

**NASA Contractor Report 181712**

**ICASE REPORT NO. 88-53**

# ICASE

IMPROVING THE ACCURACY OF CENTRAL DIFFERENCE SCHEMES

Eli Turkel

Contract No. NAS1-18107  
September 1988

(NASA-CR-181712) IMPROVING THE ACCURACY OF  
CENTRAL DIFFERENCE SCHEMES Final Report  
(NASA) 15 p OCSL 12A

N89-10577

Unclas  
G3/64 0166492

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING  
NASA Langley Research Center, Hampton, Virginia 23665

Operated by the Universities Space Research Association

**NASA**

National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23665



# IMPROVING THE ACCURACY OF CENTRAL DIFFERENCE SCHEMES

Eli Turkel<sup>1</sup>

School of Mathematical Sciences  
Sackler Faculty of Exact Sciences  
Tel-Aviv University

## ABSTRACT

Central difference approximations to the fluid dynamic equations require an artificial viscosity in order to converge to a steady state. This artificial viscosity serves two purposes. One is to suppress high frequency noise which is not damped by the central differences. The second purpose is to introduce an entropy-like condition so that shocks can be captured. These viscosities need a coefficient to measure the amount of viscosity to be added. In the standard scheme, a scalar coefficient is used based on the spectral radius of the Jacobian of the convective flux. However, this can add too much viscosity to the slower waves. Hence, we suggest using a matrix viscosity. This gives an appropriate viscosity for each wave component. With this matrix valued coefficient, the central difference scheme becomes closer to upwind biased methods.

---

<sup>1</sup>This research was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18107 while the author was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665.



## I. Introduction

In recent years, central difference schemes have been used with much success to solve transonic flow problems about aerodynamic shapes. These schemes are second order accurate for sufficiently smooth meshes and have an added artificial viscosity to stabilize the scheme and reach a steady state. This artificial viscosity is usually a blend of two terms. One is a fourth difference that stabilizes the even-odd modes that appear with central differences and constants coefficients. Without this viscosity, one cannot reduce the residual beyond some level because of a remaining high frequency mode. The second viscosity term is a nonlinear second difference that limits oscillations in the neighborhood of shocks. A nonlinear shock detector preserves the second order accuracy of the scheme in smooth regions.

An advantage of the artificial viscosity approach is that it allows the user control over the amount of dissipation. Nevertheless, one sometimes finds that there is too much dissipation in the numerical solution. Changing the global constants that appear in the formulas is not sufficient to construct an artificial viscosity that is appropriate in both the shocked and smooth regions of the flow. For some problems, we need to severely limit the viscosity in some smooth regions, e.g., near the trailing edge, while still maintaining stability near the shocks. Hence, although the standard artificial viscosity works well in most cases, it is not sufficiently flexible to handle more delicate problems.

In order to improve the existing artificial viscosity, we shall make use of ideas used in the construction of upwind schemes. In particular, we shall replace the scalar coefficient in the artificial viscosity by a matrix. To prevent difficulties near stagnation points, a cutoff is introduced that depends on the spectral radius of the matrix. By varying the cutoff, one can obtain an appropriate average between the original scalar viscosity and the new matrix viscosity. Since the matrix viscosity reduces the amount of smoothing on the slower waves, it will improve the total accuracy of the scheme.

## II. Finite Volume Formulation and Artificial Viscosity

The Euler equations for an inviscid compressible flow can be written in divergence form as

$$\frac{\partial Q}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = 0 \quad (1)$$

where

$$Q = (\rho, \rho u, \rho v, \rho w, E)^t \quad (2a)$$

$$f = (\rho u, \rho u^2 + p, \rho uv, \rho uw, (E + p)u)^t \quad (2b)$$

$$g = (\rho v, \rho uv, \rho v^2 + p, \rho vw, (E + p)v)^t \quad (2c)$$

$$h = (\rho w, \rho uw, \rho vw, \rho w^2 + p, (E + p)w)^t \quad (2d)$$

and for an ideal gas

$$p = (\gamma - 1)[E - \rho(u^2 + v^2 + w^2)/2]. \quad (2e)$$

We can also write (1) in the form

$$\frac{\partial Q}{\partial t} + \text{div}(F) = 0. \quad (1b)$$

We integrate (1) over a three dimensional cell and consider  $Q_{i,f,k}$  as an approximation to the average of  $Q$  over the cell. Hence,

$$\frac{\partial Q_{i,j,k}}{\partial t} + \frac{\int \int \int \text{div} F dV}{\int \int \int dV} = 0$$

or using the divergence theorem,

$$\frac{\partial}{\partial t}(VQ)_{ijk} + \int \int \vec{F} \cdot n dS = 0. \quad (3)$$

Hence, the time change of the average  $Q$  is governed by the fluxes entering and leaving the cell.

This finite volume approach leads to a pure central difference method for Cartesian grids. Though this scheme is stable for constant coefficient hyperbolic equations it is subject to instabilities that will prevent the convergence to a steady state. To enhance this convergence a fourth difference viscosity is added to the scheme. The fourth difference causes oscillations in the neighborhood of shocks. Hence, a shock detector is constructed and near the shocks the fourth difference is turned off and only the nonlinear second difference is operative. The total artificial viscosity,  $\bar{V}$ , is the sum of such second and fourth differences in each coordinate direction.

$$\begin{aligned} \bar{V}_{tot} = & \bar{V}_{i+\frac{1}{2},j,k}^{\xi} - \bar{V}_{i-\frac{1}{2},j,k}^{\xi} + \bar{V}_{i,j+\frac{1}{2},k}^{\eta} - \bar{V}_{i,j-\frac{1}{2},k}^{\eta} \\ & + \bar{V}_{i,j,k+\frac{1}{2}}^{\zeta} - \bar{V}_{i,j,k-\frac{1}{2}}^{\zeta} \end{aligned} \quad (4)$$

Hence it is sufficient to describe these terms in the  $\xi$  direction. Since we only take differences at neighboring points the artificial viscosity is always in conservation form.

The first difference is defined as

$$D_{i+\frac{1}{2},j,k} = Q_{i+1,j,k} - Q_{i,j,k} \quad (5a)$$

and the second  $\xi$  difference is defined as

$$E_{i,j,k} = D_{i+\frac{1}{2},j,k} - D_{i-\frac{1}{2},j,k}. \quad (5b)$$

We then form the second and fourth differences. In particular the fourth difference is formed as a second difference of a second difference with positive weights [3,8]. Hence,

$$\bar{V}_{i+\frac{1}{2},j,k}^{\xi} = \epsilon_{i+\frac{1}{2},j,k}^{(2)} D_{i+\frac{1}{2},j,k} - (\epsilon_{i+1,j,k}^{(4)} E_{i+1,j,k} - \epsilon_{i,j,k}^{(4)} E_{i,j,k}). \quad (6)$$

Let,

$$\nu_{i,j,k} = \left| \frac{p_{i+1,j,k} - 2p_{i,j,k} + p_{i-1,j,k}}{p_{i+1,j,k} + 2p_{i,j,k} + p_{i-1,j,k}} \right|. \quad (7a)$$

$\nu_{i,j,k}$  is used to detect the location of shocks. When  $\nu_{i,j,k}$  is large then the fourth difference is reduced. Let,

$$\sigma_{i+\frac{1}{2},j,k} = K^{(2)} \max(\nu_{i-1,j,k}, \nu_{i,j,k}, \nu_{i+1,j,k}, \nu_{i+2,j,k}). \quad (8)$$

We also multiply  $\sigma$  by a function of the Mach number to reduce  $\sigma$  near the surface. Let,  $A = \frac{\partial \bar{F}}{\partial Q}$ ,  $B = \frac{\partial \bar{G}}{\partial Q}$ ,  $C = \frac{\partial \bar{H}}{\partial Q}$ , where  $\bar{F}$ ,  $\bar{G}$ ,  $\bar{H}$  are the fluxes in the coordinate system  $(\xi, \eta, \zeta)$ . Let  $\lambda$  be a measure of the fluxes. The original code chose  $\lambda$  as

$$\lambda^{\xi} = \lambda^{\eta} = \lambda^{\zeta} = \rho(A) + \rho(B) + \rho(C) \quad (9a)$$

where  $\rho$  is the the spectral radius of the matrix. For problems with a highly stretched mesh it was found [2,3,8] that for increased accuracy one should choose

$$\lambda^{\xi} = \rho(A), \quad \lambda^{\eta} = \rho(B), \quad \lambda^{\zeta} = \rho(C). \quad (9b)$$

$K^{(2)}$ ,  $K^{(4)}$  are constants that determine the level of the second and fourth differences. These constants are given as input to the code. Then

$$\epsilon_{i+\frac{1}{2},j,k}^{(2)} = \lambda_{i+\frac{1}{2},j,k} \sigma_{i+\frac{1}{2},j,k} \quad (10a)$$

$$\epsilon_{i,j,k}^{(4)} = \lambda_{i,j,k} \max(0, K^{(4)} - \sigma_{i,j,k}). \quad (10b)$$

In order to imitate the upwind type [7] algorithms we now replace the scalars in (9b) by matrices. Hence,

$$\lambda^{\xi} = |A|, \quad \lambda^{\eta} = |B|, \quad \lambda^{\zeta} = |C| \quad (9c)$$

where  $|A| = T|\Lambda_{\xi}|T^{-1}$  when  $A = T\Lambda_{\xi}T^{-1}$  and  $\Lambda_{\xi}$  is a diagonal matrix with the eigenvalues of  $A$  as its entries. This definition of  $\lambda$  can lead to difficulties when an eigenvalue is near zero. Hence, we modified  $\Lambda_{\xi}$  to be

$$\bar{\Lambda}_{\xi} = \text{diag}(\max(a_i, q\rho(A))) \quad a_i = \text{e.v. of } A \quad (11)$$

where  $q$  is a specified constant. When  $q = 1$ , then  $\Lambda_\xi = \rho(A) \cdot I$  and so (9b) is recovered. When  $q = 0$  then no modification of  $\Lambda_\xi$  is done. In general, we found that  $q = 0.2$  gives good results.

We point out that the use of (9c) does not allow for a constant enthalpy solution and so enthalpy damping cannot be used [5].

### III. Results

We consider the central difference code with Runge-Kutta time stepping [3,5]. As described above we use a matrix valued artificial viscosity which approximates TVD type schemes [6,7,9]. The fourth difference viscosity is still needed to allow the multigrid acceleration to quickly reach a steady state. We consider inviscid flow about a NACA0012 with  $M_\infty = 0.8$ ,  $\alpha = 1.25^\circ$ . A  $192 \times 32$  C mesh is used with 128 points on the airfoil. In [1] it is pointed out that the standard code smears the weak shock on the lower surface. In Figure 1, we plot the result for the standard scheme, but without enthalpy damping. In Figure 2, we show the same case but using the matrix viscosity. The convergence rate is slowed down since the fourth order viscosity is not as strong but the shock on the lower surface is sharper. There is an overshoot on the shock on the upper surface. This is due to fact that the cutoff (10b) is not sufficiently sharp. One way to improve this is to replace (7) by

$$\nu_{i,j,k} = \frac{|p_{i+1,j,k} - p_{i,j,k}| - |p_{i,j,k} - p_{i-1,j,k}|}{|p_{i+1,j,k} - p_{i,j,k}| + |p_{i,j,k} - p_{i-1,j,k}| + \epsilon} \quad (7b)$$

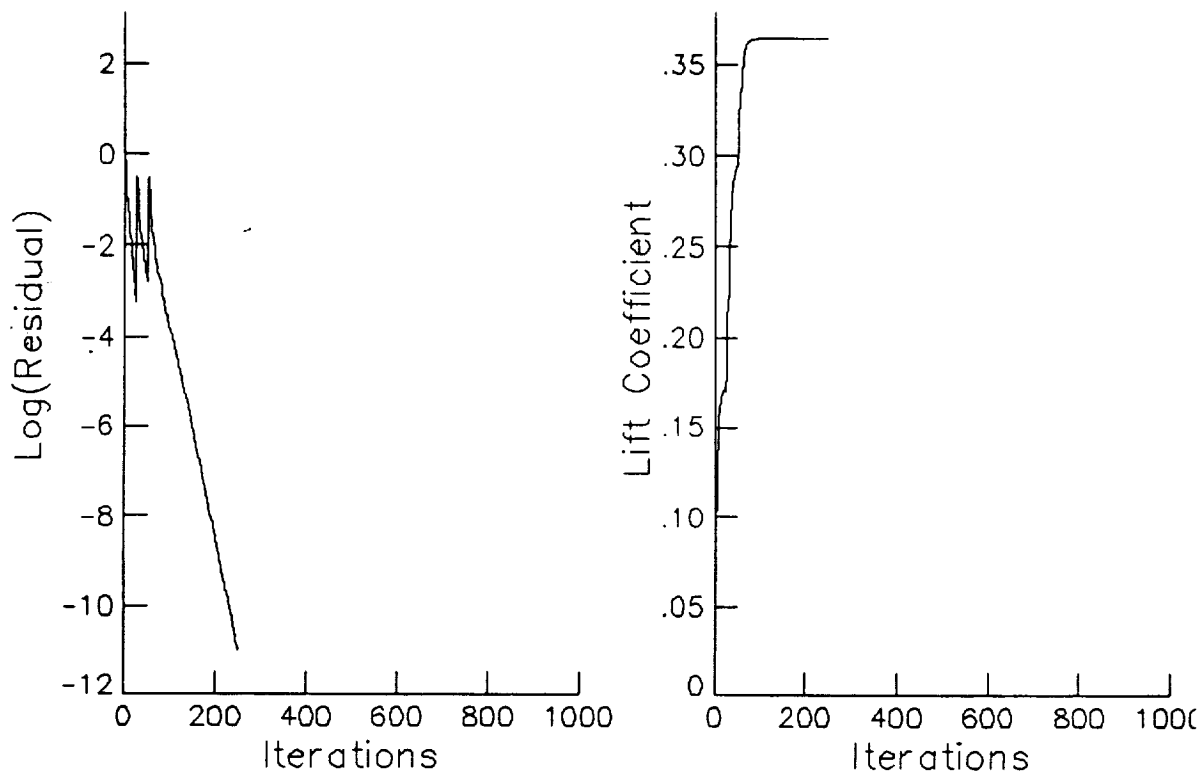
so that  $\nu_{i,j,k}$  is one at discontinuities. One can also use the matrix coefficient only for the second difference but use a scalar coefficient for the fourth difference viscosity. This accelerates the convergence to a steady state but smears the shocks. The results presented used a four step Runge-Kutta algorithm with the artificial viscosity frozen after the first stage using a matrix viscosity rather than a scalar viscosity adds about 60% to the total CPU time.

## References

- [1] Anderson, W., Kyle, Thomas, J. L., van Leer, B., Comparison of Finite Volume Flux Vector Splittings for the Euler Equations, AIAA J., Vol. 24, pp. 1453-1460 (1986).
- [2] Caughey, D. A., Turkel, E., Effects of Numerical Dissipation on Finite-Volume Solutions of Compressible Flow Problems, AIAA-88-0621 (1988).
- [3] Chima, R. B., Turkel, E., Schaffer, S., Comparison of Three Explicit Multigrid Methods for the Euler and Navier-Stokes Equations, AIAA-87-0602 (1987).
- [4] Harten, A., The Artificial Compression Method for Computation of Shocks and Contact Discontinuities III. Self-Adjusting Hybrid Schemes, Math. Comput., Vol. 32, pp. 363-389 (1978).



- [5] Jameson, A., Schmidt, W., Turkel, E., Numerical Solutions of the Euler Equations by Finite Volume Methods using Runge-Kutta Time-Stepping Schemes, AIAA-81-1259 (1981).
- [6] Osher, S., Tadmor, E., On the Convergence of Difference Approximations to Scalar Conservation Laws, *Math. Comput.*, Vol. 50, pp. 19-51 (1988).
- [7] Roe, P. L., Approximate Riemann Solvers Parametric Vectors and Difference Schemes, *J. Comput. Phys.*, Vol. 43, pp. 357-372 (1981).
- [8] Swanson, R. C., Turkel, E., Artificial Dissipation and Central Difference Schemes for the Euler and Navier-Stokes Equations, AIAA 8th CFD Conference, AIAA-87-1107-CP (1987).
- [9] Tadmor, E., Convenient Total Variation Diminishing Conditions for Nonlinear Differences Schemes, *SIAM J. Numer. Anal.*, to appear.
- [10] Turkel, E., Acceleration to a Steady State for the Euler Equations, *Numerical Methods for the Euler equations of Fluid Dynamics*, pp. 281-311, F. Angrand, et al. (editors), SIAM, Philadelphia (1985).
- [11] Turkel, E., van Leer, B., Flux Vector Splitting and Runge-Kutta Methods for the Euler Equations, 9th Inter. Conf. Numer. Methods Fluid Dynamics, Springer-Verlag Lecture Notes Physics, Vol. 208, pp. 566-570 (1984).
- [12] van Leer, B., Towards the Ultimate Conservative Difference Scheme, II. Monotonicity and Conservation Combined in a Second-Order Scheme, *J. Comput. Phys.*, Vol. 14, pp. 361-370 (1974).
- [13] Yee, H. C., Upwind and Symmetric Shock-Capturing Schemes, NASA TM 89464 (1987).



NACA 0012 FLO53  
 192 X 33 GRID      M = .800      ALPHA = 1.250      CD, CL = .02365      .365

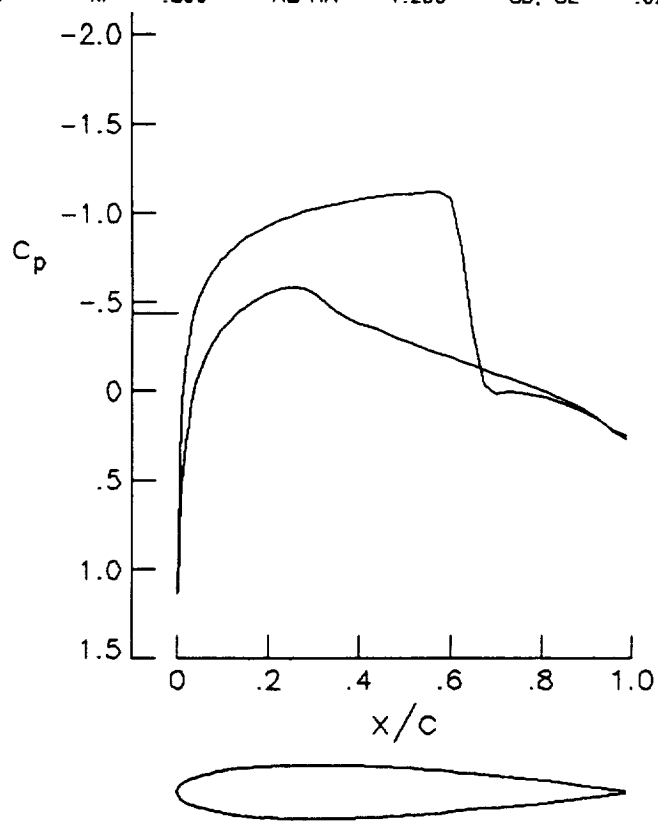
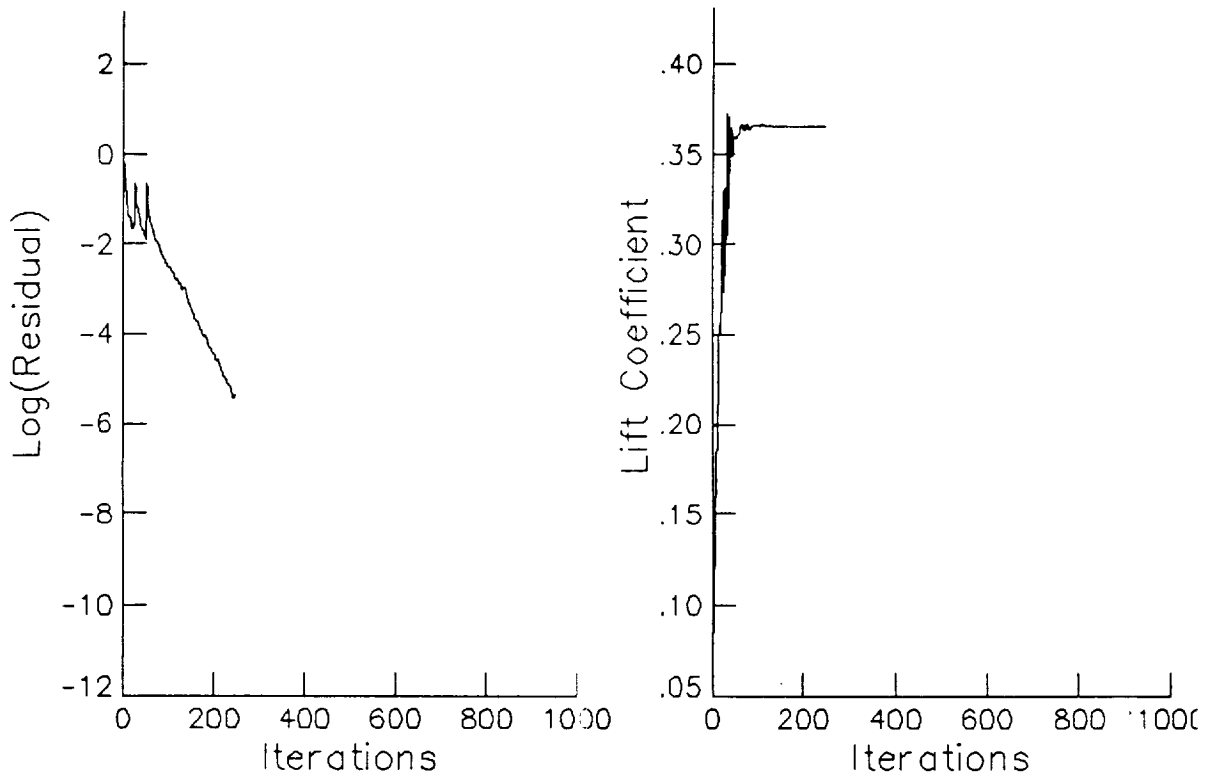


Figure 1: Scalar viscosity



NACA 0012 FLO53-VEPS  
 192 X 33 GRID M = .800 ALPHA = 1.250 CD. CL = .02338 .365

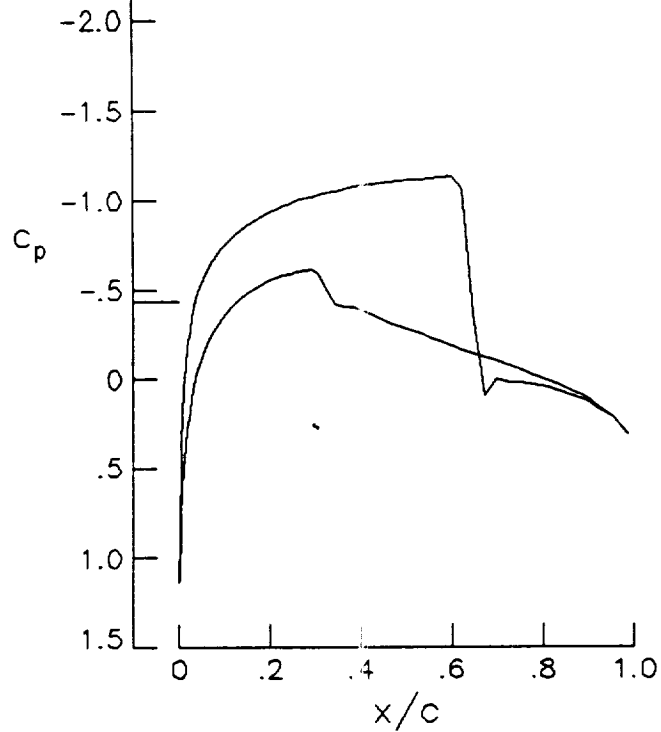


Figure 2: Matrix viscosity

## Appendix

We present the matrix viscosity in explicit form for three dimensions in general curvilinear form. Let

$$\xi = \xi(x, y, z), \quad \eta = \eta(x, y, z), \quad \zeta = \zeta(x, y, z)$$

be a change of variables from physical space  $(x, y, z)$  to a computational space  $(\xi, \eta, \zeta)$  such that the curvilinear mesh is mapped to a cube. Then (1b) can be rewritten as

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} + \frac{\partial H}{\partial \zeta} = 0 \quad (A1)$$

where

$$\begin{aligned} F &= \xi_x f + \xi_y g + \xi_z h \\ G &= \eta_x f + \eta_y g + \eta_z h \\ H &= \zeta_x f + \zeta_y g + \zeta_z h. \end{aligned} \quad (A2)$$

We next express (A1) in quasilinear form

$$\frac{\partial Q}{\partial t} + A \frac{\partial Q}{\partial \xi} + B \frac{\partial Q}{\partial \eta} + C \frac{\partial Q}{\partial \zeta} = 0 \quad (A3)$$

$$A = \frac{\partial F}{\partial Q} \quad B = \frac{\partial G}{\partial Q} \quad C = \frac{\partial H}{\partial Q}.$$

Let

$$\begin{aligned} q &= \xi_x u + \xi_y v + \xi_z w \\ r &= \eta_x u + \eta_y v + \eta_z w \\ s &= \zeta_x u + \zeta_y v + \zeta_z w \end{aligned}$$

be the three contravariant velocities. Also define  $h = \frac{E+p}{\rho}$  as the total enthalpy and let  $V^2 = \frac{(\gamma-1)}{2}(u^2 + v^2 + w^2)$  then

$$A = \begin{pmatrix} 0 & a_1 & a_2 & a_3 & 0 \\ a_1 V^2 - uq & q - (\gamma - 2)a_1 u & a_2 u - (\gamma - 1)a_1 v & a_3 u - (\gamma - 1)a_1 w & (\gamma - 1)a_1 \\ a_2 V^2 - vq & a_1 v - (\gamma - 1)a_2 u & q - (\gamma - 2)a_2 v & a_3 v - (\gamma - 1)a_2 w & (\gamma - 1)a_2 \\ a_3 V^2 - wq & a_1 w - (\gamma - 1)a_3 u & a_2 w - (\gamma - 1)a_3 v & q - (\gamma - 2)a_3 w & (\gamma - 1)a_3 \\ -q(h - V^2) & a_1 h - (\gamma - 1)qh & a_2 h - (\gamma - 1)qv & a_3 h - (\gamma - 1)qw & \gamma q \end{pmatrix} \quad (A4)$$

$$a_1 = \xi_x, \quad a_2 = \xi_y, \quad a_3 = \xi_z.$$

For  $B$  we get a similar matrix with  $a_1 = \eta_x$ ,  $a_2 = \eta_y$ ,  $a_3 = \eta_z$  and  $q$  replaced by  $r$  while for  $C$  we have  $a_1 = \zeta_x$ ,  $a_2 = \zeta_y$ ,  $a_3 = \zeta_z$  and  $q$  replaced by  $s$ .

Hence, we can find the absolute value of all three matrices in the same way. Let us assume that  $A$  has eigenvalues

$$A = \begin{pmatrix} \lambda_1 & & & & \\ & \lambda_2 & & 0 & \\ & & \lambda_3 & & \\ & 0 & & \lambda_3 & \\ & & & & \lambda_3 \end{pmatrix} \quad (A5)$$

where  $\lambda_1 = q + \sqrt{a_1^2 + a_2^2 + a_3^2}c$ ,  $\lambda_2 = q - \sqrt{a_1^2 + a_2^2 + a_3^2}c$  and  $\lambda_3 = q$ . In practice  $\lambda_j$  are redefined by (11) to prevent  $\lambda_j$  from approaching zero.

In order to find  $|A|$  it is easier to use a two step procedure. Let

$$A_1 = T_1 A T_1^{-1} \quad (A6)$$

be a symmetric matrix. Since we can symmetrize  $A, B, C$  simultaneously the same  $T_1$  will work for all three matrices.

$$T_1 = \begin{pmatrix} \frac{v^2}{c} & -(\gamma-1)u/c & -(\gamma-1)v/c & -(\gamma-1)w/c & (\gamma-1)/c \\ -u & 1 & 0 & 0 & 0 \\ -v & 0 & 1 & 0 & 0 \\ -w & 0 & 0 & 1 & 0 \\ -c^2 + V^2 & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-1)w & \gamma-1 \end{pmatrix} \quad (A7)$$

$$T_1^{-1} = \begin{pmatrix} \frac{1}{c} & 0 & 0 & 0 & -\frac{1}{c^2} \\ \frac{u}{c} & 1 & 0 & 0 & -\frac{u}{c^2} \\ \frac{v}{c} & 0 & 1 & 0 & -\frac{v}{c^2} \\ \frac{w}{c} & 0 & 0 & 1 & -\frac{w}{c^2} \\ \frac{h}{c} & u & v & w & \frac{u^2 + v^2 + w^2}{2c^2} \end{pmatrix}.$$

As expected  $T_1, T_1^{-1}$  do not depend on the matrix  $a_1, a_2, a - 3$ .

$$A_1 = \begin{pmatrix} q & a_1c & a_2c & a_3c & 0 \\ a_1c & q & 0 & 0 & 0 \\ a_2c & 0 & q & 0 & 0 \\ a_3c & 0 & 0 & q & 0 \\ 0 & 0 & 0 & 0 & q \end{pmatrix}. \quad (A8)$$

Since  $A_1$  (and  $B_1, C_1$ ) is symmetric we can diagonalize it with a unitary matrix  $T_2$ .  $T_2$  will depend explicitly on  $a_1, a_2, a_3$  and so is different for  $A_1, B_1, C_1$ . Let  $\mathcal{A} = \sqrt{a_1^2 + a_2^2 + a_3^2} \neq 0$ , then

$$T_2 = \frac{1}{\sqrt{2\mathcal{A}}} \begin{pmatrix} \mathcal{A} & a_1 & a_2 & a_3 & 0 \\ -\mathcal{A} & a_1 & a_2 & a_3 & 0 \\ 0 & x_1 & x_2 & x_3 & 0 \\ 0 & y_1 & y_2 & y_3 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2\mathcal{A}} \end{pmatrix} \quad (A9)$$

and

$$T_2^{-1} = T_2^t.$$

The  $(x_j, y_j)$  are numbers that satisfy the following equations

$$\begin{aligned} x_1^2 + x_2^2 + x_3^2 &= y_1^2 + y_2^2 + y_3^2 = 2\mathcal{A}^2 \\ a_1x_1 + a_2x_2 + a_3x_3 &= a_1y_1 + a_2y_2 + a_3y_3 = 0 \\ x_1y_1 + x_2y_2 + x_3y_3 &= 0 \\ x_1y_1 + x_2y_2 + x_3y_3 &= 0 \\ x_1x_2 + y_1y_2 &= -2a_1a_2 \\ x_1x_3 + y_1y_3 &= -2a_1a_3 \\ x_2x_3 + y_2y_3 &= -2a_2a_3. \end{aligned} \quad (A10)$$

It is difficult to give explicit formula for the  $x_j, y_j$  in all cases since some of the  $a_j$  may be zero as long as  $a_1^2 + a_2^2 + a_3^2 \neq 0$ . However, the final formula does not depend on explicitly knowing the  $a_j$ . Given  $T_2$  we find that  $\Lambda$  (A5) is given by

$$\Lambda = T_2 A_1 T_2^{-1} = T_2 T_1 A (T_2 T_1)^{-1}. \quad (A11)$$

We now reverse the process and define

$$|A| = \begin{pmatrix} |\lambda_1| & & & & \\ & |\lambda_2| & & & \\ & & |\lambda_3| & & \\ & & & |\lambda_3| & \\ & & & & |\lambda_3| \end{pmatrix} \quad (A12)$$

where  $\lambda_j$  can be modified eigenvalues of  $A$ . Then

$$|A| = (T_2 T_1)^{-1} |A| (T_2 T_1). \quad (A13)$$

Let

$$\sigma_1 = \frac{\lambda_1 + \lambda_2}{2} \quad \sigma_2 = \frac{\lambda_1 - \lambda_2}{2} \quad (A14)$$

and define the row vectors

$$\ell_1 = (\gamma - 1) \left( \frac{u^2 + v^2 + w^2}{2}, -u, -v, -w, 1 \right) \quad (A15)$$

$$\ell_2 = (-q, a_1, a_2, a_3, 0).$$

We then have the matrices

$$I = 5 \times 5 \text{ identity member}$$

$$Z_1 = (\ell_1, u\ell_1, v\ell_1, w\ell_1, h\ell_1)^t$$

$$Z_2 = (\ell_2, u\ell_2 + a_1\ell_1, v\ell_2 + a_2\ell_1, w\ell_2 + a_3\ell_1, h\ell_2 + q\ell_1)^t$$

$$Z_3 = (0, a_1\ell_2, a_2\ell_2, a_3\ell_2, q\ell_2)^t$$

and finally,

$$|A| = \lambda_3 I + \left( \frac{\sigma_1 - \lambda_3}{c^2} \right) Z_1 + \frac{\sigma_2}{AC} Z_2 + \left( \frac{\sigma_1 - \lambda_3}{A^2} \right) Z_3 \quad (A17)$$

Because of the simple nature of the matrices  $Z_j$  it is easy to multiply  $|A|$  times a vector. Define,

$$r_1 = (1, u, v, w, h)^t$$

$$r_2 = (0, a_1, a_2, a_3, q)^t.$$

Let  $(, )$  denote the standard inner product, then

$$(\ell_1, r_1) = c^2$$

$$(\ell_1, r_2) = 0$$

$$(\ell_2, r_1) = 0$$

$$(\ell_2, r_2) = \mathcal{A}^2$$

and if  $x$  is any column vector, then

$$|A|x = \lambda_3 x + \left[ \left( \frac{\sigma_1 - \lambda_3}{c^2} \right) (\ell_1, x) + \frac{\sigma_2}{\mathcal{A}c} (\ell_2, x) \right] r_1 + \left[ \frac{\sigma_2}{\mathcal{A}c} (\ell_1, x) + \frac{(\sigma_1 - \lambda_3)}{\mathcal{A}^2} (\ell_2, x) \right] r_2. \quad (A18)$$







# Report Documentation Page

1. Report No. NASA CR-181712 ICASE Report No. 88-53		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle IMPROVING THE ACCURACY OF CENTRAL DIFFERENCE SCHEMES				5. Report Date September 1988	
				6. Performing Organization Code	
7. Author(s) Eli Turkel				8. Performing Organization Report No. 88-53	
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225				10. Work Unit No. 505-90-21-01	
				11. Contract or Grant No. NAS1-18107	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225				13. Type of Report and Period Covered Contractor Report	
				14. Sponsoring Agency Code	
15. Supplementary Notes Langley Technical Monitor: Richard W. Barnwell  Final Report  Submitted to the 11th International Conference on Numerical Methods and Fluid Dynamics					
16. Abstract <p>Central difference approximations to the fluid dynamic equations require an artificial viscosity in order to converge to a steady state. This artificial viscosity serves two purposes. One is to suppress high frequency noise which is not damped by the central differences. The second purpose is to introduce an entropy-like condition so that shocks can be captured. These viscosities need a coefficient to measure the amount of viscosity to be added. In the standard scheme, a scalar coefficient is used based on the spectral radius of the Jacobian of the convective flux. However, this can add too much viscosity to the slower waves. Hence, we suggest using a matrix viscosity. This gives an appropriate viscosity for each wave component. With this matrix valued coefficient, the central difference scheme becomes closer to upwind biased methods.</p>					
17. Key Words (Suggested by Author(s)) artificial viscosity, Runge-Kutta, Euler equations			18. Distribution Statement 64 - Numerical Analysis  Unclassified - unlimited		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of pages 14	22. Price A02