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Aerodynamic Maneuvering Hypersonic Flight Mechanics

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1. Introduction

The emergence of current high-interest missions involving aeromaneuvering hypersonic flight has given rise to the corresponding need for preliminary design and performance analyses of such vehicles. This need in turn has motivated efforts to develop simplified analytical and computational methods for parametric analysis of maneuvering hypersonic flight under conditions appropriate to the mission involved. The missions of interest include those of aircraft, e.g., the Orient Express, and hybrid or "spaceplane" vehicles, e.g., the single-stage-to-orbit or transatmospheric Vehicle (TAV), and the aeroassisted spacecraft such as the Aeroassisted Orbital Transfer Vehicle (AOTV). These missions feature aeromaneuvering flight at high altitudes and high Mach number, and are characterized by protracted global-scale maneuvers. Under these conditions, the trajectory may be significantly affected by rotating-Earth and Earth-curvature effects in addition to the primary balance of gravity, thrust, and aerodynamic forces. In addition, the characteristic trajectory parameters as well as the associated coordinate system in which the trajectory is described require evaluation since aircraft related parameters and frames of reference are typcally different from those of orbiting spacecraft and reentry trajectories.

The purpose of this study was to develop simplified analytical methods for parametric analysis of hypersonic maneuvering flight. The effort included a review of different formulations of the general equations of motion, their associated coordinate frames, various simplifications of the equations, and previously achieved analytical solutions. This study sought to both extend previous solution methods and to develop new ones. In addition, evaluation of the literature and developing a systematic perspective on the knowledge it represents proved to be a major portion of the effort.

2. Background

2.1 Literature Categorization

A review of the hypersonic maneuvering-flight literature involves categorization into planar/nonplanar studies, studies including/excluding propulsive thrust, and studies using an Earth-fixed wind axes coordinate frame or Earth-centered spherical coordinate frame or orbital axes/elements frame. Also, the varied assumptions of maneuver constants and equation transformations employed in the previous studies are of interest. Excluded from this review are earlier studies involving ballistic reentry vehicles and studies focussing on the guidance/navigation/control optimization of hypersonic maneuvering flight. References 1 - 3 are examples of general studies presenting full derivation of the equations of motion with both analytical and computational solutions. Shkadov, et al[1] treat propelled aircraft as well as thrustless maneuvering descent from orbit, both with Coriolis effects excluded. Loh[2] treats the various cases of thrustless lifting and nonlifting reentry, again with Coriolis effects excluded from analytical solutions. Vinh, et al[3] derive the full equations of motion, and then exclude from analytic solutions Coriolis effects as well as thrusting during reentry. The scarce literature dealing with thrusting maneuvering hypersonic flight is found in References 4 through 13. References 4 - 10 focus on the nonplanar (turning) "aerocruise" maneuver where thrust continually cancels drag. References 11 - 13 deal with simplified analytical solutions for planar maneuvering Transatmospheric Vehicle (TAV) flight. The prolific nonthrusting or "aerogliding" literature includes References 13 through 27 and 36 for nonplanar flight and References 28 through 34 for planar flight. Reference 35 provides a general theoretical formulation applicable to all kinds of missions.

By far the majority of the literature employs a wind axis coordinate frame in which the equations of motion are expressed. As exceptions, equations are expressed in a spherical coordinate frame in References 5, 8, 15 and 29, and in an orbital element frame in References 9 and 10; the frames employed are not obvious in References 12 and 24. Finally, pure computational solution techniques are used in References 6, 7, 12, 19 through 24, 29, 30, 33 through 36. All other references to some extent provide analytic solutions, some with computational solutions for comparison. None of the references employing wind axis coordinates and providing analytical solutions include Coriolis effects on the trajectory and vehicle performance. Further, analytic-solution references show a bewildering variety of parameters chosen as the independent variable of differentiation in the transformed differential equations of motion that facilitate analytical solutions. Variables are used such as flight path angle γ, time t, and distance along the trajectory s. These analytic references also show a large variety of assumptions on constant parameters during maneuvering flight: References 4 and 7 hold velocity V and altitude (and thus density) constant during the turn, References 4, 6 and 7 bank angle σ, and References 4, 7, 11 and 13 dynamic pressure q.

3. General Equations of Motion - Different Formulations

3.1 Earth-fixed Geographical Frame

Vinh, et al[1] provide a general formulation for the nonplanar equations of motion in an Earth-fixed rotating frame where thrust and aerodynamic forces (lift and drag) are included. The reference plane is the equatorial plane, so that vehicle position is described in latitude/longitude/altitude coordinates (geographical coordinates). These equations comprise a determinate set of three dynamical and three kinematical equations in six unknowns as follows:

$$\frac{dV}{dt} = \frac{T\cos\varepsilon - D}{m} - g\sin\gamma + \omega^2 r\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\sin\psi) \tag{1}$$

$$V\frac{d\gamma}{dt} = \frac{T\sin\varepsilon + L}{m}\cos\sigma - \left(g - \frac{V^2}{r}\right)\cos\gamma + 2\omega V\cos\phi\cos\psi + \omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi) \tag{2}$$

$$V\frac{d\psi}{dt} = \frac{T\sin\varepsilon + L}{m}\frac{\sin\sigma}{\cos\gamma} - \frac{V^2}{r}\cos\gamma\cos\psi\tan\phi + 2\omega V(\tan\gamma\cos\phi\sin\psi - \sin\phi) - \frac{\omega^2 r}{\cos\gamma}\sin\phi\cos\phi\cos\psi \tag{3}$$

$$\frac{dr}{dt} = V\sin\gamma$$

$$\frac{d\theta}{dt} = \frac{V\cos\gamma\cos\psi}{r\cos\phi}$$

$$\frac{d\phi}{dt} = \frac{V\cos\gamma\sin\psi}{r}$$

where

T, D, L = thrust, drag, lift forces
ε = thrust angle from flight path
m = mass
σ = roll (bank) angle
ω = Earth rotation rate

unknowns:
V = speed
γ = flight path angle (+ is up)
ψ = azimuth or heading (from East)
r = radius position
θ = latitude
ϕ = longitude

The equations assume a spherical, rotating Earth, and out-of-plane force is generated by banking the lift force. In the discussion of these equations, the various contributing "acceleration" terms or effects are referred to as lift (L/m), drag (D/m), thrust (T/m), gravity (g), centrifugal (V^2/r), Coriolis ($2\omega V$), and Earth-centrifugal ($r\omega^2$). The last two are "Earth-rotation" effects, while the centrifugal term V^2/r is due to flight path curvature.

Table 1 estimates the order of magnitude (in g units) of each of these contributing terms for a variety of missions. It becomes obvious that although the Earth centrifugal terms ($r\omega^2$) may be neglected for all missions, the Coriolis term should be retained for high-altitude, hypersonic global-scale missions such as the TAV and the AOTV. This term is not retained in any of the earlier studies reporting analytic solutions to the equations. The development of corresponding analytical solutions where the Coriolis term is retained has been a chief focus of the present work. It is of particular interest to compare accelerations in g units as a function of Mach number for (gravity + centrifugal), Coriolis, and Earth-centrifugal effects (Table 2) It is seen that the Earth-centrifugal effect is always neglectable, but Coriolis is 10% or more of the sum (gravity + centrifugal) at Mach numbers of 15 and above (about 5% at M = 8).

Finally, Equations 1 - 3 degenerate to those employed in much of the planar-flight analytic-solution literature (References 11 - 13) when all the Earth-rotation effects are neglected (and $\varepsilon, \sigma = 0$):

$$(4) \quad \frac{dV}{dt} = \frac{T}{m} - \frac{D}{m} - g \sin\gamma$$

$$(5) \quad V\frac{d\gamma}{dt} = \frac{L}{m} - g\left(1 - \frac{V^2}{gr}\right)\cos\gamma$$

The Vinh formulation involves Earth-fixed latitude/longitude, i.e., geographical coordinates, and is a natural formulation for Earth-relative motion in nonmaneuvering reentry. The equatorial plane is the reference plane, making downrange/crossrange maneuvers very difficult to express and visualize by means of this formulation.

3.2 Orbital Frame

Cervisi[10] presents a succinct summary of the equations for motion assuming a spherical nonrotating Earth and expressed in terms of the variation of orbital elements caused by aerodynamic forces. These equations are coupled, first-order, nonlinear differential equations in terms of time derivatives of the orbital elements:

Table 1 Acceleration Terms – Relative Size for Various Missions (g units)

Mission	$\frac{dV}{dt}$ (Equation ①): $\frac{1}{g}\frac{F_T}{m}$	$\frac{W}{mg}$	$\frac{1}{g}\omega^2 r$	$V\frac{d\gamma}{dt}$ (Equation ②): $\frac{1}{g}\frac{F_N}{m}$	$\frac{W}{mg}$	$\frac{1}{g}\frac{V^2}{r}$	$\frac{1}{g}(2\omega V)$	$\frac{1}{g}\omega^2 r$	$V\frac{d\psi}{dt}$ (Equation ③): $\frac{1}{g}\frac{F_N}{m}$	$\frac{1}{g}\frac{V^2}{r}$	$\frac{1}{g}(2\omega V)$	$\frac{1}{g}\omega^2 r$
Hypersonic Aircraft	3	1	0.0035	3	1	0.0133	0.0136	0.0035	3	0.0133	0.0136	0.0035
Rocket Launch Ascent	4	1	0.0035	4	1	0.331	0.0679	0.0035	4	0.331	0.068	0.0035
ICBM Reentry	30	1	0.0035	1	1	0.59	0.0906	0.0035	1	0.59	0.091	0.0035
Spacecraft (Apollo) Reentry	8	1	0.0035	1	1	2.36	0.181	0.0035	1	2.36	0.181	0.0035
Earth Orbit	0	1	0.0035	0	1	0.98	0.118	0.0035	0	0.98	0.118	0.0035

Notes: $F_T = \underline{T\cos\varepsilon - D}$

$F_N = T\sin\varepsilon + L$

W = Vehicle weight

ω = Earth's rotation rate

V = Vehicle velocity

r = Vehicle distance from Earth's center

γ, ψ = trajectory flight path angle, heading (azimuth)

Table 2

Comparison of Acceleration Terms as a Function of Mach Number

Mach Number	Approx. Speed (m/s)	gravity + centri. $\left(1 - \frac{V^2}{V_c^2}\right)$	Coriolis $\frac{2\omega V}{g}$	Earth-Centri. $\left(\frac{r\omega^2}{g}\right)_{max}$
1	330	0.998	0.005	0.0035
5	1650	0.956	0.025	0.0035
10	3330	0.823	0.050	0.0035
15	4950	0.608	0.074	0.0035
20	6660	0.290	0.100	0.0035

Note: V_c = orbital velocity $\sim$ 7906 m/s $= \sqrt{rg}$

$$\text{(6)} \quad \frac{da}{dt} = \frac{2e\sin\nu}{n\sqrt{1-e^2}} A_R + \frac{2a\sqrt{1-e^2}}{nr} A_S$$

$$\text{(7)} \quad \frac{de}{dt} = \frac{\sqrt{1-e^2}\sin\nu}{na} A_R + \frac{\sqrt{1-e^2}}{na^2e}\left[\frac{a^2(1-e^2)}{r} - r\right] A_S$$

$$\text{(8)} \quad \frac{di}{dt} = \frac{r\cos u}{na^2\sqrt{1-e^2}} A_W \qquad \text{(9)} \quad \frac{d\Omega}{dt} = \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i} A_W$$

$$\text{(10)} \quad \frac{d\omega}{dt} = -\frac{\sqrt{1-e^2}\cos\nu}{nae} A_R + \frac{p}{eh}\left[\sin\nu\left(1+\frac{1}{1+e\cos\nu}\right)\right] A_S - \frac{r\cot i\sin u}{na^2\sqrt{1-e^2}} A_W$$

$$\text{(11)} \quad \frac{du}{dt} = \frac{na^2\sqrt{1-e^2}}{r^2} - \frac{r\cot i\sin u}{na^2\sqrt{1-e^2}} A_W$$

where a = semimajor axis
e = eccentricity
i = inclination
Ω = longitude of ascending node
h = angular momentum
$n = \sqrt{\frac{\mu}{a^3}}$, mean orbital motion
ω = argument of perigee
u = argument of latitude
ν = true anomaly
r = radius from Earth's center
p = semilatus rectum

and

$$A_S = \frac{T}{m}\cos\varepsilon - \frac{D}{m}$$
$$A_R = \left(\frac{L}{m} + \frac{T}{m}\sin\varepsilon\right)\cos\sigma$$
$$A_W = \left(\frac{L}{m} + \frac{T}{m}\sin\varepsilon\right)\sin\sigma$$

ε = thrust angle from flight path
σ = bank angle

In the absence of lift, drag, and thrust forces, these equations describe a conic orbit. Analytic and computational solutions built on this framework have largely been restricted to aerocruise and gliding plane-change (inclination change) maneuvers whose application of interest is to AOTV-like missions.

Clearly, this formulation expresses the modification of orbital motion produced by transient aerodynamic forces. It is a natural formulation for spacecraft maneuvers involving skip-trajectory passage through the atmosphere. It uses as the reference plane the original orbital plane, which is Earth-centered (and equivalent to a great-circle ground track). Thus, maneuver-produced downrange/crossrange deviations are more easily expressed with this formulation than with the previous one. However, the motion is inertially referenced, so that Earth-relative motion is not easily expressed or visualized in this case.

3.3 Crossrange/Downrange Earth-Fixed Formulation

Ikawa[35] has provided a useful formulation of the equations of motion that expresses Earth-relative motion in coordinates that use as the reference plane the initial orbital plane (or heading plane for atmospheric vehicles). This formulation leads naturally to downrange/crossrange motion solutions and visualization. As such, it is probably the most useful formulation for aeromaneuvering missions. The formulation describes orbital (conic) motion in the absence of thrust and aerodynamic forces. It was implemented on an IBM PC host microcomputer, and Ikawa describes accurate solutions for several distinctly different missions: 1) orbital motion and ground track for two different orbits including a 24-hour, 60° inclination orbit; 2) reentry missions including a synergetic orbit plane-change maneuver, and a more general mission of deorbit-to-land from elliptic LEO using aeromaneuvering; and 3) a cross-country subsonic commercial aircraft flight.

The general equations of motion developed in this formulation are as follows:

$$\text{(12)}\quad \frac{dV}{dt} = \frac{T\cos\epsilon - D}{m} - \frac{\mu\sin\gamma}{r^2} + r\omega^2 F_1$$

$$\text{(13)}\quad V\frac{d\gamma}{dt} = \frac{T\sin\epsilon + L}{m}\cos\sigma - \left(\frac{\mu}{r^2} - \frac{V^2}{r}\right)\cos\gamma + 2V\omega C_2 + r\omega^2 F_2$$

$$\text{(14)}\quad V\cos\gamma\frac{d\psi}{dt} = \frac{T\sin\epsilon + L}{m}\sin\sigma - \frac{V^2\cos^2\gamma\cos\psi\tan\phi}{r} - 2V\omega C_3 - r\omega^2 F_3$$

and $\frac{dr}{dt} = V\sin\gamma \qquad \frac{d\theta}{dt} = \frac{V\cos\gamma\cos\psi}{r\cos\phi} \qquad \frac{d\phi}{dt} = \frac{V\cos\gamma\sin\psi}{r}$

where

Coriolis terms

$$C_2 = \cos i_0\cos\phi\cos\psi - \sin i_0\left[\sin\psi\cos(\theta_0+\theta) + \cos\psi\sin\phi\sin(\theta_0+\theta)\right]$$

$$C_3 = \cos i_0(\cos\gamma\sin\phi - \sin\gamma\cos\phi\sin\psi) + \sin i_0\left[(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi)\sin(\theta_0+\theta) - \sin\gamma\cos\psi\cos(\theta_0+\theta)\right]$$

Earth centrifugal terms

$$\begin{aligned}F_1 = {} & \left[\cos^2 i_0 + \sin^2 i_0\cos^2(\theta_0+\theta)\right]\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\sin\psi) + \sin^2 i_0\left[\sin\phi(\sin\gamma\sin\phi + \cos\gamma\cos\phi\sin\psi)\right. \\ & \left. - \cos\gamma\cos\phi\cos\psi\cos(\theta_0+\theta)\sin(\theta_0+\theta)\right] - \sin i_0\cos i_0\left[2\sin\gamma\sin\phi\cos\phi + \cos\gamma\sin\psi\cos 2\phi\right) \\ & \left.(\sin(\theta_0+\theta) + \cos\gamma\cos\psi\sin\phi\cos(\theta_0+\theta)\right]\end{aligned}$$

$$\begin{aligned}F_2 = {} & \left[\cos^2 i_0 + \sin^2 i_0\cos^2(\theta_0+\theta)\right]\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi) \\ & + \sin^2 i_0\left[\sin\phi(\cos\gamma\sin\phi - \sin\gamma\cos\phi\sin\psi) + \sin\gamma\cos\phi\cos\psi\cos(\theta_0+\theta)\sin(\theta_0+\theta)\right] \\ & + \sin i_0\cos i_0\left\{\sin\gamma\sin\phi\cos\psi\cos(\theta_0+\theta) + \left[\sin\gamma\sin\psi - 2\sin\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi)\right]\sin(\theta_0+\theta)\right.\end{aligned}$$

$$\begin{aligned}F_3 = {} & \cos^2 i_0\sin\phi\cos\phi\cos\psi - \sin^2 i_0\cos\phi\left[\sin\psi\sin(\theta_0+\theta)\cos(\theta_0+\theta) + \sin\phi\cos\psi\sin^2(\theta_0+\theta)\right] \\ & + \sin i_0\cos i_0\left[\cos\psi\cos 2\phi\sin(\theta_0+\theta) - \sin\phi\sin\psi\cos(\theta_0+\theta)\right]\end{aligned}$$

4. Solutions to Forms of the General Equations

4.1 General Solutions

Trajectory solutions have been achieved computationally in the past for all types of missions - orbital motion and maneuvers, aeromaneuvering skip trajectories (orbit-to-orbit), nonmaneuvering and maneuvering orbit-to-ground (reentry), single and multi-stage ascent to orbit (maneuvering and nonmaneuvering), and hypersonic maneuvering aircraft flight. Hypersonic aircraft, ascent, and orbit-to-ground (reentry) simulations typically utilize the wind-axis, Earth-relative geographical coordinates formulation, e.g., Equations 1 - 3. Orbit-to-orbit simulations either use this same formulation or the perturbation-in-orbital-elements formulation, e.g., Equations 6 - 11.

It would appear that all maneuverable misssions, except perhaps orbit-to-orbit, would more easily be visualized and studied parametrically when expressed in downrange/crossrange motion coordinates, and this is the utility afforded by the formulation of Ikawa.[35]

Analytic solutions to simplified forms of the general equations have also been developed in many studies over the years. The most prevalent (and most relevant) solutions are those developed for forms of the planar wind-axes equations under various flight conditions. To describe and summarize these literature results would require a very lengthy discussion.

Instead, the few results achieved in the present study will be discussed since they represent extensions of some of the previously reported analytic solutions.

4.2 Analytic Solutions

4.2.1 Shallow-angle Hypersonic TAV Climb

Tauber and Adelman[13] studied the flight mechanics of a shallow-angle hypersonic climb of a TAV on the way to orbit. The assumptions of the analysis were 1) $\gamma \ll 1$ so that $\cos\gamma \approx 1$; 2) constant acceleration $\frac{dV}{dt}$ and dynamic pressure $q = \frac{1}{2}\rho V^2$; and 3) planar flight (ψ = constant). Under these conditions, Equations 1,2 reduce to (with $\sigma = 0$)

$$\frac{dV}{dt} = \frac{T}{m} - \frac{D}{m} \tag{15}$$

$$V\frac{d\gamma}{dt} = \frac{L}{m} - \left(g - \frac{V^2}{r}\right) + 2\omega V \cos\phi \cos\psi + \frac{T\sin\varepsilon}{m} \tag{16}$$

The Tauber and Adelman analysis neglected Coriolis terms so that Equations 15, 16 become

$$\frac{dV}{dt} = \frac{T}{m} - \frac{D}{m}$$

$$0 = \frac{L}{m} - \left(g - \frac{V^2}{r}\right) + \frac{T\sin\epsilon}{m} \qquad \left(\frac{d\gamma}{dt} \approx 0\right)$$

The study went on to determine several analytic results including the following expression for fuel fraction as a function of propulsion average specific impulse and vehicle average lift/drag ratio:

$$(17) \quad \frac{m_f}{m_i} = 1 - \exp\left[-\frac{V_s}{g\,I_{Ave}}\left(1 + \frac{2}{3(a/g)(L/D)_{ave}}\right)\right]$$

In the present study, a corresponding approximate result has been developed where the Coriolis effect is included. Consider the following equivalent assumptions to those made in the Tauber and Adelman study:1) γ =constant$\ll$1 and $\cos\gamma \approx 1$; 2) planar flight $\sigma = 0$ and constant heading ψ. In this case, Equations 15, 16 including the Coriolis effect become

$$(18) \quad \frac{dV}{dt} = \frac{T}{m} - \frac{D}{m}$$

$$(19) \quad 0 = \frac{L}{m} - \left(g - \frac{V^2}{r}\right) + \frac{T\sin\epsilon}{m} + 2\omega V\cos\Phi\cos\psi$$

From Equation 18, assuming $\epsilon \sim 0$

$$L = mg\left[1 - \frac{V^2}{rg} - \frac{2\omega V}{g}\cos\Phi\cos\psi\right]$$

or

$$(20) \quad D = \frac{mg}{(L/D)}\left[1 - \frac{V^2}{rg} - \frac{2\omega V}{g}\cos\Phi\cos\psi\right]$$

The Tauber and Adelman study derived the relation

$$\frac{dV}{a} = -g\,I\left(\frac{dm}{ma + D}\right) \qquad (I = \text{specific impulse})$$

and this, inserted into Equation 20 gives

$$(21) \quad -g\,I\,\frac{dm}{m} = \left[1 + \frac{1}{(a/g)(L/D)}\left(1 - \frac{2\omega}{g}V\cos\Phi\cos\psi - \frac{V^2}{V_c^2}\right)\right]dV \qquad \text{where } V_c^2 = rg \text{ orbital velocity}$$

As in the previous study, the resonable assumption is made of average values of L/D and specific impulse during hypersonic ascent. Further, it is here assumed that $\cos\Phi \approx$ constant, which is a restrictive assumption implying either small latitude changes or flight near the equator. With these assumptions (all but the assumption on $\cos\phi$ are also assumptions in the previous study), Equation 21 can be integrated with the result

$$(22) \quad \frac{m_f}{m_i} = 1 - \exp\left\{-\frac{V_c}{g\,I_{ave}}\left[1 + \frac{2}{3(a/g)(L/D)}\left(1 - \frac{3}{2}\frac{\omega k}{g}V_c\right)\right]\right\} \qquad \text{where } k = \cos\Phi\cos\psi \text{ (constant)}$$

This result is the extension of the earlier result (Equation 17) where an approximation to the Coriolis effect is now included. It should be noted that the approximation in Equation 22 represents a maximum Coriolis effect when $k = 1$. Figure 1 shows plots for Equations 17 and 22, and it indicates that the Coriolis effect reduces the average specific impulse requirement for fixed m_f/m_i by 4 to 7%. For flight at higher latitudes (or greater latitude changes), the reduction will be less pronounced, i.e., the corresponding plot will lie between the two

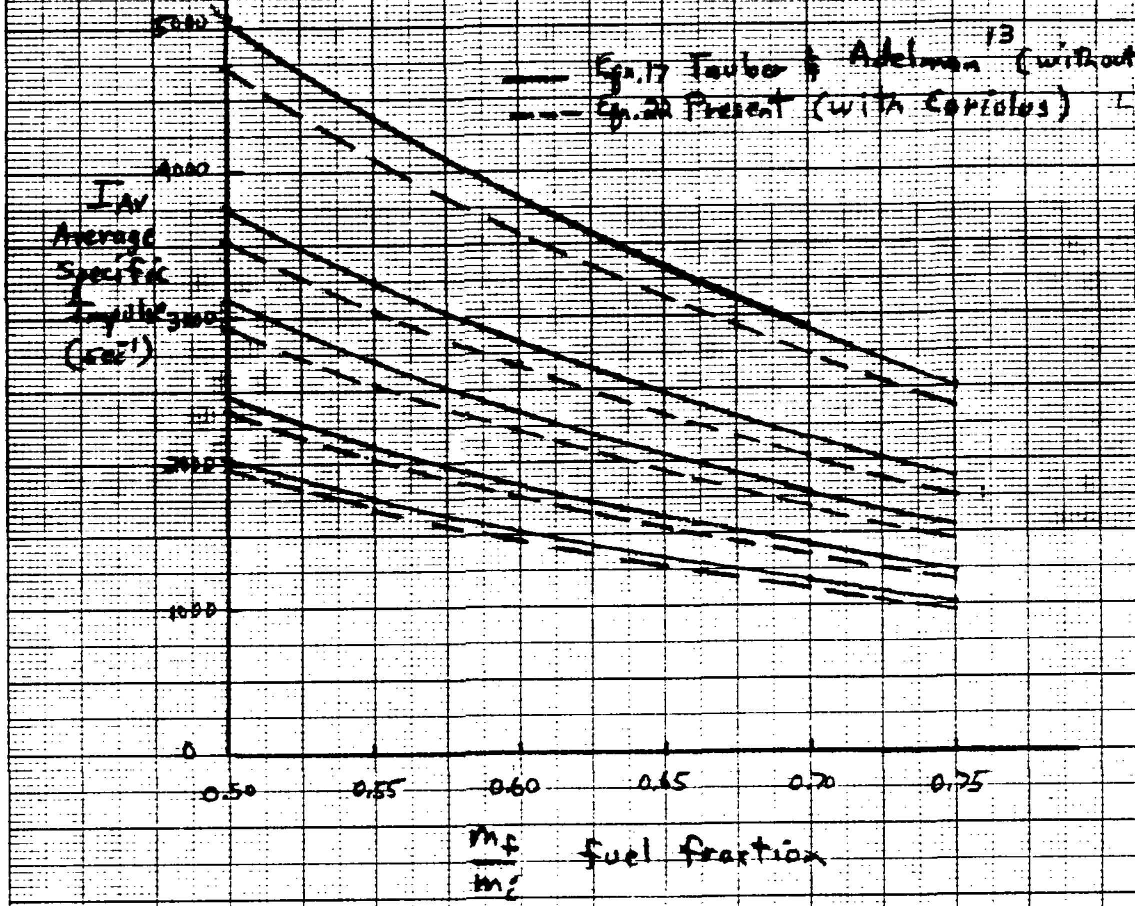

Figure 1 Required Specific Impulse as function of fuel fraction and performance

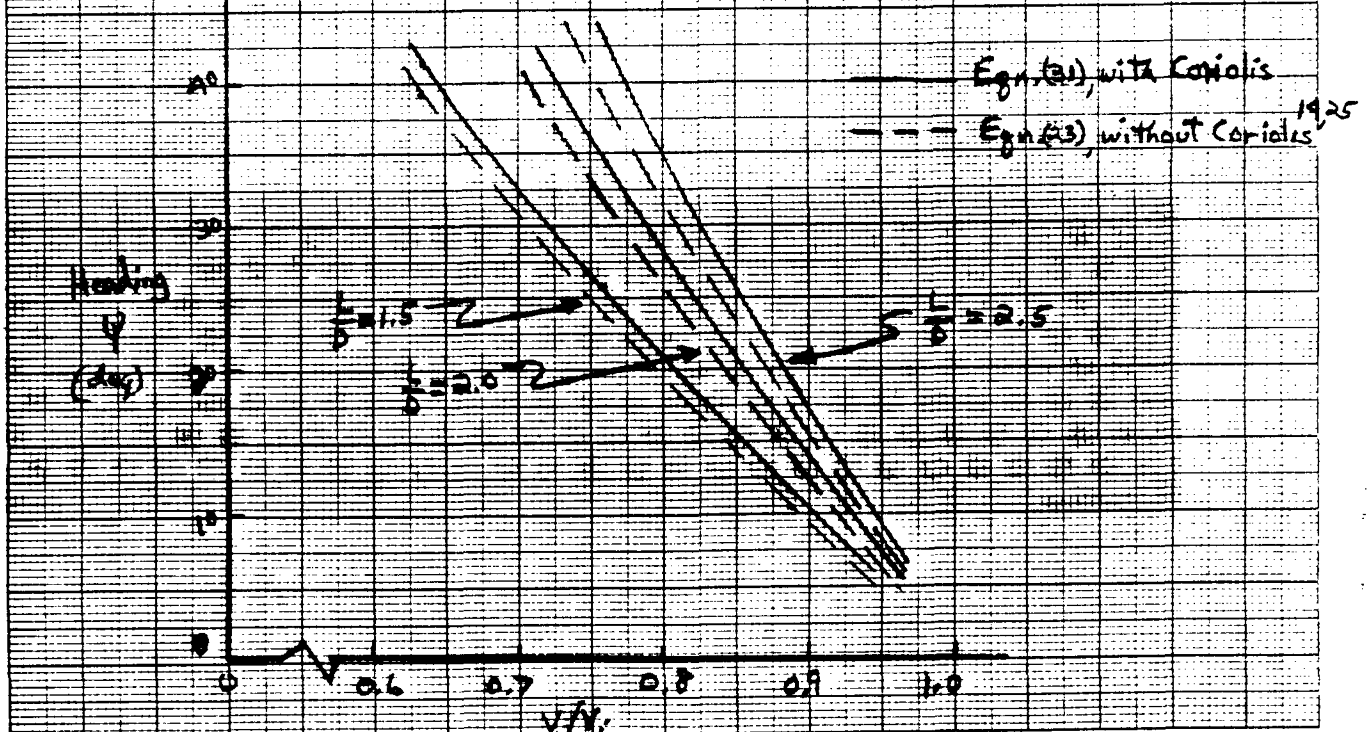

Figure 2 Equilibrium Glide Heading as a function of velocity.

4.2.2 Hypersonic Gliding Turn

A second extension to previous results was achieved in terms of including Coriolis effects (in an approximate way) in a basic relation between heading change and velocity change during a hypersonic gliding turn. Slye[14] developed the relation

$$\psi = \frac{V}{D}\ln\frac{V_i}{V} \qquad (23)$$

where heading $\psi = 0$ and velocity $V = V_i$ at the beginning of the turn, D = drag force, Y = side (lateral force) = (L/D)sinσ if produced by vehicle bank angle σ. Slye derived this result under the conditions of no thrust, shallow flight path angle ($\gamma \ll 1$), mgsin$\gamma \ll$ D, constant L/D, and constant Y/D. This relation was used by Tauber and Yang[25] to derive an equilibrium glide relation between altitude and heading, and then to produce a solution for constant bank angle which minimizes velocity loss in the turn.

The result given by Equation 23 is obtained by neglecting two terms in the complete lateral-force differential equation of motion (Equation 3). The first neglected term is the curved flight path centrifugal term ($\frac{V^2}{r}\cos\gamma\cos\psi\tan\phi$), and neglecting it amounts to assuming negligible magnitude, perhaps due to restriction to near equatorial locations (small Φ). The second neglected term is the Coriolis term [$2\omega V(\tan\gamma\cos\phi\sin\psi - \sin\phi$]. The present study obtains an approximate result containing the Coriolis term. Under the conditions assumed by Slye, Equations 1 - 3 become

$$m\frac{dV}{dt} = -D \qquad (24)$$

$$mV\frac{d\psi}{dt} = L\sin\sigma - \frac{mV^2}{r}\cos\psi\tan\phi + 2\omega Vm(\gamma\cos\Phi\sin\psi - \sin\Phi) \qquad (25)$$

$$mV\frac{d\gamma}{dt} = L\cos\sigma - \left(g - \frac{V^2}{r}\right)m + 2\omega Vm\cos\Phi\cos\psi \qquad (26)$$

If the independent variable is changed from time to distance along the trajectory s, $\frac{dV}{dt} = V\frac{dV}{ds}$ $\quad V\frac{d\gamma}{dt} = V^2\frac{d\gamma}{ds}$ $\quad V\frac{d\psi}{dt} = V^2\frac{d\psi}{ds}$

which, upon substitution into Equations 24 - 26, gives

$$mV\frac{dV}{ds} = -D \qquad (27)$$

$$mV^2\frac{d\gamma}{ds} = L\cos\sigma - \left(g - \frac{V^2}{r}\right)m + 2\omega Vm\cos\Phi\cos\psi \qquad (28)$$

$$mV^2\frac{d\psi}{ds} = L\sin\sigma - m\frac{V^2}{r}\cos\psi\tan\phi + 2\omega Vm(\gamma\cos\Phi\sin\psi - \sin\Phi) \qquad (29)$$

Dividing Equation 29 by Equation 27 gives

$$d\Psi = -\frac{dV}{V}\left[\frac{L\sin\sigma}{D} - \frac{mV^2}{Dr}\cos\Psi\tan\Phi + \frac{2m\omega V}{D}(\gamma\cos\Phi\sin\Psi - \sin\Phi)\right]$$

This equation may be integrated to give very approximate results if average values (over the integration variable velocity V) are assumed for the products/differences of trigonometric functions of heading Ψ and latitude Φ. The result is

$$\text{(30)} \quad \Psi_f - \Psi_i = \frac{L}{D}\sin\sigma \ln\frac{V_i}{V} + \frac{m}{2rD}\overline{\cos\Psi\tan\Phi}\left(V_f^2 - V_i^2\right) - \frac{2\omega m}{D}\overline{(\gamma\cos\Phi\sin\Psi - \sin\Phi)}(V_f - V_i)$$

Further, an order-of-magnitude analysis shows that in equation 30, the Coriolis term is second order, while the ($V_f^2 - V_i^2$) term is first-order, comparable to the $\ln(V_f/V_i)$ term. Thus, Equation 30 simplifies to

$$\text{(31)} \quad \Psi_f - \Psi_i = \frac{L}{D}\sin\sigma \ln\frac{V_i}{V} + \frac{m}{2rD}\overline{(\cos\Psi\tan\Phi)}\left(V_f^2 - V_i^2\right)$$

Figure 2 shows comparable plots of Equations 23 and 31 where average latitude, heading values of 30° and 20°, respectively, have been assumed in Equation 31. Unfortunately, the subsequent relationships derived from Equation 23 by Tauber and Yang[25] are difficult to parallel when beginning with Equation 31.

Ikawa[36] has examined rotating-Earth effects on AOTV trajectory simulations, and has concluded that they are signiificant (see conclusions of the present study below).

5. Conclusions and Recommendations

The research performed during this effort involved both an evaluation and organizing of a rather wide literature and the techniques and solutions presented therein, and attempts to develop new analytical solution methods for parametric analysis of vehicle performance and design. The literature evaluation was very successful, but only limited success was achieved with extending previous analytical solution methods, and no success with developing completely new methods.

It is believed the most important contribution of this research is the following set of conclusion:

1) The best formulation of the governing equations for current missions of interest appears to be that of Equations 12 - 14: the plane of reference is the initial heading plane, the motion is Earth-relative,

and the motion of the maneuvering hypersonic vehicle is easily expressed and visualized in crossrange/downrange coordinates.

2) These equations can and should be implemented (and optimized for solution speed) on modern, high-speed, high-resolution graphics engineering workstations.

3) Parametric design requires high-speed, fast-turnaround, user-interactive workstation-hosted implementation of these equations for trajectory simulations. This appears to be the only way to conveniently proceed.

4) Inclusion of Coriolis effects by the present study along with the study of Ikawa[36] indicates previous analytic results could have significant errors in design and performance concllusions, due to neglect of these effects.

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