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AN ERROR FLOOR IN TONE CALIBRATED TRANSMISSION

JAMES K. CAVERS

School of Engineering Science
SIMON FRASER UNIVERSITY,
Burnaby, B.C., V5A 1S6
Canada

ABSTRACT

Use of a low level pilot tone has been shown to eliminate the error floor in fading channels. This paper demonstrates that non-idealities in the receiver's pilot tone filter cause reappearance of the error floor. It also presents the BER in closed form, in contrast to the multidimensional numerical integration of previous work.

INTRODUCTION

Recently, a number of papers [e.g. Davarian, 1986, 1987; Simon 1986; LaRosa & Citron, 1987] have resurrected the use of a pilot tone to allow coherent reception in fading channels. The receiver extracts the pilot with a pilot bandpass filter (PBPF), and uses the result as a phase reference. The obvious costs are the non-constant envelope, the additional complexity at the receiver, and the fact that the pilot robs power from the data carrying signal. Thus we expect to see little or no improvement in the noise limited region; however, when the fade rate is high, TCT proves itself, by lowering or eliminating the error floor typical of other modulation methods.

Analyses to date have implicitly or explicitly assumed that TCT eliminates the error floor. We show here, though, that nonidealities in the PBPF lead, not just to a degraded BER, but to a nonzero error floor. Moreover, previous studies have calculated the BER by numerical integration, often in multiple dimensions. We present here a closed form solution which includes the effect of the PBPF nonidealities and a frequency offset error.

In the interest of brevity, this paper deals only with the issue of irreducible error rate, because it has not been addressed by other authors. However, the method extends in a straighforward way to include the effect of noise, and a full analysis will be published elsewhere.

SYSTEM MODEL

General Characteristics

Figure 1 is the complex envelope representation of the transmitted and received signals. The transmitted power is split between the data bearing signal and a low level pilot tone in such a manner that they do not interfere with each other. For simplicity, we will assume Manchester coding with the pilot in the center channel null, although other methods are possible; for example, the pilot could be located beside the data signal. The analysis below is restricted to two-level eyes; for convenience we assume BPSK, though the results are also valid for QPSK or OQPSK.

Data is detected with a filter matched to the transmitted pulse p(t). Samples at the filter output are denoted by u(k).

The PBPF, represented by the low pass equivalent filter $H_p(f)$, has unity dc gain; ideally, it is a rectangular lowpass filter with bandwidth $B_p/2$, made equal to the sum of frequency offset and Doppler. Since the PBPF has a greater delay than the wider bandwidth matched filter, we assume that there is delay equalization in the detector branch. The PBPF output is v(k). Feedforward phase correction is accomplished simply by making decisions on the basis of the phase-corrected decision variable:

$$d(kT) = \mathrm{Re}[u(kT)\, v^{*}(kT)] \tag{1}$$

The shape and bandwidth of the PBPF have a major effect on performance. If it is too narrow, then frequency offset and Doppler spread will prevent the filter output from following the channel fluctuations, leading to degraded performance and a nonzero error floor. If it is too wide, it will admit too much channel noise and self noise from the data signal, which again leads to degraded performance. A related issue is the fact that any phase distortion in the PBPF also results in decorrelation and a non zero error floor.

We will assume that there is no additive noise in the channel; as noted earlier, our main interest here is the irreducible BER.

The Fading Channel

We assume Rice fading, in which the ratio of specular to diffuse power is K. These components of the complex gain, when each normalized to unit power, have power spectral densities $\tilde{S}_{gs}(f)$ and $\tilde{S}_{gd}(f)$, respectively. Although our general solution does not rest on a particular spectrum for the diffuse component, the examples will use a spectrum and autocorrelation function characteristic of isotropic scattering:

$$\tilde{S}_{gd}(f) = \frac{1}{\pi}\,(f_D^2 - f^2)^{-1/2} \;; \qquad \tilde{R}_{gd}(\tau) = J_o(2\pi f_D \tau) \tag{2}$$

where $J_o()$ is the Bessel function of first kind and order zero and f_D is the Doppler frequency. For the specular component, we have:

$$\tilde{S}_{gs}(f) = \delta(f-f_L) \tag{3}$$

where f_L is its Doppler shift. Note that $|f_L| \le f_D$. We also allow a frequency offset error f_o between transmit and receive oscillators.

Signal Moments

The signal moments and their ratios are required for calculation of BER. Since we have assumed no noise, the variance in u(k) and v(k) is due to the diffuse component, and the mean is determined by the specular component. We can show that the samples at the output of the matched filter have signal to variance ratio:

$$|\mu_u|^2/\sigma_u^2 = 2K \tag{4}$$

Similarly, at the pilot tone filter output:

$$|\mu_v|^2/\sigma_v^2 = 2K\,|H_p(f_o+f_L)|^2 \left[\int \tilde{S}_{gd}(f-f_o)|H_p(f)|^2 df \right]^{-1} \tag{5}$$

Next, we calculate the normalized inner product of the means of u(k) and v(k):

$$\mu_u\mu_v^*/\sigma_u\sigma_v = 2K\, H_p^*(f_o+f_L) \left[\int \tilde{S}_{gd}(f-f_o)|H_p(f)|^2 df \right]^{-1/2} \tag{6}$$

Finally, we calculate the correlation coefficient of u(k) and v(k):

$$\rho = \int \tilde{S}_{gd}(f-f_o)H_p^*(f)df \left[\int \tilde{S}_{gd}(f-f_o)|H_p(f)|^2 df \right]^{-1/2} \tag{7}$$

DETECTION ERROR PROBABILITY

General Solution

The basic observation is that u(k) and v(k) are jointly Gaussian, non zero mean, complex random variables. The decision variable d(k) is then of a form familiar from studies of incoherent detection; we make an error if it is negative, assuming b(k) is positive. A concise derivation of the probability of this event can be found in Schwartz et al, 1966, sec8.2. Define the quantitities a and b:

$$\begin{Bmatrix} a^2 \\ b^2 \end{Bmatrix} = \frac{|\mu_u|^2/\sigma_u^2 + |\mu_v|^2/\sigma_v^2 - 2\rho_i \mathrm{Im}[\mu_u\mu_v^*]/\sigma_u\sigma_v \mp 2\sqrt{1-\rho_i^2}\,\mathrm{Re}[\mu_u\mu_v^*]/\sigma_u\sigma_v}{4\,(1-\rho_i^2)} \tag{8}$$

and the quantity C:

$$C = \left[(1-\rho_i^2)^{1/2} + \rho_r \right] / \, 2(1-\rho_i^2)^{1/2} \tag{9}$$

where $\rho = \rho_r + \rho_i$. Then the probability of bit error is:

$$P_e = Q(a,b) - C\, I_o(ab) \exp(-(a^2+b^2)/2) \tag{10}$$

where $Q(a,b)$ is the Marcum Q-function, and $I_o()$ is the modified Bessel function of first kind and order 0. Techniques for evaluating the Q function can be found in Bird & George, 1981.

In the case of Rayleigh fading, K = 0, the parameters a and b are zero, and the BER simplifies to:

$$P_{eR} = 1 - C = \left[(1-\rho_i^2)^{1/2} - \rho_r \right] / \, 2(1-\rho_i^2)^{1/2} \tag{11}$$

Moreover, if the frequency response $H_p(f)$ is real, then $\rho_i = 0$, and:

$$P_{eR} = (1-\rho)/2 \tag{12}$$

Equations (11) and (12) illustrate the requirement for extremely high correlation between the outputs of the matched filter and the PBPF. We see that filters other than the ideal rectangular lowpass produce a correlation coefficient less than 1 in magnitude, and therefore an irreducible error rate.

APPLICATIONS

Delay Mismatch

Earlier we noted that the greater delay of the narrow PBPF meant that delay equalization is required in the matched filter branch. In an implementation based on DSP, this is unlikely to be a problem. In an analog implementation, though, it is possible to have a residual delay mismatch. We can incorporate this effect by representing an otherwise perfect PBPF as:

$$H_p(f) = \exp(-j2\pi f\tau) \tag{13}$$

where the delay mismatch τ can be positive or negative. We now have:

$$\rho = \tilde{R}_{gd}(\tau)\,\exp(-j2\pi f_o\tau) \tag{14}$$

$$\begin{Bmatrix} a^2 \\ b^2 \end{Bmatrix} = K\,\frac{1 + R_S\sin(2\pi(f_o+f_L)\tau) \mp \cos(2\pi(f_o+f_L)\tau)\,\sqrt{1 - R_S^2}}{\left(1 - R_S^2\right)}$$

$$C = 0.5\left[\sqrt{1 - R_S^2} + R_C\right]\left(1 - R_S^2\right)^{-1/2} \tag{15}$$

where for notational convenience we have defined the quantities:

$$R_C = \tilde{R}_{gd}(\tau)\cos(2\pi f_o\tau)\ ; \qquad R_S = \tilde{R}_{gd}(\tau)\sin(2\pi f_o\tau) \tag{16}$$

Once again, the case of Rayleigh fading is particularly simple. We set K = 0 and find, as in (11), that the error floor is:

$$P_{eR} = 0.5\left[\sqrt{1 - R_S^2} - R_C\right]\left(1 - R_S^2\right)^{-1/2} \tag{17}$$

Figure 2 illustrates the effect of delay mismatch for K = 10 and for Rayleigh fading, respectively. The ratio of pilot power to data power is r = 0.2. We see a difference in the sensitivity to frequency errors. Rice fading is more affected by the offset frequency f_o than by the Doppler f_D. In constrast, Rayleigh fading behaviour is almost entirely determined by f_D, with little influence from f_o.

The irreducible error rate is clearly evident. Its source is the phase decorrelation introduced by the delay mismatch during intervals of rapid phase change, such as a deep fade. We can obtain a simple estimate of the error floor value in the case of Rayleigh fading, the most severe condition, by evaluating (17) for small values of $f_o\tau$ and $f_D\tau$. Using the series expansions for sin(), cos(), and $J_o()$, we obtain:

$$P_{eR} \simeq (2\pi f_D\tau)^2/8 \tag{18}$$

which agrees with Figure 3. To a first approximation, the error floor depends only on Doppler, not on frequency offset. As a numerical example, if the error floor is not to exceed 10^{-3}, and f_D = 100 Hz, then we must keep the delay mismatch $|\tau| \le 0.14$ ms. Clearly, this is not a problem in digital implementations, since this maximum delay is only 1/3 bit duration in 2400 bps BPSK. Analog implementations may find it a more difficult target.

A similar approach yields a somewhat poorer approximation to the error floor in the case of Rice fading with values of K over 5 or so. Series expansion of the transcendental functions and approximation of the integral in the Q(a,b) function yields the error floor:

$$P_e \simeq K\,(2\pi f_o \tau)^2\, e^{-K} \tag{19}$$

in the special case of $f_L = 0$.

Narrow PBPF

One can readily imagine situations in which the combined frequency offset and Doppler spread carry part of the received pilot spectrum outside the window of the PBPF. Such a situation was considered by Larosa & Citron, 1987, for a dual pilot tone system, and a number of curves were obtained by numerical integration. However, the fading spectrum was taken to be rectangular. Clearly, the U-shaped spectrum (2) is more sensitive to band edge distortion because a greater fraction of its power is concentrated near $\pm f_D$.

We will model the PBPF as a rectangular LPF with unity gain for $|f| \le B_p/2$. With amplitude distortion only, it contrasts with the delay mismatch considered earlier, which had phase distortion only.

The effect on irreducible BER can be determined by evaluating ρ, a, b, and C. We denote the normalized power of the diffuse component of the PBPF output by P_d:

$$P_d = \int \tilde{S}_{gd}(f-f_o)\,|H_p(f)|^2 df = \int \tilde{S}_{gd}(f-f_o)\,H_p^*(f)\,df$$

$$= \frac{1}{\pi}\arcsin(\min(1,(B_p-2f_o)/2f_D)) + \frac{1}{2} \tag{20}$$

As for the specular component, we observe that it is present and undistorted if it lies in the PBPF passband; otherwise, it is zero. Denote by P_s the normalized power of the specular component at the PBPF output. From (3):

$$P_s = \int \tilde{S}_{gs}(f-f_o)\,|H_p(f)|^2 df = \begin{cases} 1, & |f_o+f_L| \le B_p/2 \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

Substitution of (20,21) into (5-9) gives:

$$\rho = \sqrt{P_d}$$

$$\begin{Bmatrix} a^2 \\ b^2 \end{Bmatrix} = (K/2)\,\left(1 \mp P_s/\sqrt{P_d}\right)^2 \quad ; \qquad C = \left(1 + \sqrt{P_d}\right)/2 \tag{22}$$

for substitution into the BER expression (26).

In the case of Rayleigh fading, we can substitute ρ from (22) into (12), and obtain the error floor:

$$P_{eR} = \left(1 - \sqrt{P_d}\right)/2 \tag{23}$$

Since P_d should be close to 1 for a reasonable error floor, we can apply series expansions for arcsin() and square root to obtain:

$$P_{eR} \simeq \frac{1}{4\pi}\left(2(f_D + |f_o| - B_p/2)/f_D\right)^{1/2} \tag{24}$$

for small excursions of $f_D + |f_o|$ beyond $B_p/2$.

As a numerical example, suppose the irreducible BER is to be held to 10^{-3}. Then from (24), $(f_D+|f_o|-B_p/2)/f_D \leq 8x10^{-5}$. This is an exceedingly tight constraint; essentially it says that the slightest excursion of combined offset and Doppler frequencies beyond the PBPF window results in significant performance degradation.

Bessel PBPF

Equation (24) suggests that even moderate amounts of rolloff can do significant damage if offset and Doppler carry the "horns" of the fading spectrum into the rolloff region. Nonrectangular filters, of course, do not behave in precisely the same way, and detailed analysis requires substitution of their frequency characteristics into (5-7). We did this for a sixth order Bessel filter. Of the two integrals in (5-7), the one involving $|H_p(f)|^2$ was performed analytically; the other was evaluated numerically, after a preliminary integration by parts to remove the singularity.

The curves in Figure 3 were prepared with parameters $\alpha = f_D/R_b$ and $\beta = B_p/R_b$. Note that there is an optimum value for the pilot filter bandwidth, as we trade noise against distortion of the received pilot, and that the optimum depends on K, α, r and E_b/N_o. The values of β were near the optimum.

For K = 10, we see the onset of error saturation for $\alpha = 0.04$ and $\beta = 0.03$; that is, the noise bandwidth of the PBPF is only 0.375 times the Doppler bandwidth. However, even for this combination of parameters, saturation is a problem only at BERs far lower than we normally expect from mobile satellite communication. Rayleigh fading is somewhat different. For $\alpha = 0.04$, we see that the PBPF should be at least 1.25 times as wide as the Doppler bandwidth to prevent an unacceptable error floor. To put the results in context, $\alpha = 0.04$ represents 96 Hz fading at 2400 bps; that is, it is an extreme case. For more reasonable ranges, the sixth order Bessel filter appears to be adequate, and it is noise, rather than random FM, which determines system performance.

CONCLUSIONS

Nonidealites in the pilot tone filter produce an error floor which is especially marked in Rayleigh fading with large Doppppler frequencies. However, for normal ranges of filter parameters, it is not likely to be a problem, and TCT's claim to suppress the error floor is justified.

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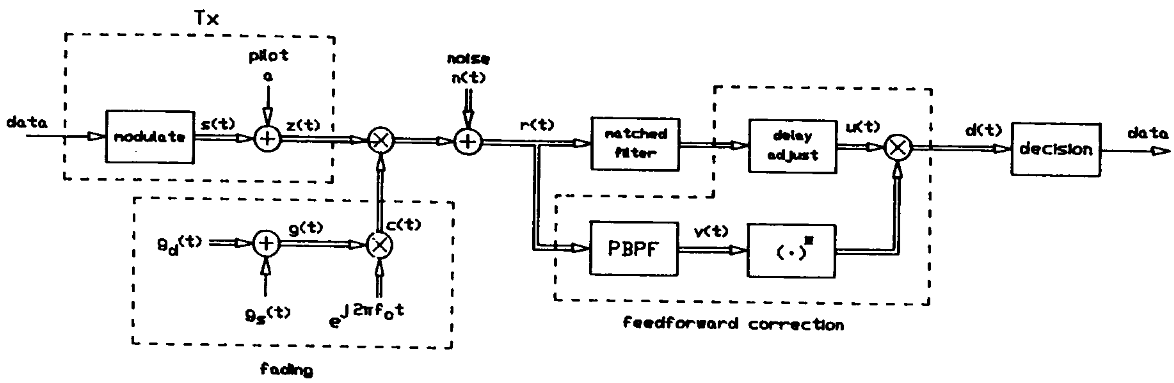

Fig. 1. Complex Baseband Model of TCT System

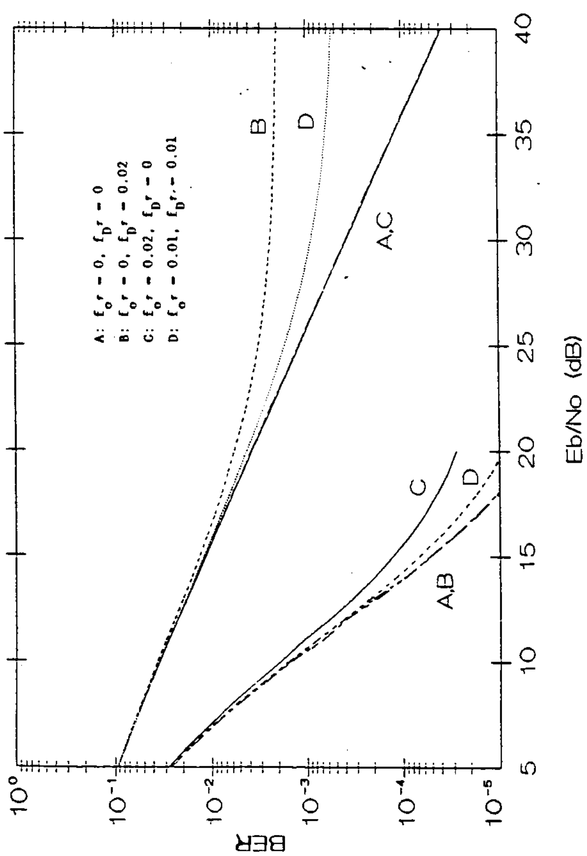

Fig. 2. Delay Mismatch, Rice (K=10) and Rayleigh Fading

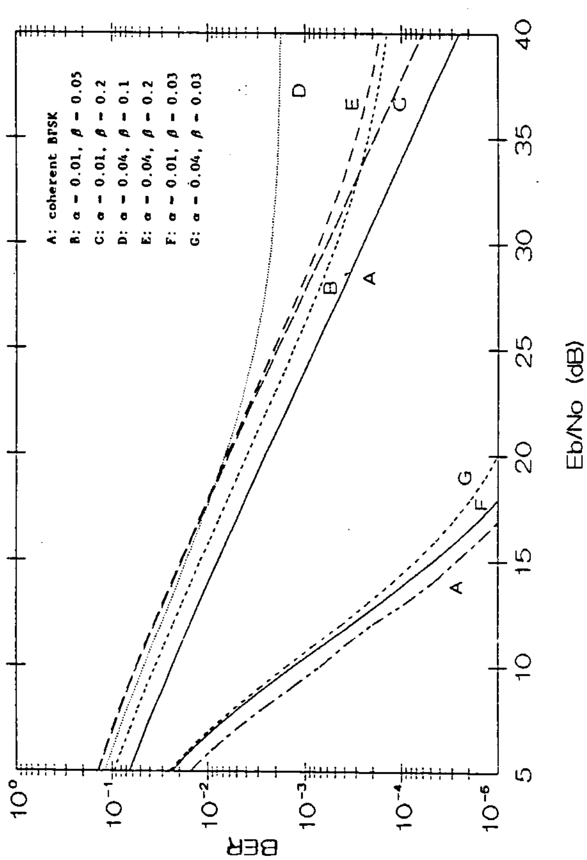

Fig. 3. Bessel PBPF, Rice (K=10) and Rayleigh Fading