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Effects of Ordinary and Superconducting Cosmic Strings on Primordial Nucleosynthesis

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Abstract

We do a precise calculation of the primordial nucleosynthesis constraint on the energy per length of ordinary and superconducting cosmic strings. For ordinary strings we provide a general formula for the constraint on the string tension. Using the current values for the various parameters that describe the evolution of loops, the constraint for ordinary strings is $G\mu < 2.2 \times 10^{-5}$. Our constraint is weaker than previously quoted limits by a factor of ≈ 5. For superconducting loops, with currents generated by primordial magnetic fields, the constraint can be less stringent or more stringent than this limit, depending upon the strength of the magnetic field. We also find, in this case, that there is a negligible amount of entropy production if the electromagnetic radiation from strings thermalizes with the radiation background.

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I. Introduction

In the past few years there has been considerable interest in cosmic strings, both the ordinary and superconducting varieties. The simplest realization of these strings is by the breaking of a $U(1)$ symmetry by a complex scalar field [1] (ordinary strings), and by the breaking of a $U(1) \otimes U(1)'$ symmetry by two complex scalar fields [2] (superconducting strings) in the early Universe. The essential parameter of the theory is the energy per length μ, which is related to the scale of symmetry breaking. Here we constrain the string tension by requiring that the inclusion of strings in our Universe leads to a primordial nucleosynthesis scenario that is observationally acceptable.

Ordinary cosmic strings are cosmologically interesting because they can produce the density fluctuations required for galaxy formation [3], provided the energy per length μ is $G\mu \approx 10^{-6}$ (a typical GUT scale) . However, loops of string radiate gravitationally at a rate that allows the energy density in gravitons to grow relative to the radiation background during the time that the Universe is radiation dominated. Primordial nucleosynthesis [4] provides an upper limit to the energy density of gravitons produced by loops at $t \approx 1$ sec [5,6,7], which translates to a constraint on the energy per length μ. Previous work [6] indicated that the constraint on the string tension from nucleosynthesis is a factor of a few away from ruling out galaxy formation by cosmic strings.

Superconducting cosmic strings (SCS) may also explain galaxy formation, yet in a very different manner. Superconducting loops, with current, can radiate electromagnetically as well as gravitationally. In the scenario of Ostriker, Thompson, and Witten [8] the currents are generated by primordial magnetic fields (*pmf*), and the electromagnetic radiation from loops of string blow bubbles in the surrounding gas, with galaxy formation taking place on the dense spherical shells of the gas. The success of their scenario depends upon the strength of the *pmf*. Therefore, it is of interest to obtain constraints on the energy per length from nucleosynthesis as a function of the strength of the *pmf*, and check for compatability with the theory of OTW.

On the face of it, since superconducting loops also radiate electromagnetically, with less energy going into gravitons (compared to ordinary strings), one might think that the constraint on μ is less stringent than that of ordinary strings. However, if there is a *pmf* that generates currents in the SCS its energy density will also affect nucleosynthesis (and the constraint on μ). Another issue arises: assuming the electromagnetic radiation from the strings thermalizes with the radiation background, there will be entropy production which will dilute the energy density in gravity waves. loops, and the pmf relative to the background. We examine the size of this effect, and find that it is negligible. Also, the baryon to photon ratio η is known at the time of nucleosynthesis, and at the start of matter domination. We therefore determine, from this knowledge, if entropy production can lead to any new constraints on the energy per length.

The paper is organized as follows: in Sec II. we do a more precise analysis than has previously been done in obtaining a constraint on μ, using primordial nucleosynthesis, for ordinary strings ; in Sec. III. we present a constraint on μ for SCS as a fuction of the pmf strength ; in Sec. IV. we consider the effects of entropy production by SCS; in Sec. V. we summarize our work.

II. Ordinary Strings

We now describe a host of relations that will be essential in the forthcoming analysis. Loops of initial radius L_f continually form by breaking off from intersecting "infinite" strings when the age of the Universe is about $t \approx L_f/\epsilon$ ($\epsilon \approx 0.2$, see Ref. [7]). The birthrate of loops of radius L_f at time t is about [9]:

$$\frac{dn}{dt} = \kappa/t^4 \tag{2.1}$$

(number per volume per time), where $\kappa \simeq \nu\epsilon^{-3/2}$ and $\nu \approx 0.01$ [10]. This birthrate is valid from the time $t_* \sim t_{planck}(G\mu)^{-2}$ when frictional effects on the string network become negligible [11] to the time that the Universe became matter dominated. After formation, the number density of loops at t formed between t_f and $t_f + dt_f$ is $dn(t,t_f) = \kappa R(t_f)^3/t_f^4 R(t)^3 dt_f$ (assuming they have not decayed), where R is the FRW scale factor. For a radiation dominated Universe $R \propto \sqrt{t}$. The loops redshift like a nonrelativistic particle specie.

The energy of a loop of radius L is:

$$M = \beta\mu L \tag{2.2}$$

with $\beta \approx 9$ [10] ($\beta \neq 2\pi$ because the loops oscillate and are not perfectly circular). Loops radiate gravitationally with a power:

$$P_{GW} = \gamma_{GW} G\mu \tag{2.3}$$

where $\gamma_{GW} \approx 50 - 100$ [12].

Using the above relations, and taking into account cosmological expansion, the energy density at time t in gravitational waves (GW) is given by:

$$G\rho_{GW}(t) = a \int_{\tau=t_*}^{\tau=t} \left(\int_{t_f=b\tau/1+b}^{t_f=\tau} \frac{R(t_f)^3 \theta(t_f - t_*)}{R(t)^3 t_f^4} dt_f \right) \frac{R(\tau)}{R(t)} d\tau \tag{2.4}$$

where θ is the unit step function, and the integrals are over the time the loops formed t_f and the time τ the radiation was emitted. We have also introduced the dimensionless parameters:

$$a = \kappa \gamma_{GW} (G\mu)^2 \tag{2.5}$$

$$b = \gamma_{GW} G\mu / \beta\epsilon \tag{2.6}$$

(loops formed at $t = t_f$ will have radiated away all their energy at $t = t_f(1+b)/b$). These parameters are very convienent, as we do not need to specify $G\mu$ or any specific loop parameters. Using the standard values for the various loop parameters and taking $G\mu \sim 10^{-6}$ yields $a \sim 6 \times 10^{-12}$ and $b \sim 3 \times 10^{-5}$. At any time t, the energy density of loops is given by:

$$G\rho_{loop}(t) = \frac{a}{b} \int_{t_f=bt/1+b}^{t_f=t} \frac{R(t_f)^3 (t_f - b(t - t_f)) \theta(t_f - t_*)}{R(t)^3 t_f^4} dt_f \tag{2.7}$$

and the integration is over the formation times t_f of the loops. A reasonable limit on $G\mu$ can be obtained because of the logarithmic growth in time of ρ_{GW}: $\rho_{GW} \sim \sqrt{G\mu} ln(t/t_*)/t^2$. The energy density in loops $\rho_{loop} \sim \sqrt{G\mu}/t^2$ does not grow relative to the radiation background $\rho_{rad} \sim 1/t^2$, and for $t >> t_*$ it is small compared to ρ_{GW} while the Universe is radiation dominated. [We shall ignore the contribution to the energy density in long strings, as the energy density is a factor $\sim \sqrt{G\mu}$ smaller than that in loops.]

The gravitons produced by the loops are noninteracting, and will not share in the entropy release of other particle species. Therefore, an accurate solution to Eqns. (2.4) and (2.7) requires knowledge of what the rest of the Universe is composed of. The bulk of the energy density is in thermal equilibrium, and the equilibrium pressure P_{eq} and energy density ρ_{eq} can easily be calculated by performing the ususal thermodynamic integrals given the masses and degeneracies of the particles (see Table 1). Through the quark-hadron phase transition (100 MeV $\lesssim T \lesssim$ 200 MeV) we use the results of Ref. [13]. The scale factor can then be calculated as a function of temperature:

$$[R(T)T]^3 g(T) = constant \tag{2.8}$$

where

$$g(T) = \frac{3(\rho_{eq} + P_{eq})}{2\rho_{photon}} \tag{2.9}$$

is the effective particle degrees of freedom. In Fig. (1) we show the evolution of $g(T)$ using the particles in Table (1). We see that $g(T)$ varies by a factor of ≈ 30 between the GUT scale and nucleosynthesis. The evolution of the energy density in gravitons created from loops is $\rho_{GW}(t) \propto g(t)^{1/3}/t^2$, so the effect of the variation in $g(t)$ is to decrease the expected amount of ρ_{GW} by roughly a half.

To do the previous integrals we need the scale factor as a function of time. This is accomplished by taking steps in temperature and calculating the corresponding time step from the FRW equation:

$$(\frac{\dot{R}}{R})^2 = \frac{8\pi G}{3}[\rho_{eq} + \rho_{GW} + \rho_{loop}] \tag{2.10}$$

We further take into account that neutrinos go out of equilibrium at $T \approx 1$ MeV, and do not share in the entropy release of e^+, e^- annihilations. For ^{4}He not to be overproduced by a faster expansion rate during nucleosynthesis requires $\rho_{extra} \lesssim 0.15\rho_{total}$ (at $T \sim 1$ MeV), and in the present case $\rho_{extra} = \rho_{loop} + \rho_{GW}$. However, because of the time variation in $(\rho_{GW} + \rho_{loop})/\rho_{total}$, we directly plug this extra contribution to the energy density into Wagoner's nucleosynthesis code and require that the mass fraction of produced ^{4}He be less than 0.254.

We take the start of the integration time to be $t_* = 10^{-32}$ sec, which corresponds to $G\mu \sim 10^{-6}$. To obtain an upper limit on the energy per length we must choose the minimum allowed values for the neutron half-life and baryon to photon ratio (at present):$\tau_{1/2} = 10.4$ min., $\eta = 3 \times 10^{-10}$. We can then determine the parameters a and b that give a helium mass fraction of 0.254. In Fig. (2) we plot the maximum allowed value for a, a_{max}, as a function of b. The curve can be fitted, to 1%, by:

$$a_{max} = 1.54 \times 10^{-4} b^{1.48} \tag{2.11}$$

This relation can be used to solve for the upper limit on $G\mu$ as a function of the loop parameters:

$$G\mu|_{max} = 4.39 \times 10^{-8} \frac{\gamma_{GW}^{0.93} \epsilon^{0.02}}{\beta^{2.86} \nu^{1.93}} \tag{2.12}$$

Using the "standard" values for these parameters ($\gamma_{GW} = 50$, $\beta = 9$, $\nu = 0.01$, $\epsilon = 0.2$) yields the limit:

$$G\mu < 2.2 \times 10^{-5} \tag{2.13}$$

It must be stressed, however, that this limit is very sensitive to the parameters ν and β, which are somewhat uncertain. [Results from another group [14] investigating the evolution of cosmic strings appear to differ quantitatively with the results of Albrecht and Turok.] Our limit differs greatly (a factor of 5) from [6] primarily because they take $\nu = 0.03$. The limit is very insensitive to the start of the integration time, i.e., changing by $\approx 7\%$ for each order of magnitude we are off. Neglecting ρ_{GW} in Eqn (2.10) results in a 23% change in the limit.

Table (1) included only known particles, and beyond the Z boson mass scale we have $g \simeq 106.75$. To investigate the effects of other possible particle species (e.g., supersymmetric particles) we vary g for $T > 500$ GeV by $s \times 106.75$. For $s = 1.5, 3, 10$ the limit on $G\mu$ is weaker by a factor of $1.2, 1.4, 2.1$, respectively. Finally, we compare our limit with that obtained analytically by assuming $g(T)$ is fixed (whence $R(t) \propto \sqrt{t}$) and taking nucleosynthesis to occur at $t \approx 1$ sec. Using the standard loop parameters, our limit is a factor ≈ 3.8 weaker than the simple estimate. Similar conclusions about the effect of the dilution of the gravity waves can be found in Refs. [6,7].

In Fig. (3) we show the evolution of $\Omega_{GW} = \rho_{GW}/\rho_{tot}$ and $\Omega_{loop} = \rho_{loop}/\rho_{tot}$ as a function of the temperature of the Universe for the choice $b = 10^{-4}$ and $a = 1.84 \times 10^{-10}$ (which results in an an extra energy density that is critically allowed). The occasional "bumps" in the plots represent the dilution of loops and gravity waves relative to the radiation background as particle species go out of equilibrium, annihilate, and heat up the background.

III. Superconducting Strings

The procedure of the last section must now be modified to allow for the additional electromagnetic decay mode of the superconducting string. The electromagnetic radiation power is:

$$P_{EM} = \gamma_{EM} J^2 \tag{3.1}$$

where J is the current, and γ_{EM} is a numerical constant (in calculations we take $\gamma_{EM} \approx \gamma_{GW}$, which is expected). [Other power laws for the EM radiation losses have been proposed [15], but a more thorough treatment [16] supports the form of Eqn. (3.1).] We use electromagnetic units here, and elsewhere, that correspond to $e^2 = \alpha_{EM}$, where $\alpha_{EM} \simeq 1/137$ is the electromagnetic coupling constant.

In the following analysis we make several restrictions. One is that the electromagnetic field energy will not be sufficient to stabilize the string tension, creating static or "floating" loops [17,18,19]. Secondly, we assume that the entropy production from the thermalization of the photons radiated from the SCS can be ignored, i.e., we assume there is no substantial dilution of gravity waves, loops, or the pmf by this effect. In Sec. IV we show that this assumption is appropriate. We assume that the pmf is a large-scale coherent field. We ignore the anisotropy introduced by the pmf. The effects of anisotropy on nucleosynthesis can be neglected since a measure of the anisotropy is the ratio of the energy density in the pmf to the total energy density, which is constrained by nucleosynthesis to be $\lesssim 0.15$. When the nucleosynthesis bound is saturated by the pmf the magnetic field strength is high enough to change the neutron half-life [20] and several important reaction rates. However, we ignore these effects which are subdominant compared to the effect of the field on the expansion rate of the Universe [21]. In what follows we reduce the many degrees of freedom by considering only the canonical loop parameters.

We now generalize the equations describing the energy density in loops and gravitational radiation:

$$G\rho_{loop}(t) = \frac{a}{b}\int_{t_f=t/u}^{t_f=t} \frac{R(t_f)^3 x(t,t_f)\theta(t_f - t_*)}{R(t)^3 t_f^3} dt_f \tag{3.2}$$

$$G\rho_{GW}(t) = a\int_{\tau=t_*}^{\tau=t}\left(\int_{t_f=\tau/u}^{t_f=\tau} \frac{R(t_f)^3\theta(t_f - t_*)}{R(t)^3 t_f^4} dt_f\right)\frac{R(\tau)}{R(t)}d\tau \tag{3.3}$$

where $x(t,t_f) = L(t,t_f)/L(t_f)$, ut_f is the lifetime of a loop, and both can be determined from the equation of motion for the loop radius.

A superconducting loop in the presence of a *pmf* of strength B will have an induced current at formation [8]:

$$J_f \approx B_f L_f / 4\pi ln(vL_f) \tag{3.4}$$

where $v \sim \sqrt{\mu}$ is the expectaion value of the Higgs field. The electromagnetic radiation power for a loop formed at t_f, at any time is then:

$$P_{EM} = \gamma_{EM}\rho_{mag}(t_f)L_f^4/2\pi ln^2(vL)L^2 \tag{3.5}$$

where the magnetic energy density $\rho_{mag} = B^2/8\pi$. It is now useful to rewrite this as

$$P_{EM} = \gamma_{GW} G\mu^2 f(t_f)\left(\frac{L_f}{L}\right)^2 \tag{3.6}$$

where we have now taken $\gamma_{EM} \approx \gamma_{GW}$ and defined

$$f(t_f) = \rho_{mag}(t_f)L_f^2/2\pi G\mu^2 ln^2(vL_f) \tag{3.7}$$

(L has been approximated by L_f in the logarithm). Now $\rho_{mag}(t_f) \propto g(t_f)^{1/3}/t_f^2$, and recalling that $L_f \propto t_f$, we see that $f(t_f)$ can vary by about two orders of magnitude from the time loops start forming to the onset of nucleosynthesis. Because of the time variation in $f(t_f)$ it becomes convienent to define a free parameter f_* at the start of integration t_*:

$$f(t_f) = f_* \left(\frac{g(t_f)}{g(t_*)}\right)^{1/3} \left(\frac{ln(\epsilon v t_*)}{ln(\epsilon v t_f)}\right)^2 \tag{3.8}$$

which takes the place of the magnetic field. To allow direct comparison with the f of OTW, we note that $f \approx f(t_f)$ after nucleosynthesis (after which there are no more particle species releasing entropy, and $f(t_f)$ becomes approximately constant).

We assume that the drain on the primordial magnetic field by the induction of loop currents is negligible. A loop formed at time t will drain an energy $W \approx \omega J_f^2/2$, where $\omega \approx 4\pi L ln(vL)$ is the inductance of the loop. In terms of the magnetic energy density:

$$W(t) \approx \rho_{mag}(t)(\epsilon t)^3/ln(\epsilon v t) \tag{3.9}$$

The evolution of the magnetic energy density is then described by:

$$\dot{\rho}_{mag} = -4\frac{\dot{R}}{R}\rho_{mag} - \kappa W(t)/t^4 \tag{3.10}$$

with solution:

$$\rho_{mag} = \rho_{mag}|_i (R_i/R)^4 (ln(\epsilon v t_i)/ln(v\epsilon t))^{\kappa\epsilon^3} \tag{3.11}$$

Because $\kappa\epsilon^3 \approx 10^{-3}$, we can safely treat the pmf as noninteracting.

The equation for the evolution of the mass of a loop is given by:

$$\dot{M} = \beta\mu\dot{L} = -\gamma_{GW} G\mu^2[f(t_f)(L_f/L)^2 + 1] \tag{3.12}$$

with the solution

$$t/t_f = (b + 1 - x + \sqrt{f}[tan^{-1}(x/\sqrt{f}) - tan^{-1}(1/\sqrt{f})])/b \tag{3.13}$$

(the electromagnetic field energy has little effect, and has been ignored here). This solution is true as long as the string remains superconducting. For bosonic strings $J_{crit} \approx e\sqrt{\mu}$ is the critical current [1], and for fermionic strings with charge carriers of vacuum mass M, $J_{crit} \approx eM$. The critical value for x, in the bosonic case is:

[1]To be consistent with OTW we take this to be the critical current. However, the results of [19,22] indicate that $J_{crit} = re\sqrt{\mu}$, where r can be much smaller than

$$x_{crit} = \sqrt{G\mu f(t_f)\gamma_{GW}/\alpha_{EM}\gamma_{EM}} \approx 11.7\sqrt{G\mu f(t_f)} \tag{3.14}$$

(for fermionic charge carriers replace μ with M^2). The superconducting lifetime of loops is then:

$$\frac{t_{crit}}{t_f} = (1 + b - x_{crit} + \sqrt{f}[tan^{-1}(x_{crit}/\sqrt{f}) - tan^{-1}(1/\sqrt{f})])/b \tag{3.15}$$

When $x \approx x_{crit}$ the EM field energy ($\sim 1/2$ of the total loop energy for $J_{crit} = e\sqrt{\mu}$) will be released in the form of high energy particles. For $x < x_{crit}$ we are left with an ordinary string which will evaporate at a time t given by:

$$u = \frac{t}{t_f} = \frac{x_{crit}}{b} + \frac{t_{crit}}{t_f} \tag{3.16}$$

The integrals can now be performed, and Eqn (2.10) is modified to include ρ_{mag}:

$$G\rho_{mag} = \frac{f_* 2\pi\gamma_{GW}(G\mu)^2 ln^2(\epsilon v t_*)R(t_*)^4}{\gamma_{EM}(\epsilon t_*)^2 R(t)^4} \approx \frac{157 f_*(G\mu)^2 ln^2(\epsilon v t_*)R(t_*)^4}{R(t)^4 t_*^2} \tag{3.17}$$

which is now expressed in terms of f_*.

We restrict our attention to bosonic superconducting strings. Since the limit on $G\mu$ may vary substantially in the superconducting case, we take the start of integration $t_* = t_{pl}/(G\mu)^2$. In Fig. (5) we show the constraint on the energy per length as a function of the parameter $f(t_f)$, evaluated at $t_f = 10^{10}$ sec. (roughly equivalent to the f of OTW). The physics is clear. For small values of f (hence small B fields) loops do not produce much EM radiation and the limit is the same as that for ordinary strings; for intermediate values of f sizeable currents are generated resulting in less gravitons and a weaker constraint on μ; and larger values of f require smaller string tensions in order that the magnetic energy density not get too large. Our nucleosynthesis constraint on $G\mu$ as a fuction of f does not restrict the ($G\mu$, f) solution space of OTW that allows galaxy formation.

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unity. This can affect the OTW solution space (f,$G\mu$) for galaxy formation, as we now describe. Their scenario assumes $L_f > L_{crit}$, which thereby constrains the range of f in their model: $f < 7.3 \times 10^{-3} r^2 \gamma_{EM}/\gamma_{GW} G\mu$. With $r = 1$, which they use, this upper limit to f is a factor ≈ 40 away from their solution space. Therefore, smaller values of r ($r \lesssim 0.15$) will require modification of the range of parameters that allow galaxy formation.

IV. Entropy Production

We now consider the effects of the long wavelength ($\sim L$) EM radiation emitted from the loops, which thermalize with the radiation background (during the radiation dominated era). Of particular interest is the time between nucleosynthesis ($t \approx 1$ sec) and matter domination ($t \approx 10^{10}$ sec) because the entropy of the Universe is approximately known at these times, during which the loops might be producing an inconsistent amount of entropy (for a given μ). Nucleosynthesis requires a baryon to photon ratio of $\eta \approx (3-10) \times 10^{-10}$ [4]. Observations of luminous matter (i.e., baryons in stars) yields a lower limit of $\eta \simeq 3.3 \times 10^{-11}$ at present. Therefore, η can at most change by a factor of $\approx 1/30$ between nucleosynthesis and matter domination. This information can be used to obtain a limit on $G\mu$.

We define $\Gamma_{EM} = \dot{\rho}_{EM}$ to be rate of electromagnetic energy per volume produced by all the loops in the Universe at some temperature T. The energy released in time dt is then $dE = \Gamma_{EM} R^3 dt$, with a corresponding release of entropy $dS = dE/T$. In the radiation dominated era, $S \approx 4\rho R^3/3T$, and the energy density $\rho \approx 3/32\pi G t^2$. We shall use an alternative measure of entropy production, the baryon to photon ratio η ($\eta \propto 1/(RT)^3 \propto 1/S$). The evolution of η, from an initial value η_i is then determined by:

$$\eta/\eta_i = exp(-8\pi G \int_{t_i}^{t} \Gamma_{EM} t^2 dt) \tag{4.1}$$

(we ignore other forms of entropy production).

We now calculate Γ_{EM}. Looking at the energy density produced at t by loops formed at t_f, we see that $d\rho_{EM}(t,t_f) = P_{EM}(t,t_f) dn_{loops}(t,t_f) dt = dt_f dt P_{EM}(t,t_f)$ $\beta/t_f^{5/2} t^{3/2}$, where we have used $R(t) \propto t^{1/2}$. Defining $y = t/t_f$, and letting $P_{EM} = \gamma_{GW} G\mu^2 K(y)$:

$$G\Gamma_{EM} = \frac{a}{t^3} \int K(y)\sqrt{y}dy \tag{4.2}$$

The integral over t in Eqn. (4.1) can now be performed,

$$\eta/\eta_i = (t_i/t)^{8\pi a \int K(y)\sqrt{y}dy} \tag{4.3}$$

However, it would be much more useful to have x instead of y as a dependent variable, and from the equation of motion for a loop we solve for y in terms of x:

$$y = 1 + \frac{1}{b}\int_x^1 \frac{dx}{1+K(x)} \tag{4.4}$$

and the entropy can now be written as:

$$\eta/\eta_i = (t_i/t)^{8\pi a I/b^{3/2}} \tag{4.5}$$

where the dimensionless coefficient I is obtained from

$$I = \int_0^1 \frac{K(x)}{1+K(x)}\sqrt{b + \int_x^1 \frac{dx}{1+K(x)}}\,dx \tag{4.6}$$

The coefficient I was calculated for two cases: (1) $K(x) = f/x^2$ for $x > x_{crit}$ and $K(x) = 0$ for $x < x_{crit}$, (2) $K(x) = f/x^2$ for $x > x_{crit}$ and $K(x) = \infty$ for $x < x_{crit}$. The first case corresponds to a loop with no electromagnetic radiation released when (and after) the critical current is reached, and the second case corresponds to all the loop energy going into photons at the critical current (the actual, physical situation should lie between these two cases). The value of I as a function of f is shown in Fig. (5) for these two cases, which give ver similar results. The coefficient I is essentially independent of μ, except for $f >> 1$ when the loops reach their critical current soon after they form.

We now check our previous assumption that the *pmf*, loops, and gravitons will not have their energy density diluted by the EM radiation from loops. From Fig. (5) the largest possible value of $G\mu$ is $\sim 10^{-4}$, and from Fig. (6) the largest possible value of I is ≈ 0.4. Using the standard loop parameters, we see that the maximum exponent in Eqn. (4.5) is $\approx 3.4 \times 10^{-3}$. For $t_i/t = 10^{-32}$ we have $\eta/\eta_i \lesssim 0.78$, and entropy production by the loops can be neglected.

Requiring η/η_i to vary by less than a factor of 1/30 over the time range $t_i/t = 10^{-10}$ yields a limit on $G\mu$:

$$G\mu \lesssim 3.5 \times 10^{-5}\frac{\gamma_{GW}}{I^2\nu^2\beta^3} \tag{4.7}$$

Using the standard string parameters we obtain the limit:

$$G\mu \lesssim 2 \times 10^{-2}/I^2 \tag{4.8}$$

Since $I \lesssim 0.4$, this is far weaker than the limits obtained from nucleosynthesis. This is just a restatement that entropy production is negligible.

V. Conclusion

We have calculated the constraint on the energy per length for ordinary cosmic loops, and given the constraint as a function of several parameters that describe the evolution of loops. The bound on $G\mu$ is clearly very sensitive to these parameters, and with recent studies [14,24] improving the accuracy of the parameters used here, previously quoted limits cannot be taken to be very accurate.

The constraint on $G\mu$ for superconducting strings, with small pmf, is the same as that for ordinary strings. Larger fields generate larger loop currents and allow a sizeable fraction of the loop energy to go into EM radiation, thus weakening the constraint. Stronger magnetic fields, with an energy density that begins to saturate the nucleosynthesis bound, will necessarily lead to stronger limits on μ.

Using the bound on $G\mu$ from nucleosynthesis we showed that there is a negligible amount of entropy production by SCS. Although gravitational waves emitted by loops are very important in constraining $G\mu$, the electromagnetic radiation is not. This situation may change, however, if the high energy, non-thermal photons resulting from superconducting loops making the catastrophic transition to ordinary loops is considered. Here, a limit is placed on $G\mu$ by requiring that light elements are not overproduced by the photodisintegration of ^{4}He [23].

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Figure Captions

1. The evolution of the particle degrees of freedom $g(T)$.

2. The parameter a that yields a critically allowed abundance of ^{4}He as a function of the parameter b.

3. The evolution of Ω_{GW} and Ω_{loop} as a function of temperature for the choice $b = 10^{-4}$ and $a = 1.84 \times 10^{-10}$. The abundance of ^{4}He in this case is critically allowed.

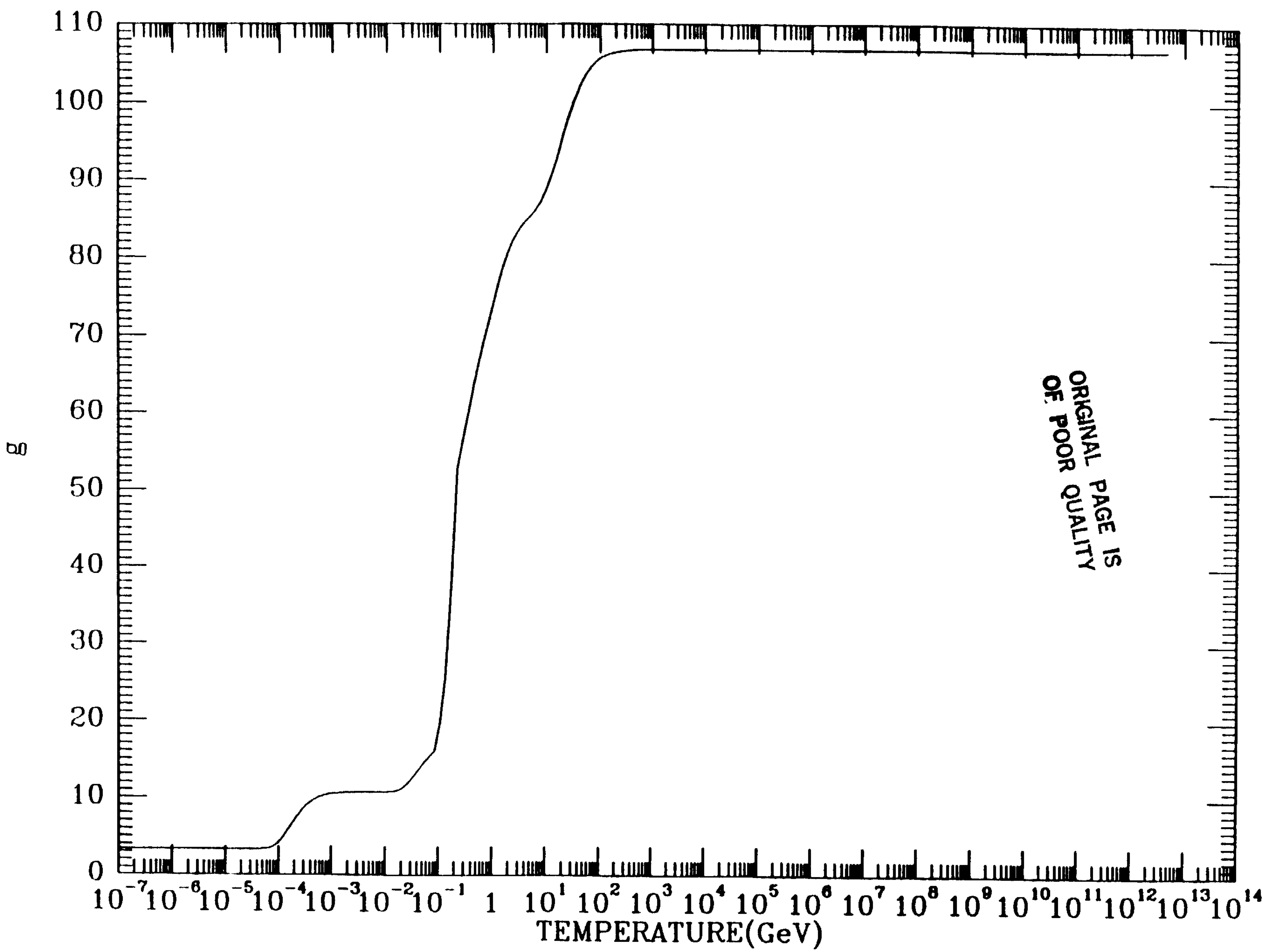
4. The evolution of Ω_{mag}, Ω_{GW}, and Ω_{loop} for the choice $f = 1$ and $G\mu = 1 \times 10^{-4}$.

5. The nucleosynthesis constraint for superconducting cosmic strings. We plot the critically allowed value of $G\mu$ as a function of f.

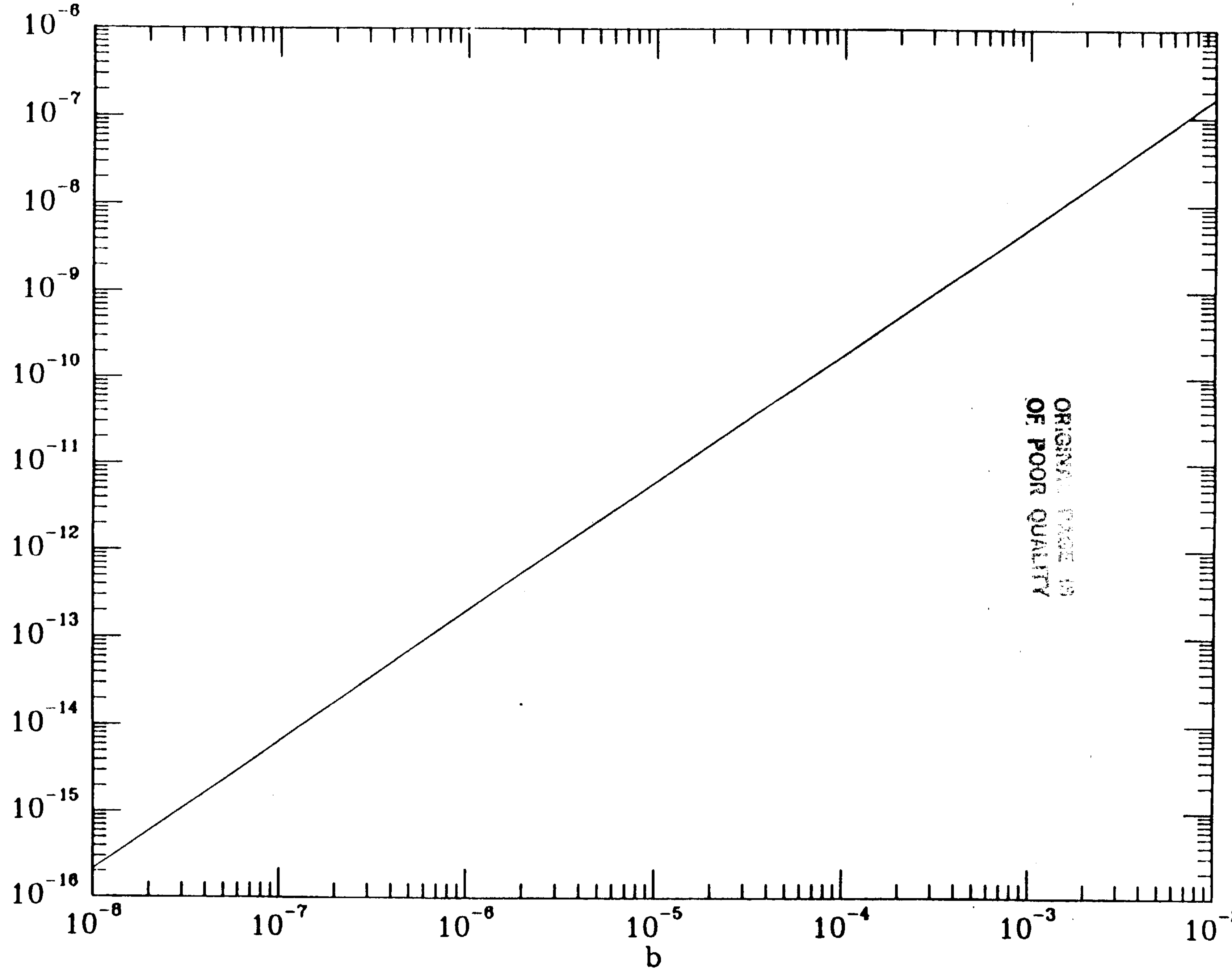
6. The parameter I as a function of f. Large values of I correspond to greater entropy production. Case (1) corresponds to no EM radiation released at the critical current, case (2) corresponds to all the loop energy being released at the critical current. Here we have taken $G\mu \approx 10^{-6}$, however, I depends strongly on μ only for $f >> 1$.

Table I. Degrees of freedom and mass assignments of particles in the early Universe.

Particle	Degeneracy	Mass (MeV)
photon	2	0
neutrinos	6	0
electron	4	0.511
muon	4	106
tau	4	1730
u quark	12	340
d quark	12	340
s quark	12	540
charm	12	1500
bottom	12	4500
top	12	40000
gluons	12	0
W boson	6	70000
Z boson	3	90000
Higgs	1	90000
charged pions	2	139
neutral pion	1	135
eta	1	548.8
charged kaons	2	493.6
neutral kaon	1	497.7

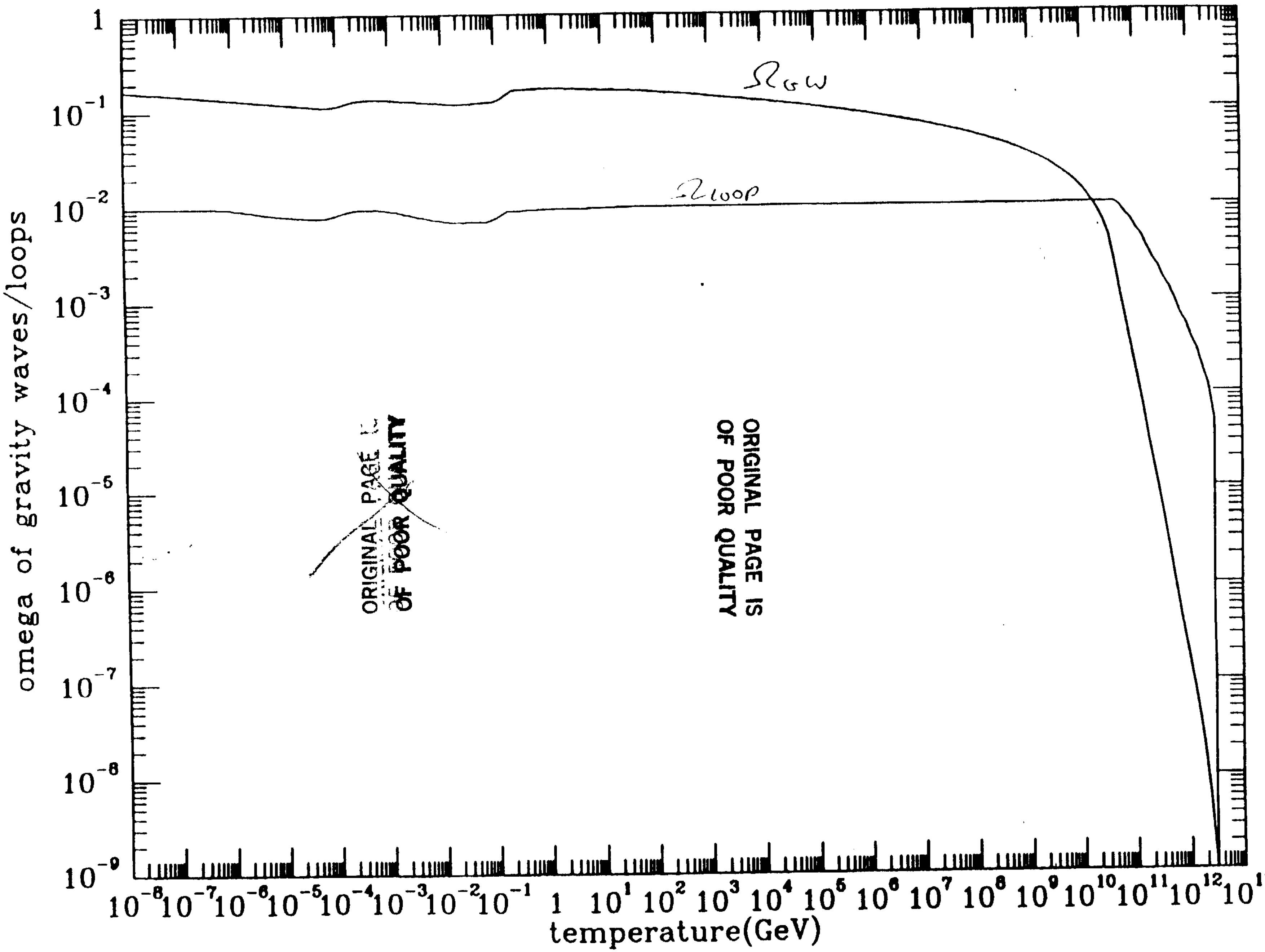

110
100
90
80
70
60
50
40
30
20
10
0
g
10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14
TEMPERATURE(GeV)

— FIGURE 1 —

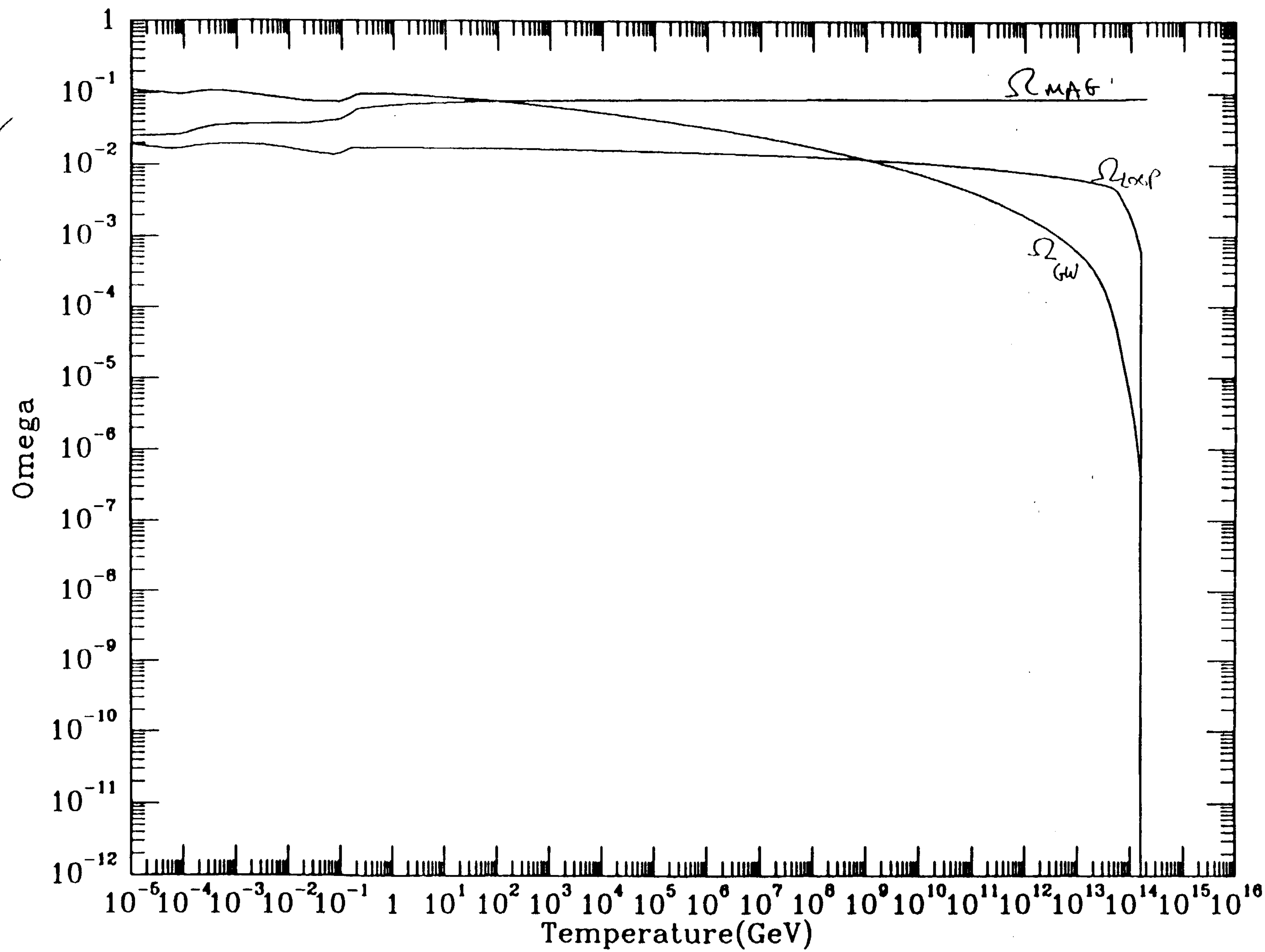


10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
10^-14
10^-15
10^-16
a
10^-8
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2
b

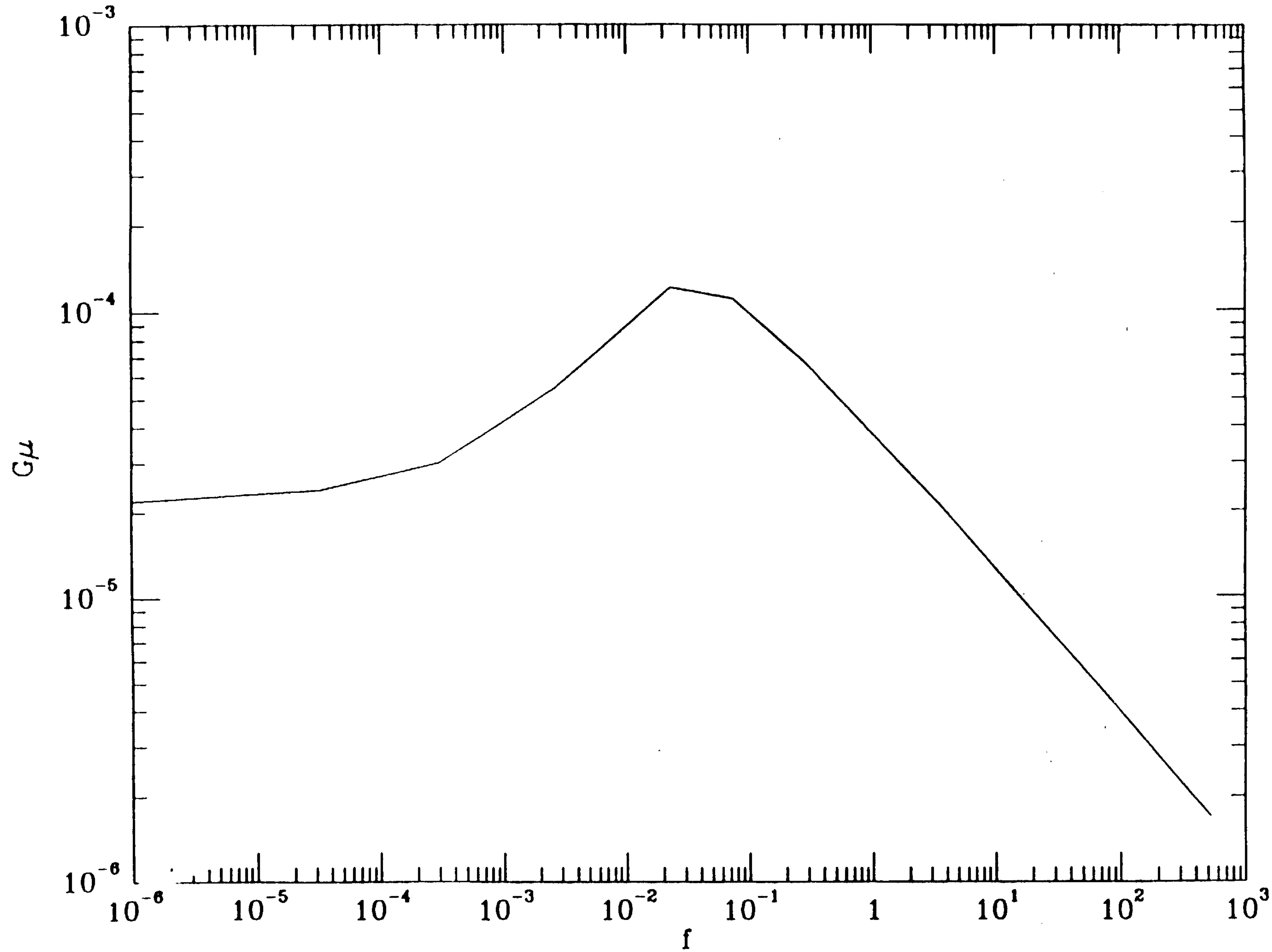
– FIGURE 2 –



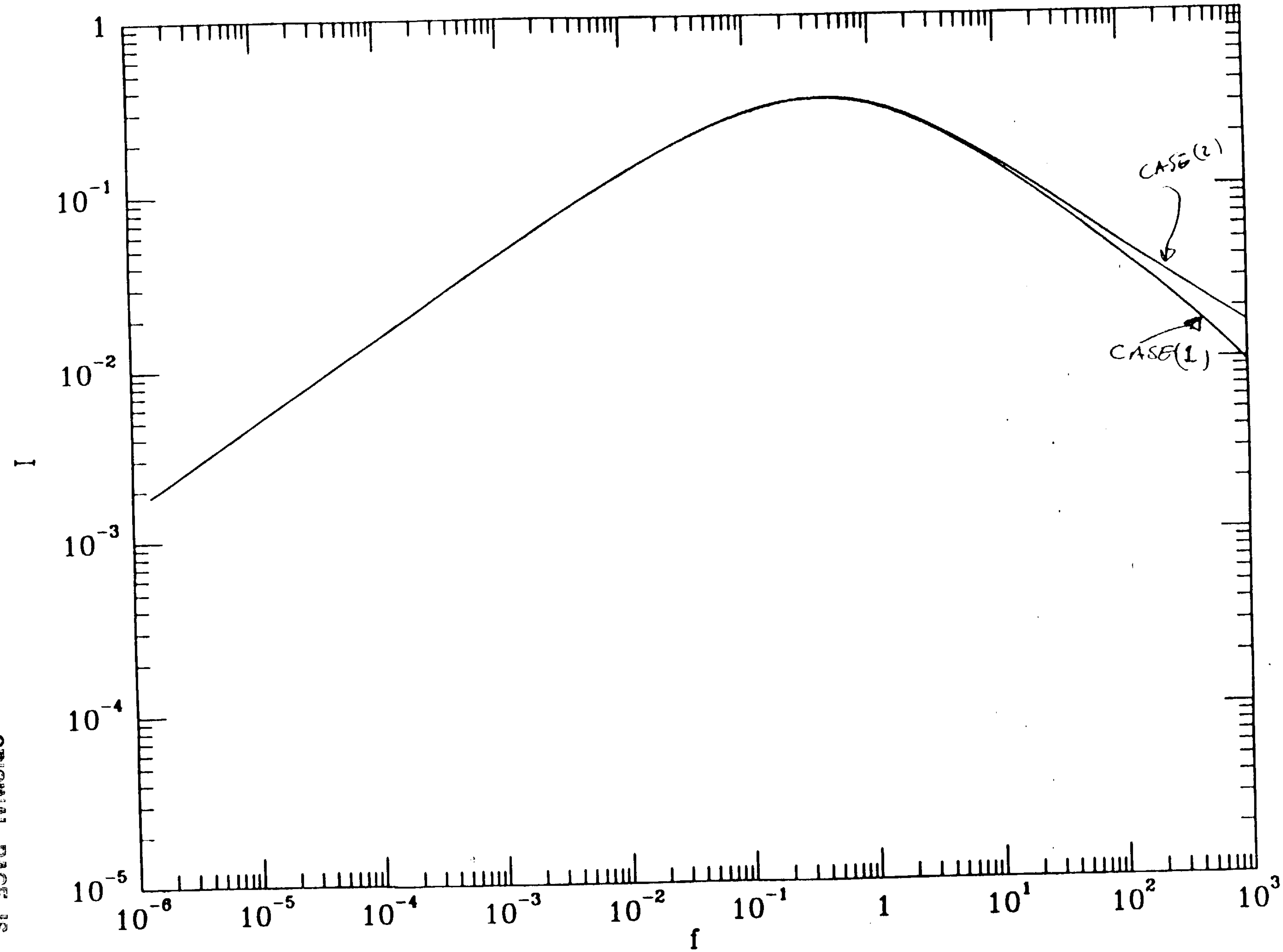
- FIGURE 3 -


Omega
1
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-5 10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16
Temperature(GeV)
Ω MAG
Ω GW

– FIGURE 4 –



— FIGURE 5 —


1
10^-1
10^-2
10^-3
10^-4
10^-5
I
10^-6
10^-5
10^-4
10^-3
10^-2
10^-1
1
10^1
10^2
10^3
f
CASE(2)
CASE(1)

– FIGURE 6 –