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Efficient Load Measurements Using Singular Value Decomposition

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Principal Investigator
Professor Kung Yao

Co-Principal Investigator
Professor A.V. Balakrishnan

NASA Technical Officer
Mr. Karl Anderson

Laboratory for Flight Systems Research
University of California
Los Angeles, CA 90024-1594

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1. Introduction

There are two fundamental operations in the load measurement problem on an aircraft structure. In the first part, we perform a system identification by measuring the responses of the strain gages mounted on different locations of the structure from a series of known applied load at various specified load points on the structure during the calibration stage on the ground. In the second part, by using some characterization of the system obtained in the first part, we can predict the actual equivalent load value and location from the gage measurements during a flight. Various known successful approaches and results have been reported in the past on the load measurement problem [1-3].

There are two fundamental and intuitively equally justifiable linear approaches (arbitrarily denoted as Approach 1 and Approach 2 in Section 2) applicable to the load measurement problem. In Approach 1, we model the load value matrix $\underline{L}$ as dependent linearly on the influence coefficient value matrix $\underline{M}$ measured by the gages. In Approach 2, we model $\underline{M}$ as dependent linearly on $\underline{L}$. In general these matrices are rectangular, thus it is not immediately clear that these two approaches are equivalent. Historically, all the work in [1-3] were based on that of Approach 1. In Section 2, we shall show that these two approaches are indeed equivalent in all cases, and can be proved by the use of the modern Singular Value Decomposition (SVD) technique. On the other hand, if we only use the more conventional and previously used normal equation technique [1-3] (also called the linear regression technique), then the limitation of this analytical technique can only show the validity of Approach 1 when the number of gages n is less or equal to the number of loads m. In addition, by using the normal equation approach, it is only possible to handle these problems with n greater or equal to m from the Approach 2 point of view. There are several theoretical, practical, and computational consequences to these observations.

At the most basic level of understanding, of course, it is theoretically important to know the equivalency of these two seemingly different approaches that yield the desired result. At the practical algorithmic operational level, the inadmissibility of having the number of gages n greater than the number of applied loads m in the calibration stage in Approach 1 is not fatal. However, as we shall show in Section 3, for the multi-stage load estimation technique (which can yield extremely accurate load predictions), we will always use more gages than the number of loads in the prediction stage. Conventional normal equation approach (i.e., Approach 1) is not possible since a crucially needed matrix involved in the processing is singular.

When the data from the gages are quite linearly independent, then there is no significant numerical difference between the use of the SVD technique or the normal equation technique. However, for highly dependent data, there can be significant advantages for the SVD technique. Detailed numerical computations based on practical observed gage measurements and load values are necessary to verify their differences. The crucial point is that in all cases, the SVD approach is always computationally more costly as well as numerically more stable. For dimensions encountered in the load measurement problems, the additional computational cost of the SVD approach is not significant to be of concern.

2. Two Equivalent Approaches to Load Measurement Evaluations

Now, we consider two possible basic equivalent approaches for load measurements. These approaches are denoted as Approach 1 and Approach 2.

2.1 Approach 1 - Linear Dependency of Load Values on Gage Values

Consider

(1) $\underline{L} = [\underline{L}_1, \underline{L}_2, \underline{L}_3]$,

the m x 3 load matrix, where

(2) $\underline{L}_1 = \underline{L}_S = [S_1, S_2, \ldots, S_m]'$,

is the shear vector at m load locations (x_i, y_i),

(3) $\underline{L}_2 = \underline{L}_B = [B_1, B_2, \ldots, B_m]'$,

is the bending moment vector with its i-th component located at y_i given by

(4) $B_i = S_i y_i$, $i=1,\ldots,m$,

and

(5) $\underline{L}_3 = \underline{L}_T = [T_i, \ldots, T_m]'$,

is the torque vector with its i-th component located at x_i given by

(6) $T_i = S_i x_i$, $i=1,\ldots,m$.

Let the m x n influence coefficient matrix M denote the response of the n gages to the m loads in the calibration process. Specifically, let

(7) $\underline{M} = [\underline{u}_1, \ldots, \underline{u}_n] = \begin{bmatrix} \underline{M}_1 \\ \vdots \\ \underline{M}_m \end{bmatrix}$,

where each u_i, $i=1,\ldots,n$, represents the normalized response of the i-th gages to the m loads. Let the n x 3 dependency coefficient matrix $\underline{b}$ consists of

(8) $\underline{b} = [\underline{b}_1, \underline{b}_2, \underline{b}_3]$,

where

(9) $\underline{L}_i = \underline{M}\ \underline{b}_i$, $i=1,2,3$,

or in matrix form

(10) $\underline{L} = \underline{M}\ \underline{b}$.

For i=1, the n x 1 vector $\underline{b}_i$ yields the dependency of $\underline{L}_1 = \underline{L}_S$, the shear vector, to the linear combinations of the influence coefficient vectors $\{\underline{u}_1, \ldots, \underline{u}_n\}$ of $\underline{M}$ in (7). Similarly, for i = 2 and 3, $\underline{b}_2$ and $\underline{b}_3$ are related to the bending moment vector $\underline{L}_2 = \underline{L}_B$ and the torque moment vector $\underline{L}_3 = \underline{L}_t$, respectively.

In the calibration process, the matrix $\underline{M}$ as well as $\underline{L}_1$, $\underline{L}_2$, and $\underline{L}_3$ are available. Define the pseudo-inverse of $\underline{M}$ as a nxm matrix from the "normal equation" point of view as

(11) $\underline{M}^+ = (\underline{M}'\underline{M})^{-1}\underline{M}'$.

We note, $\underline{M}^+$ in (11) is defined if and only if $m \geq n$ and all columns of $\underline{M}$ are linearly independent. In particular, if $n > m$, then $\underline{M}^{-1}$ in (11) is not defined.

Then (9) becomes,

(12) $\underline{b}_i = \underline{M}^+\ \underline{L}_i$ $i=1,2,3$.

By using the notation of $\underline{b}$ in (8) and $\underline{L}$ in (1), (12) can be written in matrix form as

(13) $\underline{b} = \underline{M}^{+} \underline{L}$.

In the prediction process, we observe one 1 x n dimensional gage measurement vector $\underline{\tilde{M}}$ (corresponding to the first row vector of $\underline{M}$ in (7)). From (10), the predicted 1 x 3 load vector is given by

(14) $\underline{\tilde{L}} = [\underline{\tilde{S}}, \underline{\tilde{B}}, \underline{\tilde{T}}] = \underline{\tilde{M}}\ \underline{b} = \underline{\tilde{M}}\ \underline{M}^{-1}\ \underline{L} = \underline{\tilde{M}}\ (\underline{M}'\underline{M})^{-1}\ \underline{M}'\underline{L}$.

The first component of $\hat{L}$ yields the predicted shear,

(15) $\tilde{S} = \underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_1$,

the second and third components of $\underline{\tilde{L}}$ yield the predicted bending moment $\tilde{B} = \tilde{S}\tilde{y}$ and predicted torque $\tilde{T} = \tilde{S}\tilde{x}$,

(16) $\tilde{B} = \tilde{S}\ \tilde{y} = \underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_2$,

(17) $\tilde{T} = \tilde{S}\ \tilde{x} = \underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_3$.

From (15) and (16), we can solve for $\tilde{y}$ as

(18) $$\tilde{y} = \frac{\underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_2}{\underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_1},$$

and $\tilde{x}$ as

(19) $$\tilde{x} = \frac{\underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_3}{\underline{\tilde{M}}\ \underline{M}^{+}\ \underline{L}_1}.$$

Thus, (15), (18) and (19) represent the predicted equivalent net shear, bending moment location, and torque location of the applied load that yielded the measured gage vector $\underline{\tilde{M}}$ using the normal equation approach.

Now consider the use of the SVD technique via Approach 1. Consider a general form of the SVD of the matrix $\underline{M}$ with rank p

$\leq \min(m,n)$ as given by

(20) $$\underline{M} = \underline{U}_M \, \underline{\Sigma}_M \, \underline{V}'_M \, ,$$

where $\underline{U}_M$ is a m x m orthogonal matrix, $\underline{V}'_M$ is a n x n orthogonal matrix, and $\underline{\Sigma}_M$ is a m x n matrix where the upper left p x p section is a diagonal matrix with positive singular values (S.V.) denoted by $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p > 0$, and the remaining section is the zeroth matrix. The pseudo-inverse of the matrix $\underline{M}$, is a n x m matrix denoted by $\underline{M}^{++}$, and from the SVD point of view is then given by

(21) $$\underline{M}^{++} = \underline{V}_M \, \underline{\Sigma}_M^{+} \, \underline{U}_M' ,$$

where $\underline{\Sigma}_M^{+}$ is a n x m matrix where the upper left p x p section is a diagonal matrix with reciprocal of the positive singular values of $\underline{M}$, and the remaining section is the zeroth matrix. Then by using (21) in (10) , we have

(22) $$\underline{M}^{++} \, \underline{L} = \underline{b} \, .$$

We note, (22) corresponds to (13) in the calibration stage of the previously considered normal equation technique. Then in the prediction stage, we have

(23) $$\underline{\tilde{L}} = \underline{\tilde{M}} \, \underline{b} = \underline{\tilde{M}} \, \underline{M}^{++} \, \underline{L} \, .$$

It is most interesting to note, that the predicted load vector in (23) based on the SVD technique has the same form as the predicted load vector in (14) based on the normal equation technique. Indeed, when $m \geq n$ (i.e., the number of loads is greater or equal to the number of gages), and when the gage measurements are quite linearly independent, the pseudo-inverse given by $\underline{M}^{++}$ in

(21) is equal to the pseudo-inverse given by $\underline{M}^{+}$ in (11). Thus, in those cases, either the conventional normal equation or the SVD methods will yield the same predicted load values. Of course, when the measurement values are quite linearly dependent, then the SVD approach will be better from the numerical stability point of view. As mentioned earlier, when n > m, the normal equation method is not applicable for Approach 1 since $\underline{M}^{+}$ in (11) is not defined. However, the results of (21)-(23) under the SVD method for Approach 1 are valid in all cases including n > m.

2.2 Approach 2 - Linear Dependency of Gage Values on Load Values

From a physical cause and effect point of view, it makes sense that the responses of the first gage to the m loads are given by,

(24)
$$\underline{u}_1 = \begin{bmatrix} u_{11} \\ u_{12} \\ \vdots \\ . \\ u_{1m} \end{bmatrix} = \begin{bmatrix} S_1 & S_1 y_1 & S_1 x_1 \\ . & . & . \\ . & . & . \\ . & . & . \\ S_m & S_m y_m & S_m x_m \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix}$$
$$= [\ \underline{L}_1,\ \underline{L}_2,\ \underline{L}_3\ ]\ \underline{c}_1 = \underline{L}\ \underline{c}_1 .$$

In (24), we are describing the gage measurement u_{1i} as a linear combination of $s_i c_{11} + s_i y_i c_{12} + s_i x_i c_{13}$, which depends linearly on the shear, bending moment and torque. In general, for all n gages, we have,

(25)
$$\underline{M} = [\underline{u}_1, \ldots, \underline{u}_n] = \underline{L}\ [\underline{c}_1, \ldots \underline{c}_n]$$
$$= \underline{L}\ \underline{C},$$

where the 3 x n dependency matrix $\underline{C}$ is denoted by,

(26)
$$\underline{C} = [\underline{c}_1, \ldots, \underline{c}_n].$$

In the calibration process, as before, $\underline{M}$ and $\underline{L}$ are available. In the prediction process, as before, we have a measured $\tilde{\underline{M}}$, given from (25) as

(27) $$\tilde{\underline{M}} = \tilde{\underline{L}}\,\underline{C}.$$

In order to solve for the 1 x 3 predicted load vector

(28) $$\tilde{\underline{L}} = [\ \tilde{\underline{S}},\ \tilde{\underline{B}},\ \tilde{\underline{T}}\ ],$$

we need to use (25) and (27). First, consider the use of the normal equation technique. Let the psudeo-inverse of $\underline{L}$ be denoted by

(29) $$\underline{L}^{+} = (\underline{L}'\underline{L})^{-1}\underline{L}'.$$

By using (29) in (25), we have

(30) $$\underline{C} = \underline{L}^{+}\,\underline{M}.$$

Substituting (30) in (27), we obtain

(31) $$\tilde{\underline{M}} = \tilde{\underline{L}}\,\underline{L}^{+}\,\underline{M}\ .$$

Now, in order to solve for $\tilde{\underline{L}}$, we need to multiply both sides of (31) by $\underline{M}'$ from the right and try to take inverse. Unfortunately, since $\underline{M}$ is a m x n matrix, if m > n (which is often the case), then $(\underline{MM}')$ is singular and then $(\underline{MM}')^{-1}$ does not exist. Thus, in this case, it is <u>not possible</u> to use the normal equation technique based on Approach 2. When $n \geq m$, then direct solution of (31) yields

(32) $$\tilde{\underline{L}} = \tilde{\underline{M}}\,\underline{M}'(\underline{MM}')^{-1}\,\underline{L}\ .$$

By comparing (32) of Approach 2 (valid for $n \geq m$) to (14) of Approach 1 (valid for $m \geq n$), we see that only when m = n are these two approaches yield identical results.

Now, consider solving for $\underline{C}$ in (25) by using the pseudo-inverse of $\underline{L}$ based on the SVD representation of $\underline{L}$. Specifically, consider the SVD of $\underline{L}$ as given by

(33) $\underline{L} = \underline{U}_L \, \underline{\Sigma}_L \, \underline{V}_L{}'$,

where $\underline{U}_L$ is a m x m orthogonal matrix, $\underline{V}_L$ is a 3 x 3 orthogonal matrix, and $\underline{\Sigma}_L$ is a m x 3 matrix of the form of

(34) $$\underline{\Sigma}_L = \begin{bmatrix} \sigma_{L1} & 0 & 0 \\ 0 & \sigma_{L2} & 0 \\ & & \sigma_{L3} \\ \cdots & & \\ 0 & 0 & 0 \end{bmatrix},$$

where the top 3x3 sub-matrix is a diagonal matrix of singular values $\sigma_{L1} \geq \sigma_{L2} \geq \sigma_{L3} > 0$ and the remaining (m-3) x3 sub-matrix is an all zero matrix. Then (25) becomes

(35) $\underline{M} = \underline{U}_L \, \underline{\Sigma}_L \, \underline{V}_L{}' \, \underline{C}$.

By using (30) and (33) in (35), we have

(36) $\underline{V}_L \, \underline{\Sigma}_L{}^+ \, \underline{U}_L{}'\underline{M} = \underline{C}$,

(37) $$\underline{\Sigma}_L{}^+ = \begin{bmatrix} 1/\sigma_1 & 0 & 0 & . & 0 \\ 0 & 1/\sigma_2 & 0 & . & 0 \\ 0 & 0 & 1/\sigma_3 & . & 0 \end{bmatrix}.$$

In particular, we note

(38) $\underline{\Sigma}_L{}^+ \, \underline{\Sigma}_L = \underline{I}_3$.

By using (36) in (27), we have

(39) $\tilde{\underline{M}} = \tilde{\underline{L}} \, \underline{V}_L \, \underline{\Sigma}_L{}^+ \, \underline{U}_L{}' \, \underline{M}$.

Now consider the use of the SVD representation of $\underline{M}$ given by (20) in (39). Then we have

(42) $\underline{M}_1 = \tilde{\underline{L}} \, \underline{V}_L \, \underline{\Sigma}_L{}^+ \, \underline{U}_L{}' \, \underline{U}_M \, \underline{\Sigma}_M \, \underline{V}_M$,

Direct solution of $\underline{\tilde{L}}$ in (42) yields

(43) $$\underline{\tilde{L}} = \underline{\tilde{M}}\, \underline{M}^{++}\, \underline{L} .$$

By comparing (43) to that of (23), we see that by using the SVD technique, both Approaches 1 and 2 yield the same predicted load vector, for all cases of m and n (i.e., $m \geq n$ or $n > m$). But as discussed earlier, by using the normal equation technique, Approach 1 is applicable only for $m \geq n$, while Approach 2 is applicable only for $n \geq m$. Furthermore, the predicted $\underline{\tilde{L}}$ in (14) under Approach 1 and (32) under Approach 2 are equivalent only for $m = n$.

3. Multi-Stage Load Estimation Technique (MUSLET)

Now, we consider a new proposed MUlti-Stage Load EsTimation technique (which we shall denote as MUSLET) for a more accurate prediction of the load value. As considered in Sections 1 and 2, the load matrix $\underline{L}$ in equation (1) and the gage measurement matrix $\underline{M}$ in (7) are needed to estimate the dependency matrix $\underline{b}$ in Approach 1 and the dependency matrix $\underline{c}$ in Approach 2. In either case, if we actually know more accurately the true value of the load location, then we need to use only those load locations that are close (or closest) to the true load location in the calibration process to yield a more appropriate $\underline{b}$ or $\underline{c}$ which in turn can yield a more accurate predicted load value and location. An analogous argument to this technique is that in numerical analysis, only the relevant near by x-y points to the desired x location are used to perform a numerical interpolation. The set of far away x-y points may introduce more errors rather than provide the desired smoothing effects in the interpolation process. Initial numerical experiments based on the HWTS Loads Calibration Data of 8/29/76 showed that suppose the 10th calibration load is removed from the actual system identification calibration process. By using the remaining 17 load conditions and 15 gage measurements, the initial predicted bending moment y location was given as 80.3 and the predicted torque x location was given as 60.2. The true locations were 81.3 and 60 respectively. However, from this initial prediction result, we can use only these calibration points with similar or close y values to 80.3. In this example, we can use only loads labeled as {7;8;9;11} with calibration load values situated at y = 81.3. The newly predicted y value now becomes 81.29992. Similarly, the closest x values in the calibration stage are given by loads labeled as {3;4;5;9;11; 15;17;18}. The newly predicted x value is now given by 59.84374. We note the original predicted values have errors of have errors of 1.23% in y and 0.34% in x, while the iterated second stage prediction has errors of only 0.0001% in y and 0.19% in x. Thus, this MUSLET technique appears to be able to yield significant improvement in load predictions. Many variations of the above simple proposed iteration schemes are possible and will be further investigated.

4. Conclusions

In this research period from January 1987 to March 1988, we have performed various basic research on efficient load measurement estimation techniques for aircraft structure analysis. In Section 1, we presented an over view of the load measurement problem. In Section 2, we considered two basic equivalent approaches to load measurement evaluations. Under Approach 1, the load values are modeled as depending linearly on the measured values. Under Approach 2, the measured values depend linearly on the load values. By using the modern SVD method, we showed that under all conditions of the number of loads m and number of gages n, Approach 1 is equivalent to Approach 2. By using the conventional normal equation (or linear regression) approach, Approach 1 is only valid for $m \geq n$ (which is commonly encountered case), while Approach 2 is valid only for $n \geq m$. Furthermore, except for the case of $m = n$, the load prediction formulas under the two approaches are not equivalent.

In many practical flight testing situations, we may not be able to use as many gages as those used in the calibration process. Thus, there is much interest in finding the most efficient set of n gages to be used for predictions. We have also performed exhaustive tests on various subsets of the available gages and associated load measurements. Preliminary investigations show this approach also to be meaningful. Another approach based on DIStributed Load EsTimation (DISLET) performs a series expansion of the fight measured gage data vector in terms of the calibration measured gage data vectors. Coefficients of this expansion indicates the amount of load distributions during flight relative to the m calibration load points. Further analytical and computational investigations are under study. Another multi-dimensional cluster analysis relevant in the efficient reduction of gages, also based on the use of SVD technique, has also been under consideration and will be reported on in the future.

Finally, in [4], we have published a basic research paper on the effective use of singular values in estimation problems for data contaminated by Gaussian noise. The results in [4] yielded tighter performance bounds as compared to all previously known results in this field.

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