N88-25720

CODES FOR QPSK MODULATION WITH INVARIANCE UNDER 90° ROTATION

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ABSTRACT

When QPSK modulation is combined with linear rate-1/2 convolutional encoding, the resulting coded QPSK signal sequences are, at most, code invariant under 180° signal rotation. In this paper, we show that invariance under 90° rotation (and multiples thereof) is obtained by using nonlinear convolutional codes which satisfy a parity-check equation of a particular form. Tables of optimum codes in this class are presented, and the realization of encoders with 90°-differential information encoding is discussed. For the same code complexity, the new coded QPSK schemes exhibit smaller minimum distance between coded signal sequences than comparable schemes with linear encoding. However, there are also fewer nearest-neighbor sequences. As a result, the new schemes achieve almost the same real coding gains as the known schemes with linear encoding. For correct decoding, it is no longer necessary to demodulate the received coded QPSK signals with a particular carrier phase. This simplifies the receiver design, and allows fast recovery after losses of carrier-phase synchronization. The new codes should be especially useful for transmission over mobile-communication channels, where Rayleigh/Rice fading can lead to frequent losses of carrier-phase synchronization in the receiver.

INTRODUCTION

Rate-1/2 convolutional codes are frequently employed in combination with QPSK (four-phase) modulation and soft Viterbi decoding to achieve coding gains over uncoded BPSK (binary-phase) modulation at the same bandwidth efficiency of 1 bit/s/Hz. With uncoded BPSK modulation, information transparency to 180°-phase-ambiguous demodulation in the receiver is easily accomplished by differential encoding and decoding. The same holds for 90°-phase ambiguity in the case of uncoded QPSK modulation (2 bits/s/Hz). However, when QPSK signals are encoded with a linear rate-1/2 convolutional code, it is only possible to achieve code invariance under 180°-signal rotation with certain codes, whereas with other codes no such invariance exists. The lack of invariance under 90°-signal rotations requires that QPSK signals are demodulated with correct phase before Viterbi decoding [1].[1]

Transmission over phase-agile fading channels can lead to frequent losses of carrier-phase synchronization in a receiver. The time required to reestablish correct demodulation phase for decoding can cause channel outages during considerable periods of time. It is therefore desirable to find new 90°-rotation-invariant coded QPSK schemes, which allow almost instantaneous resynchronization after severe channel disturbances.

The idea of accomplishing full rotational invariance by nonlinear convolutional codes first arose in 1983 in connection with specifying an eight-state code for trellis-coded modulation with a 32-point signal constellation for use in the CCITT V.32 modem [2-5]. In this paper, the approach taken in [3,6] for constructing such codes on the basis of a nonlinear parity-check equation and achieving differential information encoding without requiring additional memory elements, is applied to QPSK modulation and extended to codes with more than eight states.

Figure 1 illustrates an encoder-modulator with natural binary mapping of code bits (y_n^1, y_n^0) into QPSK signals. This mapping induces a set-partitioning rule as introduced in [7]. Bit y_n^0 divides the QPSK signal set with minimum squared signal distance $\Delta_0^2 = 2$ into two subsets with squared signal distance $\Delta_1^2 = 4$ between the signals in these subsets. It can easily be verified that a 90° rotation of the QPSK signals corresponds to changing the code bits y_n^1 and y_n^0 into $y'^1_n = y_n^1 \oplus y_n^0$ and $y'^0_n = y_n^0 \oplus 1 = \overline{y}_n^0$, respectively, where '$\oplus$' denotes modulo-2 addition.

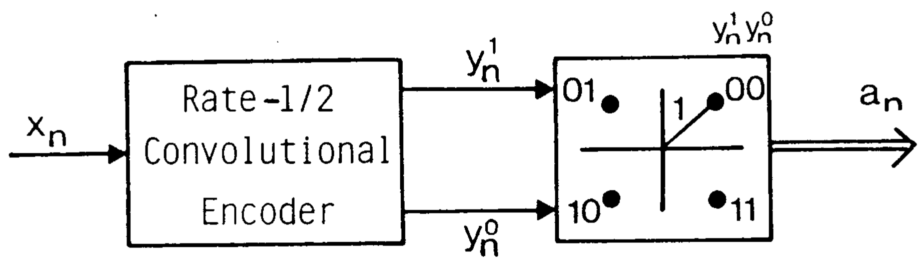

Figure 1. Encoder-modulator for rate-1/2 convolutionally encoded QPSK modulation with natural binary mapping.

An equivalent encoder-modulator for Gray-code mapping is obtained by the transformation $y_{nG}^1 = y_n^1$, $y_{nG}^0 = y_n^1 \oplus y_n^0$. With this transformation, which would normally be included in the convolutional code, Hamming distances between binary code sequences and squared signal distance between coded QPSK signal sequences become equivalent. Throughout this paper natural binary mapping is assumed. The conversion of codes to Gray-code mapping is left as an exercise to readers.

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1 The reference is general rather than specific to the topic of this paper. We are not aware of a paper available in the open literature describing the properties of QPSK modulation with linear convolutional encoding under signal rotation. Nevertheless, one can assume that the facts stated in the above paragraph are known among communication engineers working in this field.

NONLINEAR CONVOLUTIONAL CODES

In the following, binary sequences are expressed in polynomial notation with the indeterminate variable D denoting delay by one modulation interval. A convolutional code of rate 1/2 is most compactly defined as the set of pairs of binary code sequences which satisfy a scalar polynomial parity-check equation.

From the above discussion on natural binary mapping, it follows that coded QPSK sequences are invariant under 90° rotations, if for every pair $y^1(D)$ and $y^0(D)$ which satisfies the parity-check equation of a convolutional code, also the pair $y'^1(D) = y^1(D) \oplus y^0(D)$ and $y'^0(D) = \overline{y^0}(D)$ satisfies this equation. This is not possible, if the parity-check equation is linear in the sense of modulo-2 (GF-2) arithmetic. However, invariance under 90° rotation is achieved, if the code is defined by a nonlinear parity-check equation of the form

$$(D^k + D^i)y^1(D) \oplus H^0(D)y^0(D) = [D^k y^0(D)]\&[D^i y^0(D) \oplus E(D)] \ ,$$

where $H^1(D) = D^k + D^i$ with $\nu > k > i > 0$, and $H^0(D) = D^\nu + D^t + \cdots + D^s + 1$ with $\nu > t \geq s > 0$ are linear parity-check polynomials. It is important that $H^1(D)$ exhibits only two non-zero coefficients. The middle part $D^t + \cdots D^s$ in $H^0(D)$ may be absent; in this case, let $t = s = 0$. As for linear codes, the condition $\nu > k > i > 0$ guarantees that error events begin and end with the largest squared distance between QPSK signals, i.e., $\Delta_1^2 = 4$.

Without the nonlinear right-hand term, the parity-check equation would define a linear convolutional code with constraint length ν, i.e., a code with 2^ν states. The symbol '&' denotes a *logic-and* operation, which makes this term nonlinear. E(D) is the all-one sequence $1(D)$, if the coefficient sum of $H^0(D)$ is even, and the all-zero sequence $0(D)$, if this sum is odd. Modulo-2 addition of $1(D)$ to a binary sequence inverts all elements of this sequence. The rotational-invariance property is proven by substituting $y'^1(D)$ for $y^1(D)$ and $y'^0(D)$ for $y^0(D)$ in the above parity-check equation, and using the relation $u \oplus v = u\&\overline{v} \oplus \overline{u}\&v$. Henceforth, the shorter notation $D^k y^0(D) \bullet D^i y^0(D)$ will be used for the nonlinear parity-check term.

NONLINEAR ENCODER REALIZATION

It can be shown that encoders for codes characterized by a nonlinear parity-check equation of the above form can be realized with a minimum number of $\tilde{\nu} = \max[\nu, k - i + \max(k,t) - \min(i,s)]$ binary memory elements. This means that the nonlinear code could have more than 2^ν code states. In this paper we consider only codes with $\tilde{\nu} = \nu$.

As an example, we show the realization of an encoder for the code shown below in Table 1 for $\tilde{\nu} = \nu = 5$. The code is defined by the parity-check equation

$$(D^2 + D)y^1(D) \oplus (D^5 + D^3 + 1)y^0(D) = D^2 y^0(D) \bullet D y^0(D) .$$

Since the coefficient sum of $H^0(D)$ is odd, $E(D) = 0(D)$. Hence, '$\bullet$' denotes the *logic-and* operation without inversion. Figure 2(a) shows the obvious encoder realization found by expressing $y^0(D)$ explicitly in the other terms of the parity-check equation. The minimal encoder with $\nu = 5$ binary memory elements depicted in Figure 2(b) is then obtained by observing that the feedback part involving the '$\bullet$'

operation can be left-shifted into the linear part of the encoder by two memory elements.

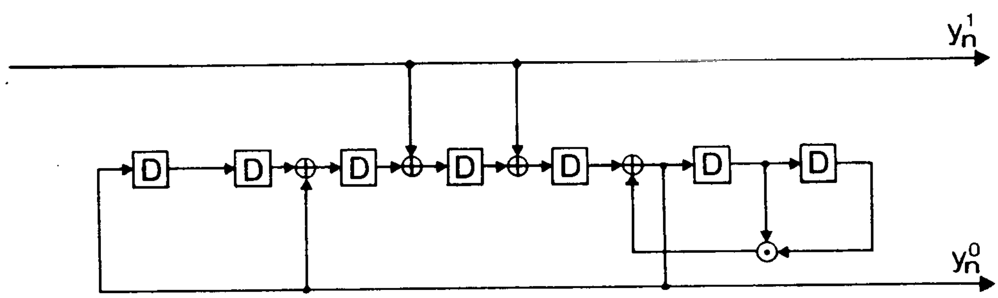
Figure 2(b) also illustrates 90°-differential encoding without additional memory elements. The operation is based on expressing the information sequence as $x(D) = D^{\alpha} y^{0}(D) \oplus D^{\beta} y^{0}(D)$ for some $\alpha \neq \beta$. A 90° rotation of the transmitted QPSK signals is equivalent to replacing $y^{0}(D)$ by the inverted sequence $\overline{y}^{0}(D)$. This inversion does not change $x(D)$. Choosing $\alpha = 0$ and $\beta = -1$, and observing the operation performed by the right-most modulo-2 adder in the minimal encoder leads to two equations

$$x(D) = y^{0}(D) \oplus D^{-1} y^{0}(D) \quad \text{and} \quad y^{1}(D) \oplus s^{1}(D) = D^{-1} y^{0}(D) \; .$$

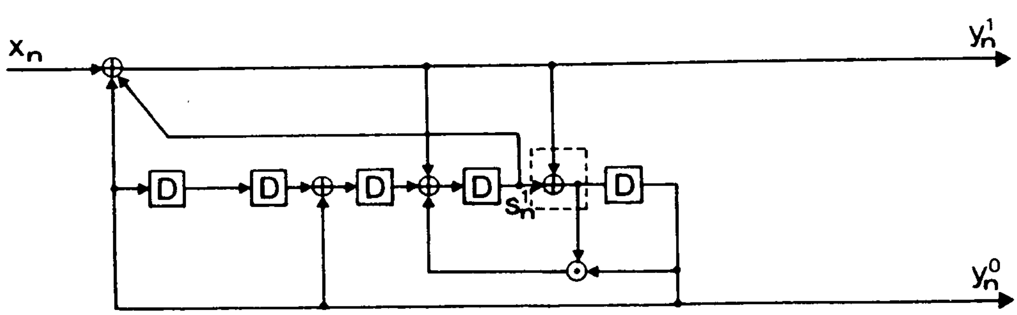
The first equation is used for differential decoding to obtain the information sequence $\hat{x}(D)$ from a received sequence $\hat{y}^{0}(D)$. The sum of the two equations defines the differential-encoding operation

$$y^{1}(D) = x(D) \oplus s^{1}(D) \oplus y^{0}(D) \; .$$

Similar minimal encoders with 90°-differential encoding can be obtained for the other nonlinear codes presented in Table 1 below.



(a) Obvious encoder realization



(b) Minimal encoder realization with 90°-differential encoding

Figure 2. Two encoder realizations for a nonlinear 32-state code.

SEARCH FOR NONLINEAR CODES AND PERFORMANCE EVALUATION

A search for codes satisfying nonlinear parity-check equations of the form considered and permitting the construction of encoders of the form shown in Figure 2(b) was performed. The search was restricted to codes with $\tilde{\nu} = \nu$. Since the codes are nonlinear, free (= minimum) distance, d_{free}, between coded QPSK signal sequences does not necessarily occur between the signal sequence corresponding to the all-zero code sequence and a sequence diverging from and remerging with this sequence. In the code search program, an efficient dynamic-programming algorithm was used which determines d_{free} among *all* possible pairs of sequences diverging in some state and remerging later in another state.

Among codes with the same free distance for a given value of ν, a code with the shortest duration of free-distance error events and the fastest minimum-distance growth between unmerged events (in this order) was selected. Then for the codes selected, the three smallest distances $d_{free} = d_1 < d_2 < d_3$ and the corresponding average multiplicities $N_{free} = N_1$, N_2, and N_3 of error events with these distances were determined. These values were then used to compute an accurate estimate of the error-event probability by

$$P_{evt} \simeq \sum_{i=1}^{3} N_i \; Q[d_i/(2\sigma)] \; ,$$

where Q(.) is the Gaussian error-integral function, and σ denotes the standard deviation of white Gaussian noise for one signal dimension. The value of σ follows from the definition of the signal-to-noise ratio, $SNR_{dB} = 10 \log[E_s/(2\sigma^2)]$. $E_s = 1$ represents the signal energy per QPSK signal and information bit transmitted, and $2\sigma^2 = N_0$ is the one-sided spectral noise density.

The results obtained for $\tilde{\nu} = \nu = 3, 4, 5,$ and 6 are summarized in Table 1. The coefficients of the parity-check polynomials $H^1(D)$ and $H^0(D)$ are specified in octal notation. For uncoded BPSK modulation, values $P_{evt} = 10^{-4}$ and 10^{-8} are achieved at $SNR_{dB} = 8.40$ and 11.97, respectively.

Table 1: New rate-1/2 nonlinear codes for QPSK modulation

Invar.	$\tilde{\nu} = \nu$	$H^1(D)$	$H^0(D)$	d^2_{free}	N_{free}	d^2_2	N_2	d^2_3	N_3	SNR_{dB}, P_{evt} 10^{-4}	10^{-8}
90°	3	006	011	10	0.50	12	1.00	14	2.00	4.33	7.83
90°	4	012	025	12	0.75	14	0.75	16	3.00	3.75	7.15
90°	5	006	051	14	1.00	16	1.75	18	3.00	3.30	6.56
90°	6	012	105	16	2.00	18	1.50	20	4.50	3.00	6.17

COMPARISON WITH LINEAR CODES

Table 2 illustrates the best linear codes and their performance. 180°-rotational invariance is achieved, if the parity-check polynomial $H^1(D)$ exhibits an even number of non-zero coefficients. For $\nu = 2$ and 4, no good codes with this property exist. For the selection of the linear codes essentially the same programs were used as for the nonlinear codes. The code with $\nu = 6$ is equivalent to the *standard* 64-state code with Hamming distance 10 ($d^2_{free} = 20$, when used with QPSK modulation and Gray-code mapping) often mentioned in recent literature. The values of N_{free}, N_2, and N_3 are exact; they do not depend on the code sequence transmitted.

Table 2: Linear rate-1/2 codes for QPSK modulation

Invar.	ν	$H^1(D)$	$H^0(D)$	d^2_{free}	N_{free}	d^2_2	N_2	d^2_3	N_3	SNR_{dB}, P_{evt} 10^{-4}	10^{-8}
360°	2	002	005	10	1	12	2	14	4	4.67	8.01
180°	3	006	013	12	2	16	10	20	49	4.20	7.39
360°	4	004	023	14	3	16	3	18	3	3.73	6.84
180°	5	014	043	16	5	20	17	24	116	3.45	6.37
180°	6	042	117	20	11	24	38	28	193	2.89	5.60

A comparison of Tables 1 and 2 shows that for the same values of ν the nonlinear codes achieve lower values of d^2_{free} than the linear codes. On the other hand, the nonlinear codes exhibit smaller multiplicities of nearest-neighbor error events than the linear codes. Consequently, at moderate signal-to-noise ratios as required to achieve $P_{evt} = 10^{-4}$, the real coding gains are nearly identical. Noticeable but still small differences are found only for very low error-event probabilities.

CONCLUSION

The new rate-1/2 nonlinear convolutional codes for QPSK modulation presented in this paper allow the achievement of full 90°-rotational invariance of coded QPSK signal sequences at no significant loss in real coding gains when compared to linear codes. For mobile-communication systems operating in a fading environment with frequent periods of low signal-to-noise ratio and the possibility of losses of carrier-phase synchronization in the receiver, the invariance to 90°-ambiguous demodulation should be a significant advantage.

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