MODIFYING PASVART TO SOLVE SINGULAR NONLINEAR 2-POINT BOUNDARY PROBLEMS

by

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To study the buckling and post-buckling behavior of shells and various other structures, one must solve a nonlinear 2-point boundary problem. Since closed-form analytic solutions for such problems are virtually nonexistent, numerical approximations are inevitable. This makes the availability of accurate and reliable software indispensable.

In a series of papers Lentini and Pereyra[4,5], expanding on the work of Keller[2], developed **PASVART**: an adaptive finite difference solver for nonlinear 2-point boundary problems. While the program does produce extremely accurate solutions with great efficiency, it is hindered by a major limitation. **PASVART** will only locate isolated solutions of the problem. In buckling problems, the solution set is not unique. It will contain singular or bifurcation points, where different branches of the solution set may intersect. Thus, **PASVART** is useless precisely when the problem becomes interesting.

To resolve this deficiency we propose a modification of **PASVART** that will enable the user to perform a more complete bifurcation analysis. **PASVART** would be combined with the Thurston bifurcation solution: an adaptation of Newton's method that was motivated by the work of Koiter[3] and reinterpreted in terms of an iterative computational method by Thurston [8,9]. Roughly speaking, Thurston's method incorporates higher order terms in the Taylor expansion to solve for approximate solutions that will converge on different branches of the solution set.

A drawback of the approach is that it requires extensive programming by the user. However, it is possible, with the use of symbolic manipulation programs, to generate the necessary computer code automatically[1,6]. Thus, we will also combine the modified version of **PASVART** with Macsyma (a symbolic manipulation program developed at MIT). This will eliminate the need for the user to perform a complex programming task. The new version of **PASVART** will be easier to use and capable of a much more sophisticated analysis.

References

[1] Gates, B. L. and Wang, P. S., "A Lisp-Based Ratfor Code Generator." In 1984 Macsyma Users' Conference, pp. 319-329.

[2] Keller, H.B., "Accurate Difference Methods for Nonlinear Two-Point Boundary Value Problems.", SIAM J. NUMER. ANAL., Vol. 11, 1974 pp. 305-320.

[3] Koiter, W. T., "Elastic Stability and Post-Buckling Behavior," Proceedings, Symposium on Nonlinear Problems, ed. Lnager, R. E., University of Wisconsin Press, 1963, pp. 257-275.

[4] Lentini, M. and Pereyra, V., "A Variable Order Finite Difference Method for Nonlinear multipoint Boundary Value Problems." Math Comput., 28 1974, pp. 981-1004.

[5] Lentini, M. and Pereyra, V., "An Adaptive Finite Difference Solver for Nonlinear Two-Point Boundary Problems with Mild Boubdary Layers," SIAM J. NUMER. ANAL., Vol. 14, No. 1 March 1977.

[6] Steinberg, S. and Roache, P., "Using Vaxima to Write Fortran Code," In 1984 Macsyma Users' Conference, pp. 1-19.

[7] Thurston, G. A., "Newton's Method Applied to Problems in Nonlinear Mechanics," Journal of Applied Mechanics, Vol. 32, Trans. ASME, Vol. 87, Series E, 1965, pp. 383-388.

[8] ______, "Continuation of Newton's Method Through Bifurcation Points," Journal of Applied Mechanics, Vol. 36, No. 3, Trans. ASME, Vol. 81, Series E, Sept 1969, pp. 425-430.

[9] ______, "A New Method for Computing Axisymmetric Buckling of Spherical Caps," Journal of Applied Mechanics, Vol. 38, No. 3, Trans ASME, March 1971, pp. 179-184.