NASA Contractor Report 4169

NASA-CR-4169 19880017474

Solution of the Orr-Sommerfeld Equation for the Blausius Boundary-Layer Documentation of Program ORRBL and a Test Case

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GRANT NAG1-798
AUGUST 1988

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Prepared for
Langley Research Center
under Grant NAG1-798

NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Division

1988

SUMMARY

In this note, a Chebyshev matrix collocation method is outlined for the solution of the Orr-Sommerfeld equation for the Blausius boundary layer. User information is provided for FORTRAN program ORRBL which solves the equation by the QR method.

1. PROBLEM FORMULATION

In this report, we summarize a numerical procedure that was implemented to develop the FORTRAN program ORRBL which solves the Orr-Sommerfeld equation for two- and three-dimensional small amplitude disturbances. The program is written for the Blausius boundary layer but the method can be easily adapted to other parallel shear flows. The current version of the program solves for the temporal eigenvalues and a new version for the solution of the spatial eigenvalues is under development.

According to the linear stability theory, the equations of continuity and momentum for parallel shear flows can be written as (Ref. 1)

$$i(\alpha\hat{u}_1 + \beta\hat{u}_3) + D\hat{u}_2 = 0, \tag{1}$$

$$[D^2 - (\alpha^2 + \beta^2) - i\alpha R(\overline{u} - c)]\hat{u}_1 = R(\overline{u}')\hat{u}_2 + i\alpha R\hat{p}, \tag{2}$$

$$[D^2 - (\alpha^2 + \beta^2) - i\alpha R(\overline{u} - c)]\hat{u}_2 = RD\hat{p}, \tag{3}$$

$$[D^2 - (\alpha^2 + \beta^2) - i\alpha R(\overline{u} - c)]\hat{u}_3 = i\beta R\hat{p}. \tag{4}$$

Note that these equations satisfy the disturbance field defined as

$$\tilde{u}_i = \hat{u}_i(y)\ exp[i(\alpha x + \beta z) - i\alpha ct]. \tag{5}$$

In eqs. (1-5), $D \equiv d/dy$, $\overline{u}$ is the mean (Blausius) velocity profile, $\overline{u}'$ is its first derivative y, α is the wave number along the streamwise (x_1) direction, β is the wave number along the spanwise (x_3) direction. Also, the complex dimensionless amplitude functions $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$ and $\hat{p}$ are functions of the cross-stream variable x_2. For temporally evolving disturbances, the eigenvalue c is complex; its real part corresponds to the phase speed of the wave and its imaginary part indicates amplification in time, i. e. for given Reynolds number (R) and wave numbers (α and β) if $c_I < 0$ for the least-damped eigenvalue, then the flow is stable. However if $c_I > 0$, then the flow is unstable. For the Blausius boundary layer, R is defined in terms of the displacement thickness and free-stream velocity.

The three-dimensional disturbance equations can be reduced to the Orr-Sommerfeld equation by the Squire transformation for which we define (Ref. 1)

$$
\begin{aligned}
k^2 &= \alpha^2 + \beta^2, \\
\hat{v} &= \hat{u}_2, \\
\alpha\hat{u}_1 + \beta\hat{u}_3 &= k\hat{u}.
\end{aligned}
\tag{6}
$$

With this substitution, the Orr-Sommerfeld equation reads

$$(D^2 - \alpha^2)^2 \hat{v} = i\alpha\ R[(\overline{u} - c)(D^2 - \alpha^2)\hat{v} - (\overline{u}'')\hat{v}]. \tag{7}$$

For the boundary layer, this equation is solved with the boundary conditions.

$$\hat{v} = D\hat{v} = 0 \quad at\ y = 0\ \ and\ \ y \rightarrow \infty. \tag{8}$$

In program ORRBL, eqs. (7,8) are solved by a Chebyshev matrix collocation method which results in a matrix equation

$$(\mathbf{A} - c\mathbf{B})\hat{v} = 0. \tag{9}$$

Here, **A** and **B** are functions of the Chebyshev collocation matrix. This matrix is explicitly given in Ref. 3. The matrix eigenvalue problem (eq. 9) is then solved by the QR method using the IMSL subroutine EIGZC. The mean velocity profile $\overline{u}$ is obtained by solving the Blausius equation which reads

$$f''' + \frac{1}{2}ff'' = 0, \tag{10}$$

by a point-by-point fifth and sixth-order Runge-Kutta-Verner method.

The current version of the program provides very high accuracy with about 100 Chebyshev collocation points; in the program this number is specified by the user. Note that Chebyshev polynomials are defined in the interval $-1 \leq y \leq 1$, so that for application to the boundary layer problem, a coordinate transformation is required. For this purpose we use (Ref. 2)

$$\eta = e^{-y/Y}, \tag{11}$$

where Y is a user defined parameter which should be $15 \leq Y \leq 25$ for maximum accuracy. Note that this transformation allows the inclusion of the boundary points and satisfies boundary conditions exactly.

2. SOLUTION METHOD

Chebyshev spectral methods have recently been very successful in boundary layer stability and transition investigations. Spectral methods are highly accurate and require much less discretization points than finite-difference and finite-element methods. The difficulty of their application to the semi-infinite integration domain of a boundary layer has been overcome with suitable mapping or transformation techniques. The matrix collocation method can be relatively easily implimented into existing codes. A brief overview of this method follows.

The semi-infinite integration domain $0 \leq y < \infty$ can be transformed into the interval $0 < \eta \leq 1$ using the mapping formula (Ref. 2)

$$\eta = exp(-\frac{y}{Y}), \tag{12}$$

with the free transformation parameter Y. In the transformed variable η, any function u can be expanded into a Chebyshev series

$$u(\eta) = \sum_{k=0}^{N} \hat{u}_k \cdot T_k(\eta), \tag{13}$$

with Chebyshev coefficients $\hat{u}_k$ (called spectrum of u) and the Chebyshev polnomials

$$T_k(\eta) = cos(k cos^{-1}\eta), \tag{14}$$

defined in the interval $-1 \leq \eta \leq 1$. Defining the 'collocation points'

$$\eta_j = cos(\frac{\pi \cdot j}{N}) \quad ; \quad j = 0...N, \tag{15}$$

the series (13) can be evaluated. With the spectrum $\hat{u}_k$ known, derivatives of u can then be evaluated.

The application of this method requires the use of the Chebyshev matrix $D^{(p)}$. The coefficients $d_{jk}^{(p)}$ of this matrix are known. This matrix relates the values of u at the collocation points η_j to the values of the p-th derivative of u at these points:

$$\frac{\partial^p u(\eta_j)}{\partial \eta^p} = \sum_{k=0}^{N} d_{kj} u(\eta_k). \tag{16}$$

We now define

$$u(\eta_j) = 0 \quad ; \quad j > N/2 + 1 = N_{half}, \tag{17}$$

(N' even) and apply the transformation (12) to the derivative relation (16) at the transformed collocation points

$$y_j = -Y ln(\eta_j) \quad ; \quad j = 0...N_{half} - 1. \tag{18}$$

Only one quarter elements of the original collocation matrix D have to be used. The boundary condition $u = 0$ at $y \to \infty$ is satisfied automatically because of the transformation (17).

REFERENCES

1. C. C. Lin: *Theory of Hydrodynamic Stability,* Cambridge University Press (1955).

2. E. Laurien *Loesung der Orr-Sommerfeld Gleichung für die Blasius'sche Grenzschichtströmung mittels Chebyshev-Kollokation*, DFVLR-Interner Bericht (1985).

3. D. Gottlieb, M. Y. Hussaini, and S. A. Orszag: *Theory and Application of Spectral Methods*, in *Spectral Methods for Partial Differential Equations* (editors R.G. Voigt, D. Gottlieb and M. Y. Hussaini) pp. 1-54, SIAM, Philadelphia (1984).

SAMPLE CASE

INPUT FILE (INPUT PARAMETERS)

The code requires an input data file named ORDATA. The assigned values for the following parameters should be included in this file.

R - (REAL) Reynolds number based on displacement thickness and the free-stream velocity.

α - (REAL) Streamwise wave number.

β - (REAL) Spanwise wave number. For two-dimensional calculations β should be equal to zero ($\beta = 0$).

Y - (REAL) This parameter is used for the coordinate transformation. A high value of Y means mesh clustering towards the boundaries. For best results: $15 \leq Y \leq 25$ (Y=20 is suggested).

N - (INTEGER) Number of the collocation points in the whole domain. It should be an even number which is ≤ 100. Due to the coordinate transformation, only half of the given N is used for the eigenvalue calculations in the code.

IOPT - (INTEGER) Option parameter.

IOPT=0 - only the eigenvalues are calculated, IOPT=1 - both the eigenvalues and the eigenvectors are calculated, but only the eigenvector corresponding to the least damped eigenvalue is printed.

EXAMPLE:

The following parameters are given,

$$R = 998, \ \alpha = 0.308021, \ \beta = 0, \ Y = 20, \ N = 100, \ IOPT = 1.$$

For this case, ORDATA has the following form,

```
998.
0.308021
0.
20.
100
1
```

Note that the input file starts at the first column of the first row.

PROGRAM OUTPUT

ORRBL creates two output files named OREIGE and ORVEL, respectively.

Output file 1: OREIGE

This file contains the following information,

- number of the collocation points used in the half domain,
- Reynolds number
- Streamwise wave number (α),
- Spanwise wave number (β),
- Y,
- Base velocity profile (Blausius solution),

column 1 - physical y-coordinates,

column 2 - mean velocity profile,

column 3 - first derivative of the mean velocity profile,

column 4 - second derivative of the mean velocity profile,

- Index of the unconverged eigenvalue (for a detailed explanation, please refer to the parameter listing of EIGZC); if this number is zero, all the eigenvalues are converged.
- EIGZC error number (for a detailed explanation, please refer to the parameter listing of EIGZC); if this number is zero, there are no errors.
- a list of the eigenvalues (Note that if one of the diagonals of **B** matrix is zero, the corresponding eigenvalue goes to infinity. Therefore, in such cases, the real part of the eigenvalue is set equal to 1×10^{30} and the imaginary part is zero.),

column 1 - index,

column 2 - real part,

column 3 - imaginary part,

- Index of the least damped eigenvalue,
- Least damped eigenvalue (CR : real part, CI : imaginary part),
- Eigenvector corresponding to the least damped eigenvalue, (printed for $y \leq 30$)

column 1 - physical y-coordinates,

column 2 - real part of the eigenvector,

column 3 - imaginary part of the eigenvector,

column 4 - (if IOPT=1) phase angle of the eigenvector.

Output file 2: ORVEL

This file is used if IOPT=1 and contains the following information,

- Real and imaginary parts of the v perturbation velocity, (printed for $y \leq 30$)

column 1 - physical y-coordinates,

column 2 - real part of the velocity,

column 3 - imaginary part of the velocity,

- Real and imaginary parts of the u perturbation velocity. (printed for $y \leq 30$) If the calculation is two-dimensional, this velocity is the same as $\hat{u}_1$ in eq. (6). Otherwise it gives $\hat{u}$ in the same equation.

column 1 - physical y-coordinates,

column 2 - real part of the velocity,

column 3 - imaginary part of the velocity,

A sample output is given on the following pages.

OUTPUT FILE 1 : OREIGE

```
NUMBER OF COLLOCATION POINTS IN THE HALF DOMAIN= 51
REYNOLDS NUMBER= .9980000E+03
STREAMWISE WAVE NUMBER= .3080210E+00
SPANWISE WAVE NUMBER= .0000000E+00
Y= .2000000E+02
```

BASE VELOCITY PROFILE (BLAUSIUS SOLUTION):

Y-COORDINATE	U	D1U	D2U
.0000000E+00	.0000000E+00	.3320573E+00	.0000000E+00
.9871228E-02	.3277814E-02	.3320573E+00	-.2686016E-05
.3950442E-01	.1311773E-01	.3320568E+00	-.4301865E-04
.8895825E-01	.2953910E-01	.3320509E+00	-.2181370E-03
.1583310E+00	.5257354E-01	.3320209E+00	-.6909488E-03
.2477615E+00	.8226237E-01	.3319176E+00	-.1691347E-02
.3574299E+00	.1186498E+00	.3316380E+00	-.3516774E-02
.4875596E+00	.1617681E+00	.3309943E+00	-.6529649E-02
.6384187E+00	.2116109E+00	.3296759E+00	-.1114651E-01
.8103224E+00	.2680876E+00	.3272077E+00	-.1780960E-01
.1003636E+01	.3309542E+00	.3229096E+00	-.2692628E-01
.1218777E+01	.3997140E+00	.3158717E+00	-.3875721E-01
.1456221E+01	.4734899E+00	.3049715E+00	-.5323555E-01
.1716504E+01	.5508795E+00	.2889696E+00	-.6971703E-01
.2000229E+01	.6298269E+00	.2667317E+00	-.8671029E-01
.2308071E+01	.7075738E+00	.2376064E+00	-.1017255E+00
.2640783E+01	.7807826E+00	.2019234E+00	-.1114798E+00
.2999209E+01	.8459167E+00	.1614495E+00	-.1127030E+00
.3384286E+01	.8998962E+00	.1194987E+00	-.1035333E+00
.3797063E+01	.9408823E+00	.8037511E-01	-.8492700E-01
.4238707E+01	.9688658E+00	.4811798E-01	-.6100169E-01
.4710523E+01	.9856925E+00	.2505886E-01	-.3755105E-01
.5213969E+01	.9944021E+00	.1106876E-01	-.1934685E-01
.5750680E+01	.9981862E+00	.4032134E-02	-.8126053E-02
.6322492E+01	.9995284E+00	.1173826E-02	-.2700904E-02
.6931472E+01	.9999051E+00	.2635151E-03	-.6865512E-03
.7579956E+01	.9999858E+00	.4379211E-04	-.1282928E-03
.8270595E+01	.9999985E+00	.5139409E-05	-.1683107E-04
.9006405E+01	.9999999E+00	.4033041E-06	-.1469160E-05
.9790840E+01	.1000000E+01	.1985682E-07	-.8012280E-07
.1062787E+02	.1000000E+01	.5665424E-09	-.2523121E-08
.1152210E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1247889E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1350452E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1460645E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1579358E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1707668E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1846893E+02	.1000000E+01	.0000000E+00	.0000000E+00
.1998668E+02	.1000000E+01	.0000000E+00	.0000000E+00
.2165057E+02	.1000000E+01	.0000000E+00	.0000000E+00
.2348718E+02	.1000000E+01	.0000000E+00	.0000000E+00
.2553151E+02	.1000000E+01	.0000000E+00	.0000000E+00
.2783097E+02	.1000000E+01	.0000000E+00	.0000000E+00
.3045207E+02	.1000000E+01	.0000000E+00	.0000000E+00
.3349219E+02	.1000000E+01	.0000000E+00	.0000000E+00
.3710236E+02	.1000000E+01	.0000000E+00	.0000000E+00
.4153558E+02	.1000000E+01	.0000000E+00	.0000000E+00
.4726618E+02	.1000000E+01	.0000000E+00	.0000000E+00
.5535902E+02	.1000000E+01	.0000000E+00	.0000000E+00
.6921210E+02	.1000000E+01	.0000000E+00	.0000000E+00
.6570835E+03	.1000000E+01	.0000000E+00	.0000000E+00

INDEX OF THE UNCONVERGED EIGENVALUE= 0
IMSL EIGENVALUE SUBROUTINE ERROR NUMBER= 0

INDEX	EIGENVALUES REAL PART	EIGENVALUES IMAGINARY PART
1	.5483498E+15	-.3065511E+14
2	-.5434929E+15	-.4661634E+14
3	-.5849787E+11	.9827289E+11
4	-.4918929E+01	-.1642465E+02
5	.4966318E+01	-.1639263E+02
6	-.8798325E-01	-.4258754E+01
7	.2771130E+00	-.3994019E+01
8	.1361247E+00	-.1875335E+01
9	.2862971E+00	-.1592164E+01
10	.2887507E+00	-.9702207E+00
11	.4100225E+00	-.7881547E+00
12	.2901083E+00	-.2766870E+00
13	.4263477E+00	-.5073704E+00
14	.3641256E+00	.7959628E-02
15	.5236208E+00	-.3776332E+00
16	.4837004E+00	-.1922256E+00
17	.5722661E+00	-.3031564E+00
18	.6726531E+00	-.2586152E+00
19	.7609321E+00	-.2370853E+00
20	.8816834E+00	-.2457643E+00
21	.8449862E+00	-.2181525E+00
22	.9081744E+00	-.1884448E+00
23	.9363043E+00	-.1638718E+00
24	.9557013E+00	-.1381578E+00
25	.9690912E+00	-.1155529E+00
26	.9784337E+00	-.9610866E-01
27	.9848149E+00	-.7975591E-01
28	.9892301E+00	-.6609220E-01
29	.9922813E+00	-.5473425E-01
30	.9944356E+00	-.4528912E-01
31	.9959518E+00	-.3743721E-01
32	.9970472E+00	-.3089302E-01
33	.9978315E+00	-.2543808E-01
34	.9984093E+00	-.2088029E-01
35	.9988281E+00	-.1707562E-01
36	.9991410E+00	-.1389604E-01
37	.9993691E+00	-.1124572E-01
38	.9995412E+00	-.9038025E-02
39	.9996666E+00	-.7207606E-02
40	.9997615E+00	-.5694493E-02
41	.9998301E+00	-.4452860E-02
42	.9998820E+00	-.3440249E-02
43	.9999188E+00	-.2623668E-02
44	.9999463E+00	-.1972373E-02
45	.9999652E+00	-.1461875E-02
46	.9999788E+00	-.1069209E-02
47	.9999877E+00	-.7753646E-03
48	.9999937E+00	-.5630750E-03
49	.9999994E+00	-.3360325E-03
50	.9999973E+00	-.4193824E-03
51	.1000000E+31	.0000000E+00

INDEX OF THE LEAST DAMPED EIGENVALUE= 14
LEAST DAMPED EIGENVALUE=> CR= .3641256E+00 CI= .7959628E-02

EIGENVECTOR CORRESPONDING TO THE LEAST DAMPED EIGENVALUE

Y-COORDINATE	REAL PART	IMAGINARY PART	PHASE ANGLE
.0000000	.5319612E-12	.2445888E-12	.4309628E+00
.0098712	.8757955E-04	-.8413862E-04	-.7653626E+00
.0395044	.1398679E-02	-.1237157E-02	-.7241957E+00
.0889583	.6994729E-02	-.5425582E-02	-.6597268E+00
.1583310	.2141506E-01	-.1395592E-01	-.5775605E+00
.2477615	.4933132E-01	-.2596601E-01	-.4845121E+00
.3574299	.9379128E-01	-.3830948E-01	-.3877735E+00
.4875596	.1551793E+00	-.4700520E-01	-.2941235E+00
.6384187	.2316440E+00	-.4921888E-01	-.2093628E+00
.8103224	.3204536E+00	-.4451928E-01	-.1380423E+00
1.0036358	.4188825E+00	-.3495078E-01	-.8324532E-01
1.2187771	.5237591E+00	-.2410007E-01	-.4598122E-01
1.4562212	.6303692E+00	-.1533483E-01	-.2432195E-01
1.7165043	.7325072E+00	-.9909034E-02	-.1352673E-01
2.0002291	.8241697E+00	-.6720111E-02	-.8153615E-02
2.3080706	.9006787E+00	-.4265912E-02	-.4736295E-02
2.6407832	.9579111E+00	-.2188051E-02	-.2284186E-02
2.9992088	.9918822E+00	-.7423556E-03	-.7484311E-03
3.3842865	.1000000E+01	.0000000E+00	.0000000E+00
3.7970632	.9824159E+00	.1766201E-03	.1797814E-03
4.2387071	.9424044E+00	.9336198E-05	.9906785E-05
4.7105227	.8856714E+00	-.2698427E-03	-.3046758E-03
5.2139687	.8188041E+00	-.4959850E-03	-.6057431E-03
5.7506798	.7476002E+00	-.6074399E-03	-.8125195E-03
6.3224917	.6761006E+00	-.6216548E-03	-.9194706E-03
6.9314718	.6065899E+00	-.5829454E-03	-.9610203E-03
7.5799564	.5401713E+00	-.5255261E-03	-.9728876E-03
8.2705948	.4773642E+00	-.4655761E-03	-.9753057E-03
9.0064048	.4184559E+00	-.4082624E-03	-.9756397E-03
9.7908396	.3636377E+00	-.3547914E-03	-.9756725E-03
10.6278723	.3130394E+00	-.3054238E-03	-.9756717E-03
11.5221016	.2667367E+00	-.2602483E-03	-.9756744E-03
12.4788863	.2247520E+00	-.2192842E-03	-.9756715E-03
13.5045180	.1870563E+00	-.1825061E-03	-.9756747E-03
14.6064469	.1535712E+00	-.1498350E-03	-.9756711E-03
15.7935801	.1241720E+00	-.1211515E-03	-.9756753E-03
17.0766832	.9869082E-01	-.9628974E-04	-.9756704E-03
18.4689309	.7692105E-01	-.7505007E-04	-.9756763E-03
19.9866788	.5862157E-01	-.5719527E-04	-.9756690E-03
21.6505713	.4352197E-01	-.4246346E-04	-.9756785E-03
23.4871801	.3132802E-01	-.3056570E-04	-.9756661E-03
25.5315075	.2172749E-01	-.2119917E-04	-.9756838E-03
27.8309718	.1439634E-01	-.1404593E-04	-.9756592E-03

OUTPUT FILE 2 : ORVEL

REAL AND IMAGINARY PARTS OF THE V PERTURBATION VELOCITY

Y-COORDINATE	REAL PART	IMAGINARY PART
.0000000	.5319612E-12	.2445888E-12
.0098712	.8757955E-04	-.8413862E-04
.0395044	.1398679E-02	-.1237157E-02
.0889583	.6994729E-02	-.5425582E-02
.1583310	.2141506E-01	-.1395592E-01
.2477615	.4933132E-01	-.2596601E-01
.3574299	.9379128E-01	-.3830948E-01
.4875596	.1551793E+00	-.4700520E-01
.6384187	.2316440E+00	-.4921888E-01
.8103224	.3204536E+00	-.4451928E-01
1.0036358	.4188825E+00	-.3495078E-01
1.2187771	.5237591E+00	-.2410007E-01
1.4562212	.6303692E+00	-.1533483E-01
1.7165043	.7325072E+00	-.9909034E-02
2.0002291	.8241697E+00	-.6720111E-02
2.3080706	.9006787E+00	-.4265912E-02
2.6407832	.9579111E+00	-.2188051E-02
2.9992088	.9918822E+00	-.7423556E-03
3.3842865	.1000000E+01	.0000000E+00
3.7970632	.9824159E+00	.1766201E-03
4.2387071	.9424044E+00	.9336198E-05
4.7105227	.8856714E+00	-.2698427E-03
5.2139687	.8188041E+00	-.4959850E-03
5.7506798	.7476002E+00	-.6074399E-03
6.3224917	.6761006E+00	-.6216548E-03
6.9314718	.6065899E+00	-.5829454E-03
7.5799564	.5401713E+00	-.5255261E-03
8.2705948	.4773642E+00	-.4655761E-03
9.0064048	.4184559E+00	-.4082624E-03
9.7908396	.3636377E+00	-.3547914E-03
10.6278723	.3130394E+00	-.3054238E-03
11.5221016	.2667367E+00	-.2602483E-03
12.4788863	.2247520E+00	-.2192842E-03
13.5045180	.1870563E+00	-.1825061E-03
14.6064469	.1535712E+00	-.1498350E-03
15.7935801	.1241720E+00	-.1211515E-03
17.0766832	.9869082E-01	-.9628974E-04
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19.9866788	.5862157E-01	-.5719527E-04
21.6505713	.4352197E-01	-.4246346E-04
23.4871801	.3132802E-01	-.3056570E-04
25.5315075	.2172749E-01	-.2119917E-04
27.8309718	.1439634E-01	-.1404593E-04

REAL AND IMAGINARY PARTS OF THE U PERTURBATION VELOCITY

Y-COORDINATE	REAL PART	IMAGINARY PART
.0000000	-.4457558E-11	.1186797E-10
.0098712	.1774305E-01	-.1680632E-01
.0395044	.7057111E-01	-.5904035E-01
.0889583	.1546010E+00	-.1058891E+00
.1583310	.2578828E+00	-.1339250E+00
.2477615	.3606521E+00	-.1290081E+00
.3574299	.4428962E+00	-.9301408E-01
.4875596	.4938284E+00	-.4093156E-01
.6384187	.5151144E+00	.8637991E-02
.8103224	.5153488E+00	.4190362E-01
1.0036358	.5009930E+00	.5319352E-01
1.2187771	.4716432E+00	.4528360E-01
1.4562212	.4239305E+00	.2842367E-01
1.7165043	.3596600E+00	.1473278E-01
2.0002291	.2867306E+00	.8976250E-02
2.3080706	.2110487E+00	.7177120E-02
2.6407832	.1337708E+00	.5209903E-02
2.9992088	.5726361E-01	.2913159E-02
3.3842865	-.1259904E-01	.1071195E-02
3.7970632	-.6928948E-01	-.8397848E-04
4.2387071	-.1083399E+00	-.5664153E-03
4.7105227	-.1290199E+00	-.5573700E-03
5.2139687	-.1344207E+00	-.3287186E-03
5.7506798	-.1297348E+00	-.1009947E-03
6.3224917	-.1199338E+00	.3301689E-04
6.9314718	-.1083584E+00	.8300407E-04
7.5799564	-.9665767E-01	.8989015E-04
8.2705948	-.8544481E-01	.8281105E-04
9.0064048	-.7490335E-01	.7300897E-04
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15.7935801	-.2222680E-01	.2169790E-04
17.0766832	-.1766564E-01	.1722526E-04
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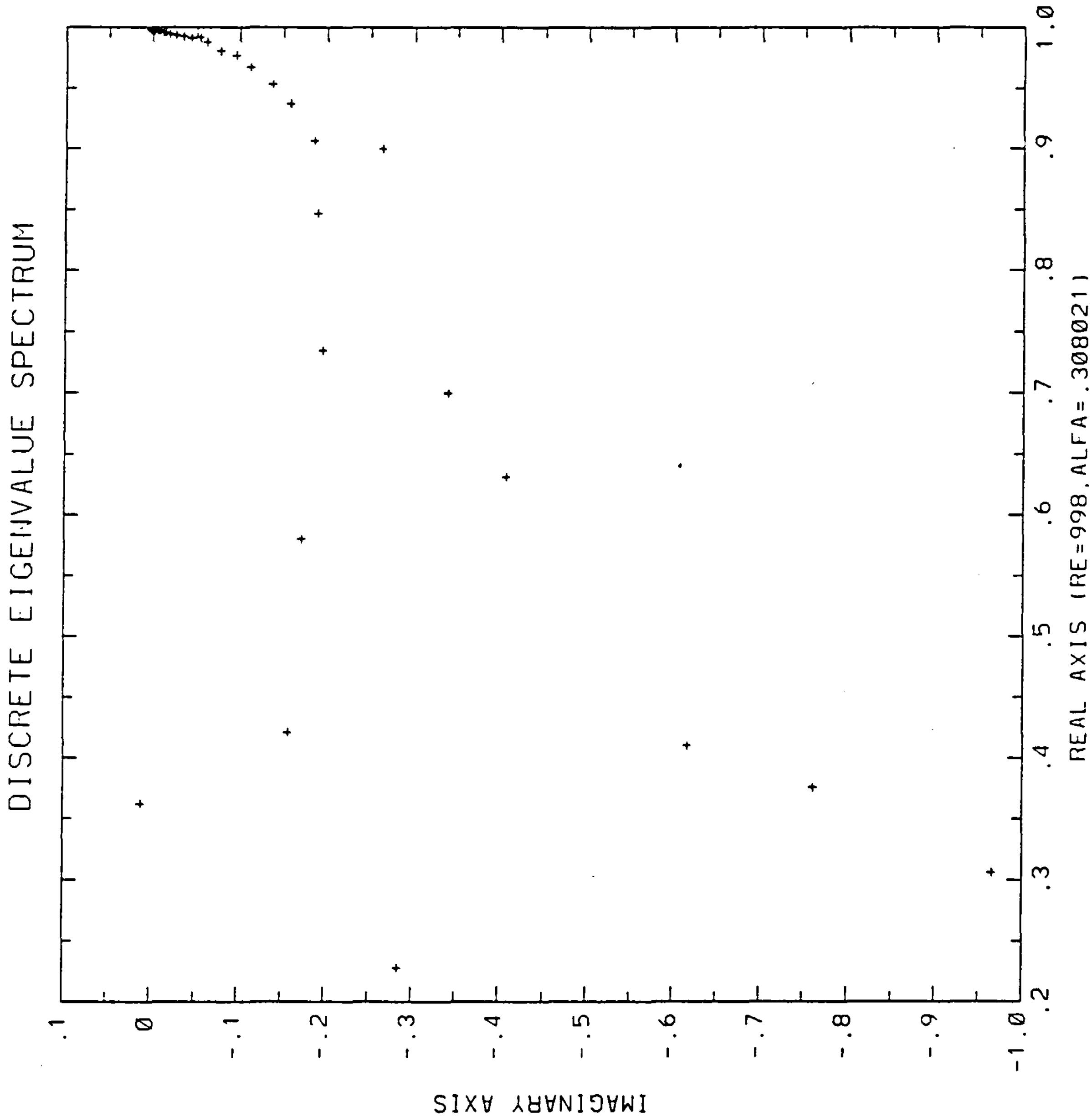
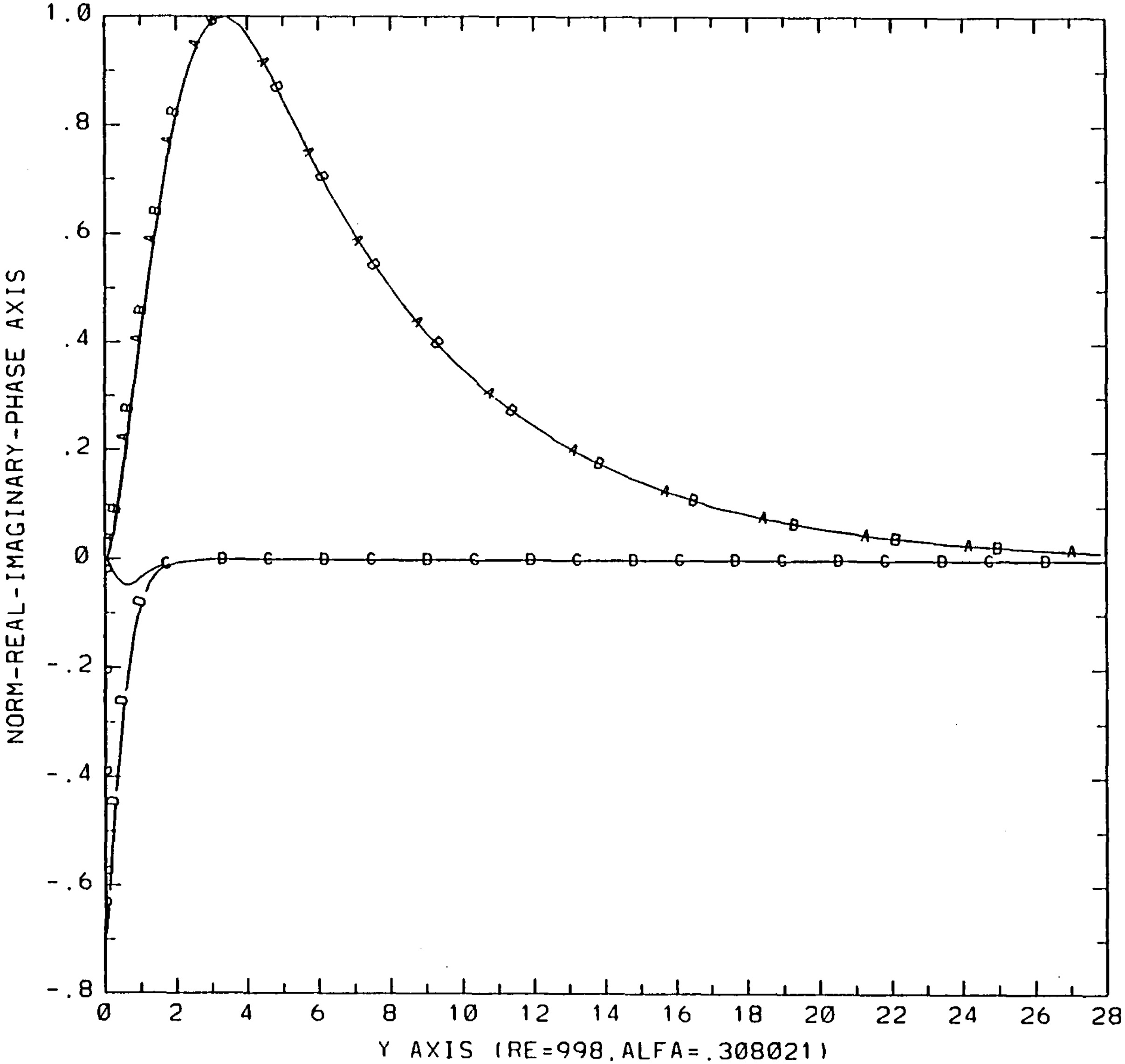
19.9866788	-.1049325E-01	.1022945E-04
21.6505713	-.7790448E-02	.7608518E-05
23.4871801	-.5607701E-02	.5464571E-05
25.5315075	-.3889235E-02	.3800519E-05
27.8309718	-.2576930E-02	.2509097E-05


DISCRETE EIGENVALUE SPECTRUM
REAL AXIS (RE=998, ALFA=.308021)
IMAGINARY AXIS

THE LEAST DAMPED EIGENVECTOR

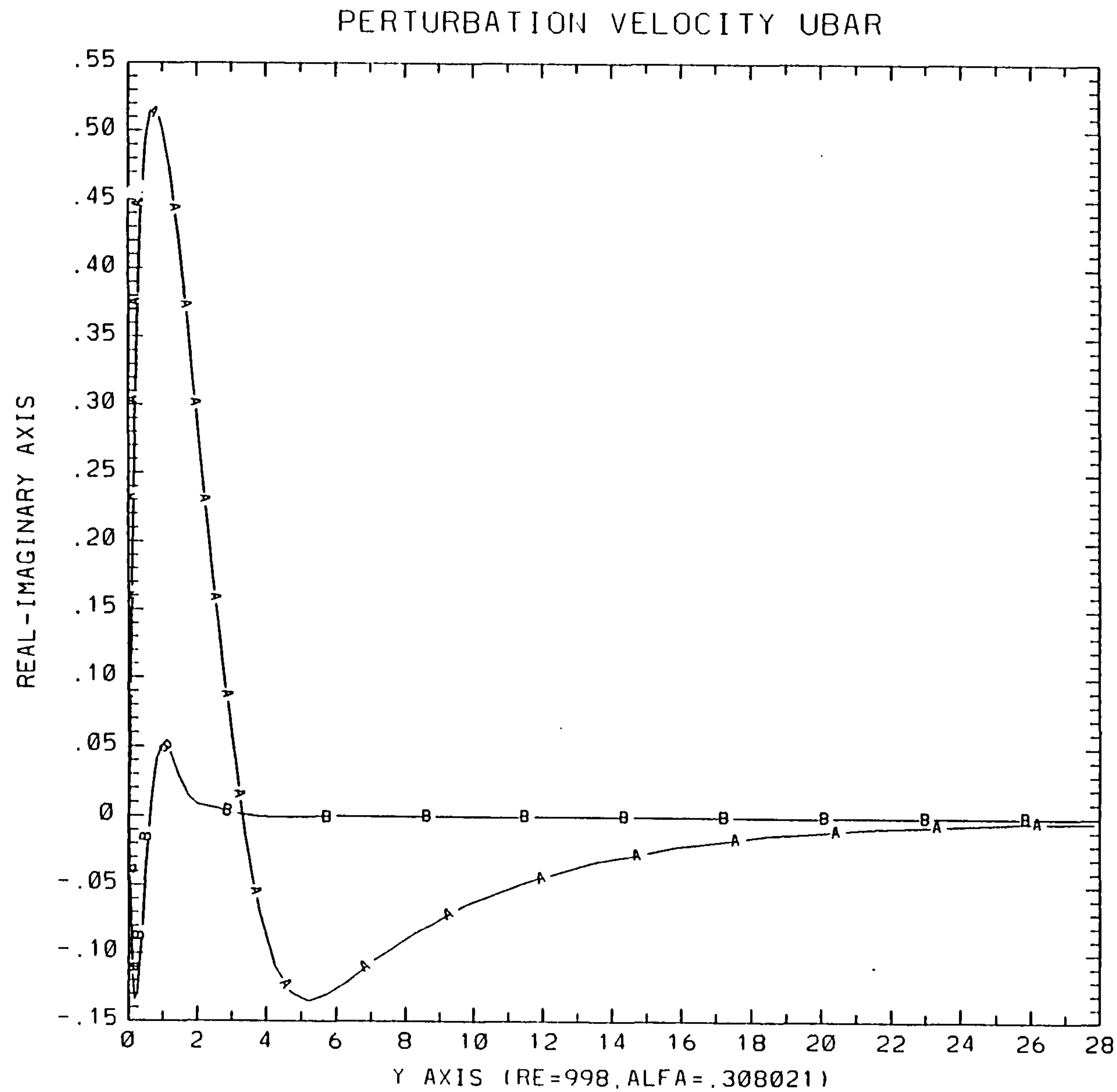


A-NORM OF THE EIGENVECTOR

B-REAL PART OF THE EIGENVECTOR

C-IMAGINARY PART OF THE EIGENVECTOR

D-PHASE ANGLE OF THE EIGENVECTOR



A-REAL PART OF THE PERTURBATION VELOCITY

B-IMAGINARY PART OF THE PERTURBATION VELOCITY

Report Documentation Page

NASA — National Aeronautics and Space Administration

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-4169		

4. Title and Subtitle	5. Report Date
Solution of the Orr-Sommerfeld Equation for the Blausius Boundary-Layer Documentation of Program ORRBL and a Test Case	August 1988
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
S. Biringen and G. Danabasoglu	
	10. Work Unit No. 505-60-21-01
9. Performing Organization Name and Address University of Colorado Department of Aerospace Engineering Sciences Box 429 Boulder, CO 80309-0429	11. Contract or Grant No. NAG1-798
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225	13. Type of Report and Period Covered Contractor Report
	14. Sponsoring Agency Code

15. Supplementary Notes

Langley Research Center Technical Monitor: W. Don Harvey

16. Abstract

In this note, a Chebyshev matrix collocation method is outlined for the solution of the Orr-Sommerfeld equation for the Blausius boundary layer. User information is provided for FORTRAN program ORRBL which solves the equation by the QR method.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Orr-Sommerfeld Blausius Spectral Solution	Unclassified - Unlimited Subject Category 61

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	20	A02

NASA FORM 1626 OCT 86 NASA-Langley, 1988

For sale by the National Technical Information Service, Springfield, Virginia 22161-2171

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