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A SIMPLE, ANALYTIC 3-DIMENSIONAL DOWNBURST  
MODEL BASED ON BOUNDARY LAYER STAGNATION  
FLOW

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BASED ON BOUNDARY LAYER STAGNATION FLOW

Rosa M. Oseguera and Roland L. Bowles  
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Page 5, first paragraph under "DISCUSSION AND RESULTS": The sentence that begins with "Horizontal wind and vertical wind profiles..." should read "Horizontal wind and vertical wind profiles in figure 7 were taken at  $h = z_m$  (maximum outflow),  $h = z^*$  (half-maximum outflow), and  $h = z_h$  (depth of outflow)." The following sentence should be deleted.

Page 7, line 3: The second equation in the left column should be

$$e_e = e^{-(h/E)}$$

Page 15, figure 7: The curve labeled " $w_x, h = z_h$ " should be " $w_x, h = z^*$ ", and the curve labeled " $w_h, h = z_m$ " should be " $w_h, h = z^*$ "

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## SUMMARY

A simple downburst model has been developed for use in batch and real-time piloted simulation studies of guidance strategies for terminal area transport aircraft operations in wind shear conditions. The model represents an axisymmetric stagnation point flow, based on velocity profiles from the Terminal Area Simulation System (TASS) model developed by Proctor [ref. 3,4] and satisfies the mass continuity equation in cylindrical coordinates. Altitude dependence, including boundary layer effects near the ground, closely matches real-world measurements, as do the increase, peak, and decay of outflow and downflow with increasing distance from the downburst center. Equations for horizontal and vertical winds were derived, and found to be infinitely differentiable, with no singular points existent in the flow field. In addition, a simple relationship exists among the ratio of maximum horizontal to vertical velocities, the downdraft radius, depth of outflow, and altitude of maximum outflow. In use, a microburst can be modeled by specifying four characteristic parameters. Velocity components in the x, y, and z directions, and the corresponding nine partial derivatives are obtained easily from the velocity equations.

## INTRODUCTION

Terminal area operation of transport aircraft in a windshear environment has been recognized as a serious problem. Studies of aircraft trajectories through downbursts show that specific guidance strategies are needed for aircraft to survive inadvertent downburst encounters. In order for guidance strategies to perform in simulations as in actual encounters, a realistic set of conditions must be present during development of the strategies. Thus, airplane and wind models that closely simulate real-world conditions are essential in obtaining useful information from the studies.

Wind models for use on personal computers, or for simulators with limited memory space availability, have been difficult to obtain because variability of downburst characteristics makes analytical models unrealistic, and large memory requirements make use of numerical models impossible on any except very large capacity computers.

Bray [ref. 1] developed a method for analytic modeling of windshear conditions in flight simulators, and applied his method in modeling a multiple downburst scenario from Joint Airport Weather Studies (JAWS) data. However, the altitude dependence of his model is not consistent with observed data, and, although flexibility in sizing the downbursts is built into the model, it does not maintain the physical relationships which are seen in real-world data among the sizing parameters. In particular, boundary layer effects should cause radial velocity to decay vertically to zero at the ground, as does the vertical velocity.

In a study conducted at NASA Langley Research Center, three different guidance strategies for a Boeing 737-100 airplane encountering a microburst on takeoff were developed [ref. 2]. These strategies were first developed using a personal computer, and then implemented in a pilot-in-the-loop simulation using a very simple wind model in both efforts [fig. 1]. This model consisted of a constant outflow outside of the downburst radius and a constant slope headwind to tailwind shear across the diameter of the downburst. It was recognized that a more realistic wind model could significantly alter the outcome of the trajectory. For the subsequent part

of this study, which involves altering the airplane model to simulate approach to landing and escape maneuvers and additional takeoff cases, a more realistic wind model was preferred. The simple analytical model outlined in this report was developed for this purpose.

#### SYMBOLS

JAWS	Joint Airport Weather Studies
NIMROD	Northern Illinois Meteorological Research On Downbursts
R	radius of downburst shaft (ft)
r	radial coordinate (distance from downburst center) (ft)
TASS	Terminal Area Simulation System
u	velocity in r-direction (or x-direction) (kts)
v	velocity in y-direction (kts)
w	velocity in z-direction (kts)
$w_{max}$	magnitude of maximum vertical velocity (kts)
$u_{max}$	magnitude of maximum horizontal velocity (kts)
x	horizontal (runway) distance, airplane to downburst center (ft)
y	horizontal (side) distance, airplane to downburst center (ft)
z	airplane altitude above ground level (ft)
$z_h$	depth of outflow (ft)
$z_m$	height of maximum U-velocity (ft)
$z_{m2}$	height of half maximum U-velocity (ft)
$z^*$	characteristic height, out of boundary layer (ft)
$\epsilon$	characteristic height, in boundary layer (ft)
$\lambda$	scaling factor ( $s^{-1}$ )

#### DEVELOPMENT OF VELOCITY EQUATIONS

Beginning with the full set of Euler and mass continuity equations, some simplifying assumptions about the downburst flow conditions were made. Effects of viscosity were parameterized explicitly, and the flow was assumed to be invariant with time. The downburst is axisymmetric in cylindrical coordinates, and characterized by a stagnation point at the ground along the axis of the downflow column. The flow is incompressible, with no external forces or moments acting on it.

The resulting mass conservation equation is

$$\nabla \cdot \mathbf{v} = 0. \quad (1)$$

Written out in full, equation 2 is

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0. \quad (2)$$

This equation is satisfied by solutions of the form

$$w = g(r^2)q(z) \quad (3a)$$

$$u = \frac{f(r^2)}{r}p(z) \quad (3b)$$

provided that

$$f'(r^2) = \frac{\lambda}{2}g(r^2) \quad (4a)$$

$$q'(z) = -\lambda p(z). \quad (4b)$$

Note that  $f'(r^2) = \frac{\partial f(r^2)}{\partial r^2}$ . To solve this system of equations, solutions were assumed for two of the functions and the other two were obtained from equations 4a and 4b.

It was desired that the velocity profiles of this analytic model exhibit the altitude and radial dependence shown in the large-scale numerical weather model TASS (Terminal Area Simulation System) [ref. 3,4]. The TASS model is based on data from the Joint Airport Weather Studies (JAWS) [ref. 5], and provides a three-dimensional velocity field, frozen in time, for given locations of an airplane within the shear [ref. 6]. Figure 2 shows dimensionless vertical profiles of horizontal velocity,  $u$ , for TASS data, laboratory data obtained by impingement of a jet on a flat plate, and data from NIMROD (Northern Illinois Meteorological Research On Downbursts) [ref. 7]. Specific points of interest are the maximum horizontal velocity (located 100 - 200 meters above the ground), below which is a decay region due to boundary layer effects, zero velocity at the stagnation point on the ground, and an exponential decay with altitude above the maximum velocity altitude. Vertical velocity profiles from TASS data are shown in figure 3, also exhibiting a decay to zero at the stagnation point.

The radially varying characteristics desired for the horizontal wind were two peaks of equal magnitude and opposite direction located at a given radius, with a smooth, nearly linear transition between the two. Beyond the peaks, the velocity should show an exponential decay to zero. The vertical velocity was required to have a peak along the axis of symmetry ( $r=0$ ), and decay exponentially at increasing radius.

A pair of shaping functions that gave velocity profiles matching TASS

data as required are given below.

$$g(r^2) = e^{-(r/R)^2}$$

$$p(z) = e^{-z/z^*} - e^{-z/\epsilon}$$

The remaining solutions were found by integrating equations 4a and 4b, yielding:

$$f(r^2) = \frac{\lambda R^2}{2} [1 - e^{-(r/R)^2}]$$

$$q(z) = -\lambda \{ \epsilon (e^{-z/\epsilon} - 1) - z^* (e^{-z/z^*} - 1) \}$$

Figures 4 and 5 show plots of these shaping functions.

Combining the functions as in equation 3, the horizontal and vertical velocities are expressed as

$$u = \frac{\lambda R^2}{2r} [1 - e^{-(r/R)^2}] (e^{-z/z^*} - e^{-z/\epsilon}) \quad (5)$$

$$w = -\lambda e^{-(r/R)^2} [\epsilon (e^{-z/\epsilon} - 1) - z^* (e^{-z/z^*} - 1)] \quad (6)$$

By taking derivatives of equations 5 and 6 with respect to  $r$  and  $z$ , respectively, and substituting in equation 2, it can be shown that the velocity distributions satisfy continuity.

The parameters  $z^*$  and  $\epsilon$  were defined as characteristic scale lengths associated with "out of boundary layer" and "in boundary layer" behavior, respectively. Analysis of TASS data indicated that  $z^* = z_{m2}$ , the altitude at which the magnitude of the horizontal velocity is half the maximum value.

It was also noted that the ratio

$$\frac{z_m}{z^*} = 0.22$$

To determine the location of the maximum horizontal velocity, the partial derivatives of  $u$  with respect to  $r$  and  $z$  were set equal to zero. The resulting equation for the  $r$ -derivative is

$$2 \left( \frac{r}{R} \right)^2 = e^{-(r/R)^2} - 1$$

The resulting equation for the  $z$ -derivative is

$$\frac{z_m}{z^*} = \frac{1}{(z^*/\epsilon) - 1} \ln(z^*/\epsilon)$$

Recalling that  $z_m/z^* = 0.22$ , the values 1.1212 and 12.5 were obtained from iteration for the ratios  $r/R$  and  $z^*/\epsilon$ , respectively.

Using these values, the maximum horizontal velocity can be expressed as  $u_{\max} = 0.2357 \lambda R$ . The maximum vertical wind is located at  $r = 0$  and  $z = z_h$ , by definition, and is given by  $w_{\max} = \lambda z^* (e^{-(z_h/z^*)} - 0.92)$ .

A ratio of maximum outflow and downflow velocities can be formed

$$\frac{u_m}{w_m} = \frac{0.2357R}{z^* (e^{-(z_h/z^*)} - 0.92)}$$

The scaling factor,  $\lambda$ , was determined by using either of equations 5 or 6 for horizontal or vertical velocity, and setting it equal to the maximum velocity,  $u_{\max}$  or  $w_{\max}$ , respectively. Solving for  $\lambda$  resulted in:

$$\lambda = \frac{w_m}{z^* (e^{-(z_h/z^*)} - 0.92)} = \frac{u_m}{0.2357R}$$

The velocity equations were easily converted to rectangular coordinates, as shown in the Appendix. Partial derivatives with respect to  $x$ ,  $y$ , and  $z$  were obtained by differentiating the velocity equations, and are also listed in the Appendix.

#### DISCUSSION AND RESULTS

Vertical and horizontal velocity profiles for  $u$  and  $w$  are shown in figures 6 and 7. Four profiles are shown for each component. The horizontal wind profiles in figure 6 were taken at the radius of peak outflow ( $r = 1.1212 R$ ) and at about one-fourth that radius ( $r = 0.3 R$ ), where the maximum outflow is approximately half the value at the peak outflow radius. The vertical wind profiles were taken at the radius of peak downflow ( $r = 0$ ) and at  $r = 0.3 R$ . Horizontal wind and vertical wind profiles in figure 7 were each taken at two altitudes,  $h = z_m$  (altitude of maximum outflow) and  $h = z_h$  (depth of outflow). The outflow velocity is reduced by half (from 37 kt to 18 kt), while the downflow velocity is increased by about 38 percent (from 21 kt to 29 kt) in the same distance.

This analytical model is compared with TASS, laboratory, and NIMROD data in figure 8. The figure shows that, when nondimensionalized by the altitude of half-maximum outflow ( $z^*$ ) and by the maximum outflow ( $u = u_{\max}$ ), the analytical model agrees closely with the other data.

Different shears can be modeled by specifying four parameters, and the location of downburst center relative to the airplane flying through it. The four parameters are: 1) a characteristic horizontal dimension; 2) maximum wind velocity; 3) altitude of maximum outflow; and 4) depth of outflow. The characteristic horizontal dimension specified is the radius of the downdraft column, noting that this is about 89 percent of the radius of peak outflow. The maximum wind velocity can be either horizontal or vertical.

## CONCLUDING REMARKS

The analytic microburst model developed for use in real-time and batch simulation studies was shown to agree well with real-world measurements for the cases studied. The functions chosen for the model showed boundary-layer effects near the ground, as well as the peak and decay of outflow at increasing altitudes, and increasing downflow with altitude. The exponential increase and decay of downflow and outflow (in the radial direction) are also characterized by the model. Equations for horizontal and vertical winds are simple and continuously differentiable, and partial derivatives in rectangular or cylindrical coordinates can be easily obtained by direct differentiation of the velocity equations. The governing equation for this system is the mass conservation law, and the analytic velocity functions developed here satisfied this condition. The model is sustained by a strong physical basis and yields high fidelity results, within the limitations of maintaining simplicity in the model, and variability of the microburst phenomenon. Parameterization of some of the characteristic dimensions allows flexibility in selecting the size and intensity of the microburst.

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APPENDIX

Define intermediate variables to simplify written equations:

$$\begin{aligned} e_r &= e^{-(r/R)^2} & e_d &= e_z - e_c \\ e_c &= e^{-(\epsilon/z^*)} & e_c &= z^* (1 - e_z) - \epsilon(1 - e_c) \\ e_z &= e^{-(h/z^*)} \end{aligned}$$

Horizontal and Vertical Velocities

$$\begin{aligned} w_x &= \frac{\lambda R^2}{2r^2} (1 - e_r) e_d x_{ad} \\ w_y &= \frac{\lambda R^2}{2r^2} (1 - e_r) e_d y_{ad} \\ w_h &= -\lambda e_r e_c \end{aligned}$$

Partial Derivatives

$$\begin{aligned} \frac{\partial w_x}{\partial x} &= \frac{\lambda R^2 e_d}{2r^2} \left[ e_r \left( \frac{2x_{ad}^2}{R^2} + \frac{2x_{ad}^2}{r^2} - 1 \right) - \frac{2x_{ad}^2}{r^2} + 1 \right] \\ \frac{\partial w_x}{\partial y} &= \frac{\lambda R^2 x_{ad} y_{ad} e_d}{r^2} \left[ e_r \left( \frac{1}{R} + \frac{1}{r^2} \right) - \frac{1}{r^2} \right] \\ \frac{\partial w_x}{\partial h} &= \frac{\lambda R^2 x_{ad}}{2r^2} (1 - e_r) \left[ \frac{e_e}{\epsilon} - \frac{e_z}{z^*} \right] \\ \frac{\partial w_y}{\partial x} &= \frac{\lambda R^2 x_{ad} y_{ad} e_d}{r^2} \left[ e_r \left( \frac{1}{R} + \frac{1}{r^2} \right) - \frac{1}{r^2} \right] \\ \frac{\partial w_y}{\partial y} &= \frac{\lambda R^2 e_d}{2r^2} \left[ e_r \left( \frac{2y_{ad}^2}{R^2} + \frac{2y_{ad}^2}{r^2} - 1 \right) - \frac{2y_{ad}^2}{r^2} + 1 \right] \\ \frac{\partial w_y}{\partial h} &= \frac{\lambda R^2 y_{ad}}{2r^2} (1 - e_r) \left[ \frac{e_c}{\epsilon} - \frac{e_z}{z^*} \right] \\ \frac{\partial w_h}{\partial x} &= \frac{2\lambda x_{ad} e_r e_c}{R^2} \\ \frac{\partial w_h}{\partial y} &= \frac{2\lambda y_{ad} e_r e_c}{R^2} \\ \frac{\partial w_h}{\partial h} &= -\lambda e_r e_d \end{aligned}$$

### Other Relationships

From TASS

$$\frac{z_m}{z^*} = 0.22$$

$$\frac{z^*}{\epsilon} = 12.5$$

Maximums

$$W_{x_{\max}} = 0.2357\lambda R$$

$$W_{y_{\max}} = W_{x_{\max}}$$

$$W_{h_{\max}} = \lambda z^* e^{-(z_h/z^*)} - 0.92$$

( $\lambda$  is determined from the above relationships)

$$\frac{W_{x_{\max}}}{W_{h_{\max}}} = \frac{0.2357R}{z^* (e^{-z_h/z^*} - 0.92)}$$

### Variable List

$z^*$  = altitude where  $W_x$  is half the value of  $W_{x_{\max}}$  (ft)  
 $\epsilon$  = characteristic height of boundary layer effects (ft)

$z_h$  = depth of outflow (ft)

$z_m$  = altitude of maximum outflow (ft)

$\lambda$  = scaling parameter ( $s^{-1}$ )

$r$  = radial distance from airplane to downburst (ft)

$h$  = altitude of airplane (ft)

$R$  = radius of downdraft (ft)

$x_{ad}, y_{ad}$  = x, y coordinates, airplane to microburst (ft)

$W_{x_{\max}}, W_{y_{\max}}, W_{h_{\max}}$  maximum winds, x, y, and h directions

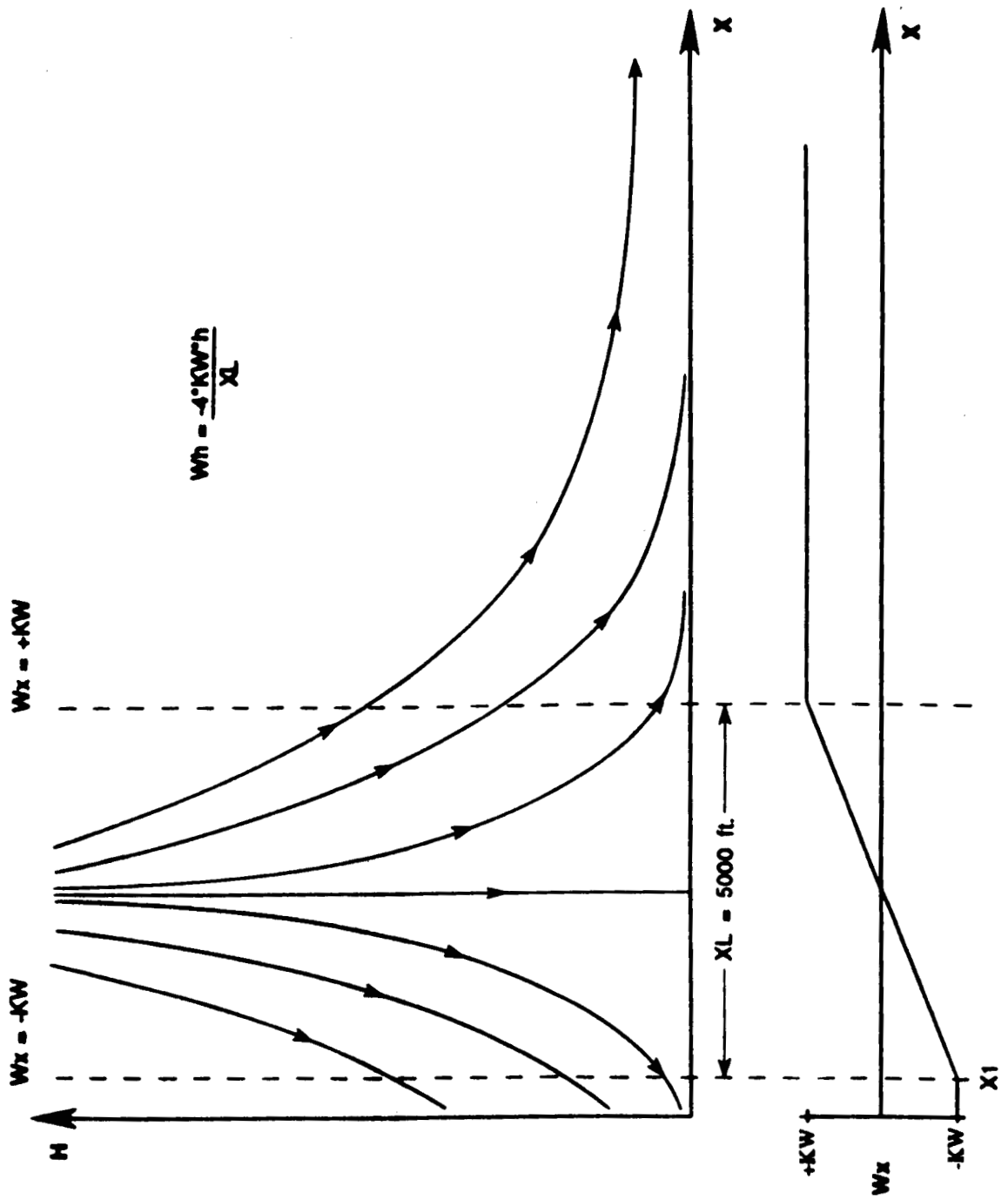


Figure 1 Wind Model Used In Guidance Studies

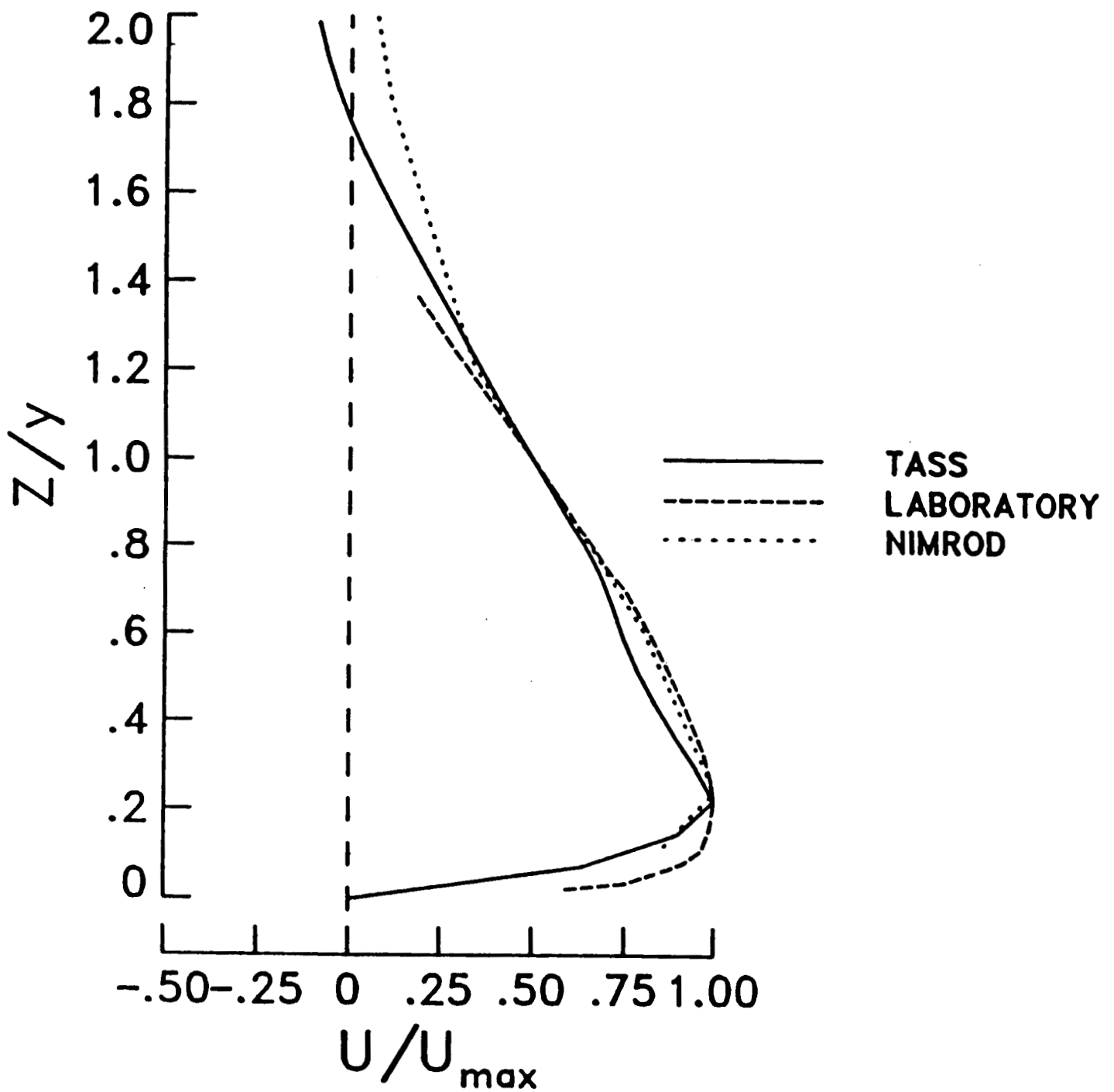


Figure 2 Vertical Profile of Microburst Outflow (Nondimensional)

VERTICAL PROFILES OF VERTICAL VELOCITY  
FOR 30 JUN 82 CASE:  
SENSITIVITY TO RADIUS OF PRECIPITATION SHAFT

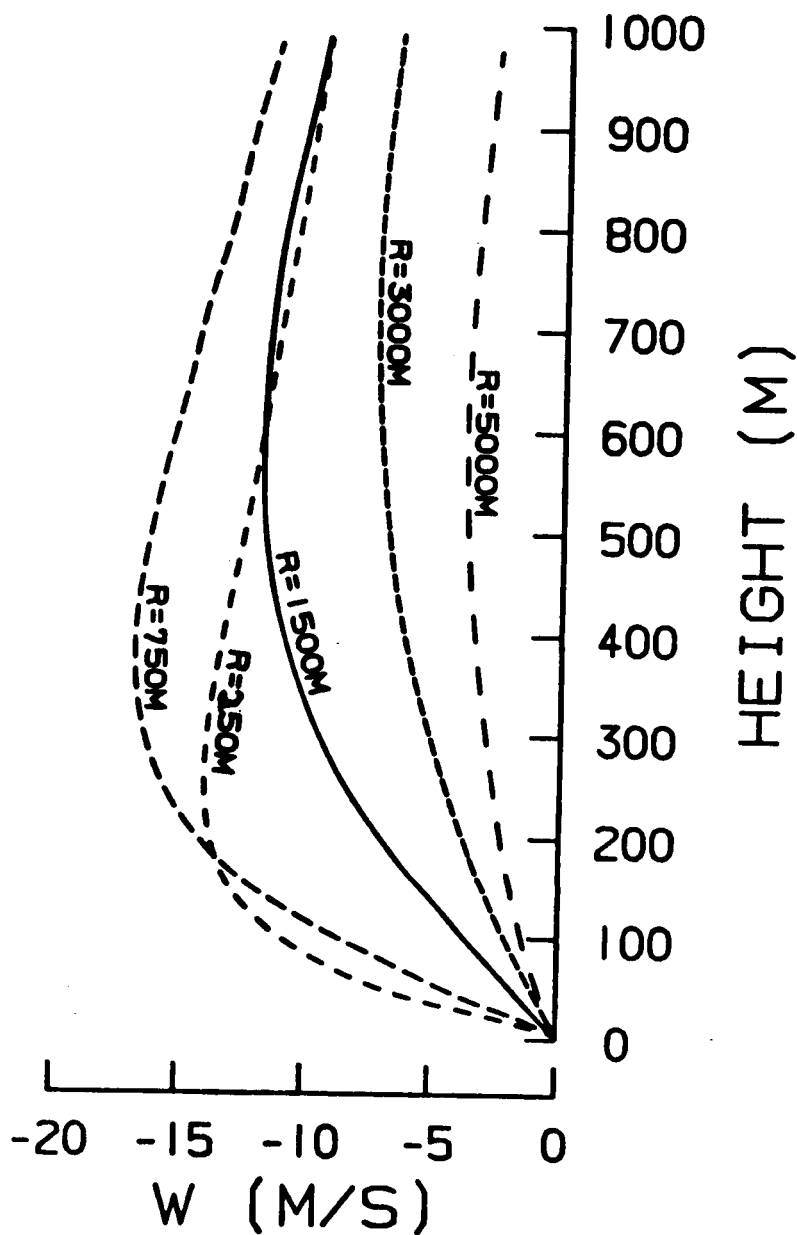


Figure 3 Vertical Profile of Microburst Downflow

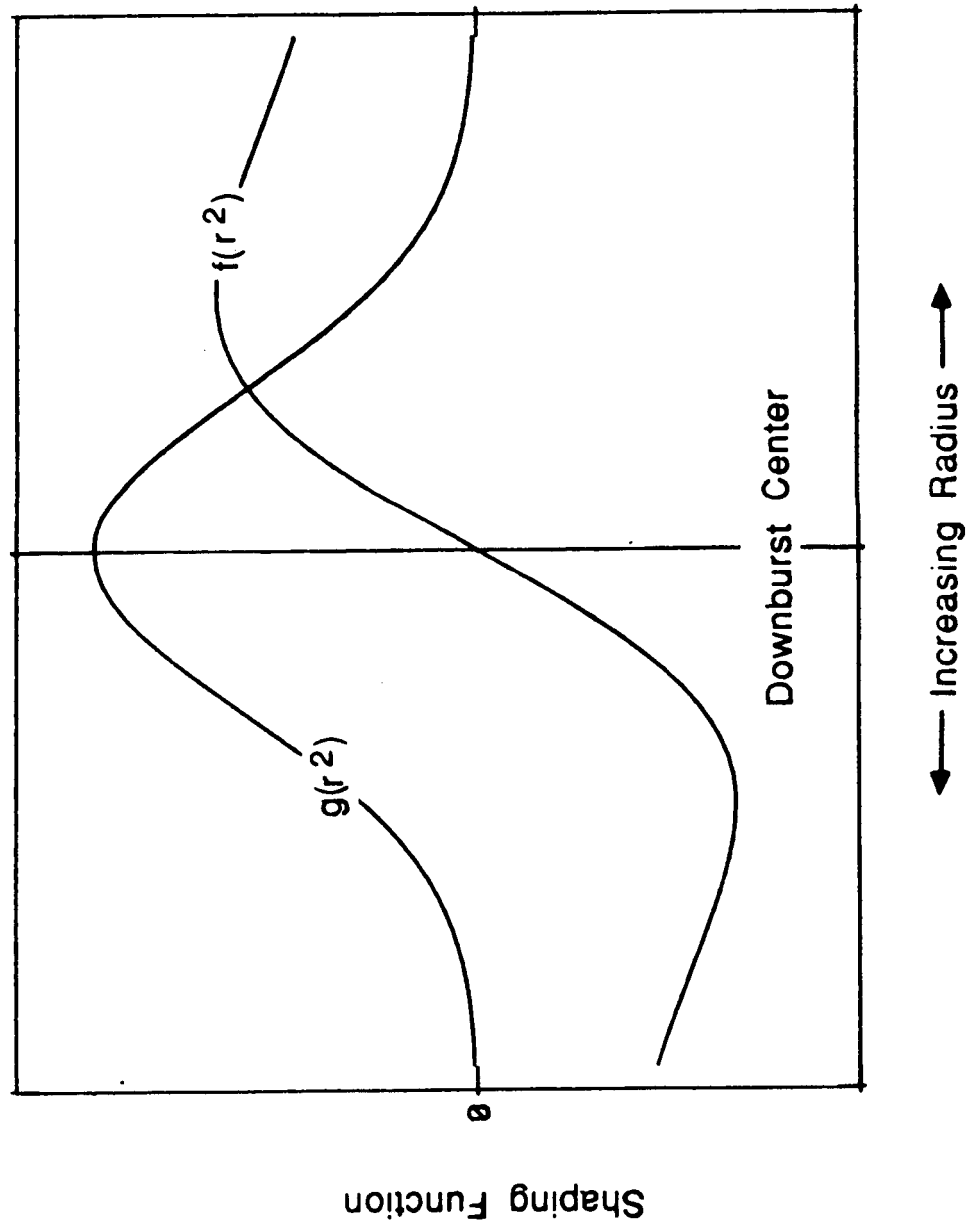


Figure 4 Characteristic Variation of Horizontal Shaping Functions

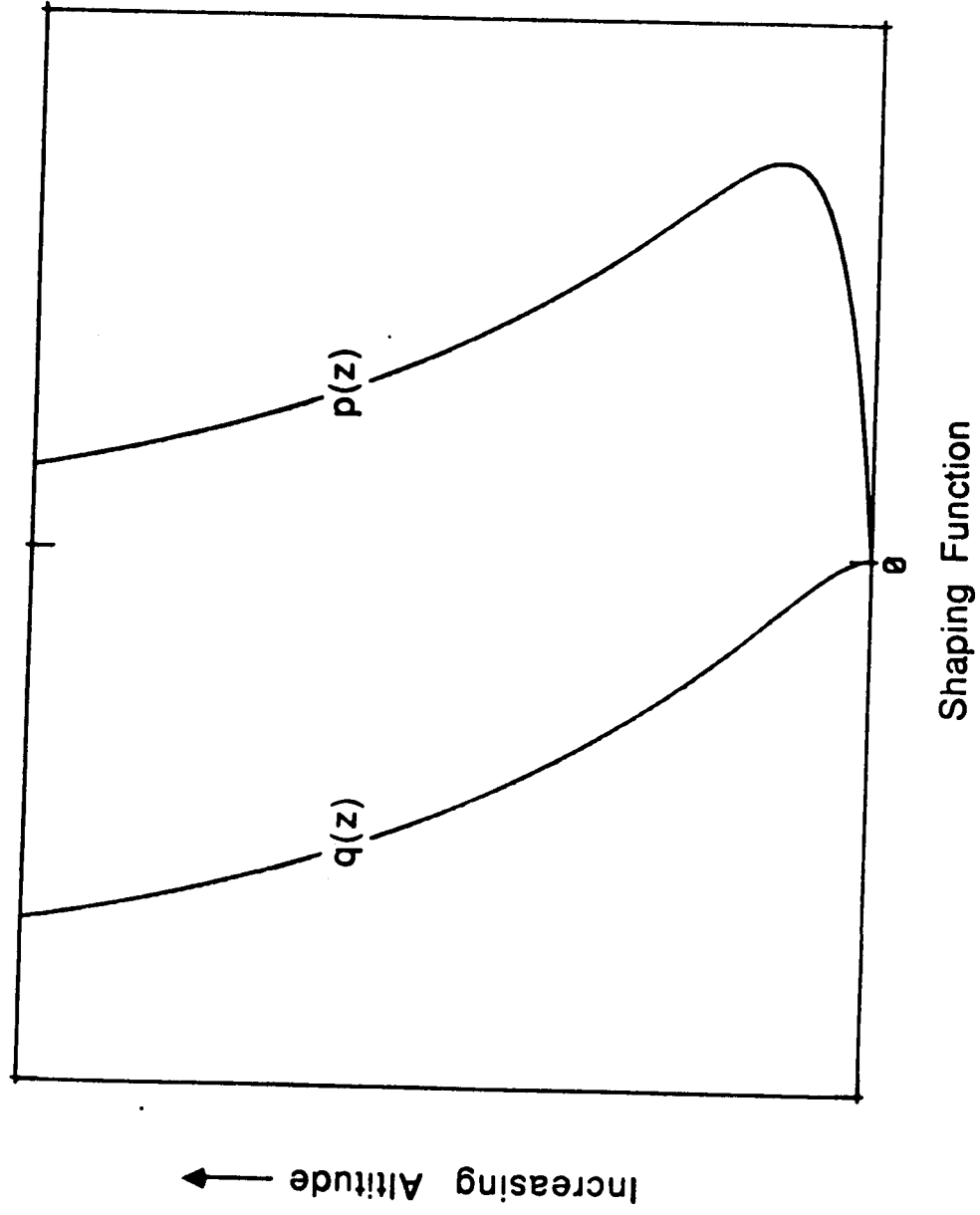


Figure 5 Characteristic Variation of Vertical Shaping Functions

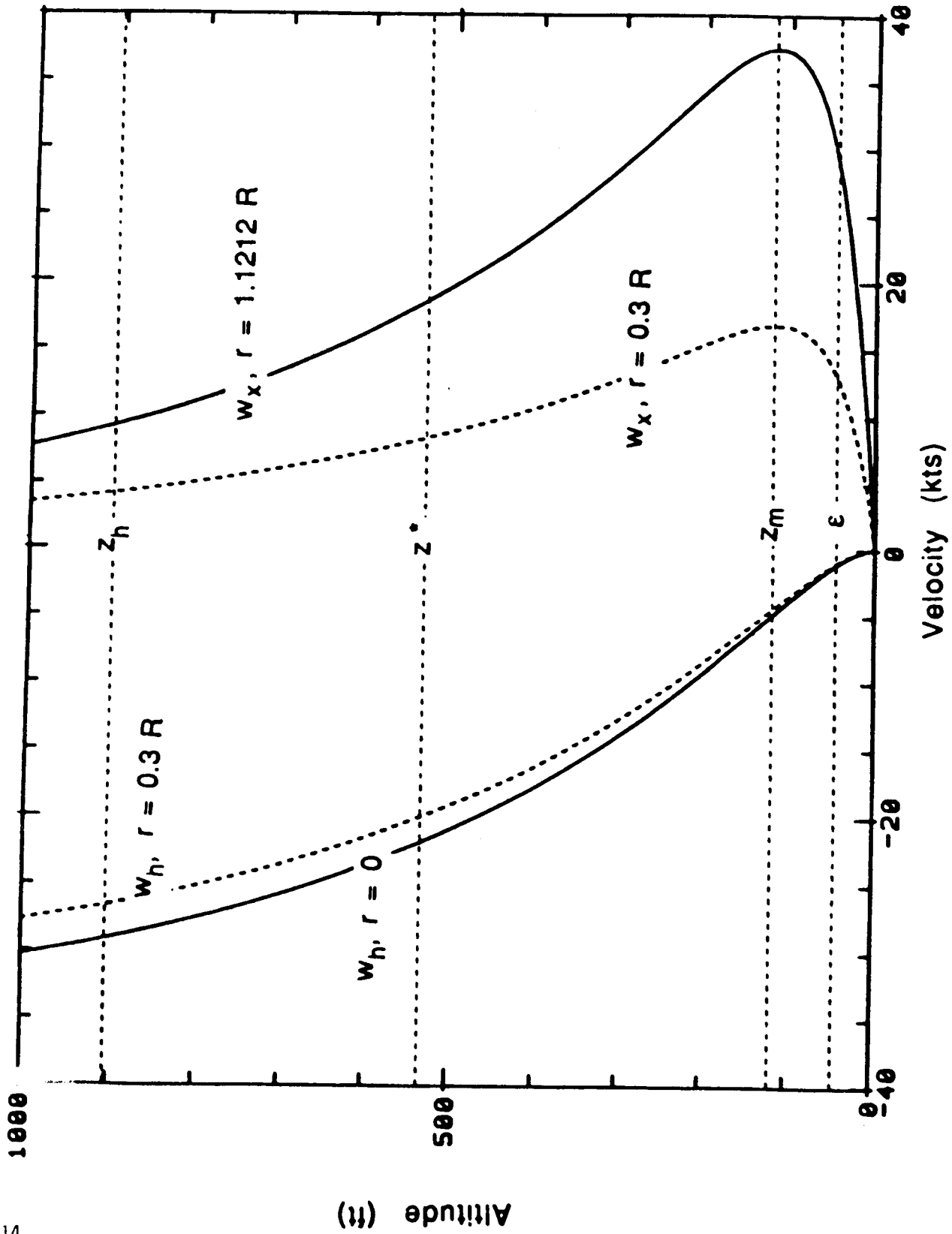


Figure 6 Vertical Velocity Profiles For Analytical Model



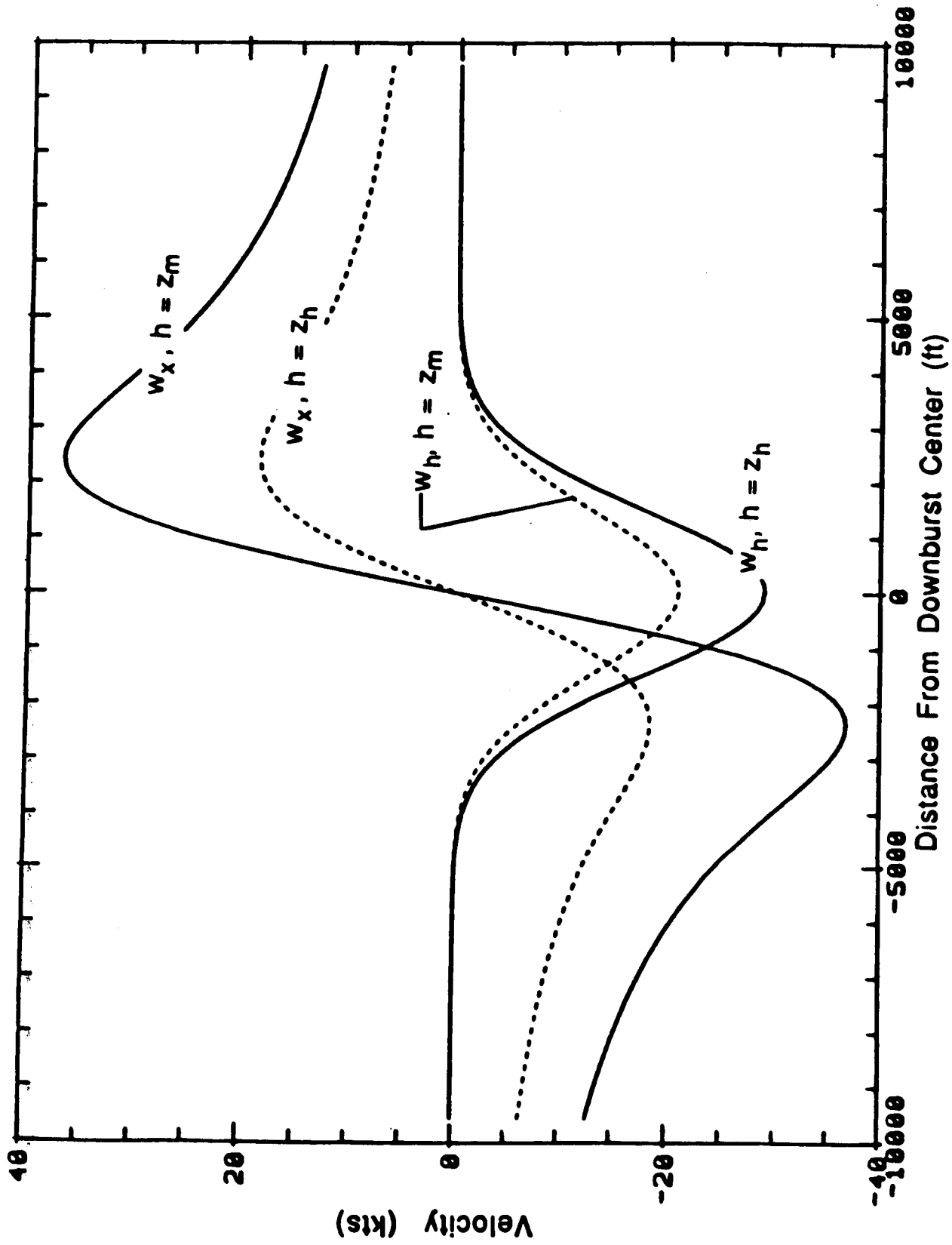


Figure 7 Radial Velocity Profiles For Analytical Model

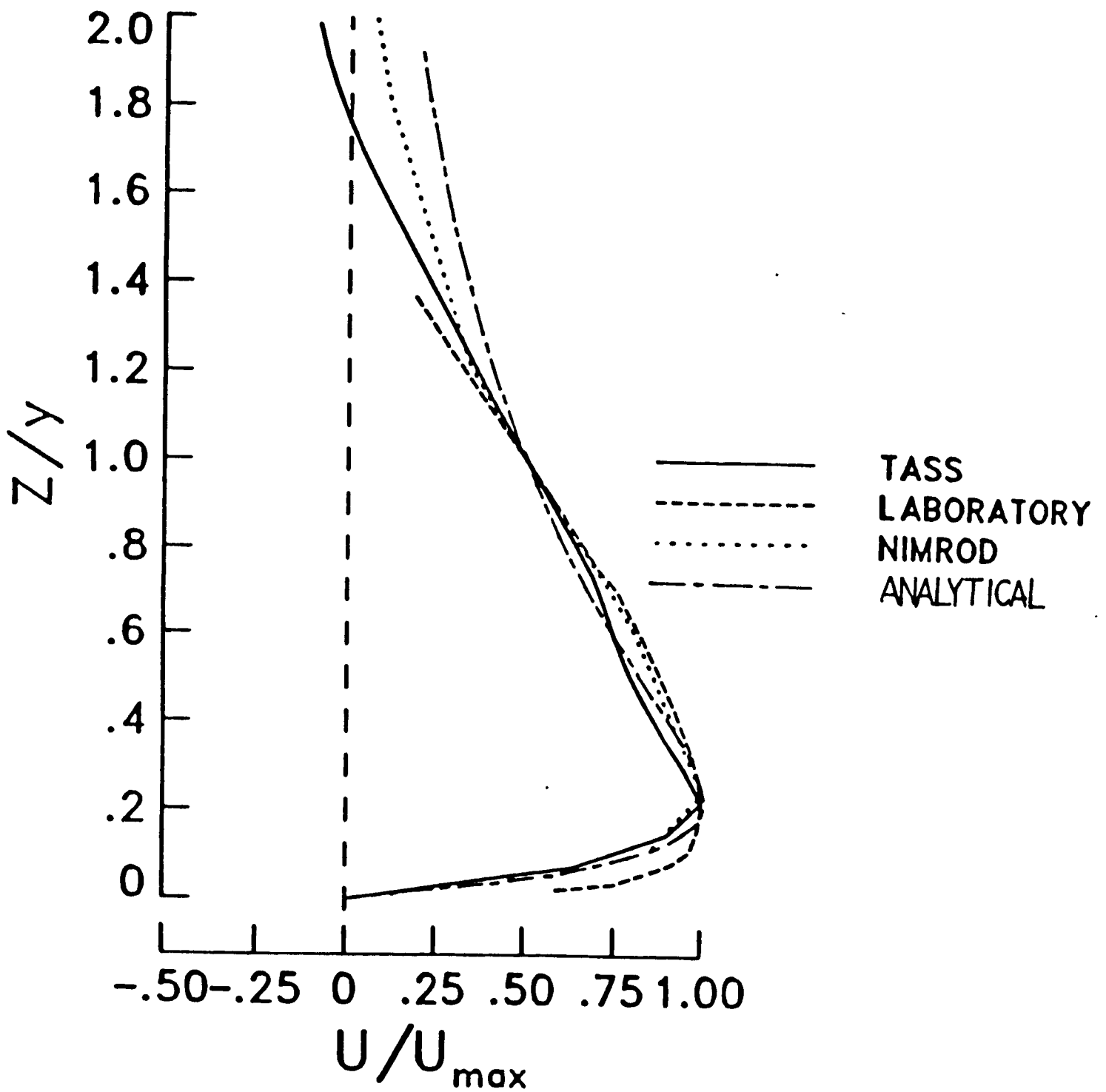


Figure 8 Comparison of Wind Model Vertical Profiles



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