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FINAL TECHNICAL REPORT ON

ROBUST DESIGN OF DISTRIBUTED CONTROLLERS FOR LARGE FLEXIBLE SPACE STRUCTURES

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SUMMARY

This report presents the research performed at the Catholic University of America on the research grant entitled: "Robust Design of Distributed Controllers for Large Flexible Space Structures," under the Grant No NAG 5-949, between June 30th, 1987 and June 30th, 1988.

Independent Modal Space Control (IMSC) method avoids control spillover generated by conventional control schemes such as Coupled Modal Control by decoupling the large flexible space structure into independent subsystems of second order and controlling each mode independently. The IMSC implementation requires that the number of actuators be equal to that of modeled modes, which is in general very huge. Consequently the number of required actuators is unrealizable.

In this report two methods are proposed for the implementation of IMSC with reduced number of actuators. In the first method, the first m modes are optimized, leaving the last (n-m) modes unchanged. In the second method, generalized inverse matrices are employed to design the feedback controller so that the control scheme is suboptimal with respect to IMSC. The performance of the proposed methods is tested by performing computer simulation on a simply supported beam. Simulation results will be presented and discussed.

1 INTRODUCTION

The development of the space shuttle has opened the possibility of constructing very large space structures (LSS) in space for space explorations. Two control problems for LSS are attitude control and shape control. Complex missions impose many stringent requirements on shape and attitude of the LSS, which lead the control researchers to the concept of distributed active control that places on the structure a number of sensor/actuator pairs in oder to optimize the LSS performance and behavior. Active control of LSS has been an active research area in the last several years [1]-[7]. A large number of control schemes has been developed for LSS, but they represent one form or another of modal control [1]. Two main modal control schemes are the Coupled Modal Control (CMC) and the Independent Modal Space Control (IMSC). The CMC employs an active controller that consists of a state estimator and a state feedback while the IMSC decouples the LSS into n independent subsystems according to n controlled modes and controls each mode independently by means of a modal filter [5] and an optimal controller. It is well-known that CMC causes control and observation spillover, which together can destabilize the LSS [1]. IMSC eliminates control and observation spillover because each mode is controlled independently. However the implementation of IMSC requires that the actuator number be equal to the number of modeled modes, which is usually very huge for the modeling of the LSS to be faithful. This fact presents a fundamental limitation of IMSC for the required number of actuators is impractical. The main objective of this report is to propose methods implementing the

IMSC with a milder requirement of the actuator number. In particular, we will develop two control schemes that uses a reduced number of actuators to control all modeled modes in such a way that the closed-loop system modes are as identical as possible to the optimal modes specified by the IMSC scheme. In the first control scheme, the first m modes are optimized, leaving the last (n-m) modes unchanged. In the second scheme, generalized inverse matrices are employed to design the feedback controller so that the control scheme is suboptimal with respect to IMSC.

Matrix notations used in this report are given below:

$$\text{Block diag}(M_1 \;\; M_2 \;, \ldots, M_n) = \begin{bmatrix} M_1 & 0 & \ldots & 0 \\ 0 & M_2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & M_n \end{bmatrix}$$

O_{mxn} = mxn null matrix

I_n = nxn identity matrix

2. SUMMARY OF INDEPENDENT MODAL SPACE CONTROL

A large flexible space structure can be described by the following partial differential equations [4]:

$$M(P)\ \partial^2 u(P,t)/\partial t^2 + Lu(P,t) = f(P,t) \tag{1}$$

that must be satisfied at every point P of the domain D, where u(P,t) is the displacement of point P, L a linear differential self-adjoint operator of order 2p, expressing the system stiffness, M(P) the distributed mass, and F(P,t) the distributed control force. The displacement u(P,t) is subject to the boundary conditions:

$$T_i u(P,t) = 0; \qquad i = 1,2,\ldots,p \tag{2}$$

where T_i, $i = 1,2,\ldots,p$ are linear differential operators of order ranging from 0 to (2p-1).

The associated eigenvalue problem is formulated by:

$$L\Phi_r(P) = \lambda_r M(P)\, \Phi_r(P); \quad r = 1,2,\ldots \tag{3}$$

with the boundary conditions:

$$T_i\Phi_r(P) = 0; \qquad i = 1,2,\ldots,p; \quad r = 1,2,\ldots \tag{4}$$

where λ_r is the rth eigenvalue and $\Phi_r(P)$ is the eigenfunction (sometimes also known as Mode Shape) associated with λ_r. Suppose the operator L is self-adjoint and positive definite, and all eigenvalues are positive and are ordered so that $\lambda_1<\lambda_2<\ldots$. Since L is self-adjoint, the eigenfunctions are orthogonal and therefore can be normalized such that:

$$\int_D M\Phi_r\Phi_s\, dD = \delta_{rs} \tag{5}$$

and $$\int_D \Phi_s L\Phi_r\, dD = \lambda_r\delta_{rs}; \quad r,s = 1,2,\ldots. \tag{6}$$

where δ_{rs} is the Kronecker Delta.

Using the expression theorem [3], the solution of u(P,t) can be obtained as:

$$u(P,t) = \sum_{r=1}^{\infty} \Phi_r(P)u_r(t) \tag{7}$$

where $u_r(t)$ is the modal coordinate. Substituting (7) into (1), multiplying both sides of the resulting expression by Φ_s, integrating over D and employing (5) and (6), we obtain

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t); \quad r = 1,2,\ldots \tag{8}$$

In (8), the mode (or natural frequency) ω_r is defined as

$$\omega_r = (\lambda_r)^{1/2} \; ; \; r = 1,2,\ldots. \tag{9}$$

and the modal control force $f_r(t)$ is computed by:

$$f_r(t) = \int_D \Phi_r(P) f(P,t)\, dD \tag{10}$$

In practice, the infinite series in (7) is truncated as

$$u(P,t) = \sum_{r=1}^{n} \Phi_r(P)\, u_r(t) \tag{11}$$

where n is chosen to be sufficiently large so that u(P,t) can be represented with good fidelity. In this case we are dealing only with the first n modes.

Eq. (8) can be transformed into state equation form as follows:

$$\dot{x}(t) = A\, x(t) + W(t) \tag{12}$$

where $x(t) = [x_1^T(t) \;\; x_2^T(t) \; \ldots . x_n^T(t)]^T$ (13)

$$W(t) = [W_1^T(t) \;\; W_2^T(t) \;\; W_n^T(t)]^T \tag{14}$$

$$A = \text{Block diag}(A_1, A_2, \ldots ., A_r) \tag{15}$$

$$x_r(t) = [u_r(t) \;\; \dot{u}_r(t)/\omega_r]^T \tag{16}$$

$$W_r(t) = [0 \;\; f_r(t)/\omega_r]^T \tag{17}$$

and $A_r = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix}$ (18)

for i=1,2,...,n.

REMARK 1:

At this point we should distinguish between the eigenvalues of A and the eigenvalues of the flexible space structure. From (15) and (18) the 2n eigenvalues of the matrix A that represent also the open-loop poles of the system consist of n pairs of imaginary numbers: $\mp\omega_1 j$, $\mp\omega_2 j, \ldots, \mp\omega_n j$. However according to (9), the eigenvalues of the flexible space structure are given by ω_1^2, ω_2^2, ω_n^2. We also note that the rth mode ω_r is the magnitude of the eigenvalues of A_r. In other words, a mode corresponds to 2 eigenvalues of one subsystem of the complete flexible system.

The system described by (12) consists of n subsystems given by

$$\dot{x}_r(t) = A_r x_r(t) + W_r(t); \quad r = 1,2,\ldots,n \qquad (19)$$

The essence of IMSC is to choose $W_r(t)$ such that it depends on $x_r(t)$ alone. Thus

$$W_r(t) = G_r x_r; \quad r = 1,2,,\ldots\ldots,n \qquad (20)$$

where $$G_r = \begin{bmatrix} g_{r11} & g_{r12} \\ g_{r21} & g_{r22} \end{bmatrix}; \qquad r = 1,2,\ldots,n \qquad (21)$$

are (2x2) gain matrices.

Substituting (16) and (17) into (20), we find that G_r must assume the following form:

$$G_r = \begin{bmatrix} 0 & 0 \\ g_{r21} & g_{r22} \end{bmatrix}; \qquad r = 1,2,\ldots,n \qquad (22)$$

For optimal control, g_{r21} and g_{r22} should be determined such that the following quadratic cost function is minimized (linear regulator problem):

$$J = \sum_{r=1}^{n} J_r \qquad (23)$$

where $$J_r = \int_0^T (x_r^T Q_r x_r + W_r^T R_r W_r)dt \qquad (24)$$

Q_r and R_r are positive semidefinite and positive definite weighting matrix, respectively, associated with the rth mode.

The form of G_r given by (22) requires that R_r assume the form given below [7]:

$$R_r = \begin{bmatrix} \infty & 0 \\ 0 & r_r \end{bmatrix}; \quad r = 1,2,\ldots,n. \qquad (25)$$

Since W_r depends on x_r alone as seen in (20), J can be minimized by minimizing each J_r, independently. From optimal control theory [15], the optimal solution for G_r is given by

$$G_r(t) = R_r^{-1} K_r(t); \; r = 1,2,\ldots,n \tag{26}$$

where $K_r(t)$ is the solution of Riccati equation:

$$\dot{K}_r(t) = -K_r A_r - A_r^T K_r + K_r R_r^{-1} - Q_r; \; r = 1,2,\ldots,n \tag{27}$$

with boundary condition $K_r(T) = 0$.

Selecting $Q_r = \omega_r^2 I_2$ and substituting (25) and (18) into (27) yield

$$\dot{K}_{11} = 2\omega_r K_{12} + r_r^{-1} K_{12}^2 - \omega_r^2 \tag{28}$$

$$\dot{K}_{12} = \omega_r(K_{22} - K_{11}) + r_r^{-1} K_{12} K_{22} \tag{29}$$

$$\dot{K}_{22} = -2\omega_r K_{12} + r_r^{-1} K_{22}^2 - \omega_r^2 \tag{30}$$

where K_{ij} (i, j = 1,2) are the elements of K_r, and $K_{12} = K_{21}$. The steady state solutions of (28)-(30) can be found by letting $\dot{K}_{11} = \dot{K}_{12} = \dot{K}_{22} = 0$. Introducing $R_r^* = (1/\omega_r^2)R_r$ where

$$R_r^* = \begin{bmatrix} \infty & 0 \\ 0 & r_r^* \end{bmatrix} \qquad r = 1,2,\ldots,n \tag{31}$$

the steady-state solutions of (28)-(30) are obtained as

$$K_{12} = K_{21} = \omega_r^2(-\omega_r r_r^* + a^{1/2}) \tag{32}$$

$$K_{22} = \omega_r^2 \, [r_r - 2\omega_r^2 r_r^{*2} + 2\omega_r r_r^* a^{1/2}]^{1/2} \tag{33}$$

$$K_{11} = \omega_r^2 \, [1/w_r^2 + (2/\omega_r r_r^*)a^{3/2} - 2r_r^* r^2 - r_r^*]^{1/2} \tag{34}$$

$$\text{where } a = \omega_r^2 r_r^{*2} + r_r^*. \tag{35}$$

Substituting (32)-(34) into (26) yields

$$G_r = \begin{bmatrix} 0 & 0 \\ \omega_r - b & -[2\omega_r(-\omega_r + b) + r_r^{*-1}]^{1/2} \end{bmatrix} \tag{36}$$

$$\text{where } b = (\omega_r^2 + r_r^{*-1})^{1/2} \tag{37}$$

From (17), (20), (22) and (35), we obtain

$$f_r(t) = (\omega_r - b)\omega_r u_r(t) - [2\omega_r(-\omega_r + b) + r_r^{*-1}]^{1/2} \dot{u}_r(t) \tag{38}$$

for $r = 1,2,\ldots,n$.

Substituting (38) into (8) yields

$$\ddot{u}_r(t) + [2\omega_r(b - \omega_r) + r_r^{*-1}]^{1/2}\,\dot{u}_r(t) + \omega_r b u_r(t) = 0 \qquad (39)$$

From (39) the closed-loop poles (eigenvalues) of the system are

$$S_{r1,r2} = =-1/2\ [2\omega_r(b-\omega_r) + r_r^{*-1}]^{1/2} \mp 1/2\ [-2\omega_r(b+w_r) + r_r^{*-1}]^{1/2} \qquad (40)$$

for $r = 1,2,\ldots,n$.

The closed-loop system is stabilized since the poles given in (40) are either complex conjugates with negative real parts, or both real and negative. Figure 1 illustrates the concept of Independent Modal Space Control in the state equation form. As we observe, each mode is controlled independently by an actuator. Therefore the number of required actuators must be equal to the number of modes that is in general very high. This facts represents a principal disadvantage of IMSC. On the other hand, one main advantage of IMSC is the elimination of control and observation spillover.

3. IMPLEMENTATION OF INDEPENDENT MODAL SPACE CONTROL

The IMSC scheme is represented by Equations (19) and (20) that however are in the modal form. In order to implement the IMSC, a finite number of discrete sensors and discrete actuators must be employed.

DISCRETE SENSORS AND MODAL FILTER

Suppose there are 2p discrete sensors consisting of p displacement sensors and p velocity sensors located at p locations $P_1, P_2, \ldots, P_p$. Using (11), the output of the displacement sensor and velocity sensor can be expressed as

$$y_i = u(P_i,t) = \sum_{r=1}^{n} \Phi_r(P_i)\, u_r(t), \quad i = 1,2,\ldots,p$$
$$\text{and} \quad \dot{y}_i = \dot{u}(P_i,t) = \sum_{r=1}^{n} \Phi_r(P_i)\, \dot{u}_r(t), \quad i = 1,2,\ldots,p \tag{41}$$

If we define an output vector y(t) containing both displacement and velocity sensor output as

$$y(t) = [y_1\ \dot{y}_1\ y_2\ \dot{y}_2\ \ldots y_n\ \dot{y}_n]^T, \tag{42}$$

then using (41), we can write

$$y(t) = C\, x(t) \tag{43}$$

where $$C = [c_{ij}],\ i,j = 1,2,\ldots,p \tag{44}$$

and $$c_{ij} = \begin{bmatrix} \Phi_j(P_i) & 0 \\ 0 & \omega_j \Phi_j(P_i) \end{bmatrix} \tag{45}$$

In order to implement (20) modal state vector $x_r(t)$ must be extracted from the sensor outputs. A device called Modal Filter [5] interpolates the discrete sensor measurements to obtain continuous displacement and velocity profiles $\hat{u}(P,t)$ and $\hat{\dot{u}}(P,t)$ and then computes the estimated modal displacement and velocity using the expansion theorem stated by (7):

$$\hat{u}_r(t) = \int_D M(P)\, \Phi_r(P)\, \hat{u}(P,t)dD \tag{46}$$

$$\hat{\dot{u}}_r(t) = \int_D M(P)\, \Phi_r(P)\, \hat{\dot{u}}(P,t)dD \tag{47}$$

for $r = 1,2, \ldots,n$

DISCRETE ACTUATORS

Since it is impossible to control force at every point in the domain D, the distributed control force is realized by m (m<n) discrete point force actuators applied at m points P_1, P_2, ..., P_m

in the domain D as given below

$$f(P,t) = \sum_{i=1}^{m} \delta(P-P_i)\ F_i(t) \tag{48}$$

where $\delta(P-P_i)$ is a spatial Dirac Delta function and $F_i(t)$ is the force applied by the ith actuator on the point P_i.

Now substituting (48) into (10) yields

$$f_r(t) = \int_D \Phi_r\ (P) \sum_{i=1}^{m} \delta(P-P_i)\ F_i(t)\ dD \tag{49}$$

From the property of the Dirac Delta function, (49) can be reduced to

$$f_r(t) = \sum_{i=1}^{m} \Phi_r(P_i)\ F_i(t) \tag{50}$$

If we define a force vector F(t) such that

$$F(t) = [\ F_1(t)\ \ F_2(t)\ldots.F_m(t)]^T, \tag{51}$$

then using (50) the relation between F(t) and W(t) can be expressed by

$$W(t) = B\ F(t) \tag{52}$$

where

$$B = [B_1^T\ B_2^T\ldots.B_{2n}^T]^T \tag{53}$$

$$B_{(2i-1)} = 0_{1xm};\ i = 1,2,\ldots.,n \tag{54}$$

$$B_{2i} = [\Phi_i(P_1)/\omega_i\ \ \Phi_i(P_2)/\omega_i\ \ldots..\ \Phi_i(P_m)/\omega_i] \tag{55}$$

for $i = 1,2,\ldots,n$.

4. PROBLEM STATEMENT

Since in this paper, the implementation problem is focused on actuators, we suppose that there exists a modal filter which takes 2p measurements of p displacement sensors and p velocity sensors and produces a "perfect" estimated state $\hat{x}(t)$ of x(t). It is obvious that if the modal displacement $u_r(t)$ and modal velocity $\dot{u}_r(t)$ can be perfectly estimated as shown by (46) and (47), then in view of (16) and (13), the state $x(t)$ can also be perfectly estimated. Here the state $x(t)$ is said to be perfectly estimated if $\hat{x}(t)$ approaches $x(t)$ as $t \to \infty$. In control theory, the corresponding state estimator is said to be asymptotically stable [10].

Figure 2 illustrates the imlementation of IMSC in which the optimal state feedback law is denoted by

$$\hat{W}(t) = G\,\hat{x}(t) \tag{56}$$

where $\hat{W}(t) = [\hat{W}_1^T(t) \;\; \hat{W}_2^T(t) \ldots \hat{W}_n^T(t)]^T$ (57)

and $G = \text{Block diag}\,(G_1, G_2, \ldots, G_n)$ (58)

In section 3, the optimal solution for IMSC is obtained in modal space, namely from (20)

$$W(t) = G\,x(t) \tag{59}$$

Under the assumption of perfect state estimation, we note that $\hat{W} \to Gx(t)$ as $t \to \infty$. Therefore the optimal solution can be achieved if (59) is satisfied. To make $W(t)$ equal to $Gx(t)$, the matrix D in Fig. 2 is chosen such that $W(t) = \hat{W}(t)$.

From Fig. 2 we note that

$$F(t) = D\,\hat{W}(t) \tag{60}$$

Substituting (60) into (52) yields

$$W(t) = B\,D\,\hat{W}(t) \tag{61}$$

Obviously to make $W(t) = \hat{W}(t)$, D is chosen such that

$$B\,D = I_{2n} \tag{62}$$

From the structure of B as given by (53)-(55), each $(2i-1)$th row (odd row) of BD, for $i=1,2,\ldots,n$ is a row of zeros. We realize that (62) can never be satisfied. However, noting that each odd row of $W(t)$ is also a row of zeros, if we define

$$\bar{B}(t) = [B_2^T\ B_4^T \ldots B_{2n}^T]^T \tag{63}$$

and $$\bar{D}(t) = [\ D_2^T\ D_4^T \ldots D_{2n}^T]^T \tag{64}$$

where B_i is the ith row of B and D_i the ith column of D, then choosing a matrix D such that

$$\bar{B}\ \bar{D} = I_n \tag{65}$$

will ensure that $W(t) = \hat{W}(t)$. It is noted that if (65) holds, then BD is a modified identity matrix of order (2nx2n) whose main diagonal elements are 0 at the (2i-1,2i-1) position and 1 at the (2i,2i) position for i = 1,2,...,n.

The solution of (65) for D needs the inverse of B. However, the existence of the inverse of B require that the number of actuators be equal to the number of modeled modes (m=n). This fact presents the principal limitation of IMSC because the number of modeled modes is usually very high, resulting in an unrealizable number of actuators.

The problem considered in this report is formulated as to develop a control scheme that allows one to use a reduced number of actuators to control all modeled modes such that the closed-loop system is as optimal as possible compared to IMSC.

In the following we will propose two control schemes that will accomplish the above objective.

5. THE FIRST CONTROL SCHEME

The development of the first control scheme is represented in the following theorem:

THEOREM 1:

Consider a large flexible space structure whose description and solution are given by (1) and (11), respectively. If the operator L is self-adjoint and the state $x(t)$ is perfectly estimated, then there exists a control scheme with m actuators, which optimizes the first m modes with respect to the cost function (24) and leaves the last (m-n) natural modes unchanged, thus ensuring the system stability.

Proof:

If the hypothesis of Theorem 1 is satisfied, then employing m actuators placed at m distinct points $P_1, P_2, \ldots, P_m$ in the domain D results in a matrix $\bar{B}$ that can be partitioned as

$$\bar{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} \tag{66}$$

where $\bar{B}_1$ is an (mxm) matrix and $\bar{B}_2$ an (n-m)xm matrix. Based on the property of the rows of $\bar{B}$ specified by (55), without loss of generality we can assume that the first m rows of $\bar{B}$ are linearly independent for the locations of m actuators are distinct. Thus $\bar{B}_1$ is a nonsingular matrix. Now if a matrix D is chosen such that

$$\bar{D} = [\bar{B}_1^{-1} \quad 0_{m\times(n-m)}] \tag{67}$$

then it is clear that

$$\bar{B}\bar{D} = \begin{bmatrix} I_m & 0_{m\times(n-m)} \\ \bar{B}_2\bar{B}_1^{-1} & 0_{(n-m)\times(n-m)} \end{bmatrix} \tag{68}$$

We proceed by considering the closed-loop feedback system given in Fig. 2 that can be described by

$$\dot{x}(t) = A\,x(t) + B\,D\,G\,\hat{x}(t) \tag{69}$$

under the assumption of perfect state estimation, in the steady state ($t \to \infty$), (69) can be rewritten as:

$$\dot{x}(t) = (A + BDG)\, x(t) \tag{70}$$

Now if $\bar{D}$ is chosen as in (67), then

$$A+BDG = \text{Block diag}(A_1, A_2, \ldots, A_n) +$$

$$+ \begin{bmatrix} I^{*}_{2m} & 0_{2m\times 2(n-m)} \\ X & 0_{2(n-m)\times 2(n-m)} \end{bmatrix} \text{Block diag}(G_1, G_2, \ldots, G_n) \tag{71}$$

where X is an $2(n-m)\times 2m$ matrix whose elements are obtained from $\bar{B}_2\bar{B}_1^{-1}$ and I^{*}_{2m} is a modified ($2m\times 2m$) identity matrix whose main diagonal elements are 0 at the $(2i-1,2i-1)$ position and 1 at the $(2i,2i)$ position for $i = 1,2,\ldots,m$.

Recalling the form of G_r for $r = 1,2,\ldots,n$ as given in (22), we can rewrite (71) as

$$A + BDG =$$

$$= \begin{bmatrix} \text{Block diag}\,(A_1 + G_1, A_2 + G_2, \ldots, A_m + G_m) & 0_{2m\times 2(n-m)} \\ X & \text{Block diag}(A_1, A_2, \ldots, A_n) \end{bmatrix} \tag{72}$$

From (72) the closed-loop eigenvalues can be computed as

$$\sigma(A + BDG) = \bigcup_{i=1}^{m} \sigma(A_i + G_i) + \bigcup_{i=1}^{m} \sigma(A_i) \tag{73}$$

where $\bigcup_{i=1}^{m} S_i$ denotes the union operation on the sets S_i, $i=1,2,\ldots,m$. and $\sigma(A)$ denotes the set of eigenvalues of the matrix A.

According to Remark 1. it is obvious from (73) that the first m modes are optimized and the last (n-m) modes are unchanged. Since optimal modes correspond to stable closed-loop poles as stated by (40) and natural modes correspond to pairs of imaginary complex conjugate eigenvalues, as pointed out in Remark 1, we conclude that the above control scheme also ensures the system stability. We therefore just complete the proof of Theorem 1.

REMARK 2:

If the control scheme proposed in Theorem 1 is implemented, then there are some unwanted excitations of the (n-m) modes, which is well-known as control spillover [1]. However the oscillations caused by exciting the (n-m) modes are insignificant because the mode amplitudes tend to decrease as the number of modes is increased because higher modes require more energy to excite. We also note that in (1), we assume the worst case where no natural damping exists. In practice, existing natural damping in the structure can suppress the unwanted oscillations of the higher modes.

6. THE SECOND CONTROL SCHEME

In this section, another control scheme is developed using the concept of generalized inverse matrices. We first present the following lemma:

LEMMA 1:

Consider the following equation:

$$\bar{W}(t) = \bar{B}\,\bar{F}(t) \qquad (74)$$

where $\bar{W}(t)$ and $\bar{F}(t)$ are matrices consisting of even rows of W(t) and F(t), respectively. If the (nxm) matrix has rank m, then the solution for (74), which minimizes the weighted norm of error

$$||e(t)||_S^2 = ||\bar{W} - \bar{B}\bar{F}||_S^2 = (\bar{W} - \bar{B}\bar{F})^T S(\bar{W} - \bar{B}\bar{F}) \qquad (75)$$

is given by

$$\bar{F}(t) = B^{*} \bar{W}(t) \qquad (76)$$

where the matrix

$$B^{*} = (\bar{B}^{T}S\bar{B})^{-1}\bar{B}^{T}S \qquad (77)$$

is the generalized inverse of $\bar{B}$. A proof of Lemma 1 can be found in [14].

The following theorem will present the development of the second control scheme:

THEOREM 2:

Consider a large flexible space structure whose description and solution are given by (1) and (11), respectively. If the operator L is self-adjoint, then there exists a control scheme with m actuators ($m < n$) that is suboptimal with respect to (24) in the sense that the closed-loop eigenvalues are assigned as closed as possible to those optimal eigenvalues specified by IMSC.

Proof:

A control scheme with a reduced number of actuators would be optimal if $\bar{D}$ could be selected to be a right inverse of $\bar{B}$ in (65). Here $\bar{B}$ is assumed to have rank m since the discrete actuators apply point forces at m distinct points $P_1, P_2, \ldots, P_m$. From [14], it is well-known that an (nxm) matrix ($m < n$) having rank m does not have any right inverse. According to Lemma 1, because $\bar{F}(t)$ as given in (76) minimizes (75), selecting a matrix $\bar{D} = B^{*}$ will minimize the difference $\bar{B}\bar{D} - I_n$, making (65) satisfied as well as possible. Therefore selecting $\bar{D} = B^{*}$ also makes the closed-loop eigenvalues as identical as possible to those specified by IMSC. Thus there exists a control scheme with a reduced number of actuators, which is suboptimal with respect to (24). We just complete the proof of Theorem 2.

7. COMPUTER SIMULATION STUDY

In order to evaluate the performance of the proposed control schemes, we consider the control of a simply-supported beam whose dynamics is given by the Euler-Bernoulli partial differential equation:

$$EI \frac{\partial^4}{\partial x^4} u(x,t) + m \frac{\partial^2}{\partial t^2} u(x,t) = f(x,t) \quad (78)$$

where for simplicity we set the mass m, the moment of inertia I, the modulus of elasticity E and the length of the beam to unity. The boundary of the simply-supported beam are

$$u(0,t) = u(1,t) = 0 \quad (79)$$

$$\frac{\partial^2}{\partial x^2} u(0,t) = \frac{\partial^2}{\partial x^2} u(1,t) = 0. \quad (80)$$

The solutions for the eigenvalue problem are given by:

$$\lambda_k = (k\pi)^2 \quad (81)$$

$$\phi_k = (2)^{1/2} \sin (k\pi x) \quad (82)$$

Computer simulation was performed to evaluate the second control scheme and to compare its performance to that of IMSC. Suppose we consider 20 modeled modes and divide the whole length of the beam into 20 sections, specified by x(k) = k/21 for k = 1,2,..., 21. Then starting with one actuator we performed computer simulation for different locations of the actuator. After that computer simulation was performed with increasing number of actuators. The simulation results are summarized and discussed below:

(a) *The Number of Actuators:*

Simulation results show that the dynamics of the closed-loop system is affected considerably by the number of the actuators. As expected, the number of stable closed-loop eigenvalues is proportional to that of the actuators. The maximum number of closed-loop eigenvalues when using 1, 2, 3, and 4 actuators is 5, 18, 19, and 20, respectively, as showed from computer simulation.

(b) *The Location of Actuators:*

Results show that the actuator location also affects the closed-loop system dynamics. Figure 2a illustrates the relationship between actuator location and the number of stable closed-loop eigenvalues for the case of 1 actuator. We note that the optimal actuator locations center around both ends of the beam. The number of the closed-loop eigenvalues decreases as the actuator moves toward the center of the beam. The above observations are consistent with the cases of 2, 3, and 4 actuators.

(c) *The Second Control Scheme versus IMSC:*

The open-loop eigenvalues of the LSS , the closed-loop eigenvalues of IMSC implemented with 20 actuators and 2 actuators and the second control scheme implemented with 2 actuators are presented in Table 1. The table shows that the second control scheme implemented with a reduced number of actuators assigns eigenvalues that are very close to those specified by the optimal IMSC.

Figures 3 and 5 represent the vertical displacements of the beam center when being excited by a unit impulse for IMSC with 20 actuators and the second control scheme with 2 actuators, respectively while Figure 4 represents the case of IMSC with 2 actuators. We observe that with a reduced number of actuators (2 actuators), the second control scheme provides with the same performance as that of IMSC when implemented by 20 actuators. On the other hand, as we see in Figure 4, when being implemented with a reduced number of actuators (2 actuators), the IMSC suffers from some oscillations. Beam movements for the case of IMSC with 20 actuators and 2 actuators are illustrated in Figures 6 and 7, respectively. The case of the second control scheme with 2 actuators is presented in Figure 8. Similar to the previous case, the performance of the second control scheme is better than that of IMSC when being implemented with a reduced number of actuators.

8. CONCLUSION AND FUTURE RESEARCH

In this report, we first reviewed the theory of Independent Modal Space Control in the context of optimal control. The requirement that the number of actuators be equal to that of modeled modes for the implementation of IMSC was pointed out as a principal limitation. Assuming that the large space structure is self-adjoint, we developed two control schemes implemented with a reduced number of actuators, which are suboptimal with respect to the IMSC performance. The first control scheme optimizes the first m modes leaving the last (n-m) modes uncontrolled. The second control scheme employs the method of generalized inverse matrices to assign closed-loop eigenvalues, which are as close as possible to those specified by IMSC. Computer simulation performed on a simply-supported beam showed that the second proposed control scheme performs better than the IMSC when both are implemented by a reduced number of actuators. Future research should focus the attention on the optimization of the number of actuators and the placement of actuators in order to minimize the control spillover. The effect of modal filter in terms of observation spillover should be investigated. The method of arbitrary eigenvalue assignment [8] should also be considered for the design of control schemes for large space structures.

The research performed under this grant were transmitted to one semiannual progress report [11], one conference paper [12] one published journal paper [9] and one technical paper to be submitted to a major journal [13].

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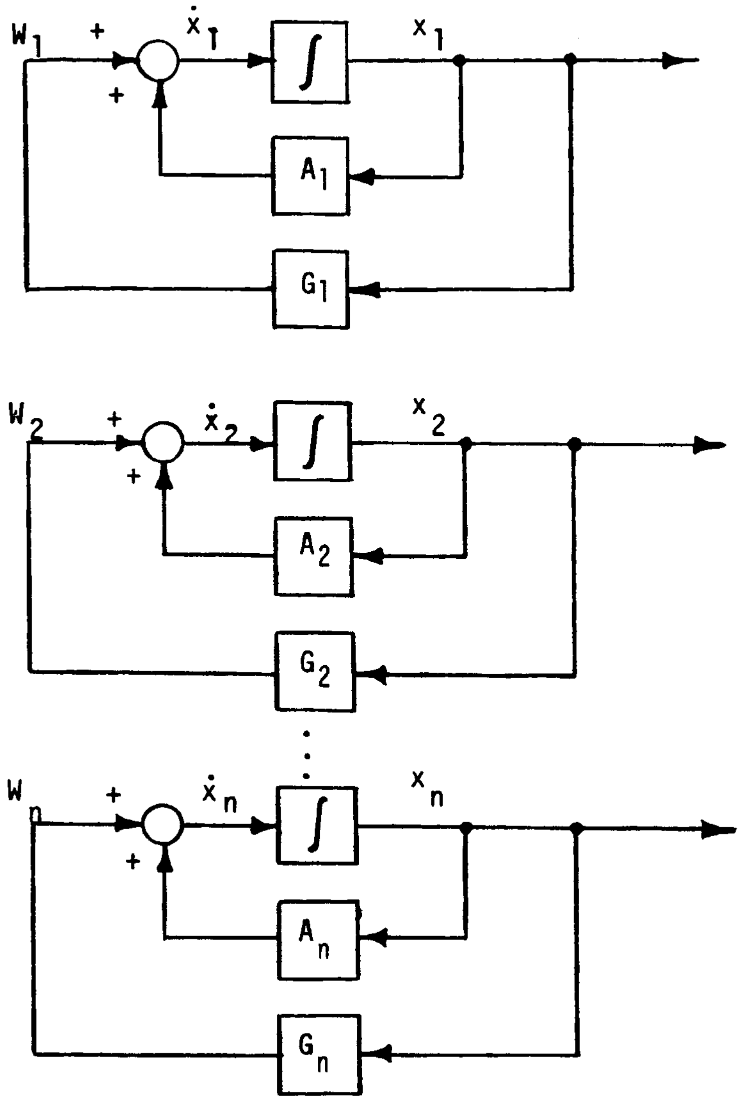


Fig. 1 : Independent Modal Space Control

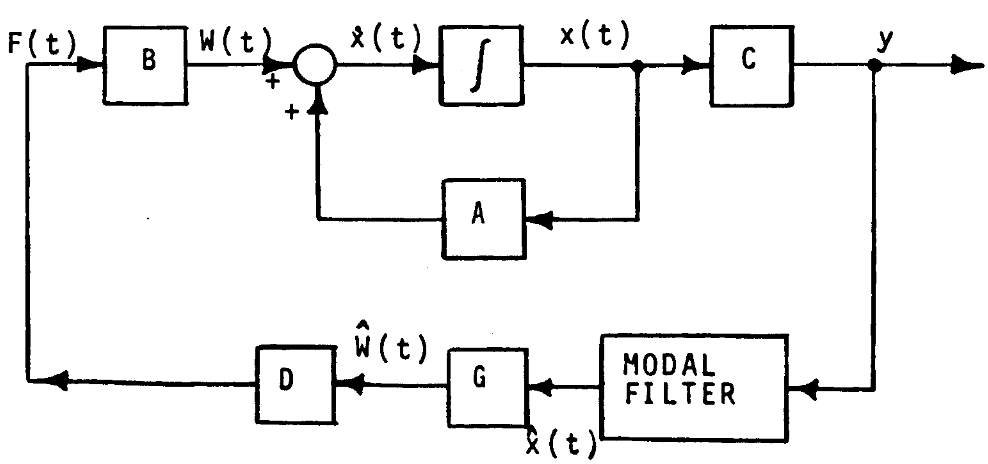
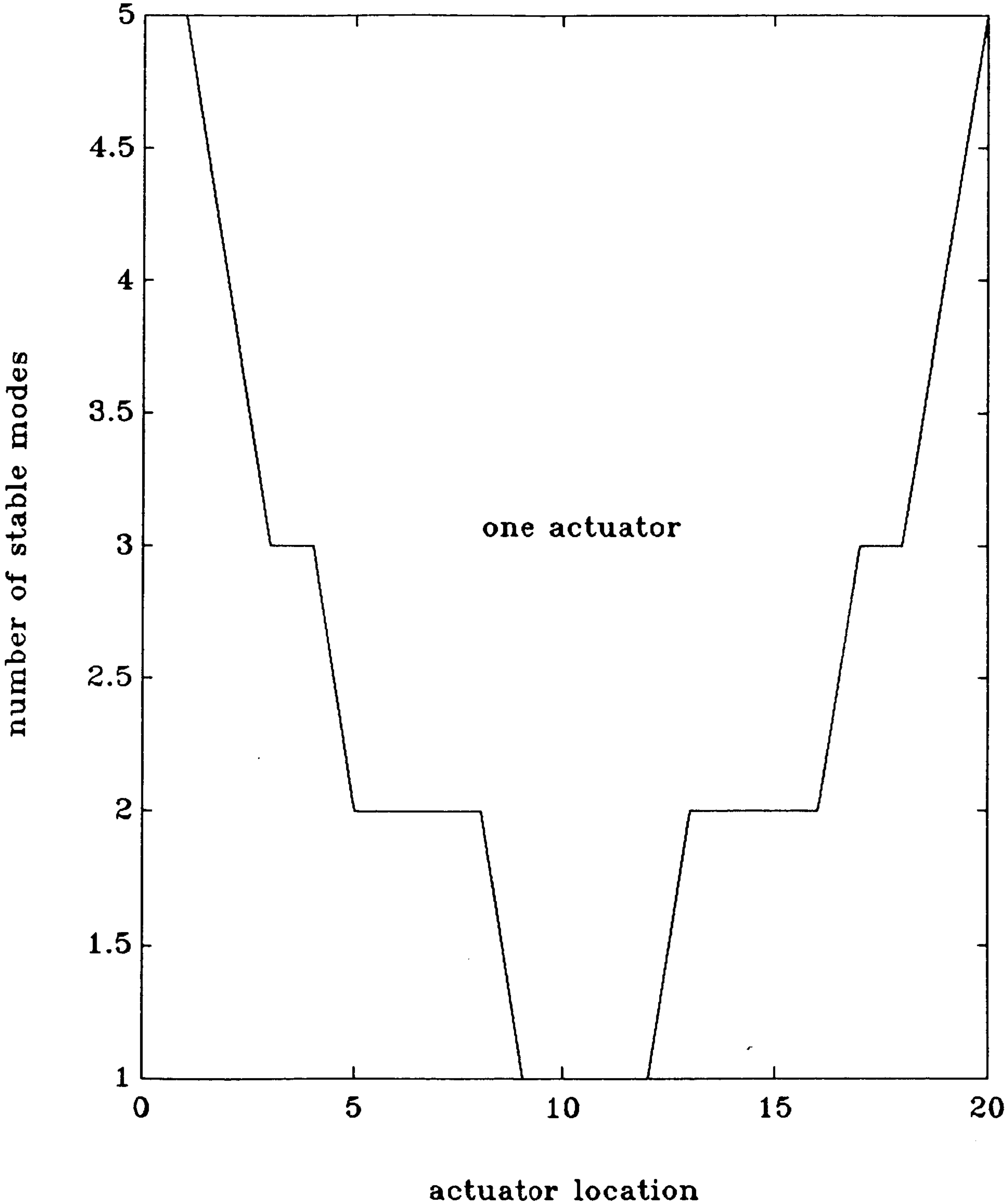


Fig. 2: Implementation of IMSC

Figure 2a: Stable Mode Number versus Actuator Location



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MODE

1.0e+003 *

	open-loop eigenvalues		IMSC (20 actuators)		IMSC (2 actuators)		New control scheme (2 actuators)	
			CLOSED-LOOP EIGENVALUES					
1	0	0.0099i	-0.0014	0.0099i	-0.0014	0.0099i	-0.0014	0.0098i
2	0	0.0395i	-0.0020	0.0395i	-0.0020	0.0395i	-0.0020	0.0378i
3	0	0.0888i	-0.0024	0.0888i	-0.0000	0.0888i	-0.0025	0.0884i
4	0	0.1579i	-0.0028	0.1579i	0.0000	0.1579i	-0.0028	0.1580i
5	0	0.2467i	-0.0032	0.2467i	0.0000	0.2467i	-0.0032	0.2466i
6	0	0.3553i	-0.0035	0.3553i	0.0000	0.3553i	-0.0035	0.3555i
7	0	0.4836i	-0.0037	0.4836i	-0.0000	0.4836i	-0.0037	0.4834i
8	0	0.6317i	-0.0040	0.6317i	-0.0000	0.6317i	-0.0040	0.6316i
9	0	0.7994i	-0.0042	0.7994i	0.0000	0.7994i	-0.0042	0.7992i
10	0	0.9870i	-0.0045	0.9870i	0.0000	0.9870i	-0.0045	0.9872i
11	0	1.1942i	-0.0047	1.1942i	-0.0000	1.1942i	-0.0047	1.1939i
12	0	1.4212i	-0.0049	1.4212i	-0.0000	1.4212i	-0.0049	1.4213i
13	0	1.6680i	-0.0051	1.6680i	-0.0000	1.6680i	-0.0051	1.6678i
14	0	1.9344i	-0.0053	1.9344i	-0.0000	1.9344i	-0.0053	1.9345i
15	0	2.2207i	-0.0055	2.2207i	-0.0000	2.2207i	-0.0055	2.2203i
16	0	2.5266i	-0.0057	2.5266i	-0.0000	2.5266i	-0.0057	2.5266i
17	0	2.8523i	-0.0058	2.8523i	-0.0000	2.8523i	-0.0058	2.8521i
18	0	3.1978i	-0.0060	3.1978i	0.0000	3.1978i	-0.0060	3.1979i
19	0	3.5629i	-0.0062	3.5629i	-0.0000	3.5629i	-0.0061	3.5624i
20	0	3.9478i	-0.0063	3.9478i	0.0000	3.9478i	-0.0063	3.9478i

Table 1: Open-Loop Eigenvalues and Closed-Loop Eigenvalues of Various Control Schemes

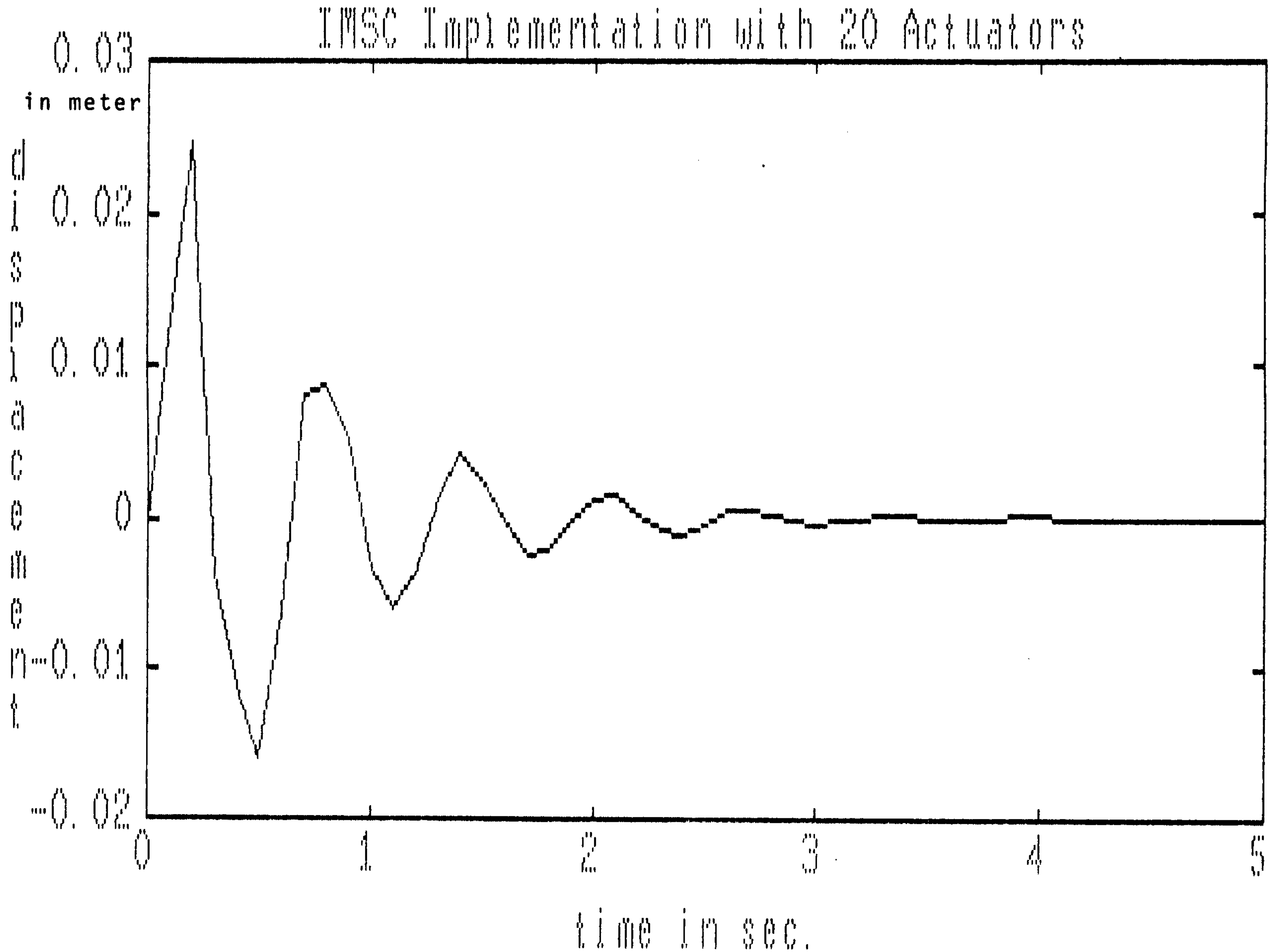


Figure 3: Time History of the Vertical Displacement of the Middle of the Beam When the IMSC is implemented by 20 Actuators.

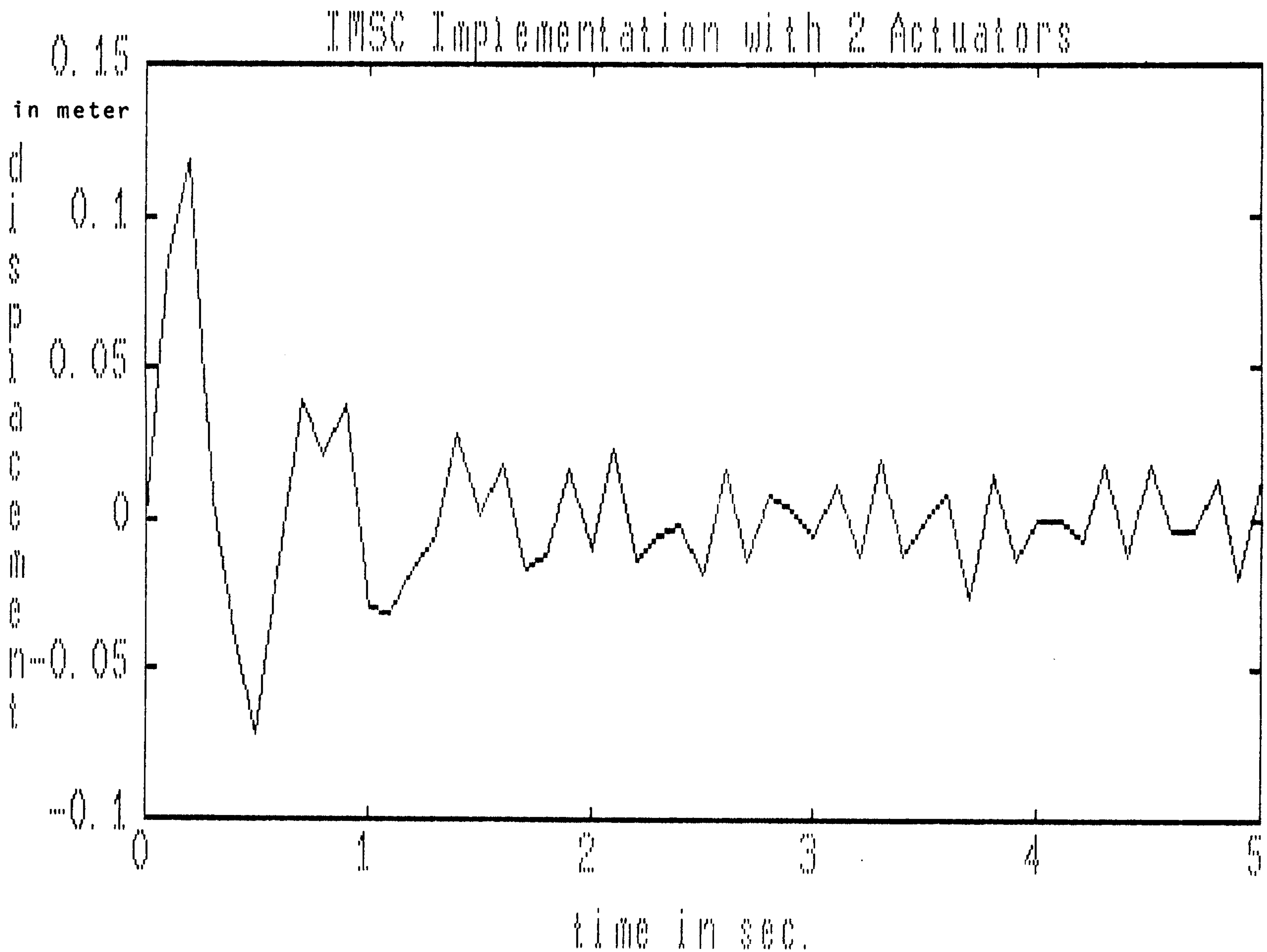
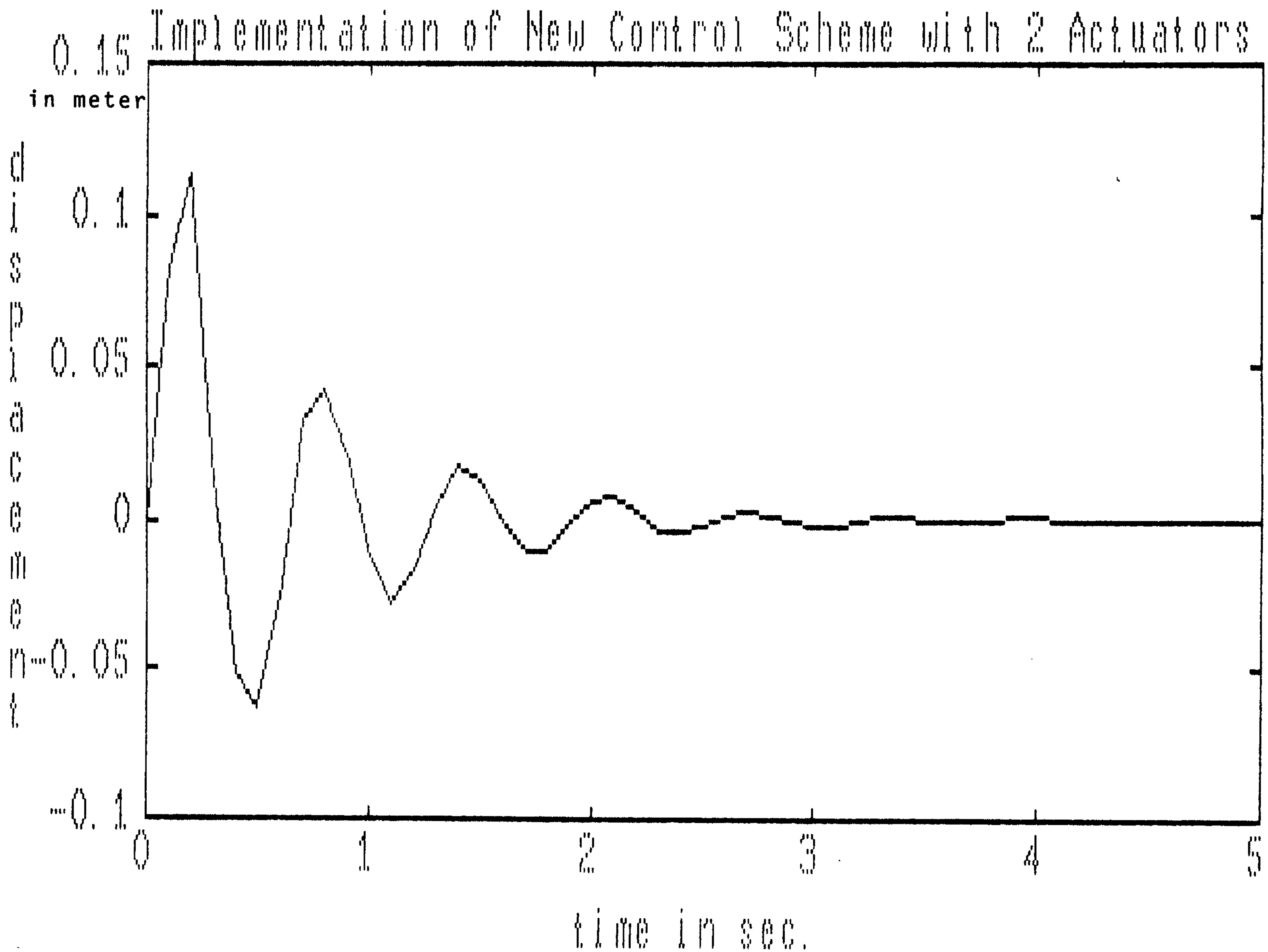


Figure 4: Time History of the Vertical Diplacement of the Middle of the Beam When the IMSC is implemented by 2 Actuators.



<u>Figure 5</u>: Time History of the Vertical Displacement of the Middle of the Beam When the new Control Scheme is implemented by 2 Actuators.

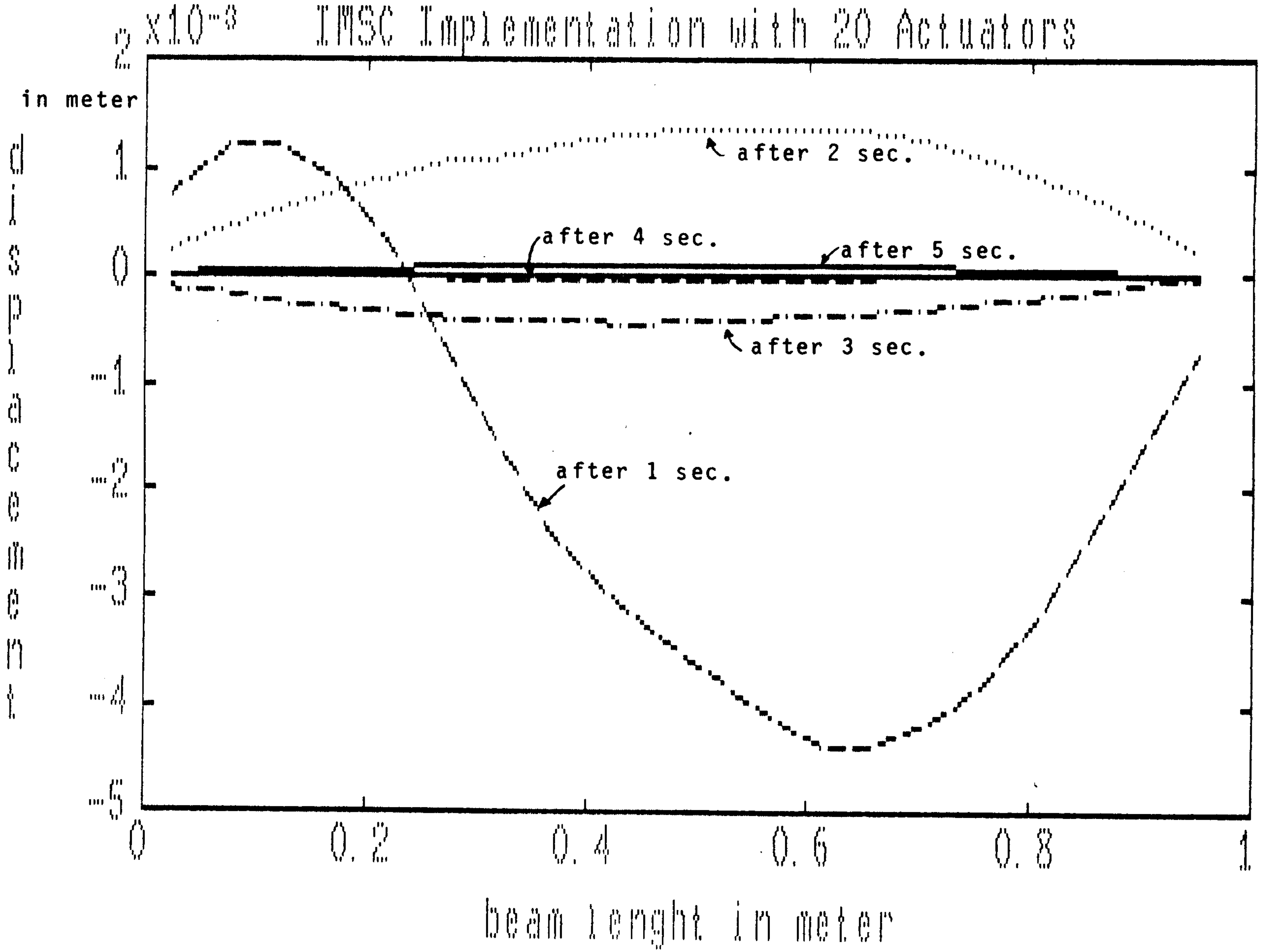


Figure 6: Beam Response When the IMSC is implemented by 20 Actuators.

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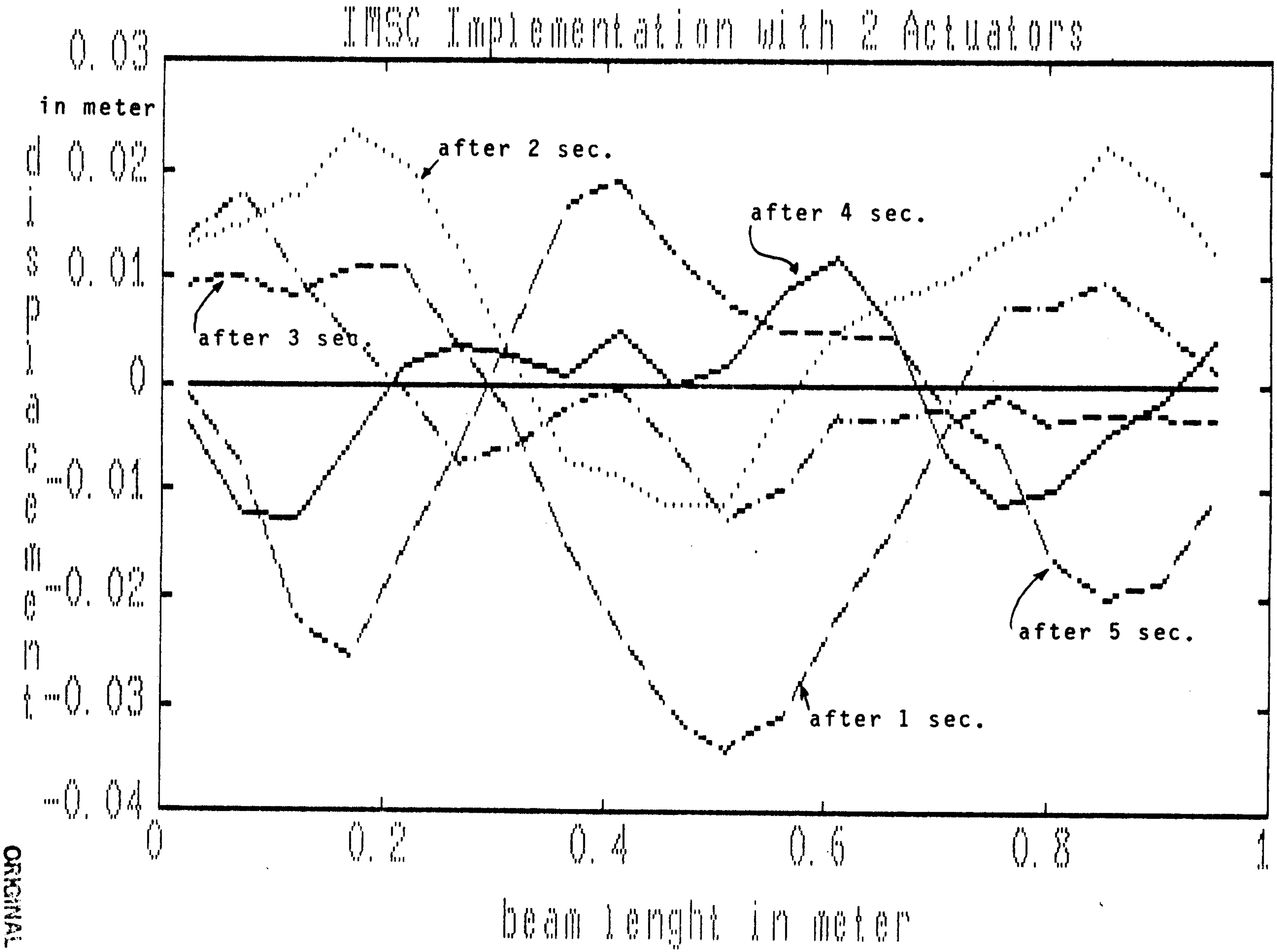

Figure 7: Beam Response When the IMSC is implemented by 2 Actuators.

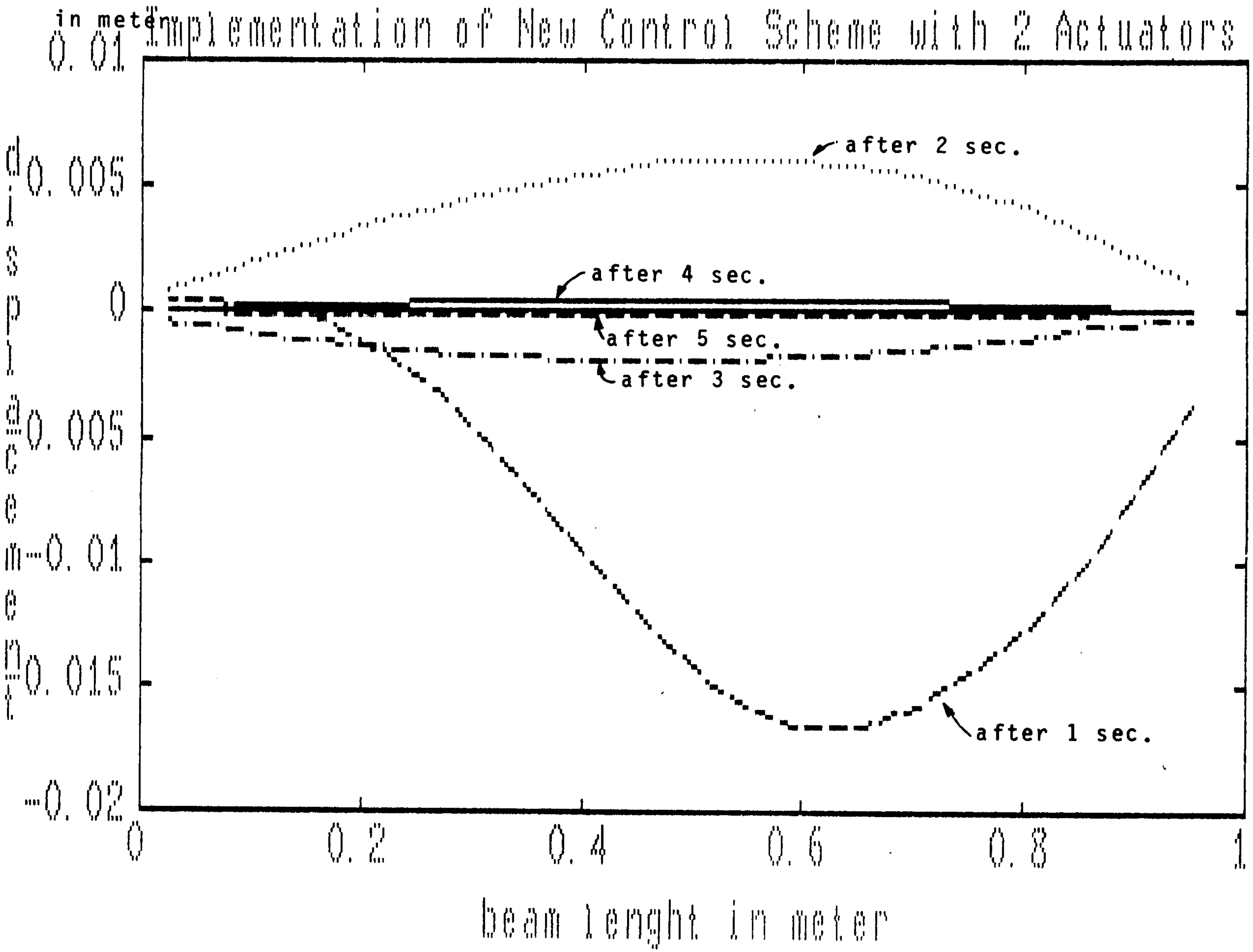

Figure 8: Beam Response When the new Control Scheme is implemented by 2 Actuators.