AERODYNAMIC PROPERTIES OF FRACTAL GRAINS: IMPLICATIONS FOR THE PRIMORDIAL SOLAR NEBULA

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ABSTRACT

Under conditions in the primordial solar nebula and dense interstellar clouds, small grains have low relative velocities. This is the condition for efficient sticking and formation of fractal aggregates. A calculation of the ratio of cross-section, σ, to number of primary particles, N, for fractal clusters yielded $\ln \sigma/N = 0.2635 + 0.5189N^{(-0.1748)}$. This ratio decreases slowly with N and approaches a constant for large N. Under the usual assumption of collisions producing spherical compact, uniform density aggregates, σ/N varies as $N^{-1/3}$ and decreases rapidly. Fractal grains are therefore much more closely coupled to the gas than are compact aggregates. This has a significant effect on the aerodynamic behavior of aggregates and consequently on their evolution and that of the nebula.

It has been known for some time (Whitlaw-Gray and Patterson, 1932; Fuchs, 1964) that aerosols are fluffy, low-density aggregates. This is also true of particulate clouds in the laboratory (Stephens and Russell, 1979; Samson *et al.*, 1987) and is illustrated in Figure 1. Micrometer-size or smaller grains in the atmosphere (Fuchs, 1964) or solar nebula (Völk *et al.*, 1980; Weidenschilling, 1984) have relative velocities about one ms^{-1}. Therefore, similar structures should be found in each case.

Recent work, both experimental (Forrest and Witten, 1979; Martin *et al.*, 1986) and theoretical (Sutherland, 1967; Witten and Sander, 1981; Kolb *et al.*, 1983; Meakin, 1983, 1984a; Sander, 1984; Kolb; Samson *et al.*, 1987) has shown that these aggregates are fractals (Mandelbrot, 1982). Figure 2 compares a soot particle and a numerical simulation of cluster-cluster growth with linear trajectories. A principal characteristic of fractal aggregates is that the number of primary particles, N, within a radius r, measured from randomly selected particle, is given by

$$N(r) = Ar^D. \tag{1}$$

A is some constant dependent on the system and D is the fractal dimension which depends upon the accretion process. For aerosols the primary process would be cluster-cluster accretion (Meakin *et al.*, 1985). This process leads to a value of D with $1.7 < D < 2.1$ (Meakin, 1984b; Julien *et al.*, 1984). For a fractal, the density $\rho(r)$ decreases with size according to the relation

$$\rho(r)\alpha\ r^{D-3}. \tag{2}$$

For aerosol aggregates, therefore, $\rho(r)$ varies approximately as r^{-1}.

This result will have a significant effect on the aerodynamic behavior of fractal aggregates. In the usual treatments of the grain collisions where it has been assumed that collisions produce compact bodies with a constant density, the gas drag decreases rapidly with size. For fractals on the other hand, drag decreases very slowly, as we now show.

We report here the first step towards a quantitative analysis of the aerodynamic behavior of fractal aggregates. A calculation of the projected cross-section as a function of size has been carried out. Numerical simulations were performed using the cluster-cluster aggregation model (Meakin *et al.*, 1985). A gas-kinetic velocity distribution in which velocities are proportional to $N^{-1/2}$ was adopted. Nine simulations were carried out starting with 200,000 particles and continued until the maximum size first exceeded 10,000 particles.

After each cluster (containing N particles) had been generated, the projected area was obtained by projecting the cluster onto a plane and picking 5N points at random in a circle of radius R_{max} enclosing the projected cluster (here R_{max} is the maximum radius of the cluster measured from its center of mass). If M of these points are in the region containing the projection, then an estimate of the projected cluster area, σ, is given by

$$\sigma = \pi R_{max}^2 \ \frac{M}{5N} \tag{3}$$

For each cluster this procedure was repeated for two other projections in directions mutually perpendicular to the first.

The continuous line in Figure 3 shows the dependence of $\ln(\sigma/N)$ on $\ln(N)$. Since the fractal dimensionality (D) is less than 2, we expect that in the asymptotic limit ($N \to \infty$) that σ/N should be independent of N (perhaps σ could depend on some power of $\ln(N)$, but not on a power of N). Consequently, the data shown in Figure 1 have been fitted to the form

$$\sigma = AN + BN^\beta. \tag{4}$$

For clusters in the size range N = 5-100 a nonlinear least squares fit give A = 0.2635, B = 0.5189 and β = 0.8252. This curve is represented by the upper line in Figure 3 which has been displaced to distinguish it from the simulation data. It is apparent that this fits the simulation results very well for all values of N. Even for single particles (N = 1) the dashed curve gives a projected area of A+B or 0.782 compared to a value of $\pi/4$ or 0.785 expected for a sphere of unit diameter. For clusters in the size range 1-2500 particles, the same procedure gave A = 0.2403, B = 0.5172 and β = 0.8465. Again, A+B = 0.768 is quite close to the expected values of $\pi/4$. These results suggest that in the limit $N \to \infty$ the projected area approaches a value of about 0.24.

For comparison, in Figure 3, we include the curve for $\sigma/N \propto N^{-1/3}$, appropriate for a compact body. This curve has been set to $\sigma = \pi/4$ at N = 1 and thus matches the fractal curve at ln N=0. Note the steep decrease in the ln σ/N compared to the fractal curve. As σ/N is the ratio of gas drag-to-particle mass, it is the dominant factor in establishing the gas-grain interaction. With the gas velocity field, it determines the motion of the aggregate.

For fractal grains, the near constancy of this ratio means that aggregates will have similar aerodynamic behavior over a wide range of sizes. In particular, larger fragments will not settle much faster than smaller ones nor will they interact very differently with turbulent eddies leading to large relative velocities (Völk *et al.*, 1980; Weidenschilling, 1984).

The optical properties of fractals also differs from those of compact particles (Berry and Percival, 1986).

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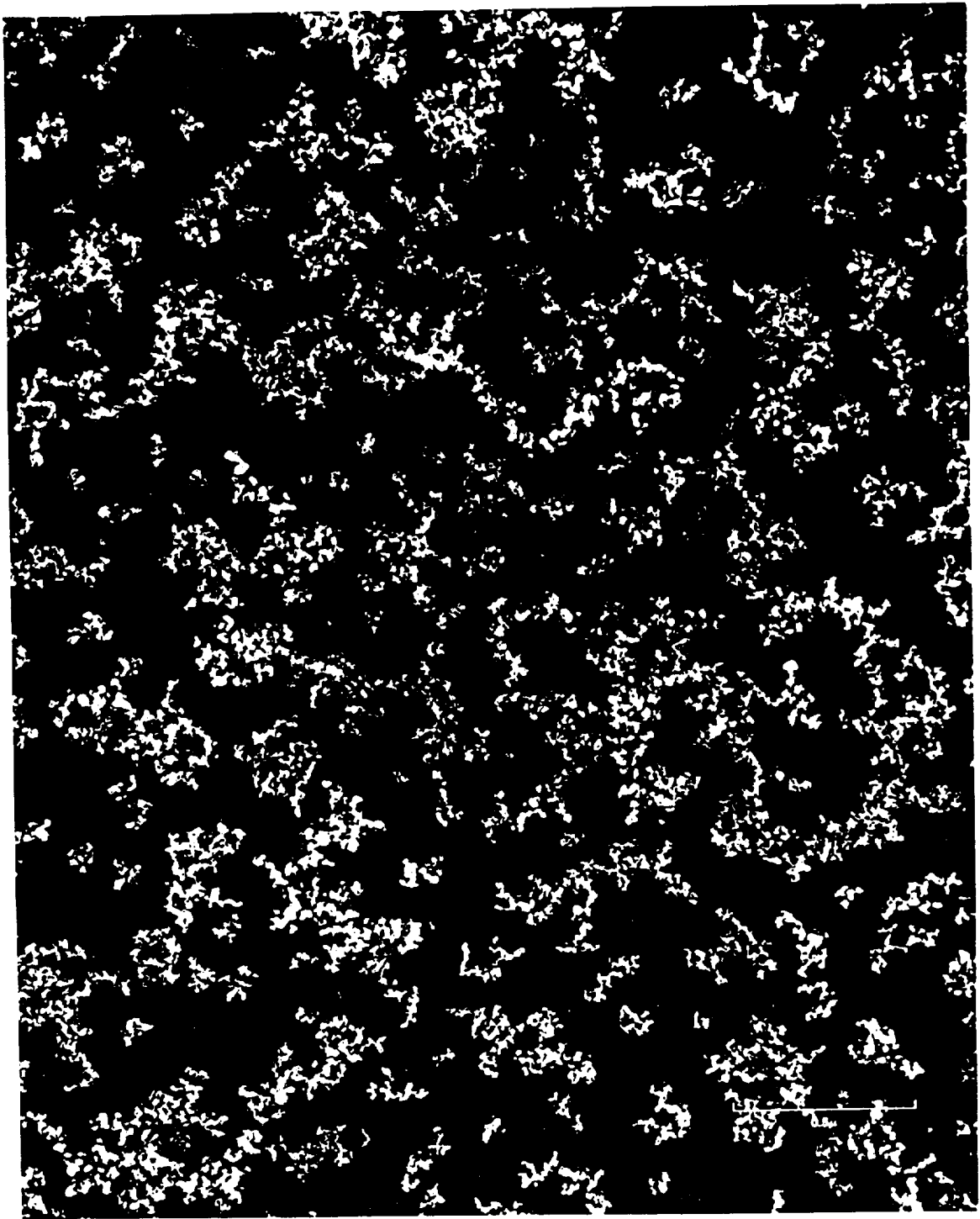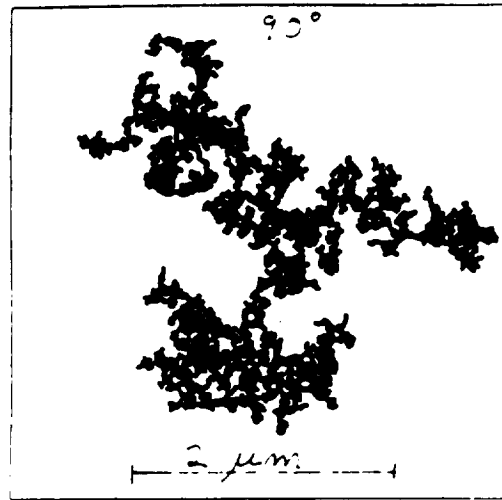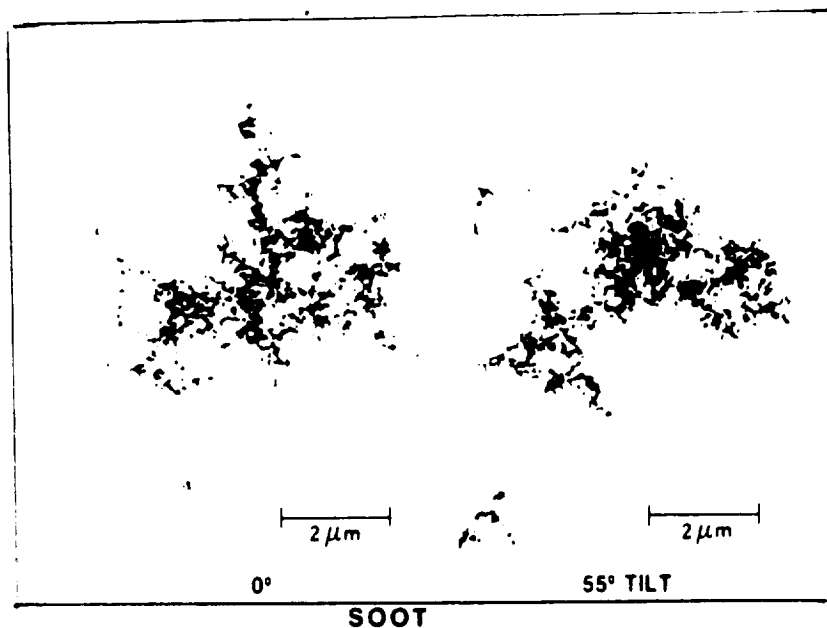Witten, T. A. and Sander, L. M., 1981, *Phys. Rev. Lett.*, **47**, 1400.

Figure 1 - Fused Silica clusters. Primary particles are ~ 10μ. Because of high-formation temperature, particles are molten and fuse on contact. (Courtesy of Cab-O-Sil Division, Cabot Corp.).

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AGGLOMERATE VIEWED AT TWO ANGLES

2 μm   2 μm

0°   55° TILT

SOOT

COMPUTER SIMULATION

Figure 2 - Comparison of soot particle and numerical simulation at two angles of view showing fractal structure. (Soot-structure courtesy of G. W. Mulholland, National Bureau of Standards).
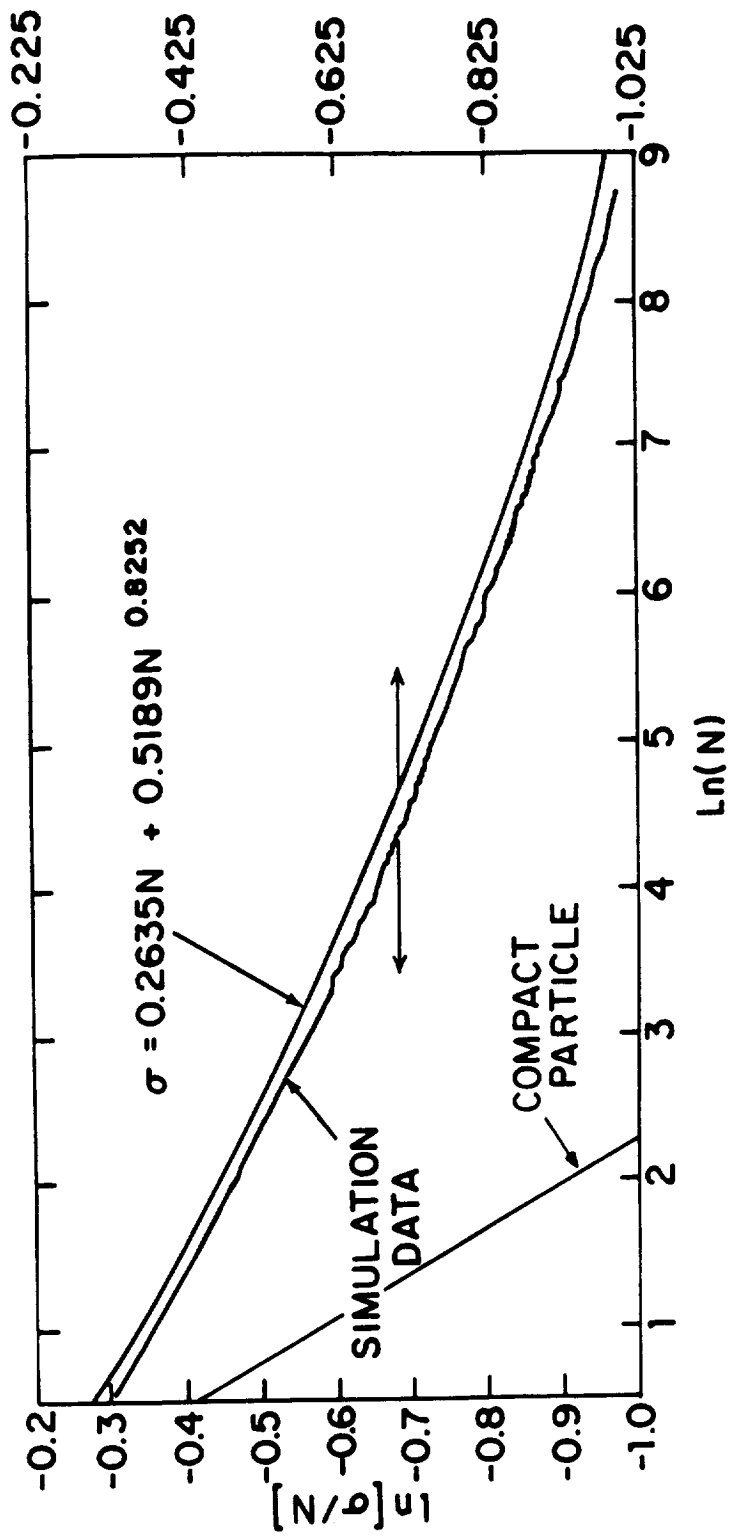
Figure 3 - Projected cross-section of fractal aggregates. Cross-section, σ, measured in terms of particles in aggregate. N is number of particles in aggregate. For terms of particle of unit diameter, $\sigma = \pi/4$. Curve for compact object decreases fractal aggregate, $(\sigma/N)_\infty = 0.24$, $\ln \sigma/N_\infty = 1.43$. Curve for compact object decreases without limit. The two fractal curves have been displaced 0.025 units in the abscissa of clarity.

In the chart:

$\sigma = 0.2635N + 0.5189N^{0.8252}$

SIMULATION DATA

COMPACT PARTICLE

Ln(N)

$\ln[\sigma/N]$