APPLICATION OF THE

MOBILITY POWER FLOW APPROACH

TO STRUCTURAL RESPONSE

FROM DISTRIBUTED LOADING

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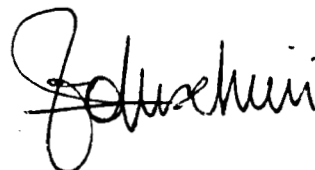
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FOREWARD

This report describes the work performed between September 1988 to December 1988 under Research Grant Number NAG-1-685, entitled, "Use of Energy Accountancy and Power Flow Techniques for Aircraft Noise Transmission". This is the fifth progress report under this research grant in which most of the work done thus far addresses the problem of the characterization of the vibrational power flow. During this latest phase of the work, a Ph.D. Thesis was completed with the title, "Vibrational Power Flow in Thick Connected Plates", the Abstract of which will be published in the Journal of the Acoustical Society of America in the Technical Notes and Research Briefs section. The work in this thesis was also partially funded by an ONR Fellowship which paid for the stipend of the graduate student.

The author would like to acknowledge the graduate students who assisted in the work on this project, the Department of Ocean Engineering and most importantly the financial support by the Structural Acoustics Branch of the NASA Langley Research Center.

Submitted by

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ABSTRACT

This report investigates the problem of the vibration power flow through coupled plate substructures when one of the substructures is subjected to a distributed load. The approach used is based on the mobility power flow method. Two types of distributed loading are considered. The first part of the report deals with a distributed mechanical load which is independent of the response of the structure. Power flow expressions are presented for this type of loading. In the second part of the report, the analysis for distributed load which is influenced by the motion of the structure, as in the case of acoustic excitation is presented. Results for power flow when the plate substructures are subjected to acoustic excitation will be published in future reports.

1. INTRODUCTION

The previous work using the mobility power flow (MPF) approach has, in general, been restricted to point excited structures [1-5]. However in the case of excitation of an aircraft fuselage distributed loading on the surface of a panel can be as important as excitation from directly applied forces at defined locations on the structure. The term "mobility power flow" is being used here to distinguish this technique from other techniques on power flow. In this approach, the vibrational power flow is determined using mobility expressions. There are other methods which are similar to either finite element analysis [6] or statistical energy analysis where the results are frequency or spatial averaged [7,8]. In this report the formalism of the power flow expressions for two coupled plate substructures with a distributed load on one of the substructures is presented.

Two types of loadings are considered. They are a distributed force load of arbitrary shape and a distributed load created by an obliquely incidence acoustic waves on the surface of one of the plate substructures. The difference between these two types of distributed loads is that in the mechanical loading case the applied loading is independent of the response of the structure while in the case of the excitation from an incident acoustic wave, the loading is dependent on the response of the structure due to the presence of the scattered acoustic waves. In this latter type of loading coupled expressions must be developed for the excitation and response. This, to some extent, can be viewed

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as an introduction into the development of expressions for the power flow (acoustical and vibrational), using a mobility power flow approach, when coupling exists between a structure and the surrounding medium as in the case of the interior of an aircraft fuselage.

2. DISTRIBUTED FORCE

The development of power flow and mobility expressions for the distributed force loading, with arbitrary dependence on the x and y coordinates, is very similar to the work reported previously for point loading [2,4]. The differences are that the presentation of the mobility functions can be generalized to apply for any loading condition. Using the configuration and axes definition as shown in figure 1, the modal input mobility function for the junction, and the transfer mobility function between the junction and any point on the plate surface, both functions being defined as the response per unit applied moment for every mode m along the junction, are given by [2,4].

$$M_{2_m} = M_{3_m} = \frac{\dot{\theta}_m}{T_m} = \frac{j}{2\sqrt{\rho h D^*}} \left[\frac{k_2}{\tan(k_2 b)} - \frac{k_1}{\tanh(k_1 b)} \right] \tag{1}$$

$$M_{12_m}(y) = \frac{\dot{u}_m(y)}{T_m} = \frac{j}{2\sqrt{\rho h D^*}} \left[\frac{\sin(k_2 y)}{\sin(k_2 b)} - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \tag{2}$$

where T_m is the mode m component of the edge moment, k_1 and k_2 are defined by:

$$k_1^2 = 2k_x^2 + k_y^2 \; ; \qquad k_2 = k_y$$

$$k_x = m\pi/a \; ; \qquad k_y^2 = \omega \sqrt{\frac{\rho h}{D^*}} - k_x^2 \qquad (3)$$

u_m is the mode m component of the plate surface displacement, θ_m is the mode m component of the angular displacement at the edge of the plate and D^* and ρh are the plate flexural rigidity and surface density respectively. In the above equations it is assumed that the two plate substructures are identical.

Note that in equation (2) the transfer mobility is a function of coordinate y. This is required since the load is distributed and defined everywhere along the y direction on the plate surface.

The above modal mobility expressions are for a system with an applied edge moment, which represents one of the configurations for which mobility expressions are derived [2]. The second configuration to be considered is the one which represents the external loading to the global structure. Considering first the distributed load, this can be decomposed into its modal components as follows:

$$F(x,y) = F_o \, f(x)f(y) \qquad (4)$$

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$$F_m = \frac{2}{a} \int_o^a F_o \; f(x) \; \sin\left(\frac{m\pi x}{a}\right) \; dx \tag{5}$$

$$F_n = \frac{2}{b} \int_o^b f(y) \; \sin\left(\frac{m\pi y}{b}\right) \; dy \tag{6}$$

$$F_{mn} = F_m \; F_n \tag{7}$$

This load decomposition will be used to define the mobility functions for the second plate configuration. Therefore, the modal mobility for the input load, M_{1mn} defined as the ratio of mode (m,n) component of the transverse velocity on the surface of the plate, to an applied distributed surface load with the same distribution as the eigenfunction for mode m,n. is given by:

$$M_{1_{mn}} = \frac{\dot{u}_{mn}}{F_{mn}} = \frac{jf}{2\pi\rho h} \; \frac{1}{\left(f_{mn}^*\right)^2 - f^2} \tag{8}$$

where

$$f_{mn}^* = \frac{1}{2\pi} \sqrt{\frac{D^*}{\rho h}} \left[\left(\frac{m\,\pi}{a}\right)^2 + \left(\frac{n\,\pi}{b}\right)^2\right] \tag{9}$$

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and u_{mn} is the mode m,n component of the plate surface displacement. The expression for the transfer mobility, M_{21m} defined as the ratio of the mode m component of a plate edge angular velocity, to the mode m component of an applied distributed load on the surface of the plate is given by;

$$M_{21_m} = \frac{\dot{\theta}_m}{F_m} = \frac{jf}{2\rho hb} \sum_n \frac{(-1)^n \, n \, F_n}{\left(f_{mn}^*\right)^2 - f^2} \qquad (10)$$

The evaluation of the above mobility functions is obtained from consideration of the equations of motion for a plate structure.

The expression for the input and transmitted power are the same as in the case of point loading [2] except that all mobility functions are modal expressions;

$$Power_{input} = \frac{1}{2} \, Real \left\{ \sum_{m,n} \frac{ab}{4} \left|F_{mn}\right|^2 M_{1_{mn}} \right.$$

$$\left. - \frac{a}{2} \sum_m \frac{M_{21_m}}{M_{2_m} + M_{3_m}} \left|F_m\right|^2 \int_0^b f(y)^* M_{12_m}(y) \, dy \right\} \qquad (11)$$

$$\text{Power}_{\text{trans}} = \frac{1}{2} \frac{a}{2} \sum_m \left| \frac{M_{21_m} F_m}{M_{2_m} + M_{3_m}} \right|^2 \text{Real}\left[M_{3_m} \right] \qquad (12)$$

m and n are respectively the mode number for the x and y directions, that is m is along the junction and n perpendicular to the junction, and a and b are the dimensions of the plates (figure 1).

For a point load, the force is in this case described mathematically by;

$$F(x,y) = F_O \ \delta(x-x_O) \ \delta(y-y_O) \qquad (13)$$

and

$$F_{mn} = \frac{4}{ab} F_O \sin\left(\frac{m\pi x_O}{a} \right) \sin\left(\frac{m\pi y_O}{b} \right) \qquad (14)$$

where x_O and y_O are the x and y coordinates of the point of application of the load. If the load is applied at the center of the plate (x_O = a/2 and y_O = b/2), then only odd values for m an n are allowed. Substituting in equations (11) and (12), the results obtained for the input and transferred power, when the load is in the center of the source plate, are shown in figure (2). Comparing these results to those obtained in reference [13], the two sets of results are identical. This verifies the

formulation of the power flow expressions in the form shown in equations (11) and (12).

For a uniform distributed load described by:

$$F(x,y) = F_o \qquad (15)$$

$$f(y) = 1.0 \qquad (16)$$

and

$$F_{mn} = \frac{16\ F_o}{\pi^2\ (2m + 1)\ (2n + 1)} \qquad (17)$$

The solution in this case follows in the same way as for the point load case. The results for the power flow are shown in figure (3).

Comparing the results for the power flow obtained for the point load with the results obtained for the distributed load, the following similarities and differences can be observed. First, the modes excited by the two types of loading are identical. This is expected since both loading conditions are symmetrical about the center of the source plate. Second, the general level of the power flow for the distributed load case decreases with frequency, while that for the point load does not decrease. The reason for this can be mathematically described by the inverse dependency of the modal components of the distributed applied load on the mode number for the case of the distributed

load (equation (17)). Physically this implies that the higher frequency modal components of the load are suppressed. The results for the distributed load would be similar to those for excitation by normal incidence acoustic plane waves, if the scattered pressure component is neglected.

3. EXCITATION BY INCIDENT ACOUSTIC WAVES

In dealing with this form of distributed excitation, the scattering of the incident acoustic wave due to the response of the receiving structure must be included in the analysis [9]. The scattered acoustic field is a function of the response of the structure, and therefore, the loading on the structure consists of two components; one component is from the incident pressure being blocked by the surface and the second component is the influence of the scattered acoustic wave. The component associated with the blocked incident pressure is independent of the motion of the structure except for the phase differences between structure response and surface motion. The component associated with the scattered pressure is a direct function of the motion of the structure and therefore the loading becomes structure response dependent. Therefore to deal with this type of distributed loading some modifications to the basic formulation of the expressions for the vibrational power input and power flow are required. To demonstrate the differences in the formulation, the case of a point load which is a function of the response velocity at the point of application for arbitrary structure configuration is discussed first.

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Consider the configuration shown in figure 4, where the input load F is a function of the response velocity. The governing equations of motion for this configuration are as follows [1];

$$V = F M_1 + F_A M_{12} \qquad\qquad 18.$$

$$V_A = - F_A M_3 \qquad\qquad 19.$$

$$= F_A M_2 + F M_{21} \qquad\qquad 20.$$

but

$$F = A + BV \qquad\qquad 21.$$

Where A and B are two constants and all the other terms have the same definition as in previous reports in this series [1]. Equations (19) and (20) can be used to solve for the junction force F_A in terms of the input load F, and then substituted into equation (18) to solve for the velocity V. In the case where F is a function of V, the result for V must be substituted back into the expression for F and then used to solve for F_A and V_A. Alternatively V and F_A are expressed in terms of the constants A and B and the mobility functions. That is,

$$\begin{bmatrix} (1-BM_1) & -M_{12} \\ BM_{21} & M_2+M_3 \end{bmatrix} \begin{bmatrix} V \\ F_A \end{bmatrix} \begin{bmatrix} A\,M_1 \\ -A\,M_{21} \end{bmatrix} \qquad\qquad 22.$$

and

$$\begin{bmatrix} V_A \end{bmatrix} = \begin{bmatrix} -M_3 \end{bmatrix} \begin{bmatrix} F_A \end{bmatrix} \qquad 23.$$

The input and transmitted power, P_i and P^t respectively, are then given by;

$$\begin{bmatrix} P_i \\ P_t \end{bmatrix} = \begin{bmatrix} (A + BV) & F_A \end{bmatrix} \begin{bmatrix} V^* \\ V_A^* \end{bmatrix} \qquad 24.$$

Applying this approach to the case of the acoustic excitation, the expressions for the input and transfer modal mobility functions for an input edge moment (equations 1 and 2) are still applicable. For the input mobility of an acoustic excited plate structure, consider an incident acoustic wave at an angle of ϕ to the normal to the plate, where the trace of the acoustic wave on the plane of the plate makes an angle θ to the x-axis (figure 5). The trace of the incident pressure on the surface of the panel is given by;

$$p(x,y) = P_i \, e^{-jk \sin \theta \sin \phi \, y} \, e^{-jk \sin \phi \cos \theta \, x} \qquad 25.$$

where k is the acoustic wavenumber, and p_i is the magnitude of the incident pressure.

The component of the input load associated with the blocked incident pressure is equal to twice the incident pressure [9] at the surface of the plate. The scattered component can then be

formulated based on the in vacuo response of the plate and is controlled by the amplitude of this response. The response that must be considered in determining the scattered component is the response of the global structure that is a simultaneous solution must be obtained for the forcing and the response of the global structure.

Let the surface velocity of the receiver plate be described by;

$$v(x,y) = \sum_q V(y)_q \sin\left[\frac{q\pi x}{a}\right] \qquad 26.$$

The scattered pressure can be obtained by equating the plate surface velocity to the particle velocity of the acoustic medium at the interface. Using a spatial transform to represent the response of the whole plate, the scattered pressure component for mode q is given by;

$$P_s(\alpha,\beta)_q = V(\beta)_q \left[\frac{q\pi}{a}\right] \frac{[(-1)^q e^{-j\alpha a} - 1]}{\left[\alpha^2 - \left[\frac{q\pi}{a}\right]^2\right]} \frac{\omega\rho_0}{k_z} \qquad 27.$$

where

$$k_z = \pm \sqrt{(k^2 - \alpha^2 - \beta^2)} \qquad 28.$$

where k is the acoustic wavenumber. The positive or negative sign is selected to satisfy the far field conditions. The total modal pressure decomposed into the wavenumber β, acting on the

plate surface is given by the summation of the blocked component and the scattered component.

$$p(\beta)_m = 2\ p_i(\beta)_m\ +\ p_s(\beta)_m \qquad\qquad 29.$$

where

$$P_i(\beta)_m = \int_{-\infty}^{\infty} \left[\int_0^a 2P_i\ e^{-jkx\sin\phi\ \cos\theta}\ \sin\left(\frac{m\pi x}{a}\right) dx \right] e^{-jky\ \sin\phi\ \sin\theta}\ e^{-j\beta y}\ dy$$

$$30.$$

and

$$P_s(\beta)_m = \sum_q V(\beta)_q \left(\frac{m\pi}{a}\right) \frac{[(-1)^m\ e^{j\alpha a} - 1]}{\left[\alpha^2 - \left(\frac{m\pi}{a}\right)^2\right]} \left(\frac{q\pi}{a}\right) \frac{[(-1)^q\ e^{-j\alpha a} - 1]}{\left[\alpha^2 - \left(\frac{q\pi}{a}\right)^2\right]} \frac{\omega\rho_o}{k_z}\ \frac{d\alpha}{\pi a}$$

$$31.$$

The terms of the summation for which $q \neq m$ represent intramodal coupling which is negligible for light fluid loading provided the modal density of the structure is low. Equations (30) and (31) define the terms A and B in matrix equation (22). To solve for the velocity of the source substructure;

$$V(\beta)_m = p(\beta)_m\ M\ (\beta)_m\ +\ T_m\ M_{12}\ (\beta)_m \qquad\qquad 32.$$

where

$$M(\beta)_m = \frac{j\omega}{D^*}\ \frac{1}{\left[\beta^2 + \left(\frac{m\pi}{a}\right)^2\right]^2 - \frac{\rho h\omega^2}{D^*}} \qquad\qquad 33.$$

which is obtained from the plate equations of motion, and $M_{12}(\beta)_m$ is the spatial fourier transform of equation (2). From continuity of motion at the junction,

$$-T_m \; M_{3m} = T_m \; M_{2m} + \left[\int_{-\infty}^{\infty} P(\beta)_m \; M(\beta)_m \; e^{j\beta y} \; d\beta \right]_{y=b} \qquad 34.$$

and therefore

$$T_m = \frac{-1}{M_{2m} + M_{3m}} \left[\int_{-\infty}^{\infty} P(\beta)_m \; M(\beta)_m \; e^{j\beta y} \; d\beta \right]_{y=b} \qquad 35.$$

The integrand of this last expression contains a $V(\beta)_m$ term and therefore when substituting equation (35) in equation (32) to solve for $V(\beta)$, the solution can only be obtained numerically. From the solution for $V(\beta)$, the input and transmitted vibrational power calculated. To evaluate the input power it is not necessary to inverse Fourier transform the expression for $V(\beta)_m$ and $P(\beta)_m$ since an integral over the structure surface is required. This would be equivalent, by Parceval's Theorem, to an integral over the entire β space. For the transmitted power this is obtained from the relationship;

$$\left[\text{Power transmitted} \right]_m = \left| T_m \right|^2 M_{3m} \qquad 35.$$

and then summed for all modes m.

A complete solution to these equations has not been achieved and therefore no results will be presented in this progress report. Results for the L-shaped plate structure, including a parametric evaluation and comparison to the results for the point force loading, will be presented in a following progress report.

4. CONCLUSION

The mobility power flow formalism for two coupled structures with excitation from distributed loading, including acoustic excitation, on one of the components of the coupled structure has been discussed. For a distributed loading which is independent of the motion of the structure, the mobility power flow formalism is relatively straight forward and very much similar to that of point loading. For the case of acoustic excitation, where the magnitude of the excitation is controlled by the response of the global structure, the formalism is much more complex. Because of the coupling of the response to the excitation, the only method available for obtaining a solution is through a numerical approach. This is similar to conclusions reached by other authors who investigated the problem of acoustic excitation where, unless the mode shapes of vibration of the global structure are a priori known, the solution is always obtained numerically. This does not in any way limit the application of the mobility power flow method to multiple coupled structures, since the complexities arise mainly for the source structure.

Results for the analysis presented in this report as applied to an L-shaped plate will be reported in the future together with

the experimental evaluation of the results for both point loading and distributed loading.

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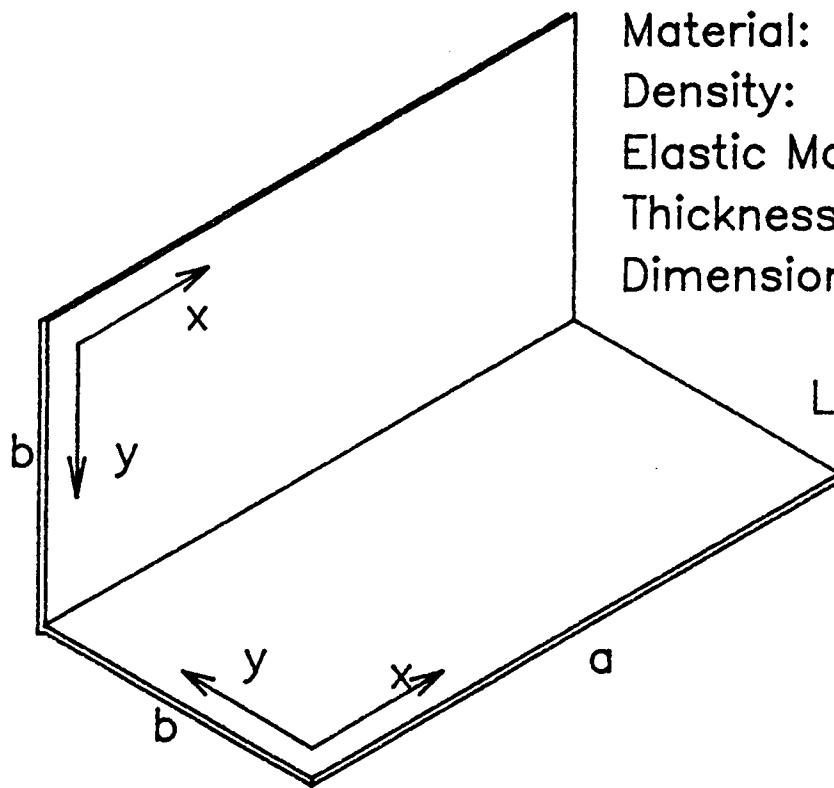
Material: Aluminium
Density: 2710 Kg/m³
Elastic Modulus: 72 GN/M²
Thickness: 0.00635m
Dimensions: a=1.0m,
            b=0.5m,
Loss Factor: 0.01

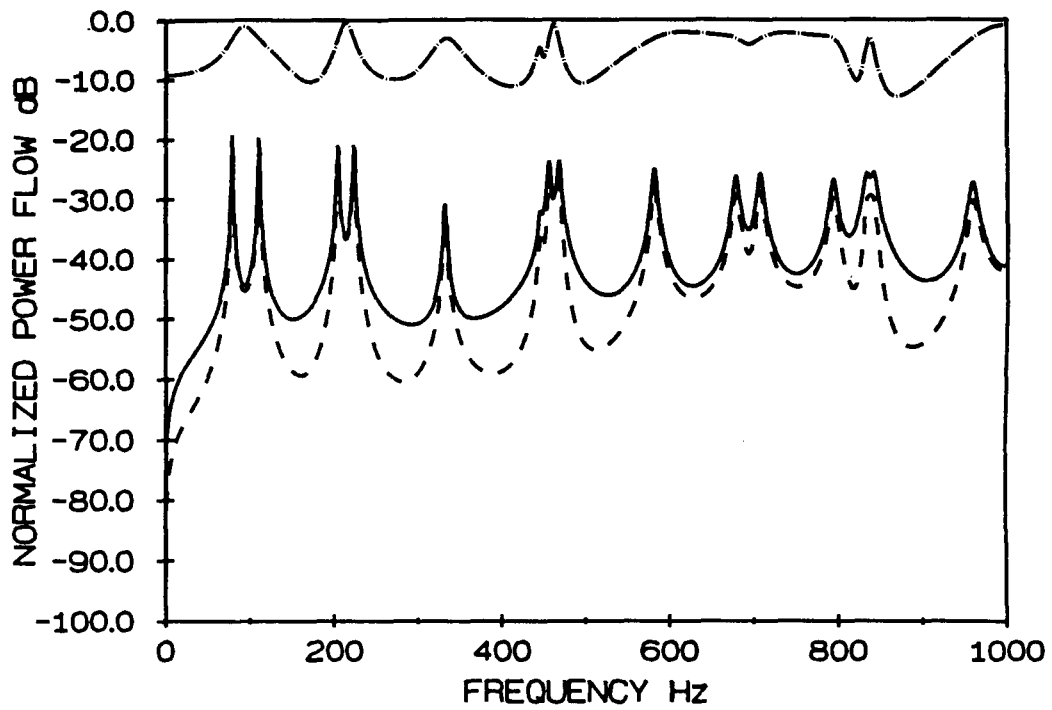Figure 1. L—shaped plate structure showing plate
characteristics.

Figure 2. Normalized power flow results for point loading.
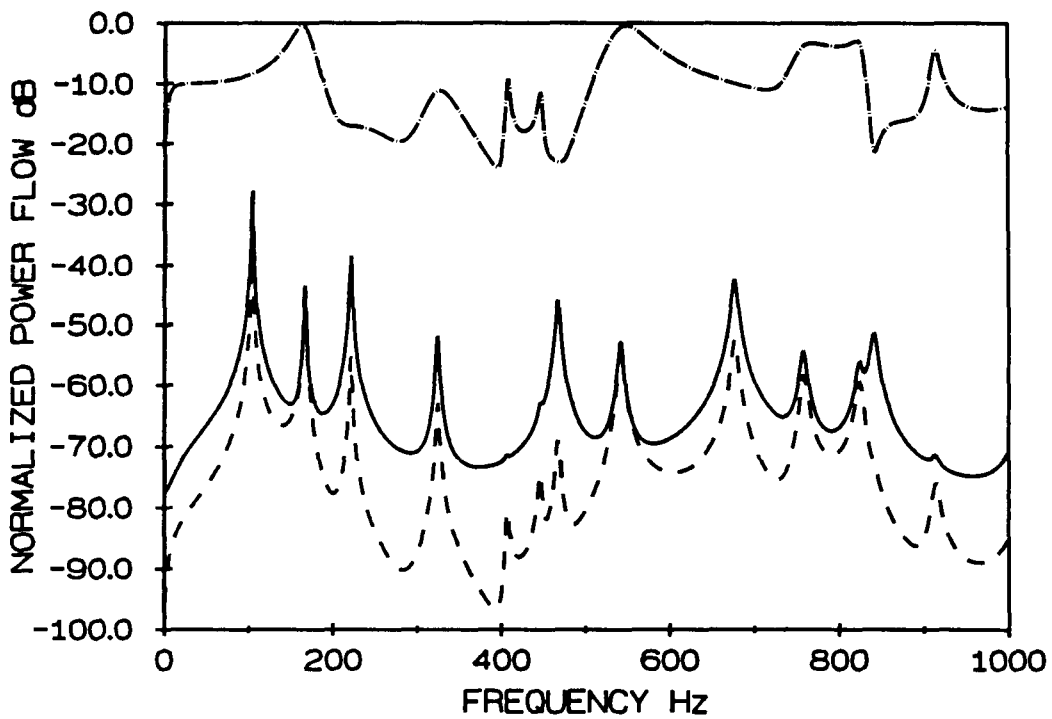___: Power input; ---: power transfer; -.-: power ratio.

Figure 3. Normalized power flow results for distributed loading.
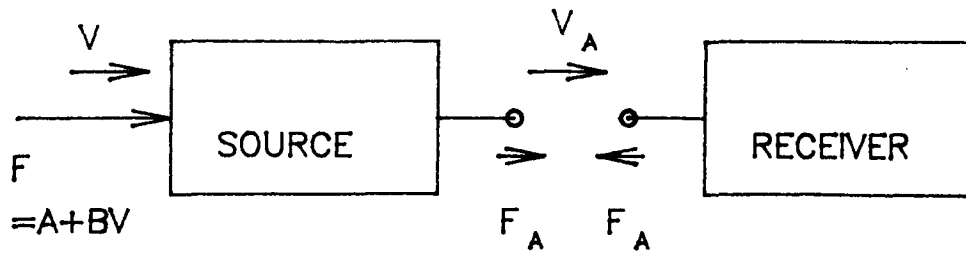___: Power input; ---: power transfer; -.-: power ratio.

Figure 4. Source—receiver model where F is a function
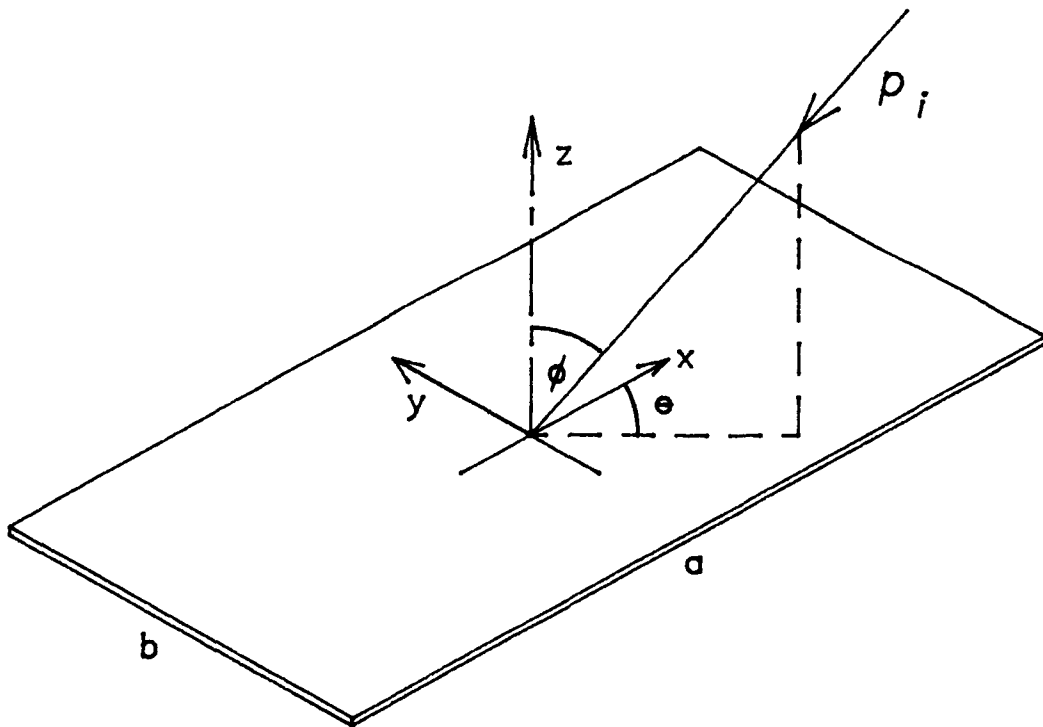of the response at the point of application.

Figure 5. Incedent acoustic pressure at oblique angle ϕ.