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SCALING LAWS FOR HOMOGENEOUS TURBULENT
SHEAR FLOWS IN A ROTATING FRAME

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the integrity of the financial system and for the ability to detect and prevent fraud.

2. The second part of the document outlines the various methods used to collect and analyze data. It describes the use of statistical techniques to identify trends and patterns in the data, and the importance of using reliable sources of information.

3. The third part of the document discusses the challenges of data collection and analysis. It notes that the volume and complexity of data have increased significantly in recent years, and that this has made it more difficult to manage and analyze the data effectively.

4. The fourth part of the document discusses the importance of data security. It notes that the collection and analysis of data often involves the use of sensitive information, and that it is essential to take appropriate measures to protect this information from unauthorized access and disclosure.

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6. The sixth part of the document discusses the importance of data quality. It notes that the accuracy and reliability of the data are essential for the integrity of the financial system, and that it is essential to take appropriate measures to ensure that the data is of high quality.

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8. The eighth part of the document discusses the importance of data accessibility. It notes that the collection and analysis of data often involves the use of large volumes of data, and that it is essential to make this data accessible to all stakeholders who need it.

9. The ninth part of the document discusses the importance of data interoperability. It notes that the collection and analysis of data often involves the use of data from multiple sources, and that it is essential to ensure that this data is interoperable and can be used together effectively.

10. The tenth part of the document discusses the importance of data innovation. It notes that the collection and analysis of data often involves the use of new and innovative techniques, and that it is essential to continue to invest in research and development to ensure that the financial system remains at the forefront of innovation.

SCALING LAWS FOR HOMOGENEOUS TURBULENT SHEAR FLOWS IN A ROTATING FRAME

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ABSTRACT

The scaling properties of plane homogeneous turbulent shear flows in a rotating frame are examined mathematically by a direct analysis of the Navier-Stokes equations. It is proven that two such shear flows are dynamically similar if and only if their initial dimensionless energy spectrum $E^*(k^*, 0)$, initial dimensionless shear rate SK_0/ε_0 , initial Reynolds number $K_0^2/\nu\varepsilon_0$, and the ratio of the rotation rate to the shear rate Ω/S are identical. Consequently, if universal equilibrium states exist, at high Reynolds numbers, they will only depend on the single parameter Ω/S . The commonly assumed dependence of such equilibrium states on Ω/S through the Richardson number $Ri = -2(\Omega/S)(1 - 2\Omega/S)$ is proven to be inconsistent with the full Navier-Stokes equations and to constitute no more than a weak approximation. To be more specific, Richardson number similarity is shown to only rigorously apply to certain low-order truncations of the Navier-Stokes equations (i.e., to certain second-order closure models) wherein closure is achieved at the second-moment level by assuming that the higher-order moments are a small perturbation of their isotropic states. The physical dependence of rotating turbulent shear flows on Ω/S is discussed in detail along with the implications for turbulence modeling.

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1. INTRODUCTION

A variety of geophysical and engineering systems of interest contain turbulent shear flows which evolve within a rotating framework (e.g., ocean currents, atmospheric boundary layers, turbomachinery, etc.). Consequently, it is essential that the effects of rigid body rotations on turbulent shear flows be understood if such physical systems are to be modeled properly. In order to gain insight into these complicated systems, a variety of researchers have considered the idealized problem of plane homogeneous turbulent shear flow in a steadily rotating frame (see Bradshaw¹, Ferziger and Shaanan² and Bardina, Ferziger, and Reynolds³). This work, which is somewhat empirical in its orientation since it is based on turbulence modeling, has shed some important new light on the physical structure of rotating turbulent shear flows. However, several fundamental questions related to the scaling of rotating turbulent shear flows remain unanswered. To be specific: (a) which dimensionless parameters must be invariant to ensure the similitude of two homogeneous turbulent shear flows in a rotating frame, (b) when will equilibrium states exist that depend only on the ratio of the rotation rate to the shear rate Ω/S , and (c) when is this dependence on Ω/S exclusively through the Richardson number $Ri = (2\Omega/S)(1 - 2\Omega/S)$? The answers to such scaling questions play a crucial role in the construction of turbulence models that have the correct physical behavior in rotating frames. The motivation for the present paper is therefore to derive scaling laws for homogeneous turbulent shear flows in a rotating frame based on a direct analysis of the Navier-Stokes equations.

It will be shown that two homogeneous turbulent shear flows in a rotating frame are dynamically similar if and only if their initial dimensionless energy spectrum, shear rate, and Reynolds number (all scaled by the initial turbulent kinetic energy and dissipation rate) are identical in addition to the ratio of the rotation rate to the shear rate Ω/S . It then follows that if an equilibrium structure exists at high Reynolds numbers which attracts all initial conditions (i.e., a universal equilibrium), its turbulence statistics will be determined by a single parameter – the ratio of the rotation rate to the shear rate Ω/S . We will examine the circumstances under which the dependence of such equilibrium states on Ω/S collapses to Richardson number similarity. It will be proven that Richardson number similarity does not hold for general solutions of the Navier-Stokes equations for homogeneous turbulent shear flow in a rotating frame. However, it will be demonstrated that low order truncations of the Navier-Stokes equations such as certain second-order closure models (which are obtained by assuming that third-order moments are a small perturbation of their isotropic state) do exhibit Richardson number similarity with respect to the turbulent kinetic energy, dissipation rate and Reynolds shear stress. This result has important implications for the modeling of rotating turbulent shear flows where it has

been a common practice to modify simpler turbulence models by the inclusion of an ad hoc Richardson number dependence to account for rotations⁴⁻⁶. The broader implications of these results for the modeling of rotating turbulent shear flows will be discussed briefly in the last section.

2. DIMENSIONAL INVARIANCE OF ROTATING TURBULENT SHEAR FLOWS

We will consider the problem of homogeneous turbulent shear flow in a rotating frame for a viscous incompressible fluid (see Figure 1). The governing equations of motion for this flow are the Navier-Stokes and continuity equations given by⁷

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \nu \nabla^2 \mathbf{v} - 2\boldsymbol{\Omega} \times \mathbf{v} \quad (1)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (2)$$

where \mathbf{v} is the velocity field, P is the modified pressure (which includes the centrifugal acceleration potential), ν is the kinematic viscosity of the fluid, and

$$\boldsymbol{\Omega} = \Omega \mathbf{k} \quad (3)$$

is the rotation rate of the reference frame relative to an inertial framing. As in the usual treatments of turbulence, the velocity field \mathbf{v} and pressure P are decomposed into ensemble mean and fluctuating parts as follows

$$\mathbf{v} = \bar{\mathbf{v}} + \mathbf{u}, \quad P = \bar{P} + p. \quad (4)$$

For those turbulence statistics which are homogeneous, the ergodic hypothesis can be invoked and ensemble averages can be replaced by spatial averages. The spatial average of a flow variable f is defined as follows⁸

$$\bar{f}(t) = \lim_{V \rightarrow \infty} \frac{1}{V} \int_V f(\mathbf{x}, t) d^3x. \quad (5)$$

For the plane turbulent shear flow under consideration,

$$\frac{\partial \bar{v}_i}{\partial x_j} = \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (6)$$

or, equivalently, in Cartesian tensor notation

$$\frac{\partial \bar{v}_i}{\partial x_j} = S \delta_{1i} \delta_{2j} \quad (7)$$

where S is the shear rate and δ_{ij} is the Kronecker delta. It is a simple matter to show that the mean velocity field given by $\bar{v}_i = S \delta_{1i} \delta_{2j} x_j$ is consistent with the Reynolds equation and mean continuity equation

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}} = -\nabla \bar{P} + \nu \nabla^2 \bar{\mathbf{v}} - 2\boldsymbol{\Omega} \times \bar{\mathbf{v}} \quad (8)$$

$$\nabla \cdot \bar{\mathbf{v}} = 0 \quad (9)$$

which are obtained by taking the ensemble mean of Eqs. (1)-(2) and making use of the fact that the divergence of the Reynolds stress tensor $\tau_{ij} \equiv -\overline{u_i u_j}$ vanishes since the turbulence is homogeneous. Specifically, $\bar{\mathbf{v}}$ identically satisfies the continuity equation (9) and the Reynolds equation (8) if

$$\bar{P} = -\Omega S y^2 + \text{constant}. \quad (10)$$

The fluctuating velocity \mathbf{u} is a solution of the momentum and continuity equations given by

$$\frac{\partial \mathbf{u}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \mathbf{u} = -\mathbf{u} \cdot \nabla \mathbf{u} - \mathbf{u} \cdot \nabla \bar{\mathbf{v}} - \nabla p + \nu \nabla^2 \mathbf{u} - 2\Omega \times \mathbf{u} \quad (11)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (12)$$

which are obtained by introducing the decomposition (4) into (1)-(2) and then differencing the resulting equations with (8)-(9). For the rotating homogeneous turbulent shear flow under consideration, equation (11) takes the component form

$$\frac{\partial u_i}{\partial t} + S \delta_{k1} \delta_{j2} x_j \frac{\partial u_i}{\partial x_k} = -u_k \frac{\partial u_i}{\partial x_k} - \frac{\partial p}{\partial x_i} - (S \delta_{i1} \delta_{k2} - 2\Omega \varepsilon_{3ik}) u_k + \nu \nabla^2 u_i \quad (13)$$

where ε_{ijk} is the permutation tensor.

Now, we will non-dimensionalize these equations by introducing the dimensionless variables

$$u_i^* = \frac{u_i}{K_0^{\frac{1}{2}}}, \quad x_i^* = \frac{x_i \varepsilon_0}{K_0^{\frac{3}{2}}} \quad (14)$$

$$p^* = \frac{p \varepsilon_0}{S K_0^2}, \quad t^* = S t \quad (15)$$

where K_0 and ε_0 are, respectively, the initial turbulent kinetic energy and dissipation rate. It should be noted that since the turbulence is homogeneous, the turbulent kinetic energy K and dissipation rate ε defined by⁸

$$K(t) = \frac{1}{2} \overline{u_i u_i}, \quad \varepsilon(t) = \nu \overline{\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}} \quad (16)$$

are functions of time alone. The introduction of (14)-(15) into (12)-(13) yields the dimensionless equations of motion

$$\frac{\partial u_i^*}{\partial t^*} + \delta_{k1} \delta_{j2} x_j^* \frac{\partial u_i^*}{\partial x_k^*} = -\frac{\varepsilon_0}{S K_0} u_k^* \frac{\partial u_i^*}{\partial x_k^*} - \frac{\partial p^*}{\partial x_i^*} - A_{ik}^* u_k^* + \frac{\varepsilon_0}{S K_0} \frac{1}{Re_0} \nabla^{*2} u_i^* \quad (17)$$

$$\frac{\partial u_k^*}{\partial x_k^*} = 0 \quad (18)$$

where

$$Re_0 = \frac{K_0^2}{\nu \varepsilon_0} \quad (19)$$

$$A_{ik}^* = \delta_{i1} \delta_{k2} - 2(\Omega/S) \varepsilon_{3ik} = \begin{pmatrix} 0 & 1 - 2\Omega/S & 0 \\ 2\Omega/S & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (20)$$

The equations of motion (17)-(18) must be solved subject to the initial condition

$$\mathbf{u}^*(\mathbf{x}^*, 0) \equiv \mathbf{u}_{(0)}^*(\mathbf{x}^*) \quad (21)$$

at time $t^* = 0$. We will restrict our attention to the case where the turbulence is initially isotropic – the same initial condition that has been used in previous physical and numerical experiments on homogeneous turbulent shear flows (c.f., Champagne, Harris, Corrsin⁹, Tavoularis and Corrsin¹⁰, Rogallo¹¹, and Bardina, Ferziger, and Reynolds³). Given an isotropic turbulence, the hierarchy of turbulence statistics are derivable from the energy spectrum $E(k, t)$. For example, the turbulent kinetic energy K and dissipation rate ε are given by

$$K(t) = \int_0^\infty E(k, t) dk \quad (22)$$

$$\varepsilon(t) = 2\nu \int_0^\infty k^2 E(k, t) dk \quad (23)$$

and the skewness S_k in the quasi-equilibrium range¹² takes the form

$$S_k = \frac{-3\sqrt{30}}{7} \nu \frac{\int_0^\infty k^4 E(k) dk}{[\int_0^\infty k^2 E(k) dk]^{\frac{5}{2}}} \quad (24)$$

Hence, if we are only interested in the time evolution of the Reynolds stress tensor τ_{ij} and turbulent dissipation rate ε , it is reasonable to believe that we need only be concerned with initial conditions on the fluctuating velocity $\mathbf{u}^*(\mathbf{x}^*, 0)$ through the initial energy spectrum $E^*(k^*, 0)$. These initial fields are related by

$$E^*(k^*, 0) = \frac{1}{4\pi^2} k^{*2} \int_{-\infty}^\infty Q_{ii}^*(\mathbf{r}^*) e^{-i\mathbf{k}^* \cdot \mathbf{r}^*} d^3 r^* \quad (25)$$

where

$$Q_{ij}^*(\mathbf{r}^*) \equiv \overline{u_i^*(\mathbf{x}^*, 0) u_j^*(\mathbf{x}^* + \mathbf{r}^*, 0)} \quad (26)$$

is the dimensionless two-point velocity correlation tensor and $k^* = kK_0^{\frac{3}{2}}/\varepsilon$ is the dimensionless wavenumber.

Thus, it has been demonstrated that two homogeneous turbulent shear flows in a rotating frame (evolving from an initially isotropic state) are dynamically similar in a statistical sense if the following flow parameters are identical:

- the initial dimensionless shear rate SK_0/ε_0
- the initial dimensionless energy spectrum $E^*(k^*, 0)$
- the initial Reynolds number $K_0^2/\nu\varepsilon_0$
- the ratio of the rotation rate to the shear rate Ω/S .

When these four parameters are the same for two different homogeneous turbulent shear flows in rotating frames, their dimensionless Reynolds stress tensor τ_{ij}^* and dissipation rate ε^* (as well as other moments constructed from \mathbf{u}^*) will evolve identically with respect to the dimensionless time $t^* = St$. Consequently, if universal equilibrium states exist (which are solutions for $t \rightarrow \infty$ that are completely independent of initial conditions), they will only depend on a single parameter – the ratio of the rotation rate to the shear rate Ω/S . In the next section, we will explore when such universal equilibrium states depend on Ω/S through the Richardson number $-2(\Omega/S)(1 - 2\Omega/S)$.

3. RICHARDSON NUMBER SIMILARITY

Bradshaw¹ presented an analogy between rotating and stratified turbulent flows. In that study, he defined a Richardson number for rotating shear flows as follows

$$Ri = \frac{-2\Omega(S - 2\Omega)}{S^2} \quad (27)$$

This is analogous to the Richardson number for density stratified flows in that (27) serves as the square of the Brunt-Väisälä frequency ω_{BV} for small amplitude inertial oscillations induced by the application of a rotation to shear flow. Although the analysis leading to Eq. (27) is straightforward, we feel that it would be instructive to include it here since Bradshaw¹ did not provide the details. If we consider small homogeneous perturbations of shear flow of the form

$$\mathbf{v} = Sy \mathbf{i} + \mathbf{u}(t) \quad (28)$$

$$P = -\Omega S y^2 + p(t) \quad (29)$$

it is a simple matter to show, by substituting Eqs. (28)-(29) into (13), that \mathbf{u} a solution of the equation

$$\dot{u}_k = -A_{k\ell} u_\ell \quad (30)$$

where $A_{k\ell} \equiv S\delta_{k1}\delta_{\ell 2} - 2\Omega\varepsilon_{3k\ell}$. Equation (30) has a general dimensionless solution of the form

$$u_1^* = a_1 e^{i\sqrt{Ri} t^*} + a_2 e^{-i\sqrt{Ri} t^*} \quad (31)$$

$$u_2^* = a_1' e^{i\sqrt{Ri} t^*} + a_2' e^{-i\sqrt{Ri} t^*} \quad (32)$$

where a_i and a_i' are related to the initial conditions. Hence, the Brunt-Väisälä frequency for this perturbation of homogeneous shear flow in a rotating frame is given by

$$\omega_{BV} = \sqrt{Ri} \quad (33)$$

as argued by Bradshaw. This motivated Bradshaw¹ to propose a modification of the mixing length, to account for rotations in turbulent shear flows, that is of the form

$$\frac{\ell}{\ell_0} = 1 - \beta Ri. \quad (34)$$

In (34), ℓ is the mixing length with rotations, ℓ_0 is the mixing length in the absence of rotations, and β is an empirical constant. Various investigators in the intervening years have modeled rotating turbulent flows with a mixing length of the same form as (34) (c.f., Lezius and Johnston⁴, Launder, et al.⁵, and Howard, et al.⁶). When such a length scale is used in a turbulent eddy viscosity model, the Reynolds shear stress, as well as the turbulent

kinetic energy and dissipation rate, will exhibit Richardson number similarity. Hence, it is critical to know in what sense such similitude is consistent with the Navier-Stokes equations if we are to properly evaluate these turbulence models.

We will now demonstrate that Richardson number similarity is *not* consistent with general solutions of the Navier-Stokes equations. In order to accomplish this task, we will eliminate the pressure from equation (17) by taking the divergence of that equation and making use of the constraint (18). This yields the Poisson equation

$$\nabla^{*2} p^* = -\frac{\varepsilon_0}{SK_0} \frac{\partial u_k^*}{\partial x_i^*} \frac{\partial u_i^*}{\partial x_k^*} - (A_{ik}^* + \delta_{i1} \delta_{k2}) \frac{\partial u_k^*}{\partial x_i^*} \quad (35)$$

which has the solution

$$p^* = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{1}{|\mathbf{x}^* - \mathbf{z}^*|} \left[\frac{\varepsilon_0}{SK_0} \frac{\partial u_k^*}{\partial z_i^*} \frac{\partial u_i^*}{\partial z_k^*} + B_{ik}^* \frac{\partial u_k^*}{\partial z_i^*} \right] d^3 z^* \quad (36)$$

in an unbounded flow domain where

$$\begin{aligned} B_{ik}^* &= A_{ik}^* + \delta_{i1} \delta_{k2} \\ &= \begin{pmatrix} 0 & 2 - 2\Omega/S & 0 \\ 2\Omega/S & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (37)$$

Hence, \mathbf{u}^* is a solution of the integro-differential equation

$$\begin{aligned} \frac{\partial u_i^*}{\partial t^*} + \delta_{k1} \delta_{j2} x_j^* \frac{\partial u_i^*}{\partial x_k^*} &= -\frac{\varepsilon_0}{SK_0} u_k^* \frac{\partial u_i^*}{\partial x_k^*} - A_{ik}^* u_k^* + \frac{\varepsilon_0}{SK_0} \frac{1}{Re_0} \nabla^{*2} u_i^* \\ &\quad - \frac{1}{4\pi} \frac{\partial}{\partial x_i^*} \int_{-\infty}^{\infty} \frac{1}{|\mathbf{x}^* - \mathbf{z}^*|} \left[\frac{\varepsilon_0}{SK_0} \frac{\partial u_k^*}{\partial z_\ell^*} \frac{\partial u_\ell^*}{\partial z_k^*} + B_{ik}^* \frac{\partial u_k^*}{\partial z_\ell^*} \right] d^3 z^* \end{aligned} \quad (38)$$

Since the eigenvalues of A_{ij}^* are $\pm i\sqrt{Ri}$ and the eigenvalues of B_{ij}^* are $\pm i\sqrt{Ri - 2\Omega/S}$ there exists no basis relative to which the dependence of (38) on Ω/S can be collapsed to a Richardson number dependence. Hence, *Richardson number similarity does not follow from the full Navier-Stokes equations*. The recent large-eddy simulations of Bardina, Ferziger, and Reynolds³ were the first to cast grave suspicions on such Richardson number scaling. In figure 2, results of these large-eddy simulations for homogeneous turbulent shear flow in a rotating frame are shown for $\Omega/S = 0$ and $\Omega/S = 0.5$ – the two values of Ω/S which correspond to a Richardson number $Ri = 0$. Clearly, the turbulent kinetic energy and dissipation rate associated with these two cases are quite different and thus indicative of a violation of Richardson number similarity (the initial conditions for both flows were identical).

We will now proceed to show that certain low-order truncations of the Navier-Stokes equations do exhibit Richardson number similarity. Second-order closure models will be considered wherein the infinite hierarchy of moments obtained from the Navier-Stokes equations are truncated at the second-moment level by modeling the higher-order moments. This second moment level consists of the Reynolds stress transport equation

$$\dot{\tau}_{ij} = \Phi_{ij} - A_{ik}\tau_{kj} - A_{jk}\tau_{ki} \quad (39)$$

where $\tau_{ij} \equiv -\overline{u_i u_j}$ is the Reynolds stress tensor and Φ_{ij} given by

$$\Phi_{ij} = -p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + 2\nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \quad (40)$$

is the sum of the pressure strain and dissipation rate correlations. Equation (39) (which is not a closed system for the determination of τ_{ij}) is obtained by taking the ensemble mean of the symmetric part of the product of equation (13) with u_j . Closure can be achieved by assuming that Φ_{ij} is a functional of the variables $\tau, \nabla \bar{\mathbf{v}}$ and ε as follows in an inertial frame[†]

$$\Phi_{ij} = \Phi_{ij}(\mathbf{b}, \nabla \bar{\mathbf{v}}, K, \varepsilon) \quad (41)$$

where $b_{ij} = -(\tau_{ij} + \frac{2}{3}K\delta_{ij})/K$ is the anisotropy tensor. Since for isotropic turbulence, Φ_{ij} is given by⁸

$$\Phi_{ij} = \frac{2}{3}\varepsilon\delta_{ij}, \quad (42)$$

the physical motivation for (41) is clear: anisotropies in Φ_{ij} arise from the mean velocity gradients $\nabla \bar{\mathbf{v}}$ which are then reflected in nonzero values of \mathbf{b} . For small perturbations of isotropy, we have

$$\mathbf{b} = \delta \mathbf{b}, \quad \nabla \bar{\mathbf{v}} = \delta(\nabla \bar{\mathbf{v}})$$

To the first order in $\delta \mathbf{b}$ and $\delta(\nabla \bar{\mathbf{v}})$, Φ_{ij} is given by

$$\begin{aligned} \Phi_{ij}(\mathbf{b}, \nabla \bar{\mathbf{v}}, K, \varepsilon) &= (\Phi_{ij})_0 + \left(\frac{\partial \Phi_{ij}}{\partial \mathbf{b}} \right)_0 \cdot \delta \mathbf{b} + \left(\frac{\partial \Phi_{ij}}{\partial \nabla \bar{\mathbf{v}}} \right)_0 \cdot \delta(\nabla \bar{\mathbf{v}}) \\ &= \frac{2}{3}\varepsilon\delta_{ij} + c_1\varepsilon b_{ij} + c_2K \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) \end{aligned} \quad (43)$$

where $(\cdot)_0$ denotes a function evaluated at $\mathbf{b} = 0, \nabla \bar{\mathbf{v}} = 0$ (i.e., for the state of isotropy) and c_1 and c_2 are dimensionless constants. Equation (43) is obtained by making a Taylor expansion about the isotropic state (see the Appendix). The modeled equation for Φ_{ij} is

[†]The reader should note that in a rotating frame $\partial \bar{v}_i / \partial x_j$ must be replaced by $\partial \bar{v}_i / \partial x_j + \varepsilon_{kji}\Omega_k$ (see Speziale¹²).

precisely the same as that used in the Rotta-Kolmogorov model¹³ if we take $c_1 = 3.21$ and $c_2 = -0.112$.

As an alternative to this approach, the pressure strain and dissipation rate correlations can be modeled separately. Since dissipation is manifested primarily at the small scales (which are not far removed from isotropy at high Reynolds numbers) the dissipation rate correlation can be approximated by its isotropic form

$$2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} = \frac{2}{3} \varepsilon \delta_{ij}. \quad (44)$$

The pressure strain correlation vanishes for isotropic turbulence and for homogeneous turbulence can be shown to be of the form (see Shih and Lumley¹⁴)

$$p \overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} = R_{ij} + M_{ijkl} \frac{\partial \bar{v}_k}{\partial x_\ell} \quad (45)$$

in an inertial framing where $R_{ij}(t)$ and $M_{ijkl}(t)$ are functionals of the energy spectrum tensor $E_{ij}(\mathbf{k}, t)$ over wave number space. It can be assumed that (see Shih and Lumley¹⁴)

$$R_{ij} = R_{ij}(\mathbf{b}, K, \varepsilon) \quad (46)$$

$$M_{ijkl} = M_{ijkl}(\mathbf{b}, K, \varepsilon) \quad (47)$$

For small anisotropies, (46)-(47) can be expanded in a Taylor series which, to the first order in \mathbf{b} , yields the following expression for the pressure strain correlation in a rotating frame^{15,16}:

$$\begin{aligned} -p \overline{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} &= C_1 \varepsilon b_{ij} + C_2 K \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) + C_3 K (b_{ik} \bar{S}_{jk} + b_{jk} \bar{S}_{ik}) \\ &\quad - \frac{2}{3} b_{k\ell} \bar{S}_{k\ell} \delta_{ij} + C_4 K (b_{ik} \bar{W}_{jk} + b_{jk} \bar{W}_{ik}) \end{aligned} \quad (48)$$

where

$$\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right), \quad \bar{W}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{v}_i}{\partial x_j} - \frac{\partial \bar{v}_j}{\partial x_i} \right) + \varepsilon_{mji} \Omega_m \quad (49)$$

and we have made use of the fact that the pressure strain correlation is traceless and must vanish in the limit as \mathbf{b} , $\nabla \bar{\mathbf{v}}$, and Ω go to zero. If we take $C_1 = 1.8$, $C_2 = -0.4$, $C_3 = -0.6$, and $C_4 = -0.6$, the Launder, Reece, and Rodi¹⁷ model is obtained.

When closure relations for Φ_{ij} of the general form (41) are substituted into (39), closure is achieved once a modeled transport equation for ε is provided. For high Reynolds number turbulence that is homogeneous, the exact transport equation for ε takes the form¹⁷

$$\frac{D\varepsilon}{Dt} = -2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_\ell} \frac{\partial u_k}{\partial x_\ell}} - 2\nu^2 \overline{\frac{\partial^2 u_i}{\partial x_k \partial x_\ell} \frac{\partial^2 u_i}{\partial x_k \partial x_\ell}} \quad (50)$$

It can be shown that the lowest-order model for (50) is of the form

$$\frac{D\varepsilon}{Dt} = c_{\varepsilon 1} \frac{\varepsilon}{K} \mathcal{P} - c_{\varepsilon 2} \frac{\varepsilon^2}{K} \quad (51)$$

where $\mathcal{P} \equiv \tau_{ij} \partial \bar{v}_i / \partial x_j$ is the turbulence production and $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are dimensionless constants (see Lumley¹⁸ and Launder, Reece, and Rodi¹⁷). Equation (51) is obtained by assuming that the higher-order correlations on the right-hand-side of (50) are functions of \mathbf{b} , $\nabla \mathbf{v}$, K , and ε which are expanded in a Taylor series to the first order in \mathbf{b} (ε and K merely set the length and time scales for dimensional consistency). For the Rotta-Kolmogorov model,[§] $C_{\varepsilon 1} = 1.8$ and $C_{\varepsilon 2} = 2.0$ whereas for the Launder, Reece, and Rodi Model, $C_{\varepsilon 1} = 1.45$ and $C_{\varepsilon 2} = 1.90$. When (51) is solved in conjunction with (39) and a model of the form of (41), a closed system of equations for the determination of the Reynolds stress tensor and the turbulent dissipation rate are obtained. This constitutes a low order truncation of the hierarchy of moments of the Navier-Stokes equations (achieved at the second moment level) by assuming that higher-order moments are a small perturbation of their isotropic state.

Equation (39) can be written in the alternative dimensionless matrix form

$$\dot{\mathbf{i}}^* = \Phi^* - \mathbf{A}^* \boldsymbol{\tau}^* - \boldsymbol{\tau}^* \mathbf{A}^{*\text{T}} \quad (52)$$

where $\boldsymbol{\tau}^* = \boldsymbol{\tau} / K_0$, $\Phi^* = \Phi / K_0 S$ and $\mathbf{A}^{*\text{T}}$ denotes the transpose of \mathbf{A}^* . If we introduce the variable

$$\boldsymbol{\tau}^{**} = \mathbf{A}^* \boldsymbol{\tau}^* + \boldsymbol{\tau}^* \mathbf{A}^{*\text{T}}, \quad (53)$$

then (52) can be written in the alternative form

$$\dot{\mathbf{i}}^{**} = \Phi^{**} - \mathbf{A}^* \boldsymbol{\tau}^{**} - \boldsymbol{\tau}^{**} \mathbf{A}^{*\text{T}} \quad (54)$$

where

$$\Phi^{**} = \mathbf{A}^* \Phi^* + \Phi^* \mathbf{A}^{*\text{T}} \quad (55)$$

Equation (54) is obtained by adding the two equations that result from the pre-multiplication of (52) with \mathbf{A}^* and the post-multiplication of (52) with $\mathbf{A}^{*\text{T}}$. Since,

$$\mathbf{A}^* = \begin{bmatrix} 0 & 1 - 2\Omega/S \\ 2\Omega/S & 0 \end{bmatrix} \quad (56)$$

it follows that

$$\boldsymbol{\tau}^{**} = \begin{bmatrix} 2(1 - 2\Omega/S)\tau_{12}^* & \tau_d^* \\ \tau_d^* & 4(\Omega/S)\tau_{12}^* \end{bmatrix} \quad (57)$$

[§]The Rotta-Kolmogorov model is actually based on a transport equation for the length scale which, for homogeneous turbulence, can be converted to a transport equation for ε of the form (51).

$$\mathbf{A}^* \boldsymbol{\tau}^{**} = \boldsymbol{\tau}^{**} \mathbf{A}^{*T} = \begin{bmatrix} (1 - 2\Omega/S)\tau_d^* & -2Ri \tau_{12}^* \\ -2Ri \tau_{12}^* & 2\Omega/S \tau_d^* \end{bmatrix} \quad (58)$$

where

$$\tau_d^* = (1 - 2\Omega/S)\tau_{22}^* + 2\Omega/S \tau_{11}^* \quad (59)$$

Taking the trace of (54) and dividing by two we obtain the equation

$$\dot{\tau}_{12}^* = \frac{1}{2} tr \Phi^{**} - \tau_d^* \quad (60)$$

whereas the 1,2 - component of (54) yields the equation

$$\dot{\tau}_d^* = \Phi_{12}^{**} - 2Ri \tau_{12}^*. \quad (61)$$

Equations (60)-(61) must be solved in conjunction with the turbulent kinetic energy and dissipation rate equations given in the dimensionless form

$$\dot{K}^* = \tau_{12}^* - \frac{\varepsilon_0}{SK_0} \varepsilon^* \quad (62)$$

$$\dot{\varepsilon}^* = c_{\varepsilon 1} \frac{\varepsilon^*}{K^*} \tau_{12}^* - c_{\varepsilon 2} \frac{\varepsilon_0}{SK_0} \frac{\varepsilon^{*2}}{K^*} \quad (63)$$

where $K^* = K/K_0$ and $\varepsilon^* = \varepsilon/\varepsilon_0$. The system (60)-(63) is solved subject to the initial conditions

$$K^*(0) = 1, \quad \varepsilon^*(0) = 1 \quad (64)$$

$$\tau_{12}^*(0) = 0, \quad \tau_d^*(0) = -\frac{2}{3} \quad (65)$$

Hence, for a given initial value of ε_0/SK_0 , (60)-(63) will yield solutions for K^* , ε^* , τ_{12}^* and τ_d^* which exhibit Richardson number similarity provided that

$$tr \Phi^{**} = f(\tau_{12}^*, \tau_d^*, K^*, \varepsilon^*, \varepsilon_0/SK_0, Ri) \quad (66)$$

$$\Phi_{12}^{**} = g(\tau_{12}^*, \tau_d^*, K^*, \varepsilon^*, \varepsilon_0/SK_0, Ri) \quad (67)$$

where f and g are arbitrary functions that need only be sufficiently smooth. For the Rotta-Kolmogorov model,

$$tr \Phi^{**} = -2c_1 \frac{\varepsilon_0}{SK_0} \frac{\varepsilon^*}{K^*} \tau_{12}^* + 2c_2 K^* \quad (68)$$

$$\Phi_{12}^{**} = \frac{2}{3} \frac{\varepsilon_0}{SK_0} \varepsilon^* - c_1 \frac{\varepsilon_0}{SK_0} \frac{\varepsilon^*}{K^*} \left(\frac{2}{3} K^* + \tau_d^* \right) \quad (69)$$

which satisfies (66)-(67). Hence, the Rotta-Kolmogorov model exhibits Richardson number similarity. However, in the Launder, Reece, and Rodi model,

$$\begin{aligned} tr \Phi^{**} = & -2C_1 \frac{\varepsilon_0}{SK_0} \frac{\varepsilon^*}{K^*} \tau_{12}^* + 2C_2 K^* - 2C_3 K^* \left(\frac{4}{3} + \frac{\tau_{11}^*}{K^*} + \frac{\tau_{22}^*}{K^*} \right) \\ & + 2C_4 (1 - 2\Omega/S) \left(\frac{\tau_{11}^*}{K^*} - \frac{\tau_{22}^*}{K^*} \right) \end{aligned} \quad (70)$$

$$\Phi_{12}^{**} = \frac{2}{3} \frac{\varepsilon_0}{SK_0} \varepsilon^* - C_1 \frac{\varepsilon_0}{SK_0} \frac{\varepsilon^*}{K^*} \left(\frac{2}{3} K^* + \tau_d^* \right) - \frac{1}{3} C_3 \tau_{12}^* - C_4 \left(1 - \frac{4\Omega}{S} \right) \left(1 - \frac{2\Omega}{S} \right) \tau_{12}^* \quad (71)$$

which clearly violates (66)-(67). Consequently, the Launder, Reece, and Rodi model violates Richardson number similarity. In figure 3, computed results for the time evolution of the turbulent kinetic energy is shown for a Richardson number $Ri = 0$ (which corresponds to the two dimensionless rotation rates of $\Omega/S = 0$ and $\Omega/S = 0.5$) for the second-order closure models considered in this study. Consistent with the proofs already presented, the Rotta-Kolmogorov model yields the same results for both cases whereas the Launder, Reece, and Rodi model yields results that are dramatically different (and which, in comparison to the large-eddy simulations, deviate *too strongly* from Richardson number similarity). Both the Launder, Reece, and Rodi model and the Rotta-Kolmogorov model predict a universal equilibrium for the dimensionless shear rate SK/ε and anisotropy tensor b_{ij} in the limit as $t \rightarrow \infty$. These results are shown in Table 1. Clearly, the results obtained from the Rotta-Kolmogorov model exhibit Richardson number similarity whereas those obtained from the Launder, Reece, and Rodi model do not.

Finally, we would like to comment in more detail on the asymptotic long time behavior of the Rotta-Kolmogorov model which constitutes the lowest-order second moment truncation of the Navier-Stokes equations. It can be shown that for dimensionless time $t^* \gg 1$, the Rotta-Kolmogorov model yields solutions such that (see Speziale and Mac Giolla Mhuiris¹⁹)

$$K^*(t^*) \sim \exp \left[\left[(b_{12})_\infty + \left(\frac{\varepsilon}{SK} \right)_\infty \right] t^* \right] \quad (72)$$

$$\varepsilon^*(t^*) \sim \exp \left[\left[(b_{12})_\infty + \left(\frac{\varepsilon}{SK} \right)_\infty \right] t^* \right] \quad (73)$$

for intermediate rotation rates $-0.11 < \Omega/S < 0.61$. For values of Ω/S outside of this range, the flow undergoes a relaminarization (i.e., K and $\varepsilon \rightarrow 0$ as $t \rightarrow \infty$). In figure 4, the time evolution of the turbulent kinetic energy computed from the Rotta-Kolmogorov model is shown for rotation rates in both characteristic regimes (i.e., for $\Omega/S = 0.25$ in the unstable regime and for $\Omega/S = 0.75$ where a relaminarization occurs). It is interesting to note that the existence of an unstable flow regime, at intermediate rotation rates, where

there is an exponential growth in time of the turbulent kinetic energy and dissipation rate has been argued based on alternative models (see Bertoglio¹¹, Rogallo²⁰, and Tavoularis²¹). Furthermore, the Rotta-Kolmogorov model predicts the existence of universal equilibrium states in this unstable regime where the anisotropy tensor and dimensionless shear rate approach the values of $(b_{ij})_\infty$ and $(SK/\varepsilon)_\infty$ which depend only on Ω/S through the Richardson number¹⁹ – a result which is a crude approximation of numerical simulations of the Navier-Stokes equations³.

4. CONCLUSION

Conditions for the dynamic similarity of homogeneous turbulent shear flows in a rotating frame have been obtained based on a direct analysis of the Navier-Stokes equations. It was proven that the dimensionless parameters which established similitude are the initial energy spectrum $E^*(k^*, 0)$, the initial shear rate SK_0/ε_0 , the initial Reynolds number $K_0^2/\nu\varepsilon_0$, and the ratio of the rotation rate to the shear rate Ω/S . It was proven that the commonly assumed dependence of the Reynolds stress tensor and dissipation rate on Ω/S through the Richardson number $Ri = -2(\Omega/S)(1 - 2\Omega/S)$ is not a rigorous consequence of the Navier-Stokes equations. In fact, only the lowest-order second-moment truncation of the Navier-Stokes equations (i.e., the Rotta-Kolmogorov model) exhibits Richardson number similarity with respect to the turbulent fields τ_{12}^* , τ_d^* , K^* , and ε^* . The second moment closure of Launder, Reece, and Rodi deviates quite strongly from Richardson number similarity.

The introduction of an ad hoc Richardson number dependence in the length scale for a one equation turbulence model can only yield a crude approximation of the turbulence structure for rotating shear flows. However, the commonly used turbulence models tend to be so empirical in nature that the errors introduced in making the assumption of Richardson number similarity may be comparatively unimportant. There is, of course, a need to develop turbulence models which can more accurately predict the turbulence structure in rotating turbulent flows. We believe that such improved models should be based on physically motivated modifications of the Reynolds stress and dissipation rate transport equations, which rationally account for rotational strains, rather than on the introduction of an ad hoc Richardson number dependence. The development and testing of such improved turbulence models will be the subject of a future paper.

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APPENDIX

The Taylor expansion (43) can be written in the form

$$\Phi_{ij}(\mathbf{b}, \nabla\bar{\mathbf{v}}, K, \epsilon) = \frac{2}{3}\epsilon\delta_{ij} + c_{ijkl}b_{kl} + d_{ijkl}\frac{\partial\bar{v}_k}{\partial x_\ell} \quad (\text{A1})$$

where we have made use of representation (42) for $(\Phi_{ij})_0$ and also of the fact that $\delta\mathbf{b} = \mathbf{b}$ while $\delta(\nabla\bar{\mathbf{v}}) = \nabla\bar{\mathbf{v}}$. In this form of the expansion, c_{ijkl} and d_{ijkl} are defined by

$$c_{ijkl} = \left(\frac{\partial\Phi_{ij}}{\partial b_{kl}} \right)_0 \quad (\text{A2})$$

$$d_{ijkl} = \left(\frac{\partial\Phi_{ij}}{\partial \bar{v}_{k,\ell}} \right)_0 \quad (\text{A3})$$

where $\bar{v}_{k,\ell} = \partial\bar{v}_k/\partial x_\ell$. We remind the reader that $(\cdot)_0$ denotes a function evaluated at $\mathbf{b} = 0, \nabla\bar{\mathbf{v}} = 0$; therefore c_{ijkl} and d_{ijkl} are functions of K and ϵ alone.

Dimensional and tensorial invariance along with the symmetry of ϕ_{ij} and b_{ij} yields the following form for c_{ijkl} :

$$c_{ijkl}(K, \epsilon) = c_1\frac{\epsilon}{2}(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) + c'_1\epsilon\delta_{ij}\delta_{kl} \quad (\text{A4})$$

where c_1 and c'_1 one dimensionless constants. Since b_{kl} is traceless it follows that

$$c_{ijkl}b_{kl} = c_1\epsilon b_{ij} \quad (\text{A5})$$

The same arguments can be used to show that

$$d_{ijkl}(K, \epsilon) = c_2\frac{K}{2}(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) + c'_2K\delta_{ij}\delta_{kl} \quad (\text{A6})$$

and, hence,

$$d_{ijkl}\frac{\partial\bar{v}_k}{\partial x_\ell} = c_2K\left(\frac{\partial\bar{v}_i}{\partial x_j} + \frac{\partial\bar{v}_j}{\partial x_i}\right). \quad (\text{A7})$$

where c_2 is a dimensionless constant. The final form of equation (43) then follows.

Rotta-Kolmogorov Model	$\Omega/S = 0$	$\Omega/S = 0.5$	Launder, Reece and Rodi Model	$\Omega/S = 0$	$\Omega/S = 0.5$
$(SK/\varepsilon)_\infty$	3.71	3.71	$(SK/\varepsilon)_\infty$	5.42	∞
$(b_{12})_\infty$	-0.337	-0.337	$(b_{12})_\infty$	-0.369	0
$(b_{11})_\infty$	0.483	-0.241	$(b_{11})_\infty$	0.381	-0.240
$(b_{22})_\infty$	-0.241	0.483	$(b_{22})_\infty$	-0.190	0.320
$(\tau_d/K)_\infty$	-0.426	-0.426	$(\tau_d/K)_\infty$	-0.477	-0.427

Table 1. Equilibrium values of the Rotta-Kolmogorov model and the Launder, Reece, and Rodi model for a Richardson number $Ri = 0$.

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- Figure 4. Time evolution of the turbulent kinetic energy obtained from the Rotta-Kolmogorov model for an initial condition $\varepsilon_0/SK_0 = 1$ ($\Omega/S = 0.25$ and $\Omega/S = 0.75$).

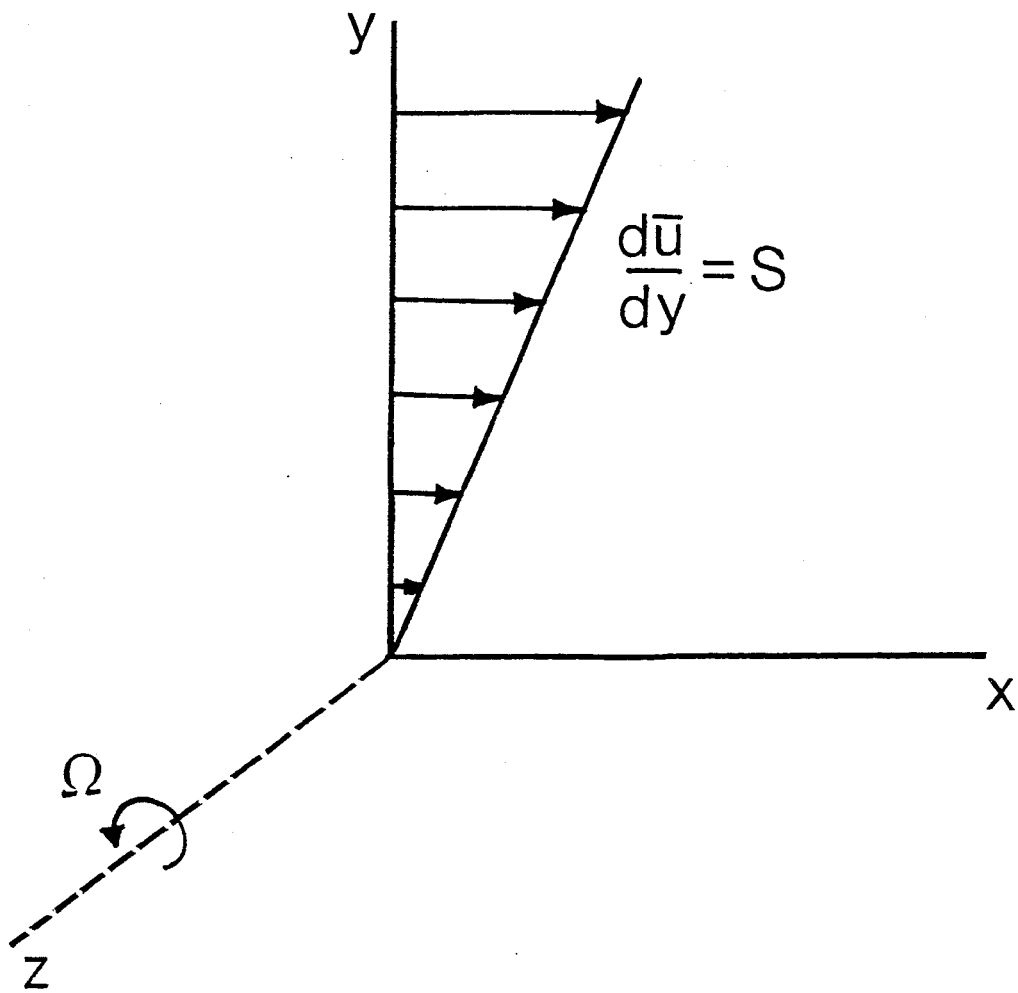


Figure 1. Homogeneous turbulent shear flow in a rotating frame.

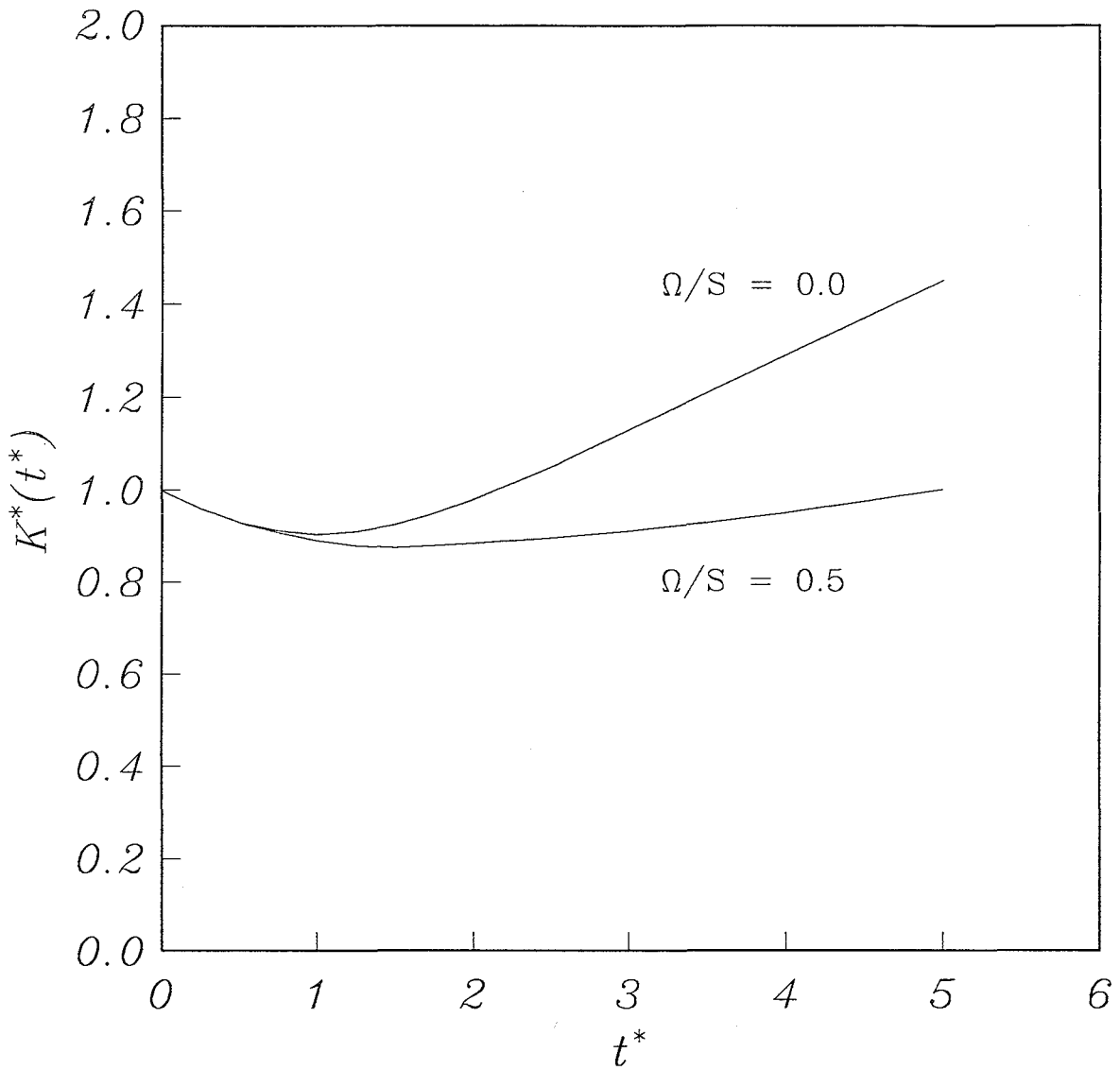


Figure 2. Time evolution of the turbulent kinetic energy for a Richardson number $Ri = 0$ taken from the large-eddy simulations of Bardina, Ferziger, and Reynolds³.

Large Eddy Simulations \circ
Rotta-Kolmogorov Model ---
Launer, Reece and Rodi Model -

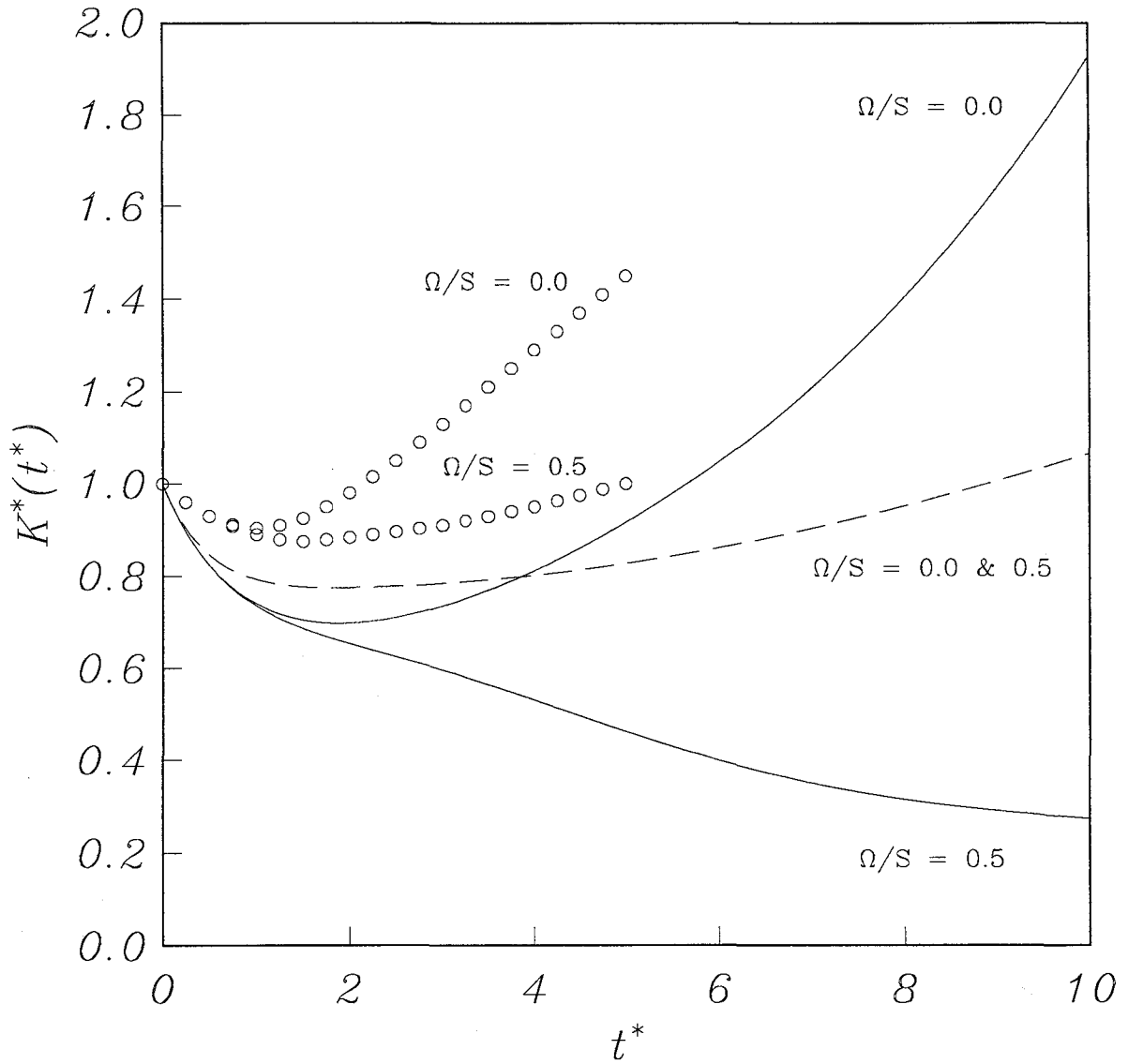


Figure 3. Time evolution of the turbulent kinetic energy for a Richardson number $Ri = 0$ and initial condition $\varepsilon_0/SK_0 = 0.496$: \circ Large-eddy simulations³, --- Rotta-Kolmogorov model, — Launder, Reece, and Rodi model.

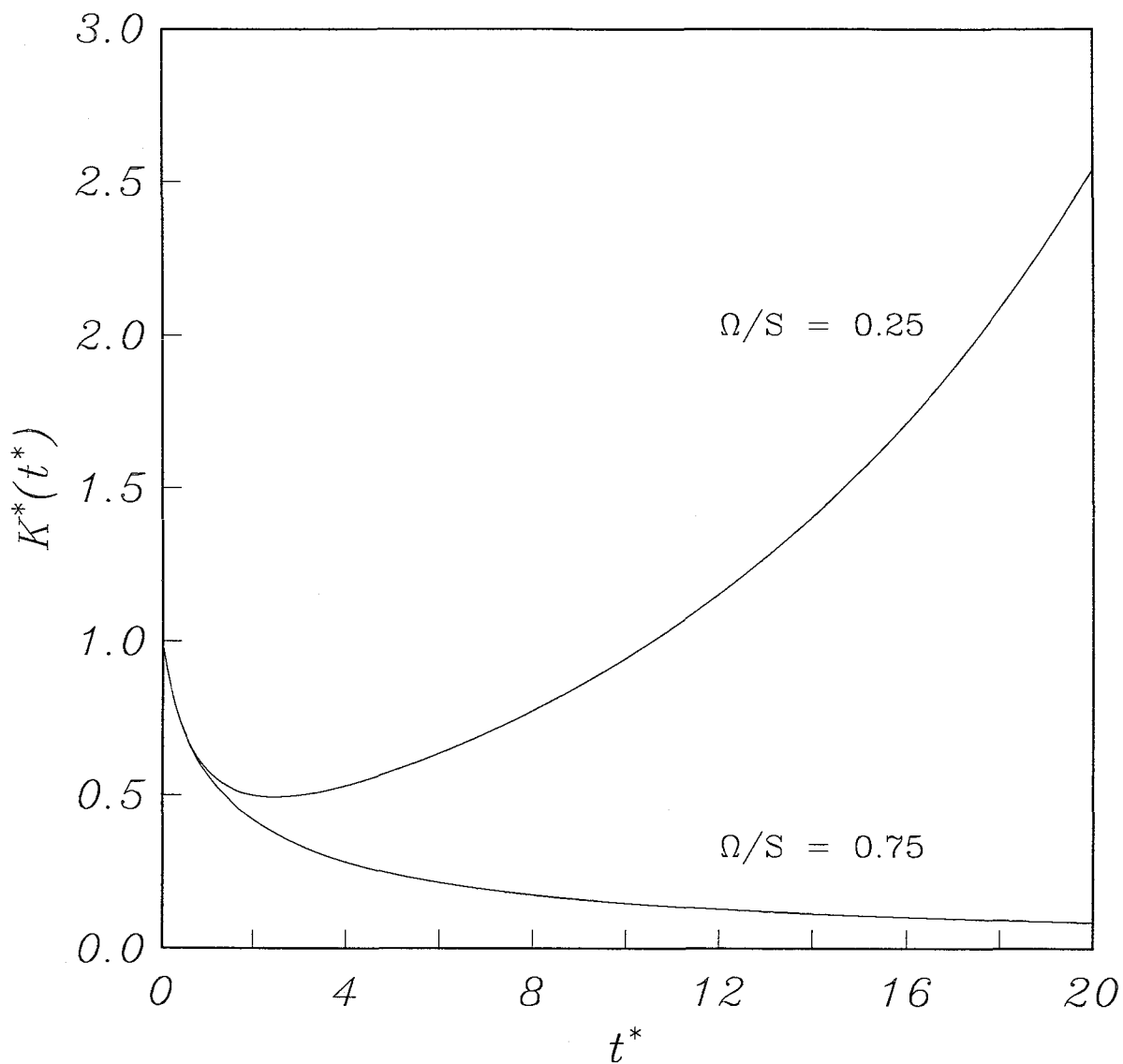


Figure 4. Time evolution of the turbulent kinetic energy obtained from the Rotta-Kolmogorov model for an initial condition $\varepsilon_0/SK_0 = 1$ ($\Omega/S = 0.25$ and $\Omega/S = 0.75$).





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16. Abstract The scaling properties of plane homogeneous turbulent shear flows in a rotating frame are examined mathematically by a direct analysis of the Navier-Stokes equations. It is proven that two such shear flows are dynamically similar if and only if their initial dimensionless energy spectrum $E^*(k^*, 0)$, initial dimensionless shear rate SK_0/ϵ_0 , initial Reynolds number $K_0^2/\nu\epsilon_0$, and the ratio of the rotation rate to the shear rate Ω/S are identical. Consequently, if universal equilibrium states exist, at high Reynolds numbers, they will only depend on the single parameter Ω/S . The commonly assumed dependence of such equilibrium states on Ω/S through the Richardson number $Ri = -2(\Omega/S)(1 - 2\Omega/S)$ is proven to be inconsistent with the full Navier-Stokes equations and to constitute no more than a weak approximation. To be more specific, Richardson number similarity is shown to only rigorously apply to certain low-order truncations of the Navier-Stokes equations (i.e., to certain second-order closure models) wherein closure is achieved at the second-moment level by assuming that the higher-order moments are a small perturbation of their isotropic states. The physical dependence of rotating turbulent shear flows on Ω/S is discussed in detail along with the implications for turbulence modeling.					
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