View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by NASA Technical Reports Server

IN-35-CR

159381

12P.

Interferometric tomography of continuous fields with incomplete projections

(NASA-CR-183181) INTERFEROMETRIC TOMOGRAPHY OF CONTINUOUS FIELDS WITH INCOMPLETE PROJECTIONS (Illinois Univ.) 12 p CSCL 14B

N88-28295

Unclas

G3/35 0159381

Soyoung S. Cha and Hogwei Sun

Department of Mechanical Engineering, University of Illinois at Chicago

Chicago, Illinois 60680

Interferometic tomography in the presence of an opaque object has been investigated. The developed iterative algorithm does not need to augment the missing information. It is based on the successive reconstruction of the difference field, the difference between the object field to be reconstructed and its estimate, only in the defined region. The application of the algorithm results in stable convergence.

The interest in tomographic reconstruction arises in various fields including interferometric flow visualization,[1-3] medical imaging,[4] electronmicroscopy,[5] and nondestructive testing.[6] The reconstruction in these fields frequently encounters ill-posed problems of incomplete projection and limited view angle, which pose formidable challenges of restoring distorted images. This is especially true in optical fluid-flow measurements,[7-9] where physical constraints (i.e., test model and test-section enclosure) limit the angular scanning or block part of the probing rays. This paper discusses a new approach for reconstructing a field from incomplete projections as depicted in Fig. 1. Discussion will be developed in terms of interferometric reconstruction of continuous flow fields; however, its extension to analogous fields is self-evident.

In interferometry, the projection data which represent fringe order numbers are the line integrals of a refractive-index field. In the case of a hollow view, as shown in Fig 1, it is feasible to obtain theoretically a unique reconstruction of the field outside the opaque object, where the information is available.[10] Cha[2] and Vest and Prikryl,[7] working on this principle, reconstructed incomplete projections by fitting only the available data with a truncated series. In practice, however, series expansion methods pose a stability problem under severe ill-posed conditions. Past practices are mostly to interpolate missing information directly or iteratively with or without incorporating a priori information.

Zien et al.[9] assigned fictitious values to the missing zone based on the zeroth-moment invariant principle of the line integral transform (Radon transform).[3] Lewitt and Bates[11] demonstrated substantial improvements in direct reconstruction when missing segments were interpolated with a simple

curve or a truncated series which satisfied consistency conditions. Vest and Prikryl[7] adopted image-projection-domain revision in their iterative convolution reconstruction, which also employed the zeroth-moment invariant principle to initally estimate the missing information. This method is in essence similar to those of Medoff et al.[4] and Choi et al.[12] except that a constant field was not assigned to the opaque object region.

Here we present a different approach for reconstruction of fields with opaque objects, which is spirited from the difference impage reconstruction by Hefferman and Robb.[6] The method, as described below, is based on iterative estimation of the object field and corresponding difference projections only in the region where the information is available.

(1) Make an initial estimation of the object field $f_a(r,\psi)$ outside the opaque object.

(2) Calculate the projection data $g_a(\rho,\theta)$ of the approximate field $f_a(r,\psi)$ only in the region where the measured data $g(\rho,\theta)$ is known.

(3) Find the projections $g_d(\rho,\theta) = g(\rho,\theta) - g_a(\rho,\theta)$ of the difference field $f_d(r,\psi) = f(r,\psi) - f_a(r,\psi)$, where $f(r,\psi)$ is the unknown exact field.

(4) Reconstruct the difference field through the inverse Radon transformation: $f_d(r,\psi) = R^{-1}\{g_d(\rho,\theta)\}$.

(5) Improve the estimate of the object field by $f_a^{i+1}(r,\psi) = f_a^i(r,\psi) + f_d(r,\psi)$. At this stage, a priori information can be incorporated to further refine the new estimate.

(6) Continue the iteration by returning to step (2) with the new estimate unless certain convergence criteria are met.

The nature of the iterative correction and the use of a priori in-

formation are similar to those of previous investigations.[4,7] However, the major difference arises in that neither assignment of a constant value in the opaque object region nor estimation of missing projections are necessary during iteration. The fundamental principle of the algorithm can be perceived in a better way by examining the reconstruction error energy.

In practice, reconstruction from projections does not lead to the original field f(x,y) due to imperfect data-aquistion and reconstruction, that is, discrete approximate data and finite numerical calculation. Most of these operations, however, can be linear or guasi-linear. By assuming a spatially-invariant point-spread fuction h(x,y),[13] the directly-reconstructed field $f_r(x,y)$ is

$$f_r(x,y) = h(x,y)** f(x,y) \tag{1}$$

where ** denotes two-dimensional convolution. The error engery E is then, by applying Rayleigh's theorem,[14]

$$\begin{aligned} E &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |f(x,y) - f_r(x,y)|^2 dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |[1-H(u,v)]F(u,v)|^2 du\ dv \end{aligned} \tag{2}$$

where the upper case denotes the Fourier transform. In the iterative algorithm, the difference field is reconstructed. Consequently, the error energy is

$$E = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |[1-H(u,v)]\ [F(u,v) - F_a(u,v)]|^2 du\ dv. \tag{3}$$

By comparing Eqs. (2) and (3), we can see that reasonable estimation of the object field can substantially reduce the error energy in the difference field method. Fundamentally the proposed algorithm is based on the successive estimation of the object field. The application of a priori information in step (5) can further reduce the error energy as demonstrated by Vest and Prikryl.[7]

The iterative algorithm has been tested through computer simulation of experiments. The initial estimation of the field (step (1)) was made through direct reconstruction of the field. For reconstruction, the convolution method with the Shepp-Logan filter[15] was adopted. The convolution method requires full projection data. Linear interpolation was employed for the completion of data. Only the boundary constraint, that is, $f(x,y) = 0$ for $|r| > 1$, was utilized as a priori information.

Figure 2 in the plot of a four-hump test field given below.

$$f(x,y) = \exp\left\{\frac{-6[(x-0.6)^2 + y^2]}{1-(x^2+y^2)}\right\} + 0.5 \exp\left\{\frac{-6[(x+0.6)^2+y^2]}{1-(x^2+y^2)}\right\} \tag{4}$$

$$+ \exp\left\{\frac{-6[x^2+(y-0.6)^2]}{1-(x^2+y^2)}\right\} + 0.5 \exp\left\{\frac{-6[(x^2+(y+0.6)^2]}{1-(x^2+y^2)}\right\}$$

This object field is continuous and defined only in the domain of $r > 0.4$, having an opaque object of radius 0.4 at the center. 32 equally-spaced data points were generated, excluding the opaque region, for each of 28 projections which scanned the 180° viewing angle. The reconstruction error was defined by $\left|[f(x,y) - f_a(x,y)]/\ \text{Max}\left\{f(x,y)\right\}\right|$. As shown in Fig. 3, the maximum and

average errors were reduced from the initial values of 20.9 and 3.6 percent to the limiting values of 13.9 and 2.1 percent, respectively, after 12 iterations. The errors were calculated at 60 x 60 equally-spaced grid points in the defined area. The convergence of the errors was relatively slow. Figure 4 is the plot of the reconstructed field after 12 iterations. Also shown in Fig. 3 are the reconstruction results of the conventional method by Vest and Prikryl,[7] which iteratively estimates missing projection data through projection-object field corrections. The error reduction of the conventional method is initially faster than that of the difference field method; however, it diverges. This phenomenon was also observed in the previous investigation.[7]

Reconstruction results from other continuous fields with a simple shape also demonstrated a steady but rather slow convergence behavior. The convergence is believed to be expedited by employing an appropriate relaxation parameter in finding a new approximate object field. The relaxation parameter employed in this study is unity.

In conclusion, this study presented a limited reconstruction example. More extensive tests, which cover a broad scope of object fields and reconstruction conditions, are due in order to clearly define the behavior of the method. As seen in Eq. (3), the convergence of the method depends on the initial estimate of th field. One possible way for choosing an appropriate estimate is to use the reconstruction from other methods.

This work was supported by ARO under grant 87-K-0098 and NASA under grant NAG 2-388.

References

1. D. W. Sweeney and C. M. Vest, Int. J. Heat Mass Transfer 17, 1443(1974).
2. H. Tan and D. Modarress, Opt. Eng. 24, 435(1985).
3. S. Cha, Opt. Eng. 27, 557(1988).
4. B. P. Medoff, W. R. Brody, M. Nassi, and A. Macovski, J. Opt. Soc. Am. 73, 1493(1983).
5. G. N. Ramachandran and A.V. Lakshminarayanan, Indian J. Pure Appl. Phys. 9, 997(1971).
6. P. B. Heffernan and R. A. Robb, Appl. Opt. 24, 4105(1985).
7. C. M. Vest and I. Prikryl, Appl. Opt. 23, 2433(1984).
8. S. Parenfed and P. V. Farrell, Opt. Lett. 10, 203(1985).
9. T. F. Zien, W. C. Rapsdale, and W. C. Spring, AIAA J. 13, 841(1975).
10. S. Cha, Ph.D. thesis (University of Michigan, Ann Arbor, Michigan, 1980).
11. R. M. Lewitt and R. H. T. Bates, Optik 50, 269(1978).
12. J. S. Choi, K. Ogawa, M. Nakajima, and S. Yata, IEEE Trans. Sonics Ultrason. 29, 143(1982).
13. R. C. Gonzalez and P. Wintz, Digital Image Processing (Addison-Wesley, Reading, Mass., 1977).
14. R. Bracewell, The Fourier Transform and Its Applications (McGraw-Hill, New York, 1965).
15. L. A. Shepp and B. F. Logan, IEEE Trans. Nucl. Sci. 21, 21(1974).

Figure Captions

Fig. 1 Projection of an object field with an opaque object.

Fig. 2 Profile of the test object field.

Fig. 3 Reconstruction errors for two different iteration methods.

Fig. 4 Profile of the reconstructed field after 12 iterations.

Fig. 1

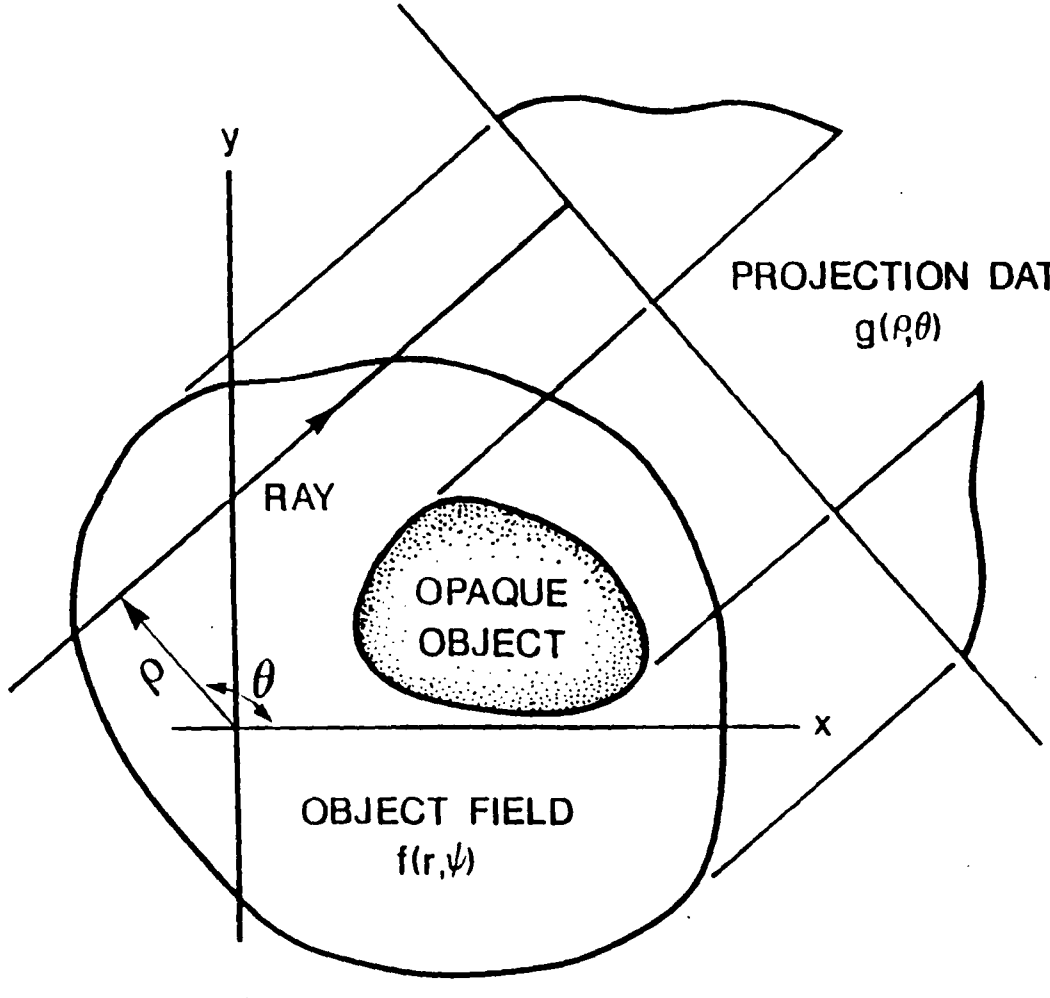

y
PROJECTION DATA
g(ρ,θ)
RAY
OPAQUE
OBJECT
ρ
θ
x
OBJECT FIELD
f(r,ψ)

Fig. 2

ORIGINAL PAGE IS
OF POOR QUALITY

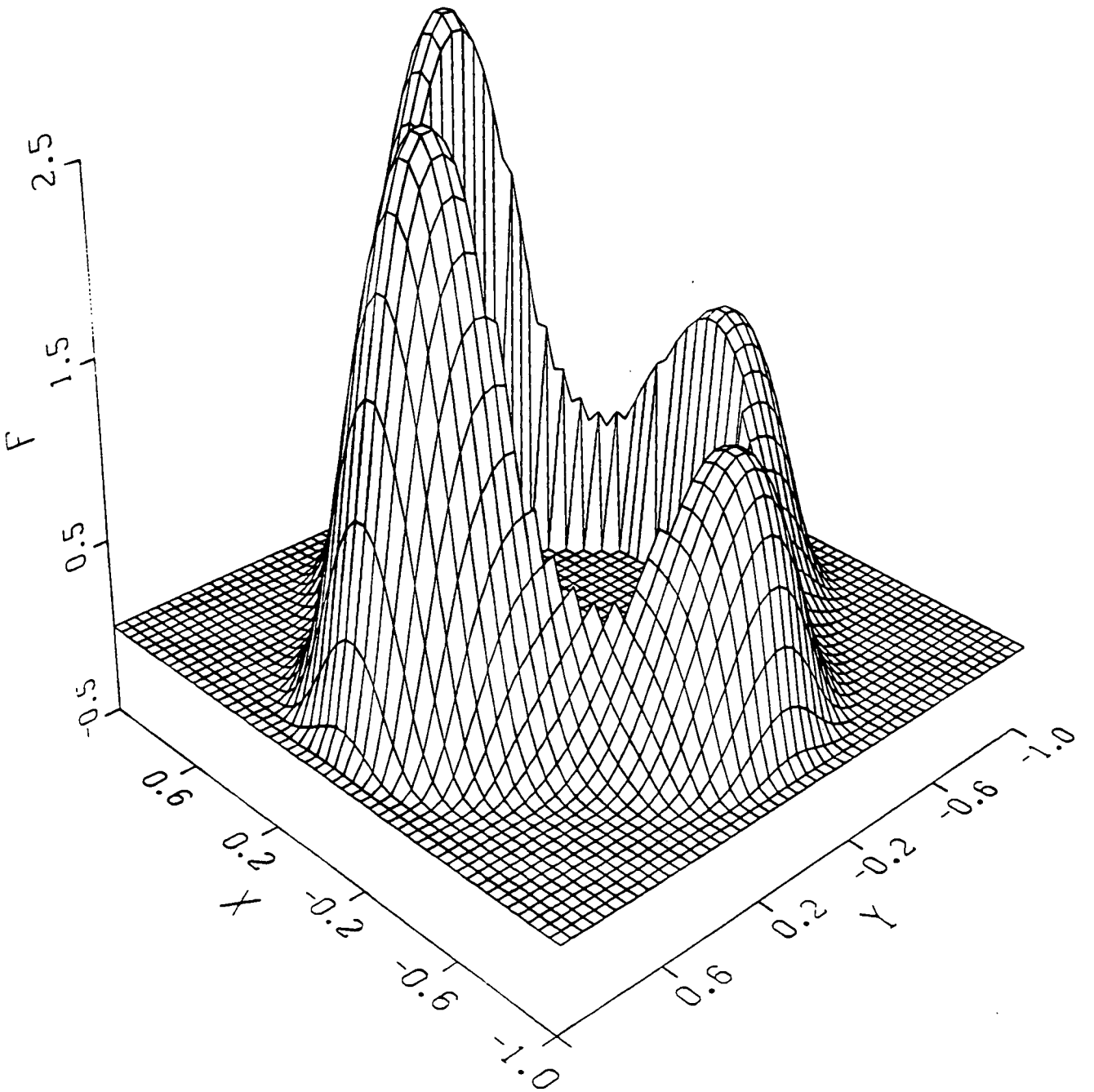
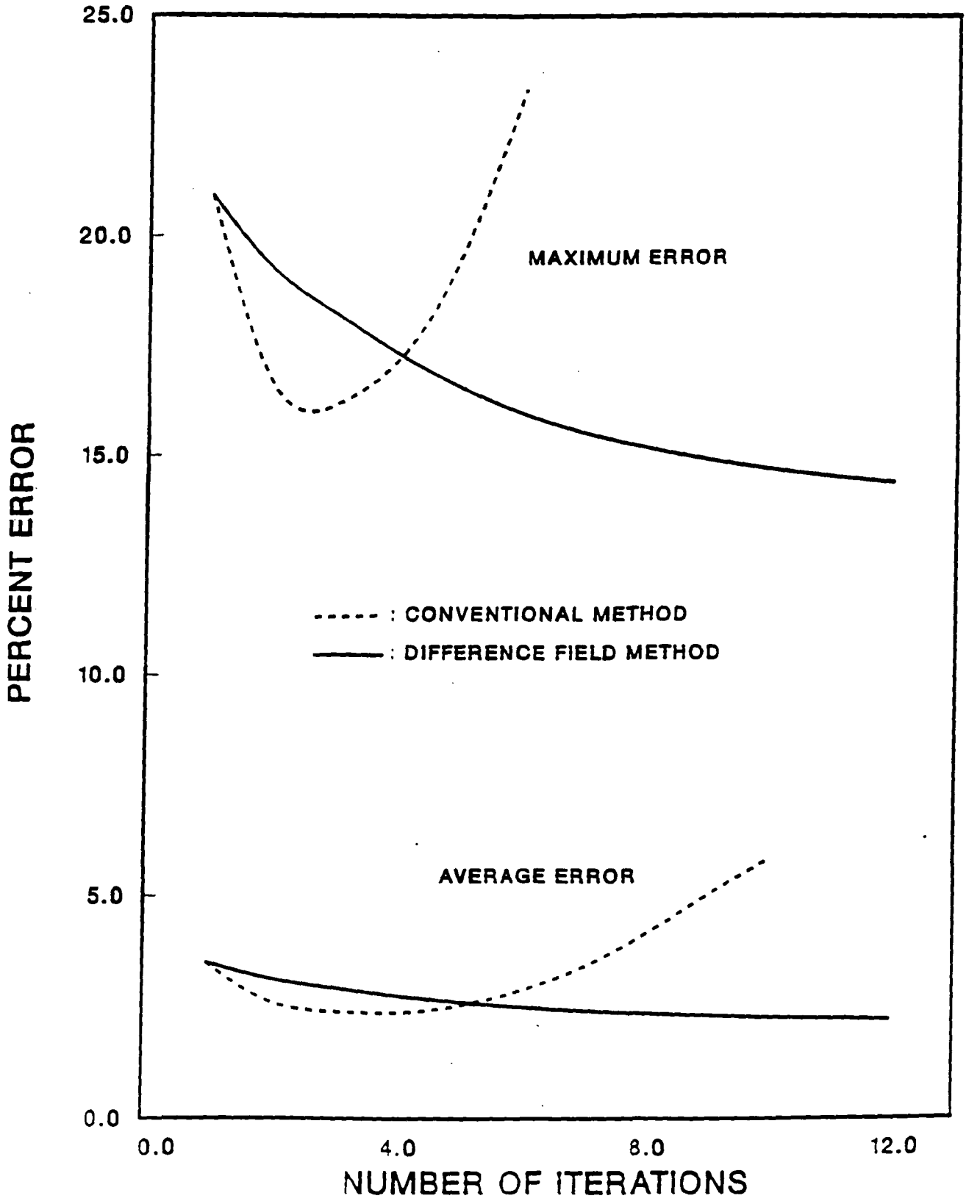


Fig. 3


25.0
20.0
15.0
10.0
5.0
0.0
PERCENT ERROR
MAXIMUM ERROR
----- : CONVENTIONAL METHOD
——— : DIFFERENCE FIELD METHOD
AVERAGE ERROR
0.0
4.0
8.0
12.0
NUMBER OF ITERATIONS

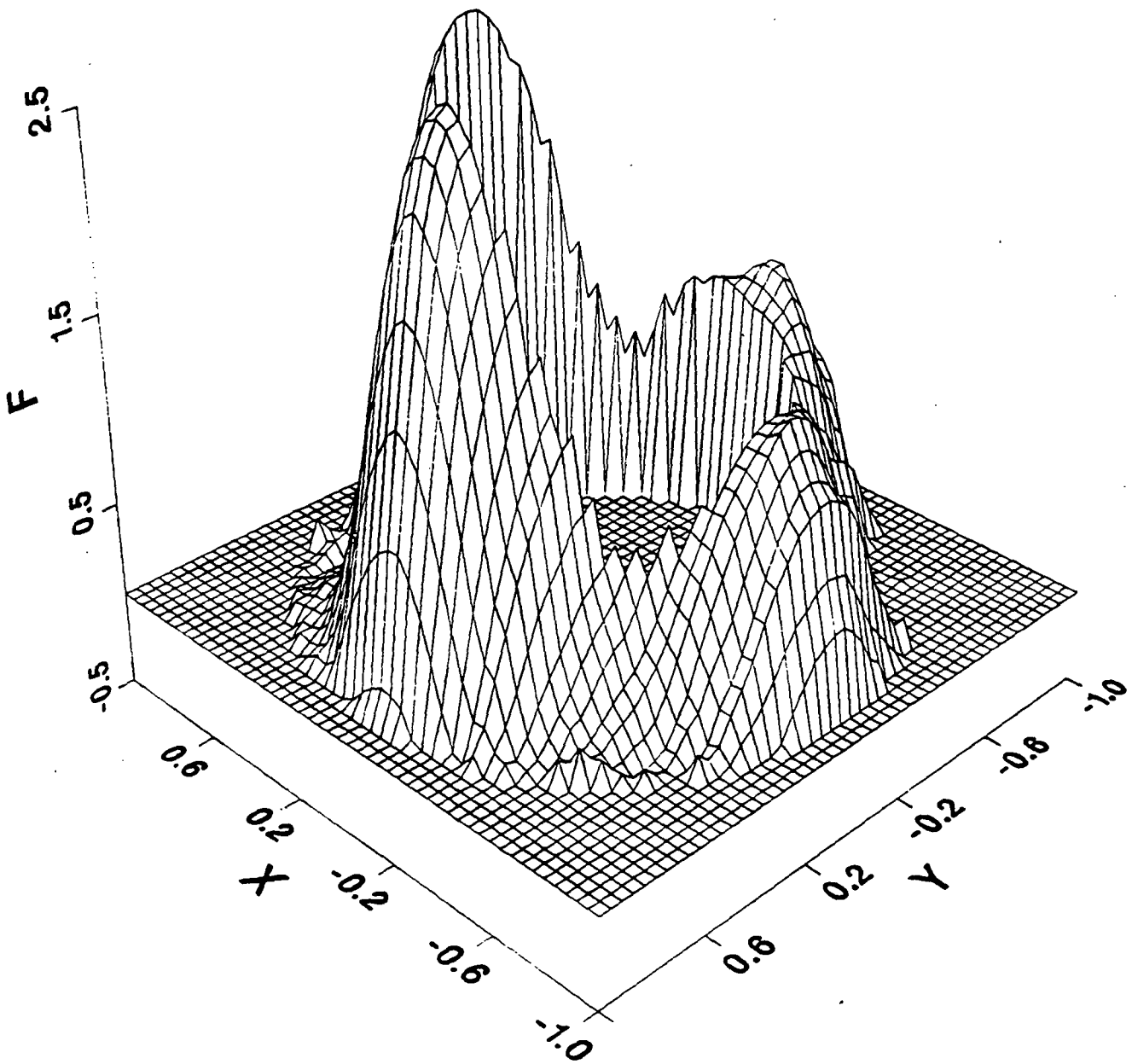



Fig. 4