

# A CONSIDERATION OF THE USE OF OPTICAL FIBERS TO REMOTELY COUPLE PHOTOMETERS TO TELESCOPES

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Short title: Coupling Photometers to Telescopes with Optical  
Fibers

## ABSTRACT

The possible use of optical fibers to remotely couple photometers to telescopes is considered. Such an application offers the apparent prospect of enhancing photometric stability as a consequence of the benefits of remote operation and decreased sensitivity to image details. A properly designed fiber coupler will probably show no significant changes in optical transmission due to normal variations in the fiber configuration. It may be more difficult to eliminate configuration-dependent effects on the pupil of the transmitted beam, and thus achieve photometric stability to guiding and seeing errors. In addition, there is some evidence for significant changes in the optical throughputs of fibers over the temperature range normally encountered in astronomical observatories. Until these issues are resolved by better laboratory measurements than are currently available, it may be imprudent to utilize optical fiber couplers in astronomical instruments intended for high photometric precision.

## INTRODUCTION

The motivations for the consideration of optical fibers to optically couple astronomical instruments to telescopes are easy to appreciate. The flexibility of optical fibers allows the coupled instrument to be removed from the telescope and remotely operated in essentially an "optical bench" configuration, thereby eliminating the well-known problems associated with mounting sensitive instruments to moving telescopes (Figure 1).

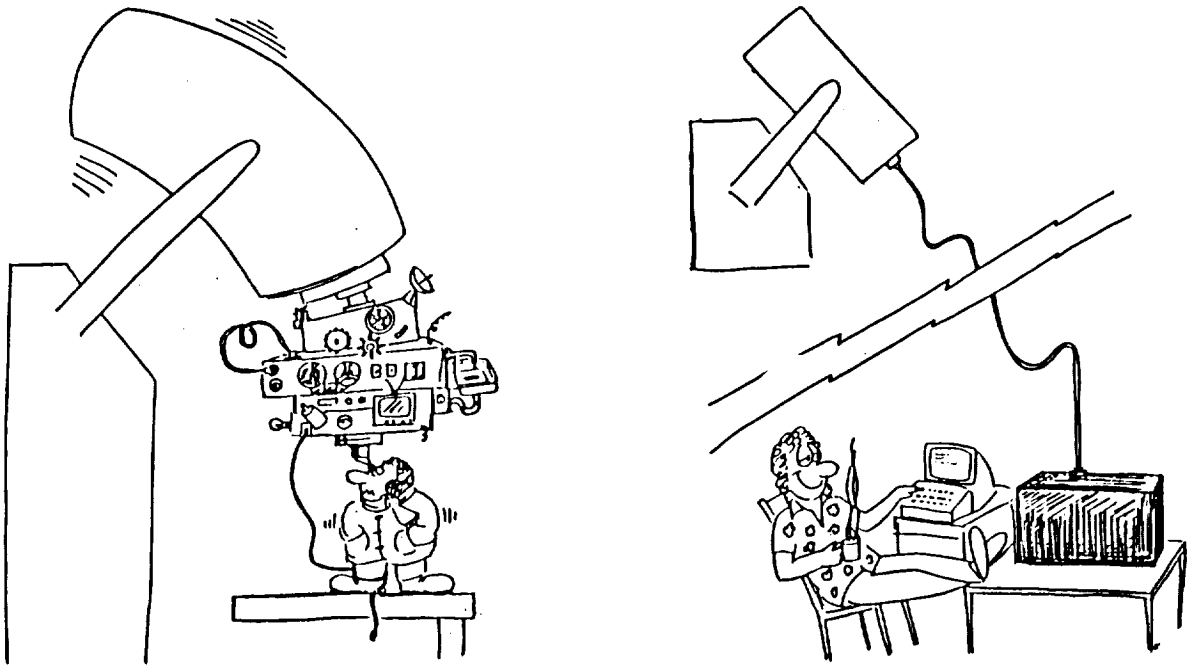


Fig. 1. In which the joys of remotely coupling an instrument to a telescope are contrasted with the tribulations encountered in traditional astronomical photometry (cartoons by Kitty Heacox).

The flexibility of fibers also allows optical multiplexing: several objects and/or the sky background can be simultaneously monitored in separate channels. Finally, the image scrambling produced by the variety of optical path lengths within a step-index fiber can reduce the extent of photometric variations due to image detail and/or instability in conjunction with non-uniform detector response. For all these reasons, optical fibers have been applied in recent years to optically couple a variety of spectrographic instruments to astronomical telescopes.

Application to photometric instruments may seem to be a straightforward extension of the spectrographic experience, but careful consideration of the matter reveals several possible sources of difficulty, especially if high photometric precision is required. The second section of this paper discusses the relevant physics of analog optical transmission by optical fibers, and the third section applies the results to the possible application of fibers to remotely couple photometers to telescopes.

Such an application presents two types of problems. The first arises from the fact that optical fibers represent a class of optical waveguides whose entropic characteristics fall somewhere between those of a perfect imaging device (e.g., a train of confocal lenses) and a complete image scrambler (integrating sphere). A beam guided by an optical

fiber will lose some structure both in the image present on the input end of the fiber, and in the pupil of the beam. The extent of this increase in optical entropy will depend on several factors, among them being the physical configuration of the fiber. An optical fiber used to remotely couple an instrument to a telescope will consequently yield an output beam whose characteristics vary to a greater or lesser degree as the telescope moves, a behavior with obvious implications for photometric applications.

The second class of difficulties concerns the stability of the optical transmission of fibers. Short-term stability to configuration changes may be adequate for photometric applications, since the transmission of a properly designed optical fiber coupler does not appear to be as sensitive as optical entropy to the physical configuration of the fiber. The question of the long-term photometric stability of optical fibers remains unstudied, but presumably no difficulties should arise that cannot be resolved by contemporaneous calibration. More worrisome is the prospect of temperature variations inducing significant transmission variations, possibly by altering a fiber's configuration on a small scale.

## MODAL PROPERTIES OF ANALOG TRANSMISSION BY OPTICAL FIBERS

### Fundamental Properties of Cylindrical Dielectric Waveguides

The theory of the optical properties of fibers is a subset of that of electromagnetic propagation in dielectric waveguides, and is extensively treated by Marcuse [1974] and by Kapany and Burke [1972]. The gross properties of the realization of cylindrical optical waveguides in optical fibers (and rods) are as follows.

A step-index fiber guides electromagnetic radiation by total internal reflection at the boundary between the fiber core (index of refraction  $n$ ) and the surrounding cladding material (index  $n'$ ,  $n' < n$ ). There is thus a maximum angle of incidence that will propagate, and this angle is usually specified (in vacuum) in terms of the fiber's numerical aperture (NA):

$$NA = \sin(\theta_{\max}) = \{(n)^2 - (n')^2\}^{\frac{1}{2}} \quad (1)$$

Typically,  $n - n' \ll n$  and the NA lies in the range 0.2 to 0.4. The minimum focal ratio that will propagate without loss is approximately  $1/(2NA)$ , although as we shall see it is not prudent to allow the beam pupil to approach the limit set by the fiber NA. The optical path length experienced by a meridional ray of angle of incidence  $\theta$  is  $(L)(n)\sec(\theta)$ , where  $L$  is the fiber length; the optical path length and, hence, attenuation within the fiber core varies across the pupil of the beam.

This is undesirable for most digital telecommunications purposes, a fact that has led to the development of gradient-index fibers in which the index of refraction has a radial gradient--decreasing from the center of the core outward, and usually of parabolic profile--that causes all guided rays to (ideally) experience equal optical path lengths. Like their macroscopic analog, gradient-index rod ("Selfoc") lenses, such fibers form images of the input end at each integral multiple of the waveguides's "pitch", a characteristic distance along the guide determined by the refractive index profile. The concept of numerical aperture applies also to gradient-index waveguides, but with a more complex expression for its numerical evaluation. Gradient-index fibers appear to offer no advantages over step-index fibers for analog transmission; indeed, as discussed below, they are probably inferior to step-index fibers for astronomical applications, and the remaining discussion will concentrate on step-index fibers.

#### Modal Description of Step-Index Fiber Propagation

The modal nature of guided waves within optical fibers is easily visualized by analogy with propagation within a planar waveguide, one consisting of two infinite, reflecting planes. A wave propagating down such a guide at a given angle of incidence can be decomposed into a longitudinal wave and, due to reflections off the two plane surfaces that bound the guide, a standing wave set up by transverse waves in both directions perpendicular to the reflecting planes. For loss-free propagation, this standing wave must have a node at each boundary plane, which is equivalent to requiring the propagating wave to experience a phase shift of an integral multiple of pi radians between successive internal reflections. As a result, the beam propagates as a set of discrete modes, each with its corresponding angle of incidence. Exact solution of the wave equation for such boundary conditions produces essentially this result, but with electromagnetic fields that decay exponentially into the boundary surfaces rather than go to zero there. The result is a set of evanescent waves outside the waveguide that ideally carry no power, but that can lead to power loss from a perturbed waveguide.

Cylindrical waveguides have more complicated modal characteristics that can be adequately approximated in step-index fibers by a set of two integer mode numbers, radial (m) and azimuthal (l); highly skew rays have large values of l [Gloge, 1972a]. The meridional angle of incidence of a ray is approximated by

$$\theta \approx \frac{M \cdot \lambda}{2dn} \quad (2)$$

where d is the fiber core diameter and M is the combined mode number:

$$M = 2m + 1 \quad (3)$$

Note that, for highly skew rays, the angle of incidence at the core/cladding interface will be less than the meridional angle given by Equation 2. A similar analysis applies to gradient-index fibers.

#### Image Transfer by Step-Index Fibers

The image transfer properties of step-index optical fibers have been analyzed by Heacox [1987]. In transmission through a perfect, cylindrical, step-index waveguide the azimuthal structure of the image formed on the input end is essentially destroyed, while the radial structure is transformed according to

$$R_{out}(\rho) = \frac{2}{\pi} \int_{t=0}^{\rho} \int_{a=t}^r \frac{\rho R_{in}(a) da dt}{[(r^2 - t^2)(a^2 - t^2)(\rho^2 - t^2)]^{1/2}} \quad (4)$$

In this expression,  $R(\rho)$  is the annular flux density as a function of radial distance  $\rho$  on the waveguide cross-section:

$$R(\rho) = \rho \int_{\alpha=0}^{2\pi} I(\rho, \alpha) d\alpha. \quad (5)$$

where  $I(\rho, \alpha)$  is the image flux density in polar coordinates. Examples of the output annular image structure for point sources imaged at varying radial distances on the input end are shown in Figure 2.

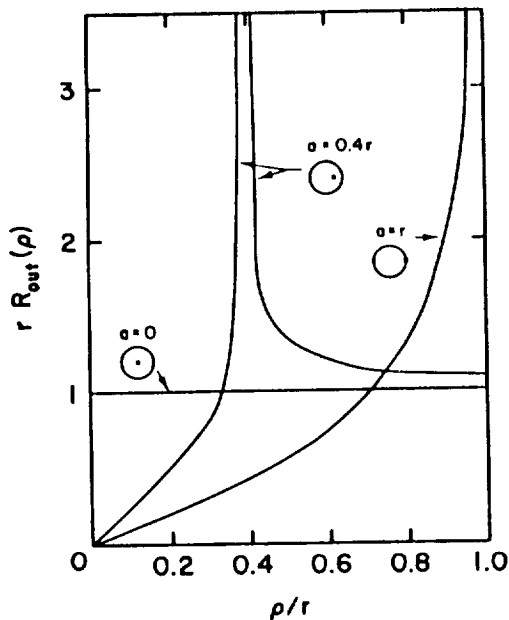


Fig. 2. Output annular flux distributions of an idealized optical fiber for point sources imaged (a) at the center of the input end of the fiber; (b) offset from the center by 40% of the fiber core radius; and (c) at the edge of the input end.

Rather clearly a good deal of image detail survives passage through such a waveguide. Whether this conclusion also applies to real optical fibers remains to be seen, since departures from a perfect cylindrical form in fibers will probably increase the image scrambling by some (currently unknown) amount. It seems clear that in the situation likely to be encountered in astronomical photometry--an input image much smaller in diameter than the fiber core--the image scrambling produced by a cylindrical, step-index waveguide is much less than that achieved by such conventional means as Fabry imaging or integrating spheres. Gradient-index waveguides, being designed essentially as imaging elements, produce even less image scrambling than do step-index waveguides.

#### Effects of Departures from Cylindrical Form

It is conventional to distinguish two scales of imperfections in optical fiber geometry: **microbends** are irregularities in fiber shape on a scale (along the fiber length) that is not large in comparison with the distance between successive internal reflections by most rays in the guided beam; **macrobends**, in contrast, are departures from cylindrical form on a much larger scale. Defects on the two scales are somewhat interdependent, and both serve to redistribute flux in the pupil of the guided beam.

Microbends arise mostly from stress at the core/cladding interface. Since a variety of materials and manufacturing techniques are used in the optical fiber industry, the degree of microbending exhibited by commercially available fibers can vary dramatically from one product to the next. Periodic microbends are observed to strongly couple optical power in modes whose longitudinal propagation constants vary by an amount corresponding to the wavenumber of the disturbance [Marcuse 1974]. The behavior of real fibers is apparently best modeled by assuming random disturbances and analyzing the effects on modal structure in terms of power diffusion. Gloge [1972a] thus derives a diffusion equation on the assumption that coupling occurs only between adjacent modes; this equation has been solved in closed form by Gambling, Payne and Matsumura [1975] for the boundary conditions corresponding to a collimated input beam. For the asymptotic cases of very small and very large input angles of incidence, their solution is approximated as follows [Heacock 1986]: after traversing a fiber of length  $L$ , a collimated beam in input angle of incidence  $\theta_0$  has its power distributed among angles of incidence according to

$$P(\theta|\theta_0) \sim \begin{cases} \frac{1}{4L(AD)^{1/2}} \exp\left[-\frac{\theta^2 + \theta_0^2}{4DL}\right], & \frac{2\theta\theta_0}{4DL} < 1 \\ \frac{1}{4(\pi AL\theta\theta_0)^{1/2}} \exp\left[-\frac{(\theta - \theta_0)^2}{4DL}\right], & \frac{2\theta\theta_0}{4DL} > 1. \end{cases} \quad (6)$$

In these expressions,  $P(\theta)$  is the optical power in angle of incidence  $\theta$ ,  $A$  is an absorption coefficient,

$$D = (\lambda/2dn)^2 d_0 \quad (7)$$

$d_0$  is a modal diffusion constant, and the remaining symbols have their previously assigned meanings. These expressions appear to be qualitatively correct; in particular, for a sufficiently large input angle of incidence  $\theta_0$ , the output beam is observed to be a cone (due to azimuthal scrambling of skew rays) of opening semi-angle  $\theta_0$  and Gaussian cross-section. We can combine the second equation of (6) with the expected mode coupling due to diffraction to yield the following useful result: for a collimated input beam of sufficiently large angle of incidence  $\theta_0$ , the output beam has a distribution of power among angles of incidence proportional to

$$P(\theta|\theta_0) \propto \exp \left[ -\frac{1}{2} \left( \frac{\theta - \theta_0}{\sigma} \right)^2 \right] \quad (8)$$

where

$$\sigma \simeq \frac{\lambda}{d} \left[ \frac{1}{2n^2} \frac{L}{L_D} + 0.19 \right]^{1/2} \quad (9)$$

[Heacox 1986]. The parameter  $L_D$  in equation 9 is just the inverse of Gloge's modal diffusion constant  $d_0$  and carries the units of length (per square radian); it is consequently called the modal diffusion length and is usefully thought of as the length of fiber required for microbending to produce about as much modal power redistribution as does diffraction.

Since the maximum angle of incidence in the beam is roughly  $1/(2f)$ , where  $f$  is the input focal ratio, the change in focal ratio due to microbending-induced modal diffusion is approximately

$$\delta f \simeq -2f^2 \sigma \quad (10)$$

This is only a crude estimate since a beam propagated through a fiber will show a Gaussian tail at the margin of the pupil rather than a sharp cutoff.

This model appears to describe the behavior of real fibers fairly well. The observations of Gambling, Payne and Matsumura [1975] verify the  $L^2$  behavior predicted by Equation 9 and yield estimates for the parameters  $A$  and  $d_0$ . In addition, the laboratory measurements reported by Heacox [1986] imply values for  $L_D$  that are consistent over the approximate focal ratio range of  $f/7$  to  $f/3$ , once the effects of macrobending have been accounted for (see below).

As a result, the parameter  $L_D$  (or, equivalently,  $d_0$ ) appears to be the best parameter with which to characterize microbending in optical fibers, at least for the propagation of relatively fast beams. Some fibers appear to have values of  $L_D$  in the tens or hundreds of meters [Heacock 1986]; such fibers can probably be used in optical couplers with little fear of significant power loss due to microbending-induced scattering of rays outside the fiber's NA, providing that the input pupil does not nearly fill the NA.

On the other end of the scale are the macrobends corresponding to departures from a linear fiber configuration in a given application. A crude analysis of the effects of such bends is as follows: A ray encountering a bend of radius of curvature  $R$  will have its meridional angle of incidence  $\theta$  changed by roughly

$$|\delta\theta| \approx \frac{d}{R} \cdot \text{ctn}(\theta) \approx d/(R\theta) \quad (11)$$

for small  $\theta$ . The cumulative effects of  $N$  such bends will then be to change the effective focal ratio by about

$$\delta f \approx -2f^3 \cdot d \cdot N^{1/2} \langle R^{-1} \rangle \quad (12)$$

where  $\langle * \rangle$  denotes an average. In most 'normal' configurations--e.g., the fiber allowed to drape loosely between telescope and instrument--the mean value of  $d/R$  will be so small that the beam size will not approach the fiber's NA and no light loss should consequently be experienced from this mechanism alone.

A much more thoughtful analysis by Gloge [1972b] takes into account the evanescent waves that propagate in the cladding and the fact that bending entails stretching the fiber core and consequent reduction of its index of refraction. Of more practical interest are the attenuation measurements on bent fibers reported by Engelsrath, Danielson and Franzen [1986]. In brief summary of both these works, it appears that radii of curvature as small as a few mm, or at most a few cm, are required to produce measurable excess loss in fibers with beams whose pupils nearly fill the fibers' NA. It thus appears to be easy to configure fibers so that no significant power loss will result from macrobends alone.

There is an important caveat to this conclusion, however. Fibers that show large amounts of microbending induced modal diffusion typically exhibit greatly enhanced levels of microbending when they are disturbed in any manner, including macrobends, twisting, and applied pressure. The mechanism is probably one of induced stress on the core/cladding interface and, partially as a result, fibers with large amounts of microbending are probably not suitable for photometric applications in which they may be subject to perturbations.



### Optical Transmission

A full discussion of spectral transmission properties of optical fibers is beyond the scope of this paper. The matter is discussed in, among others, Midwinter [1979]. Of interest here is the variation in optical attenuation across the system pupil due to the distribution of optical path lengths within step-index waveguides. For a beam of focal ratio  $f$  focused on the input end of a cylindrical waveguide of length  $L$ , integration of Beer's Law over the distribution of optical path lengths yields (to first order) this attenuation:

$$A \approx \{1 + \alpha L / (4nf)^2\} \exp(-\alpha L) \quad (13)$$

where  $\alpha$  is the core material absorption coefficient and  $n$  its index of refraction. The sensitivity of optical transmission to focal ratio changes from this mechanism alone is thus

$$\frac{\delta A}{\delta f} \approx \frac{A \cdot \ln(A)}{16f^3 n^2} \quad (14)$$

where  $A$  is the attenuation of an axial ray; i.e.,  $\exp(-\alpha L)$ .

## APPLICATION TO PRECISE PHOTOMETRY

Of principle interest to photometric applications are the stability of optical attenuation in the system and of the distribution of flux on the detector. The chief cause of concern regarding the use of optical fibers arises from the varying fiber configurations entailed by coupling a stationary instrument to a moving telescope. Of course, in those applications in which the fiber configuration does not change--e.g., multi-aperture, focal plane mounted devices--these concerns do not arise, and the following discussion is largely (but not entirely) irrelevant.

### Stability of Optical Attenuation

As the fiber configuration varies so does the distribution of optical power in the pupil of the guided beam, a consequence of the geometrical optics of macrobends and of the induced microbending. To first order the result is a decrease in focal ratio given crudely by Equations 10 and 12. If this change is sufficiently large, the pupil may carry significant power outside the fiber's NA, power which is immediately lost to the beam by scattering into the fiber cladding. Fortunately, available fibers have sufficiently large NA's, are sufficiently resistant to small radii of curvature in non-pathological applications (especially when jacketed), and (in some cases) show sufficiently small

amounts of microbending that fiber couplers can be designed to avoid this source of photometric error. From Equation 12, it is important to keep the beam fast, but not so fast as to risk exceeding the maximum angle of incidence for total internal reflection. Since most fibers' NA's correspond to beams faster than about  $f/2$ , an input beam of  $f/3$  to  $f/5$  will usually suffice.

Even if such losses to the cladding are avoided, a redistribution of flux in the pupil will affect the attenuation within the core material. The extent of the effect can be roughly estimated from Equation 14 in conjunction with Equations 10 and 12, and is generally found to be small; smaller still will be the change in attenuation as the fiber configuration varies during observations. It is important to this conclusion that a fiber be used whose microbending is so small that it is not significantly increased by macrobends. If this is the case, changes in fiber core attenuation with fiber configuration will usually be insignificant.

There remains one ominous possible source of inconstant optical transmission in fibers used for astronomical applications. Since microbending apparently arises from stress at the core/cladding interface, and core and cladding materials typically have different thermal expansion coefficients, the extent of microbending-induced modal diffusion might be expected to vary with temperature. Observations of this effect have recently been reported in the literature [Grebel and Herskowitz 1986], in which dramatic changes in optical transmission were observed in a stressed fiber over a temperature range of  $+25$  to  $-30$  C. The stress was produced by clamping the fiber between two corrugated plates, thereby inducing perturbations of amplitude about 100 microns every 15 mm along a length of about 150 mm of fiber. While it is not clear how these results will translate to astronomical applications, the extent of the excess loss with lower temperatures--typically several db over the temperature range of 55 C--should induce caution in plans for the use of fibers in precise astronomical photometry.

#### Stability of Flux Distribution

Aside from stability of optical throughput, one is also concerned with the maintenance of an invariant distribution of flux on the detector in order to avoid photometric errors arising from a combination of guiding/seeing errors and nonuniform responsivity. The traditional way to accomplish this is to image the pupil onto the detector with a Fabry lens, but as we have seen the size and shape of the pupil of a guided beam will be sensitive to changes in the fiber configuration. The extent of the resulting photometric error may not be so large as to create difficulty: the enlargement of the beam produced by macrobending, as estimated from Equation 12, will typically be less than 10%, and the focal ratio will consequently change by at most

a few percent in the course of a series of observations. If the detector used is sufficiently uniform in response, or if the desired precision is not too high, this may be a situation one can live with.

Alternatively, one could image the fiber end directly onto the detector and trust the image scrambling of the fiber to eliminate the changes in flux distribution created by image motion and seeing variations. But as we have seen, the image scrambling of a perfect fiber is far from complete, and it is possible that the scrambling produced by real fibers may be inadequate. In addition, direct imaging onto the fiber end can lead to photometric errors arising from imperfections--perhaps even dust motes--at the input. In compensation, one could employ Fabry imaging at the input end of the fiber so that the output end reflects a scrambled image of the telescope pupil. But this is potentially dangerous to the photometric stability of the system, since the focal ratio input to the fiber will then be inversely proportional to the effective image diameter, a quantity reflecting both the seeing disc diameter and the axial offset of the image, and consequently sensitive to guiding errors. As we have seen, the optical performance of fibers depends sensitively on focal ratio, so that Fabry imaging at the input end of the coupling fiber would probably introduce more photometric problems than it would cure.

A more prudent approach--one suggested by Norman Walker at this workshop--is to illuminate the input end of the fiber with a defocused image of the star, thus achieving something of a compromise between direct and Fabry imaging. In combination with the image scrambling produced by optical fibers, this strategy may well yield a distribution of illumination across the output end of the coupling fibers that is sufficiently stable to guiding and seeing errors at the input end. Of course, a fairly large core diameter will be required in order to accept the entire defocused image and still leave judicious room for guiding errors. Silica fibers of core diameters as large as 1.5 mm are apparently available, and liquid-core fibers can be obtained with diameters of 5 mm or more. The optical properties of such fibers are not well understood, at least not by the author, and it is unclear how useful they would be in this application.

In practice, the extent of the problem depends on the actual performance of fiber couplers, the uniformity of the detector used, and the precision desired; the only completely safe solutions may be the use of detectors of highly uniform responsivity or of additional levels of image scrambling, such as integrating spheres, at the fiber output. One suspects there may be difficulties with either of these approaches.

### Final Remarks

However one chooses to get light into and out of the coupling fiber, it is unwise to leave the fiber ends uncovered. Whether polished or cleaved, fiber ends seldom constitute good optical surfaces and are often inclined to the fiber axis by a few degrees. For the best optical performance, fiber ends should be covered with high quality plane-parallel, fused silica plates and intervening index matching fluid or cement. This strategy also allows the designer to achieve higher optical efficiency by the use of anti-reflection coatings on the exterior surfaces of the silica plates.

This paper has concentrated on step-index fibers for two reasons. The first is that gradient-index fibers almost certainly yield far less image scrambling than do step-index fibers; the second is that the use of a gradient index does not appear to mitigate the effects of micro- and macrobending. Indeed, available gradient-index fibers may, on average, show more microbending-induced modal diffusion than do step-index fibers. For these reasons it appears that step-index fibers are superior for all analog optical waveguide applications, including astronomical ones.

### SUMMARY

The situation can now be summarized as follows. It may well be possible to design optical fiber photometer-telescope couplers whose optical throughputs remain sensibly invariant as the telescope moves. Configuration-dependent effects on the pupil are potentially more troublesome, but may be amenable to the use of defocused input images on large-diameter fibers. In addition, there is some evidence of a sensitivity of optical performance of fibers to the temperature range typically experienced within astronomical observatories.

What are now needed are good laboratory measurements of many aspects of analog transmission by step-index fibers, but principally of these two: image scrambling and temperature dependence of optical throughput, possibly as functions of fiber configuration and perturbation. Until these analog properties of optical fiber transmission are better understood, their suitability for coupling photometers to telescopes remains uncertain.

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