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## INTERLEAVER DESIGN FOR TRELIS-CODED DIFFERENTIAL 8-PSK MODULATION WITH NON-COHERENT DETECTION

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### ABSTRACT

Trellis-coded differential 8-DPSK with non-coherent detection is considered to be an efficient modulation scheme for satellite mobile communication. Theoretical analysis of such systems often assumes perfect, i.e., infinite interleaving. In this paper the effect of finite interleaver size on bit error rate (BER) performance of coded 8-DPSK is determined by means of computer simulations. The losses evaluated in this way include the SNR-degradation due to the timing- and frequency errors of the symbol synchronizer and the AFC (automatic frequency control) of the receiver. BER-measurements are presented using a conventional 2/3-rate convolutional 8-state trellis code for typical Rayleigh- and Rician fading channels. It is shown that for a Rician channel with Rician parameter of 7 dB, a Doppler spread of 100 Hz and a data rate of 2400 bps, an interleaver with size 16x16 symbols performs nearly as well as a very large interleaver. It is also shown that for moderately fast Rayleigh channels the BER-curves flatten out at large SNR. This is due to the fact, that the random phases of the fading channel in rare cases coincide with valid phase sequences of coded symbols, thus producing error events, even without any additive noise disturbance. For such channels an additional code design criterion would be to minimize the error floor.

### INTRODUCTION

Trellis-coded modulation provides coding gain without expanding bandwidth [Ungerböck, 1982]. Most applications in the past have been concerned with coherent systems. For fading channels non-coherent modulation schemes like trellis-coded differential octal phase shift keying (coded 8-DPSK) with differentially coherent detection are probably more suitable since they do not require the derivation of a coherent reference carrier. Coded 8-DPSK has been investigated by [Simon and Divsalar 1987, McLane et al. 1987, Edbauer 1987, Makrakis et al. 1987]. Theoretical analysis of interleaved systems often assumes ideal, i.e., infinite interleaving, to make the channel to appear memoryless. In this paper the effect of finite interleaver size on BER-performance is determined by means of computer simulations for a Rayleigh and a Rician channel, which are considered to be typical of mobile satellite communications.

### MODULATOR FOR CODED 8-DPSK SIGNALS

The modulator is shown in Fig. 1. The input bits are passed through a rate 2/3 trellis encoder which generates at its output coded 3-bit tuples. These are written into a block interleaver row by row. The interleaver consists of  $l$  rows and  $r$  columns and its size is denoted by  $l \times r$ . The 3-bit tuples are read out column by column and mapped into coded 8-PSK symbols  $a'_n$  ( $|a'_n| = 1$ ) in the complex signal plane according to the set partitioning method [Ungerböck, 1982]. The symbols  $a'_n$  are then differentially phase encoded producing coded 8-DPSK symbols  $c_n$ .



Fig. 1. Generation of Interleaved Trellis-Coded 8-DPSK Signals

$$c_n = c_{n-1} a'_n. \quad (1)$$

These are filtered by a pulse shape filter with impulse response  $h(t)$ , supplying

$$x(t) = \sum_i c_i h(t - iT), \quad (2)$$

where  $T$  is the symbol period.  $x(t)$  is converted to the radio frequency and transmitted.

#### DIGITAL RECEIVER WITH SOFT DECISION 8-PSK VITERBI DECODER FOR CODED 8-DPSK SIGNALS

A simplified block diagram of the receiver in the complex baseband is drawn in Fig. 2. Coarse AFC, AGC, filtering and interleaver synchronisation are not shown. We assume Nyquist signaling with zeros of the filter impulse response at multiples of the symbol period, satisfying Nyquist's first criterion. For the computer simulation the full Nyquist filter is placed on the transmitter side.

The receiver input signal is written in the complex baseband in the form

$$y(t) = A(t)x(t) + w(t). \quad (3)$$

$A(t)$  is a complex-valued stochastic process which describes multiplicative frequency non-selective fading. For Rayleigh- and Rician channels  $A(t)$  is a complex Gaussian process. The envelope  $|A(t)|$  has a Rayleigh- or Rician distribution.  $w(t)$  is additive complex Gaussian noise. The input signal is sampled once per modulation interval by an A/D-converter providing the complex sample

$$y_n = A_n x_n + w_n. \quad (4)$$

$x_n$  is the sample of  $x(t)$  at time  $t_n = nT - \tau$ , where  $n = 0, 1, 2, \dots$ , and  $\tau$ , with  $-T/2 \leq \tau < T/2$ , determines the sampling point within a symbol period. The noise samples  $w_n$  can be assumed to be complex Gaussian and uncorrelated. The average bit energy to spectral noise power density ratio is

$$\frac{E_b}{N_o} = \frac{E\{|A|^2\}}{4\sigma^2}, \quad (5)$$

where  $E\{\}$  denotes expectation and  $\sigma^2$  is the variance of the noise  $w_n$  in each dimension. The average signal power  $E\{|A|^2\}$  can be determined by the AGC of the receiver whose time constant is large with respect to the fading period. In the following the normalization  $E\{|A|^2\} = 1$  is assumed.

The receiver performs the following signal processing functions: differentially coherent 8-DPSK symbol detection, hard 8-PSK detection, automatic frequency control (AFC), symbol synchronisation (timing) and soft decision Viterbi decoding.

Differential symbol detection is performed by a one-delay symbol detector whose output is

$$z_n = y'_n y'^*_{n-1} = h(\tau) a'_n + n_n + v_n. \quad (6)$$

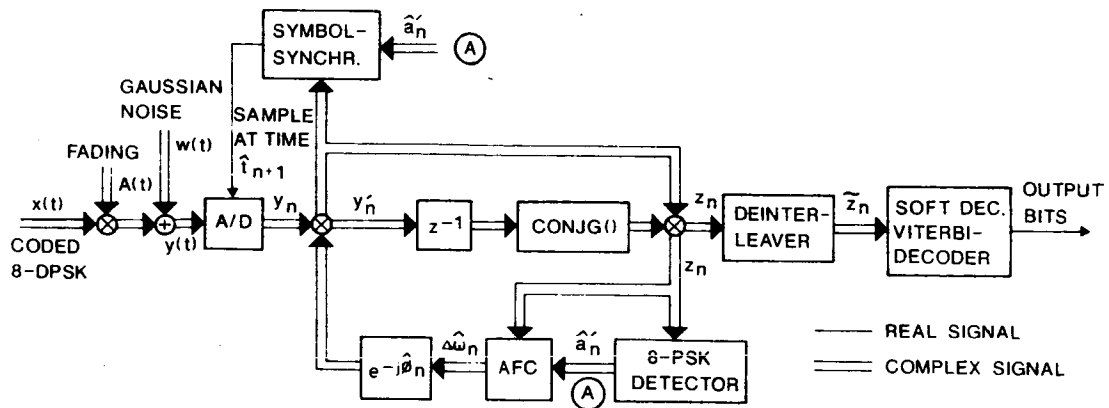


Fig. 2. Simplified Block Diagram of the Digital Receiver with Soft Decision Viterbi Decoder for Interleaved Trellis-Coded 8-DPSK Modulation

where  $y'_n$  is the back-rotated version of the received 8-DPSK symbol  $y_n$  after frequency offset compensation (see Eq. (10)), and  $(\cdot)$  denotes the conjugate complex value.  $n_n$  is additive complex noise and  $v_n$  is the intersymbol interference term which vanishes for exact timing ( $\tau = 0$ ).

The hard 8-PSK decision  $\hat{a}'_n$  of  $z_n$  is given by that symbol  $\tilde{a}_n$  which is closest to  $z_n$ :

$$\hat{a}'_n \leftarrow \min_{\tilde{a}_n} |z_n - \tilde{a}_n|. \quad (7)$$

$\hat{a}'_n$  is needed for decision-directed AFC and decision-directed symbol synchronisation.

An error signal for the radian frequency offset  $\Delta\omega_n$  of the receiver input signal  $y_n$  is given by

$$e_{afc,n} = \text{Im}\{z_n \cdot \hat{a}'_n\}. \quad (8)$$

$\hat{\Delta\omega}_n$ , being the estimate of  $\Delta\omega_n$ , is updated by a first order loop

$$\hat{\Delta\omega}_n = \hat{\Delta\omega}_{n-1} + \frac{e_{afc,n}}{T_{afc}}, \quad (9)$$

where  $T_{afc}$  is the AFC time constant for high SNR and small frequency errors. The frequency offset is compensated by back-rotating  $y_n$  with angle  $\hat{\phi}_n$ , yielding  $y'_n$ . The rotated signal then reads

$$y'_n = y_n \exp(-j\hat{\phi}_n) \quad (10)$$

with

$$\hat{\phi}_n = \hat{\phi}_{n-1} + \hat{\Delta\omega}_n \cdot T. \quad (11)$$

Symbol synchronisation for coded 8-DPSK [Edbauer, 1987] is achieved by using an approach similar to [Mueller, 1976]. It can be shown, that an error signal for the timing error  $\tau = nT - t_n$ , where  $t_n$  is the sampling time of the A/D-converter for symbol  $n$ , is given by

$$e_{t,n} = T \cdot \text{Re}\{y'_n y'^*_{n-1} - \hat{a}'_n |y'_{n-1}|^2\}. \quad (12)$$

Eq. (12) has the property that it is invariant against a constant phase rotation, thus enabling non-coherent operation. For a first order loop one obtains

$$\hat{t}_{n+1} = \hat{t}_n + T + \frac{4\alpha e_{t,n}}{\lambda}, \quad (13)$$

where  $\alpha$  is the ratio of loop noise bandwidth to symbol rate, and  $\lambda$  is the slope of the average loop S-curve at the origin  $\tau = 0$ .

Decoding of the trellis code is performed by a Viterbi decoder which uses the unquantized deinterleaved symbol detector outputs  $\tilde{z}_n$  of Eq. (6) for metric computation. The Viterbi algorithm is used to determine that coded 8-PSK symbol sequence  $\{\tilde{a}_n\}$  whose quadratic Euclidean distance is minimum with respect to the differentially detected noisy 8-PSK symbol sequence  $\{\tilde{z}_n\}$ . The metric increment over one trellis branch therefore is

$$q_n = -|\tilde{z}_n - \tilde{a}_n|^2, \quad (14)$$

where  $\tilde{a}_n$  is the coded 8-PSK symbol attributed to that branch. We note that Eq. (14) is not the optimum metric in the maximum likelihood sense, since the noise  $n_n$  of the 8-DPSK detector output (Eq. (6)) is non-Gaussian and correlated.

The digital signal processing and decoding methods, as applied here, are explained in more detail in [Edbauer, 1987].

## COMPUTER SIMULATION RESULTS

Computer simulations of the proposed coded 8-DPSK system have been performed to determine the effect of finite interleaver size on BER-performance. Symbol synchronisation and AFC have been included in the simulation programs as described above. Two typical mobil satellite fading channels, a Rician and a Rayleigh channel with a Doppler spread of 100 Hz, have been chosen for the simulation. The data rate, having been used, was 2400 bps, which may allow sufficient digital speech quality for aeronautical applications. The Rician parameter  $K$ , i.e., the power ratio of the direct line of sight signal to the scattered non-coherent component, was 7 dB. The choice of channel parameters and of the data rate was mainly motivated by aeronautical channel measurements [Neul et al., 1987]. In the simulation, samples  $A_n$  (Eq. (4)) of the complex multiplicative fading were computed by digitally low-pass filtering independent Gaussian random numbers. The Doppler spread of channels, generated in this way, is twice the bandwidth of these filters. Pulse shape filtering was performed by a raised cosine Nyquist filter with roll-off of 40 % and impulse response length of 9 symbol periods. A conventional 8-state Ungerböck type convolutional code [Ungerböck 1982, Fig. 9] without feed-back was used as trellis code. Since the interleaver size should be the only varying parameter the decision delay of the Viterbi decoder was kept constant and equal to  $M = 16$  symbols.

Usually the block interleaver size is chosen in that way that the time for transmission of a column - i.e. the number of rows - is of the order of expected fade durations, and that the number of columns is equal to the decoding decision delay. In the simulation even smaller interleaver sizes were selected since short delays are desirable especially for speech transmission.

The BER results for the Rician channel with the interleaver size as parameter are shown in Fig. 3. It can be seen that a small interleaver with size 16x16 is only 0.5 dB worse than the large 100x30 interleaver at all  $E_b/N_0$  values of the figure. The smaller 8x8 interleaver would also be acceptable giving a total delay of 120 ms between transmitter and receiver for digital 2400 bps speech transmission. No further improvement was possible by using an interleaver larger than 100x30. If no interleaver were used the losses would be 12 dB and 6 dB at BER of  $P_b = 10^{-4}$  and  $P_b = 10^{-3}$  with respect to the largest interleaver. For comparison reasons the theoretical and the computer simulation BER-curves of uncoded 4-DPSK with differentially coherent detection are also drawn in Fig. 3.

BER-measurements for a Rayleigh channel with the same Doppler spread of 100 Hz and interleaver sizes 8x16 and 8x8 are shown to the right of Fig. 3. It is seen that the BER-curves flatten out at large  $E_b/N_0$ . This error floor is caused by the phase variations of the fading channel which in rare cases coincide with the phases of coded symbols of an incorrect path, even without any additive noise disturbance. The error floor depends on the trellis code, the

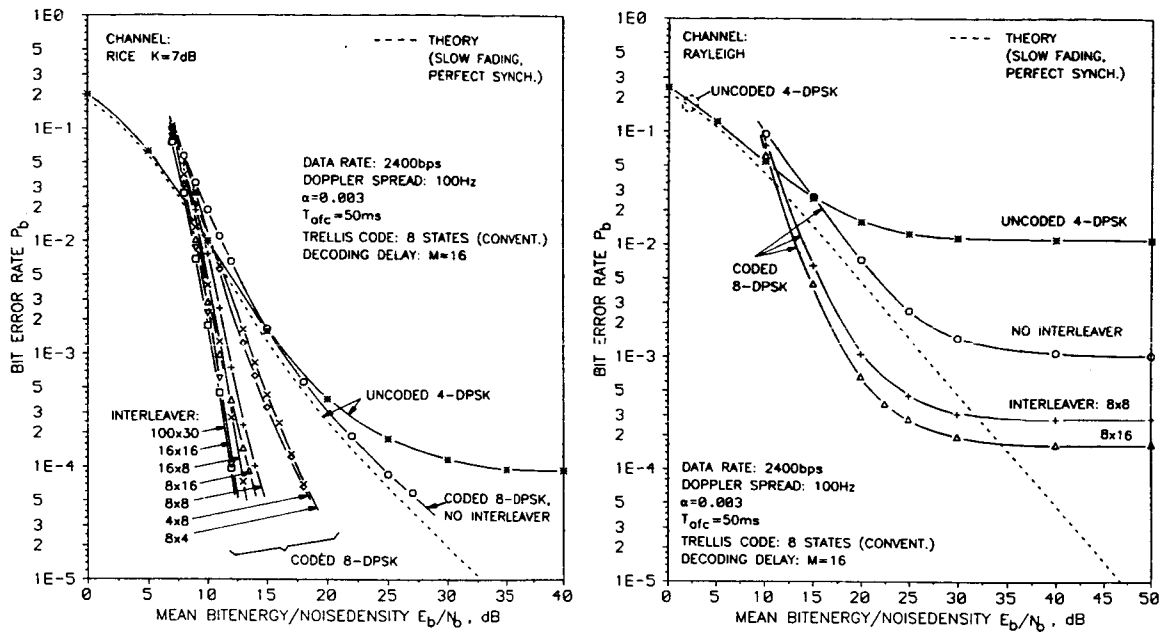


Fig. 3. Computer Simulation Results of BER-Performance of Coded 8-DPSK Modulation for Various Interleaver Sizes over a Rician and a Rayleigh Channel

interleaver size and, most significantly, on the ratio of the Doppler spread to the symbol rate. For the slower Rayleigh channel with Doppler spread of 20 Hz and similar signal processing no error floor was observed above  $P_b = 10^{-5}$  [Edbauer, 1987]. It was shown by computer simulation that the effect of the AFC on the error floor is negligible. This was done by setting the AFC output to a constant which corresponded to the Doppler frequency of a vehicle moving with constant speed.

An important aspect in system design for a fading environment is the proper choice of the trellis code. [Simon and Divsalar, 1987] have analytically derived the following code design criteria for the slow fading fully interleaved Rayleigh channel with differential detection. The code should have:

- a.) large length of the shortest error event path, where equal symbols of the correct and the incorrect path do not contribute to the length,
- b.) large product of the branch distances of this path,
- c.) large minimum free Euclidean distance.

A criterion concerning the error floor does not yet exist, since this would require consideration not only of amplitude- but also of phase variations of the channel.

In view of these criteria the convolutional 8-state trellis code, used in this work, is not optimum. It was designed for the Gaussian channel [Ungerböck, 1982], mainly by considering criterion c.). The code has two minimum free distance paths of lengths 3 and 4 with distance 2.14 for the unit radius constellation. But the code also has one path with length 2 and distance 2.45, which does not meet criterion a.) very well. If an 8-state code with shortest path length of 3 would exist, some performance improvement would be possible. We therefore intend to extend this work to other codes, e.g. those proposed by [Simon and Divsalar, Globecom'87].

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