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OPTIMIZATION OF SLENDER WINGS FOR CENTER-OF-PRESSURE SHIFT DUE TO CHANGE IN MACH NUMBER

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It is observed that the center of pressure on a wing shifts as the Mach number is changed. Such shifts are in general undesirable and are sometimes compensated for by actively shifting the center of gravity of the aircraft or by using active stability controls. To avoid this complication, it is desirable to design the wings of a high speed aircraft so as to minimize the extent of the center-of-pressure shifts. This, together with a desire to minimize the center-of-pressure shifts in missile control surfaces, provides the motivation for this project.

There are many design parameters which affect center-of-pressure shifts, but it is expected that the largest effects are due to the wing planform. Thus, for the sake of simplicity, this study is confined to an investigation of thin, flat (i.e., no camber or twist), relatively slender, pointed wings flying at a small angle of attack, α_∞ . Once the dependence of the center of pressure on planform and Mach number is understood, we can expect to investigate the sensitivity of the center-of-pressure shifts to various other parameters.

The planform of the wing is specified by functions $\pm g(x)$ with $0 < x < c$ where c is the chord length. The fact that the wing is pointed requires $g(0) = 0$. It is further assumed that $g'(x) \geq 0$, $g'(c) = 0$ (streamwise tips), and that the trailing edge is an unswept straight line. Viscosity, vorticity and nonlinear compressibility effects are assumed to be negligible. Therefore, the problem is formulated in terms of a velocity potential ϕ which satisfies the Prandtl-Glauert equation with a Kutta-Joukowski condition imposed on the trailing edge. Solutions are sought for both the supersonic and subsonic regimes.

The simplest approximate analytic solutions are given by Jones' slender wing theory¹ which is valid in the limiting case that the aspect ratio, $A = \frac{(\text{span})^2}{\text{Area}}$, goes to zero. Then the location of the center-of-pressure is given as a function only of the planform function $g(x)$. In particular, there is no dependence on the Mach number, M .

To see a dependence on M , we turn to not-so-slender wing theory, which was developed by Adams and Sears² using classical transforms and recast into a modern form by Wang³ using matched asymptotic expansions. Here the expansion parameter is $\beta A = \sqrt{|M^2 - 1|}A$. Two different expansions of ϕ are given by Wang depending on whether the flow is supersonic or subsonic, and both expansions include $\beta^2 A^2$ and $\beta^2 A^2 \log(\beta A)$ terms. Jones' results are independent of βA .

Wang's expressions for ϕ involve derivatives of integrals which in general cannot be evaluated in closed form. However, if $g(x)g'(x)$ can be expressed as a polynomial,

$g(x)g'(x) = \sum a_i x^i$, Squires⁴ developed for the supersonic case closed-form analytic expressions for the chordwise loading and for the total lift. The comparable integral for the subsonic case is a little more complicated, but under the same condition comparable analytic expressions were found by the author this summer.

The expressions for the center-of-pressure involve another integration step, but this step requires some numerical quadrature. Center of pressure shifts for a number of plan-forms are being examined.

References

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