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## Non-Isothermal Buckling Behavior Of Viscoplastic Shell Structures

by

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### ABSTRACT

The paper describes the mathematical model and solution methodologies for analyzing the structural response of thin, metallic elasto-viscoplastic shell structures under large thermomechanical loads. Among the system responses associated with these loads and conditions are snap-through, buckling, thermal buckling and creep buckling. Thus geometric and material nonlinearities (of high order) can be anticipated and are considered in the model and the numerical treatment.

The problem of inelastic analysis of shell structures has been investigated recently by Kojic<sup>1</sup> and Bathe<sup>1</sup>. They used the "effective-stress-function" algorithm to compute plastic and creep effects on the behavior of shell like structures. The effects of inelastic material behavior on stability of shells found their way into the literature since the late 1970's. The paper by Miyazaki and Ando<sup>2</sup> deals with creep buckling of perfect spherical shells subjected to pressure loading and considers only the effects of steady-state creep. Xirochakis and Jones<sup>3</sup> have studied axisymmetric and bifurcation creep buckling of externally pressurized spherical shells under the condition of secondary creep only. Botros and Bieniek<sup>4</sup> presented a numerical treatment of the creep buckling of these configurations. Their work includes the effects of elastic strain, primary and secondary creep strains and creep recovery. The influence of temperature and viscous effects on dynamic buckling of shells has been considered by Wojewodzki and Bukowski<sup>5,6</sup>. The book by Owen and Hinton<sup>7</sup> gives a list of references for the applications of finite element methods to the problem of creep buckling of structures.

As far as the authors know no work has been reported on the non-isothermal buckling behavior of elasto-viscoplastic shell structures.

The prediction of inelastic behavior of metallic materials at elevated temperatures has increased in importance in recent years. The operating conditions within the hot section of a rocket motor or a modern gas turbine engine present an extremely harsh thermo-

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mechanical environment. Large thermal transients are induced each time the engine is started or shut down. Additional thermal transient from an elevated ambient, occur whenever the engine power level is adjusted to meet flight requirements. The structural elements employed to construct such hot sections, as well as any engine components located therein, must be capable of withstanding such extreme conditions. Failure of a component would, due to the critical nature of the hot section, lead to an immediate and catastrophic loss in power and thus cannot be tolerated. Consequently, assuring satisfactory long term performance for such components is a major concern for the designer.

In previous papers<sup>8,9,10</sup> the authors have presented a constitutive law for thermo-elasto-viscoplastic behavior of metallic materials, in which the main features are: (a) unconstrained strain and deformation kinematics, (b) selection of reference space and configuration for the stress tensor, bearing in mind the rheologies of real materials, (c) an intrinsic relation which satisfies material objectivity, (d) thermodynamic consistency, and (e) proper choice of external and internal thermodynamic variables.

The problems of buckling of shallow arches under static thermo-mechanical loads was investigated in Ref. 10. It was shown there that the material constitutive equations are capable of capturing all non-isothermal, elasto-viscoplastic characteristics. Furthermore, the method is capable of predicting response which includes pre and postbuckling behavior due to creep and plastic effects.

The non-isothermal inelastic analysis of shell structures represent some major challenges. In order to perform an effective and accurate analysis, efficient formulation and solution strategies must be employed.

A complete true ab-initio rate theory of kinematics and kinetics for curved thin structures, without any restriction on the magnitude of the strains or the deformation, was formulated. The time dependence and large strain behavior are incorporated through the introduction of the time rates of the metric and curvature in two coordinate systems; a fixed (spatial) one and a convected (material) coordinate system. The relations between the time derivative and the covariant derivatives (gradients) have been developed for curved space and motion, so that the velocity components supply the connection between the equations of motion and the time rate of change of the metric and curvature tensors.

A time and temperature dependent elasto-viscoplastic model was formulated<sup>8-10</sup> in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by phenomenological thermodynamic considerations<sup>8</sup>.

The obtained complete rate field equations consist of the principles of the rate of the virtual power and the rate of conservation of energy, of the constitutive relations, and of boundary and initial conditions. These equations provide a sound basis for the formulation of the adopted finite element solution procedures.

The derived shell theory, in the least restricted form, before any simplifying assumptions are imposed, has the following desirable features:

- (a) The two-dimensional, impulse-integral form of the equations of motions and the Second Law of Thermodynamics (Clausius-Duhem inequality) for a shell follow naturally and exactly from their three-dimensional counterparts.
- (b) Unique and concrete definitions of shell variables such as stress resultants and couples, rate of deformation, spin and entropy resultants can be obtained in terms of weighted integrals of the three-dimensional quantities through the thickness.
- (c) There are no series expansions in the thickness direction.
- (d) There is no need for making use of the Kirchhoff Hypotheses in the kinematics.
- (e) All approximations can be postponed until the discretization process of the integral forms of the First Law of Thermodynamics.
- (f) A by-product of the descent from three-dimensional theory is that the two-dimensional temperature field (that emerges) is not a through-the-thickness average, but a true point by point distribution. This is contrary to what one finds in the literature concerning thermal stresses in the shell.

To develop geometrically nonlinear, doubly curved finite shell elements the basic equations of nonlinear shell theories have to be transferred into the finite element model. As these equations in general are written in tensor notation, their implementation into the finite element matrix formulation requires considerable effort. The nonlinear element matrices are directly derived from the incrementally formulated nonlinear shell equations, by using a tensor-oriented procedure. For this formulation, we are using the "natural" degrees of freedom per mid-surface shell node: three incremental velocities and the rates of rotations about the material coordinates in a mixed form.

The quasi-linear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain  $V$ . In the present instance, the choice has been made in favor of a simple expansion in time for the construction of incremental solutions from the results of finite element spatial integration of the governing equations.

The procedure employed permits the rates of the field formulation to be interpreted as increments in the numerical solution. This is particularly convenient for the construction of incremental boundary condition histories.

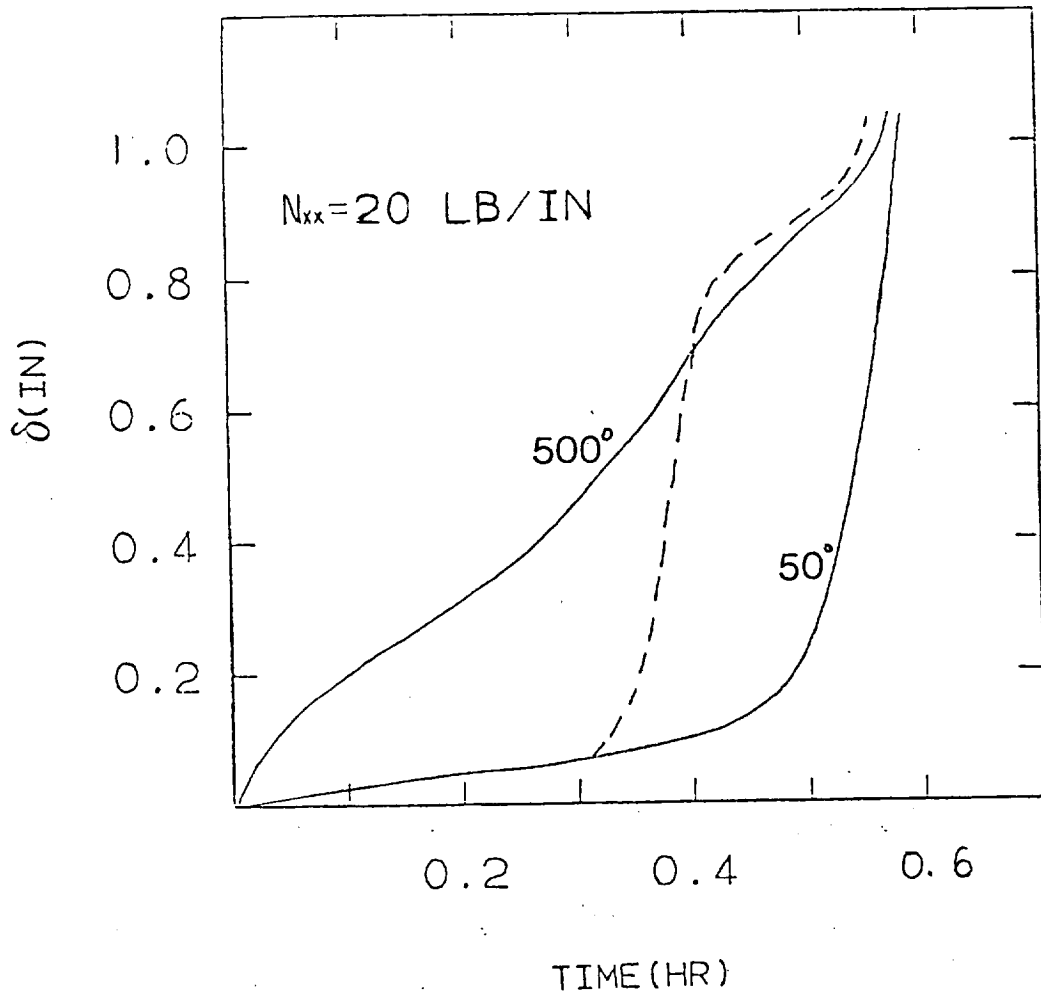
Finite element solution of any boundary-value problem involves the solution of the equilibrium equations (global) together with the constitutive equations (local). Both sets of equations are solved simultaneously in a step by step manner. The incremental form of the global and local equations can be achieved by taking

the integration over the incremental time step  $\Delta t = t_{j+1} - t_j$ . The rectangular rule has been applied to execute the resulting time integration.

Clearly, the numerical solution involves iteration. A simplified version of the Riks-Wempner constant-arc-length method has been utilized. This iteration procedure which is a generalization of the displacement control method also allows to trace the non-linear response beyond bifurcation points. In contrast to the conventional Newton-Raphson techniques, the iteration of the method takes place in the velocity and load rate space. The load step of the first solution in each increment is limited by controlling the length  $ds$  of the tangent. Either the length is kept constant in each step or it is adapted to the characteristics of the solution. In each step the triangular-size stiffness matrix has to be checked for negative diagonal terms, indicating that a critical point is reached.

Several examples of shell cylindrical panels and shallow spherical caps of different materials subjected to different loading and temperature conditions have been analyzed. As an example of the results the central deflection time history and the influence of temperature change of a thin, imperfect, cylindrical shell panel made of carbon steel C-45 is shown in the figure shown herein. The panel is simply supported on all sides, and subjected to inplane loads along the generator. The applied load of 20 lbs/in. is well below the linear critical (buckling) load for this geometry, which is 42.15 lbs/in. At a temperature of 50° F the shell is in a primary creep state for the first 28 minutes, reaching a deflection of 0.2 in. and the critical time for creep buckling (this implies that the deflection becomes unbounded) is 35 minutes. At temperature of 500° F the shell 'snaps' into its post-buckled configuration almost immediately but the critical time for creep buckling remains almost unchanged. The dashed line in the figure represents a non-isothermal process where the temperature was suddenly increased from 50° F to 500° F after 0.3 hrs. As a result the shell snaps-through to its post buckled position at 500° F with small over-shoot and reaches its critical time of creep buckling two minutes sooner (33 min.).

The study shows that in the presence of high temperature and viscoplasticity, the process of shell buckling assume an entirely new character. While the stability phenomena still exist under sufficiently large loads, buckling develops, as a time and temperature dependent deformation process under constant or variable thermomechanical loads of magnitude smaller than the purely elastic critical values. If the elastic behavior of a structure displays limit points and snap-through phenomena, the deformation process of creep buckling become even more complicated and it usually exhibits a combination of snapping and creep responses.



CYLINDER PANEL CREEP RESPONSE

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