transport measurements with which to compare code predictions, other methods of validation must be considered. As noted in references (2) and (4), there are several different versions of HZE transport codes available. When used with the same input spectra, interaction parameters, and boundary conditions, all should yield comparable results. The history of transport code development, however, suggest otherwise. For this reason, a realistic, nontrivial, exact, analytic solution to the simplified Boltzmann equation used to describe HZE transport has been formulated as an absolute standard for code comparison purposes.

GCR TRANSPORT BENCHMARK

In passing through bulk matter, heavy ions lose energy through interactions with atomic orbital electrons along their trajectories. On occasion there is a violent collision with nuclei of the target medium. These collisions produce projectile fragments moving in the forward direction and low-energy fragments of the struck target nucleus which are nearly isotropically distributed. The transport equations for these short-range target fragments can be solved in closed form in terms of collision density (8); therefore, projectile fragment transport is the main subject of current interest in HZE transport.

For the GCR transport problem, one typically uses the straight ahead approximation and neglects target secondary fragments enabling the transport equation to be written as (4,5)

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} \bar{S}_j(E) + \sigma_j \right] \phi_j(x, E) = \sum_{k>j} m_{jk} \sigma_k \phi_k(x, E) \quad (1)$$

where $\phi_j(x, E)$ is the flux of ions of type j with atomic mass A_j at x moving along the x axis at energy E in units of MeV/amu, σ_j is the corresponding macroscopic nuclear absorption cross section, $\bar{S}_j(E)$ is the change in E per unit distance, and m_{jk} is the multiplicity of ion j produced in collision by ion k . The range of the ion is given as

$$R_j(E) = \int_0^E \frac{dE'}{\bar{S}_j(E')} \quad (2)$$

The solution to Eq. (1) is found to be subject to boundary specification at $x = 0$ and arbitrary E as

$$\phi_j(0, E) = F_j(E) \quad (3)$$

where $F_j(E)$ is called the incident beam spectrum.

It follows from Bethe's theory that

$$\bar{S}_j(E) = \frac{A_p Z_j^2}{A_j Z_p^2} \bar{S}_p(E) \quad (4)$$

which holds for all energies above 10 MeV/amu provided the ions remain fully stripped. We can then write for the j^{th} ion

$$\frac{Z_j^2}{A_j} R_j(E) = \frac{Z_p^2}{A_p} R_p(E) \quad (5)$$

where the subscript p refers to protons. Equation (5) is accurate for high energies but only approximately correct at low energy because of : (a) electron capture by the ion which effectively reduces its charge, (b) higher order Born corrections to Bethe's theory, and (c) nuclear stopping at the lowest energies. The range scale parameters v_j are obtained from

$$v_j R_j(E) = v_k R_k(E) \quad (6)$$

and are generally energy-dependent. When the ion velocity is large compared to the velocity of the orbital electrons the v_j approach

$$v_j = \frac{Z_j^2}{A_j} \quad (7)$$

For the benchmark problem, the incident spectrum is limited to a single ion type ($j = J$). Since the GCR spectrum for a typical ion is of the form

$$F(E) \sim E^{-\alpha} \quad (8)$$

where $\alpha \approx 2.5$, we choose the energy spectrum to be of similar functional form as

$$F_j(E) = \delta_{jJ} / \{ [R_j(E)]^2 \tilde{S}_j(E) \} \quad (9)$$

Defining the characteristic variables

$$\eta_j = x - R_j(E) \quad (10)$$

and

$$\xi_j = x + R_j(E) \quad (11)$$

equation (1) can be solved by the method of characteristics (4,5) to give

$$\bar{\phi}_j(x, E) = e^{-\hat{\sigma}_j x} / [v_j x + R_j(E)]^2 \quad (12)$$

where

$$\bar{\phi}_j(x, E) \equiv \tilde{S}_j(E) \phi_j(x, E) \quad (13)$$

and

$$\hat{\sigma}_j = \sigma_j(1 - m_{jJ}) \quad (14)$$

This is the trivial solution for the incident beam species. For $j < J$ (secondary fragments) it can be shown that

$$\bar{\phi}_j(x, E) = \sigma_J M_{jJ} \frac{v_j}{v_J} I_j(x, E) e^{-(\hat{\sigma}_j \eta_j + \hat{\sigma}_j \xi_j)/2} \quad (15)$$

where, in terms of the exponential integral function $E_2(x)$,

$$I_j(x, E) = \frac{e^{-b(v_J + v_j)\xi_j/2}}{(v_J - v_j)} \left[\frac{E_2(b v_j \xi_j)}{v_j \xi_j} - \frac{E_2(b v_J \xi_J)}{v_J \xi_J} \right] \quad (16)$$

for $j = J-1$ and

$$b = \frac{(\hat{\sigma}_J - \hat{\sigma}_j)}{(v_J - v_j)} \quad (17)$$

Clearly, equations (16) and (17) are true for all j if $m_{kj} = 0$ for all $j < J$ (i.e. the secondary fragments themselves do not fragment).

BENCHMARK RESULTS

The benchmark solution was calculated for an incident iron beam ($J = 26$) in an aluminum target, for which the input parameters are

$$\hat{\sigma}_{26} = .04568 \text{ cm/g}^2$$

$$\hat{\sigma}_{25} = .04260 \text{ cm/g}^2$$

$$M_{25,26} \sigma_{26} = .00403 \text{ cm/g}^2$$

Results of the GCR transport code simulation of this benchmark for the propagating incident iron beam and secondary manganese ($j = 25$) ions are displayed in Tables 1 and 2 where they are compared to the exact analytic predictions obtained from equations (12) and (15). It is clear from these tabulated results that the numerical solution methods developed previously (4,5) are accurate in solving equation (1) for GCR transport to within one percent. This indicates that any limitations to accurately solving GCR transport problems must focus upon the simplifying approximations used to obtain equation (1), as well as upon unresolved issues concerning the need to include multiple-Coulomb scattering effects, fragment momentum dispersion effects, and perhaps most importantly, the nature and quality of the input cross section data bases. To illustrate this last point, we are aware of only one heavy ion transport code (2) which uses energy-dependent cross sections. Recent studies, however, suggest that fully energy-dependent cross sections may be important for some transport code applications (9).

CONCLUDING REMARKS

The need to develop suitable benchmarks for use in validating and comparing existing galactic cosmic ray transport codes has been described and an exact nontrivial analytic benchmark solution presented. This benchmark solution was then used to establish computational accuracy for a previously-published cosmic ray transport code to within one percent. Finally, remaining unresolved issues in GCR transport were briefly described.

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TABLE 1

Comparison of the Benchmark Numerical Simulation to the Analytic Solution for Iron Ions as a Function of Ion Energy and Depth into the Aluminum Absorber.

E, MeV/amu	$\bar{\Phi}_{\text{Fe}}(0, E)$		$\bar{\Phi}_{\text{Fe}}(10\text{g/cm}^2, E)$		$\bar{\Phi}_{\text{Fe}}(20\text{g/cm}^2, E)$	
	numerical	analytic	numerical	analytic	numerical	analytic
.0198	1.394E5	1.394E5	4.334E-5	4.382E-5	6.942E-6	7.044E-6
.1147	1.692E4	1.692E4	4.334E-5	4.381E-5	6.942E-6	7.044E-6
1.090	9.217E2	9.217E2	4.333E-5	4.379E-5	6.942E-6	7.043E-6
10.07	1.062E1	1.062E1	4.321E-5	4.360E-5	6.932E-6	7.027E-6
100.1	9.310E-3	9.310E-3	3.699E-5	3.718E-5	6.400E-6	6.478E-6
1059.	5.089E-6	5.089E-6	2.014E-6	2.019E-6	8.741E-7	8.799E-7
10490.	2.970E-8	2.970E-8	1.833E-8	1.833E-8	1.132E-8	1.132E-8

TABLE 2

Comparison of the Benchmark Numerical Simulation to the Analytic Solution for Secondary Manganese Ions as a Function of Ion Energy and Depth into the Aluminum Absorber.

E, MeV/amu	$\bar{\phi}_{Mn}(10g/cm^2, E)$		$\bar{\phi}_{Mn}(20g/cm^2, E)$	
	numerical	analytic	numerical	analytic
.0198	1.772E-6	1.780E-6	5.704E-7	5.768E-7
.1147	1.772E-6	1.780E-6	5.704E-7	5.768E-7
1.090	1.772E-6	1.779E-6	5.704E-7	5.767E-7
10.07	1.767E-6	1.771E-6	5.696E-7	5.753E-7
100.1	1.504E-6	1.503E-6	5.242E-7	5.291E-7
1059.	7.797E-8	7.806E-8	6.880E-8	6.918E-7
10490.	7.004E-10	7.004E-10	8.728E-10	8.728E-10