GUIDANCE AND CONTROL STRATEGIES FOR AEROSPACE VEHICLES

## By

Desineni S. Naidu, Research Associate

Joseph L. Hibey, Principal Investigator

Progress Report
For the period July 1, 1988 through December 31, 1988

Prepared for the
National Aeronautics and Space Administration Langley Research Center
Hampton, Virginia 23665-5225

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January 1989

## GUIDANCE AND CONTROL STRATEGIES

## FOR AEROSPACE VEHICLES

## By

Desineni S. Naidul ${ }^{1}$ and Joseph L. Hibey ${ }^{2}$


#### Abstract

SUMMARY Enclosed is a List of Publications/Reports, a conference paper and a report on the above titled project for the period July 1, 1988 through December 31, 1988.


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## List of Publications/Reports

(i) D. S. Naidu and D. B. Price, "Singular Perturbation and Time Scale Approaches in Discrete Control Systems", Journal of Guidance, Control and Dynamics, Vol., 11, no. 5, pp. 592-594, Nov.-Dec., 1988.
(ii) D. S. Naidu and D. B. Price, "Singular Perturbations and Time Scales in the Design of Digital Flight Control Systems", NASA Technical Paper 2844, Langley Research Center, Hampton, December 1988

* (iii) D. S. Naidu, J. L. Hibey, and C. Charalambous, "Optimal Control of Aeroassisted, Coplanar, Orbital Transfer Vehicles", 26 IEEE Conference on Decision and Control, Austin, Texas, December 7-9, 1988.
(iv) D. S. Naidu and D. B. Price, "On the Method of Matched Asymptotic Expansions", accepted for publication in Journal of Guidance, Control and Dynamics, 1989 (in press).
(v) D. S. Naidu, "There-Dimensional Atmospheric Entry Problem using Method of Matched Asymptotic Expansions", accepted for publication in IEEE Transactions on Aerospace and Electronic Systems, 1989.
* (vi) D. S. Naidu, "Fuel-Optimal Trajectorles of Aeroassisted Orbital Transfer Vehicles", Report, ODU Research Foundation, Norfolk, VA, January, 1989.
(vii) D. S. Naidu, and D. B. Price, "Fuel-Optimal Trajectories of Aeroassisted Orbital Transfer with Plane Change," Submitted for AIAA Guidance, Navigation, and Control Conference, Boston, MA, August 14-16, 1989.
* coples enclosed


# FUEL-OPTIMAL TRAJECTORIES FOR AEROASSISTED COPLANAR ORBITAL TRANSFER PROBLEM 

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THE 27th IEEE CONFERENCE ON DECISION AND CONTROL Austin, Texas, December 7-9, 1988

- Associate Professor (Research), © Assoclate Professor \# Graduate Student


# FUEI-OPTIMAL TRAJECTORIES FOR AEPRASSISTED COPLANAR ORBITAL TRANSFER PROBLEM 

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## ABSTRACT

The optimal control problem arising in coplanar, orbital transfer employing aeroassist technology is addressed. The maneuver involves the transfer from high Earth orbit to low Earth orbit with minimum fuel consumption. Simulations are carried out for obtaining a corridor of entry conditions which are sultable for flying the spacecraft through the atmosphere. A highlight of the paper is the application of an efficient multiple shooting method for taming the notorious non-linear, two-polnt, boundary value problem resulting from the optimization procedure.

## NOMENCLATURE

| A $\mathbf{a}_{\text {c }}$ | $\begin{aligned} & S p_{d} / 2 m ; A_{1}=C_{D 0} S p_{D} H / 2 m ; A_{2}=C_{L R} S \rho_{d} H / 2 m \\ & R_{c} / R_{d} ; G_{d}=R_{d} / R_{d} ; b=R_{d} / H_{d} ; C_{D}=C_{D O}+K C_{L}^{2} \end{aligned}$ |
| :---: | :---: |
| $\mathrm{C}_{\text {D }}$ : | zero-lift drag coefficient |
| $C_{L}$ : | 1ift coefficient: $C_{L R}=\sqrt{C_{D O} / X} ; \quad c=C_{L} / C_{L R}$ |
| E | (L/D) $\mathrm{max}^{\text {; }} \mathrm{g}$ : gravitational acceleration |
| H | altitude; $\mathrm{h}=\mathrm{H} / \mathrm{H}_{\mathbf{a}}$; J : performance index |
| K | Induced drag factor: $\quad$ : vehicle mass |
| R | distance from Earth center |
| $\mathrm{R}_{\text {a }}$ : | radius of the atmosphere; $R_{E}$ : radlus of Earth |
| S | aerodynamic reference area |
|  | time; $v=V / \sqrt{\mu R_{a}} ; \beta:$ inverse scale heigh |
| $\boldsymbol{\gamma}$ | flight path angle; $\delta=\exp (-\mathrm{h} \beta \mathrm{H}$ |
| $\lambda$ | costate variable; $\mu$ : gravitational constant |
| $\rho$ | $\text { density; } \quad \tau=t / \sqrt{R^{3} / \mu}$ |
| $\Delta v:$ | characteristic velocity |

Subseripts
$c$ : circularization or reorbit; $d$ : deorbit
e : entry to atmosphere; $f$ : exit from atmosphere

## 1. INIRODUCTION

In this paper, we address the fuel-optimal control problem arising in coplanar orbital transfer employing aeroasslst technology [1-3]. The maneuver involves a transfer from high Earth orbit (HEO) to low Earth orbit (LEO) with inimum fuel consumption. A sultable performance index is the sum of the characteristic velocitles for deorbit and reorbit (or circularization) [4]. Use of fontryagin minimum principle leads to a two-point boundrary value problem (TPBVP) in state and costate variables. This problem 18 solved by using an efficient multiple shooting method [5] in preference to the sequential gradient-restoration algorithm [4]. In addition, simulations are carried out for obtaining a spectrum of entry conditions which are suitable for flying the spacecraft through the atmosphere.

## II. AEROASSISTED COPLANAR TRANSFER

In an aeroassisted, coplanar transfer, the vehicle is transferred from HEO at $R_{d}$ to LEO at $R_{c}$, by flying deep Into the atmosphere to achleve the necessary velocity reduction (Figure 1). We start with a tangential propulsive burn, having a characteristic velocity $\Delta V_{d}$
for deorbitting from HEO and entering into an elliptical transfer orbit. At point $E$ the spacecraft enters the atmosphere with flight path angle $\gamma_{0}$ and undergoes reduction in velocity due to atmospheric drag. At point $F$, the spacecraft leaves the atmosphere with flight path angle $\boldsymbol{\gamma}_{\boldsymbol{f}}$. Once again, the transfer orbit is elliptical with the corresponding apogee at $R_{c}$. Finally, the maneuver ends with a circularizing or reorbit burn having a characteristic velocity $\Delta V_{c}$ to make the vehicle enter Into the low Earth orbit. Thus, the maneuver consists of two impulses $\Delta V_{d}$ for deorblt, and $\Delta V_{e}$ for circularization and is assumed to take place right at the perigee itself.

## Equations of Motion

Consider a vehicle with constant point mass, moving about a nonrotating spherical planet. The atmosphere surrounding the planet is assumed to be at rest, and the central gravitational field obeys the usual inverse square law. The equations of motion are given by
$\frac{d H}{d t}=V s i n y$
$\frac{d V}{d t}=-A C_{D} V^{2} \exp (-H \beta)-\left(\mu / R^{2}\right) \sin \gamma$
$\frac{d y}{d t}=A C_{L} V \exp (-H \beta)+\left[V / R-\mu\left(R^{2} V\right)\right] \cos \gamma$
Using normalized values,
$\frac{d h}{d \tau}=b v s i n g$
$\frac{d v}{d \tau}=-A, b\left(1+c^{2}\right) \delta v^{2}-\frac{b^{2} s i n y}{(b-1+h)^{2}}$
$\frac{d \gamma}{d \tau}=A_{2} b c \delta v+\frac{b v \cos \gamma}{(b-1+h)}-\frac{b^{2} \cos \gamma}{(b-1+h)^{2} v}$
Optimal Control
For minlmum fuel consumption, the performance Index is glven by
$J=\Delta v=\Delta v_{d}+\Delta v_{c}$
where,
$\Delta v_{d}=\sqrt{1 / a_{d}}-\left(v_{\bullet} / a_{d}\right) \cos \left(-\gamma_{0}\right)$
$\Delta v_{c}=\sqrt{1 / a_{c}}-\left(v_{f} / a_{c}\right) \cos \left(\gamma_{f}\right)$
We are interested in finding the minimization of the fuel with respect to the control $C_{L}$. Using Pontryagin's principle [4], the unconstrained optimal control is obtalned as
$c=E_{V} \lambda_{\gamma} / v \lambda_{V}$

Realistically, the control $C_{L}$ is bounded by the aerodynamic characteristics of the vehlcle. The initial and final boundary conditions are given as $h(r=0)=$ $1.0 ; h\left(\tau=\tau_{r}\right)=1.0$. and
$\left(2-v^{2}\right) a_{d}^{2}-2 a_{d}+v^{2} \cos ^{2} \gamma_{0}=0$
$\left(2-v_{f}^{2}\right) a_{c}^{2}-2 a_{c}+v_{f}^{2} \cos ^{2} \gamma_{f}=0$
The remaining boundary conditions are obtalned from the transversality conditions on the costates. Thus, the optimization procedure, requiring the solution of state and costate equations along with the boundary conditions leads us to the formation of a TPBVP, which is solved by using a multiple shooting method [5].

## [II. NUNERICAL DATA AND RESULTS

A typical set of numerical values used for simulation purposes is given below [3]. $C_{D O}=0.21 ; \quad K=1.67$;
$\Delta S=300 \mathrm{~kg} / \mathrm{m}^{2} ; \rho_{\mathrm{E}}=1.225 \mathrm{~kg} / \mathrm{m}^{3} ; \mu=3.986 \times 10^{14}$ $\mathrm{m}^{3} / \mathrm{sec}^{2}: \beta=1 / 6900 \mathrm{~m}^{-1}: \mathrm{R}_{\mathrm{E}}=6378 \mathrm{~km} ; \mathrm{H}_{\mathrm{c}}=120 \mathrm{~km} ; \mathrm{R}_{\mathrm{d}}$ - $12996 \mathrm{~km} ; \mathrm{R}_{\mathrm{c}}=6558 \mathrm{~km}$. Flgure $2(\mathrm{a})$ shows the time history of altitude. The spacecraft enters and exits the atmosphere at an altitude of 120 km . The minimum altitude reached is 55.58 km . The velocity versus time is shown in figure $2(b)$. The vehicle enters the atmosphere with a velocity of $9029 \mathrm{~m} / \mathrm{sec}$ and leaves the atmosphere with a speed of $7795 \mathrm{~m} / \mathrm{sec}$, thus giving a velocity reduction of $1234 \mathrm{~m} / \mathrm{sec}$. The profile of filight path angle with time is shown in Figure 2(c). The spacecraft enters the atmosphere with an Inclination of -5.665 degrees and exits with +0.927 degrees. The control history is shown in figure $2(d)$. The vehicle enters the atmosphere with maximum lift capabllity and switches to the minimum lift coefficient and then gradually increases during the remalining flight.

The minimum-fuel transfer requires a deorbit (impulse) characteristic velocity $\Delta V_{d}$ of $1034.29 \mathrm{~m} / \mathrm{sec}$ and a reorbit characteristic velocity $\Delta V_{c}$ of 73.25 $\mathrm{m} / \mathrm{sec}$, with a total characteristic velocity of 1107.54 $\mathrm{m} / \mathrm{sec}$. Let us compare this aeroasslsted transfer with the Hohmann transfer, which is maneuvered entirely in outer space, and has a total characteristic velocity of $2194.64 \mathrm{~m} / \mathrm{sec}$. This shows that the saving due to coplanar, aeroassisted transfer over Hohmann transfer Is 49.54 percent. In the case of idealized transfer which follows a grazing trajectory along the atmospheric boundary, the total characterlstic velocity is $1034.18 \mathrm{~m} / \mathrm{sec}$. The optimal transfer requires only 6.63 percent more fuel than that of the Ideallzed transfer. The heating rate shows a peak value of 129.2 $\mathrm{W} / \mathrm{cm}^{2}$. and the total integrated heating load is found to be 15.536 KW -sec/cm ${ }^{2}$. The peak dynamic pressure is $26.73 \mathrm{kN} / \mathrm{sq}$. $\mathrm{m}_{3}$ and the density attalns a maximum value of $0.3902 \times 10^{-3} \rightarrow \mathrm{~kg}^{\mathrm{g}} / \mathrm{m}^{3}$.

## Entry Corridor

A given vehicle cannot fly an acceptable atmospheric flight for arbitrary initial conditions at the entry point. If the flight path angle $r$ is too steep, the vehicle will later suffer excessive aerodynamic and aerothermodynamic loadings even if the maximum lift is directed upward. This also may lead to "crash" condition. Or if the entry flight path angle is too shallow, the vehicle will exit the atmosphere again with an orbital velocity even if the maximum lift is directed downward. This leads to "escape" or uncontrolled skip-out condition. These boundarles of entry flight path angle are often taken to define the
corridor of acceptable entry conditions.
The entry corridor is the entry interface undershoot and overshoot and is usually specified by the entry flight path angle $\gamma_{0}$. as dictated by the entry dynamics. In the present-case of fuel-optimal, coplanar, orbital transfer, four slmulations are carrled out as shown below.

| No | ${ }^{\boldsymbol{\gamma}}$ |  | $\overline{\mathrm{C}}$. | filaht time seconds |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -7. 240 | 9020 | 0.5381 | 1510 |
| 2 | -6. 412 | 9025 | 0.5786 | 6 600 |
| 3 | -5.665 | 9029 | 0.6299 | 540 |
| 4 | -5.485 | 9030 | 0.6502 | 2600 |

Figure 3 shows the successive approximations of the altitude $H$, during the course of 0,5 , and 14 iterations in using the multiple shooting method [5]. For the sake of clarity only 4 out of 20 intervals are shown. The initial guessed value for the altitude is 120 km at every interval. It can be seen how the Initially large Jumps at the subdivision points of the multiple shooting method are "flattened out" with the Increase of iterations.

The strategy for the atmospheric portion of the minimum-fuel transfer is to fly at the maximum L/D Inttially in order to recover from the downward plunge, and then to fly at a negative $L / D$ to level off the flight such that the vehicle skips out of the atmosphere with a flight path angle near zero degrees.

## ACKNOWLEDGEHIDNTS

This research work was supported by grant NAG1-736 from NASA Langley Research Center, Hampton, under the technical monltorship of Dr Douglas B. Price, Assistant Head, Spacecraft Control Branch.

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Figure 1 Aeroassisted coplanar orbital transfer


Figure $2(b)$ Time history of veloctly


Figure 2(d) Time history of lift coefficient


Figure $2(a)$ Time history of altitude


Figure 2(c) Time history of night path angle


Figure 3 Successive approximations for altitude

# FUEL-OPTIMAL TRAJECTORIES <br> FOR AEROASSISTED <br> <br> COPLANAR ORBITAL TRANSFER PROBLEM 

 <br> <br> COPLANAR ORBITAL TRANSFER PROBLEM}

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27th IEEE Conference on Decision and Control December 7-9, 1988 Austin, Texas

## OUTLINE

* Aeroassist Technology
* Mission Description
* Problem Formulation
* Problem Solution
* Results
* Concluding Remarks



## SPACE TRANSPORTATION SYSTEMS

Low Cost Transportation is the Key to Exploration and Utilization of Space

Orbital Transfer Vehicle (OTV) is an advanced upper stage concept for transportation of payloads from LEO such as Space Shuttle, Space Station to HEO such as GEO and other planetary excursions and return to a specified low earth parking orbit for reusability.

Aeroassist Technology is a technical capability for substantial reduction in propellant requirements by using the atmospheric (aeroassisted) maneuver on the return journey of the mission.

An orbital transfer vehicle utilizing the aeroassist technology becomes an Aeroassisted Orbital Transfer Vehicle (AOTV).

Advantages (1) High Performance, (2) Reusability



Aeroassisted coplanar orbilal transfer

## -6-

## PROBLEM FORMULATION

Equations of Motion
$\mathrm{dH} / \mathrm{dt}=\mathrm{V} \sin \gamma$
$\mathrm{dV} / \mathrm{dt}=-\mathrm{AC}_{\mathrm{D}} \mathrm{V}^{2} \exp (-\beta \mathrm{H})-\left(\mu / \mathrm{R}^{2}\right) \sin \gamma$
$\mathrm{d} \gamma / \mathrm{dt}=\mathrm{AC}_{\mathrm{L}} \operatorname{Vexp}(-\beta \mathrm{H})-\left\{\left(\mathrm{V} / \mathrm{R}-\mu /\left(\mathrm{R}^{2} \mathrm{~V}\right)\right\} \cos \gamma\right.$

H : altitude; V : velocity; $\gamma$ : flight path angle

$$
\mathrm{A}=\mathrm{S} \rho_{\mathrm{s}} / 2 \mathrm{~m} ; \quad \mathrm{C}_{\mathrm{D}}=\mathrm{C}_{\mathrm{D} 0}+\mathrm{KC}_{\mathrm{L}}{ }^{2} ; \quad \rho=\rho_{\mathrm{s}} \exp (-\beta \mathrm{H})
$$

## PROBLEM FORMULATION (Cont.)

## Performance Index

$$
\begin{aligned}
\mathrm{J} & =\Delta \mathrm{V}=\Delta \mathrm{V}_{\mathrm{d}}+\Delta \mathrm{V}_{\mathrm{c}} \\
\Delta \mathrm{~V}_{\mathrm{d}} & =\sqrt{\mu / \mathrm{R}_{\mathrm{d}}}-\left(\mathrm{R}_{\mathrm{a}} / \mathrm{R}_{\mathrm{d}}\right) \mathrm{V}_{\mathrm{e}} \cos \left(-\gamma_{\mathrm{e}}\right) \\
\Delta \mathrm{V}_{\mathrm{c}} & =\sqrt{\mu / \mathrm{R}_{\mathrm{c}}}-\left(\mathrm{R}_{\mathrm{a}} / \mathrm{R}_{\mathrm{c}}\right) \mathrm{V}_{\mathrm{f}} \cos \gamma_{\mathrm{f}}
\end{aligned}
$$

$\Delta \mathrm{V}_{\mathrm{d}}$ : deorbit characteristic velocity at HEO
$\Delta \mathrm{V}_{\mathrm{c}}$ : circularizing characteristic velocity at LEO

## PROBLEM SOLUTION

The application of optimal control theory leads to a nonlinear, two-point, boundary value problem (TPBVP) with appropriate boundary conditions on state and costate variables.

For the entry and exit altitudes

$$
\mathrm{H}(\mathrm{t}=0)=120 \mathrm{~km}=\mathrm{H}(\mathrm{t}=\mathrm{T})
$$

For the HEO-to-entry elliptic transfer orbit

$$
\left(2-v_{e}^{2}\right) a_{d}^{2}-2 a_{d}+v_{e}^{2} \cos ^{2} \gamma_{e}=0
$$

For the exit-to-LEO elliptic transfer orbit

$$
\left(2-v_{f}^{2}\right) a_{c}^{2}-2 a_{c}+v_{f}^{2} \cos ^{2} \gamma_{f}=0
$$

## PROBLEM SOLUTION (Cont.)

## Multiple Shooting Method

In the conventional (or simple shooting) method of solving TPBVP, we assume additional initial data and integrate forward so that the solution satisfies the given final conditions. The convergence of the solutions is highly sensitive to the assumed data.

In the multiple shooting method, the whole interval is subdivided into smaller intervals with simultaneous application of simple shooting method over these subintervals. Here, the trajectory may be restarted at intermediate points using new guesses and finally reducing all the discontinuties at internal grid points to zero.

The corresponding OPTSOL code was developed by DFVLR at Oberpfaffenhofen, West Germany.




$$
\therefore \text { Successive approximations for altitude }
$$

## CONCLUDING REMARKS

1. Fuel optimal trajectories for noncoplanar orbital transfer problem have been obtained.
2. The strategy for the atmospheric portion of the minimum-fuel transfer is to fly at maximum L/D initially in order to recover from the downward plunge, and the vehicle skips out of the atmosphere with flight path angle near zero.
3. An efficient multiple shooting method has been used to solve the resulting TPBVP.
4. The multiple shooting method can be applied to solve the noncoplanar (plane change) orbital transfer problems.

# FUEL-OPTIMAL TRAJECTORIES OF <br> AFROASSISTED ORBLTAI. TRANSFFR VFIIICI.F.S <br> (summary) 

Dr. D. S. Naidu<br>Old Dominion University<br>Norfolk, VA<br>January 1989

ABSTRACT: The fuel-optimal control problem arising in orbital transfer vehicles employing aeroassist technology is addressed. The maneuver involves the transfer from high Earth orbit to low Earth orbit with plane change being performed during the atmospheric pass. A performance criterion is chosen to minimize the total fuel consumption for the transfer. The simulations are carried out using the industry standard POST program.

## NOMENCLATURE

```
g : gravitational acceleration
g
H : altitude
J : performance criterion
m : vehicle mass
R : distance from Earth center to vehicle center of gravity
Ra}: radius of the atmospheric boundary
R : radius of the low Earth orbit
R
R : radius of Earth
S : aerodynamic reference area
L : aerodynamic reference length
t : time
V : velocity
\gamma: flight path angle
\sigma : bank angle
\mu : gravitational constant of Earth
\DeltaV : characteristic velocity
```


## Subscripts

```
c : subscript for circularization or reorbit
d : subscript for deorbit
s : subscript for surface level
```


## 1. INTRODUCTION

In space transportation system, the concept of aeroassisted orbital transfer opens new mission opportunities, especially with regard to the initiation of a permanent space station [1]. The use of aeroassisted maneuvers to affect a transfer from high Earth orbit (HEO) to low Earth orbit (LEO) has
been recommended to provide high performance leverage to future space transportation systems. The space-based, orbit transfer vehicle (OTV) is planned as a system for transporting payloads between LEO and other locations in space. The OTV, on its return journey from HEO, dissipates orbital energy through atmospheric drag to slow down to LEO velocity. In a synergetic maneuver for aeroassisted, orbital transfer vehicles (AOTV's), the basic idea is to employ a hybrid combination of propulsive maneuvers in space and aerodynamic maneuvers in sensible atmosphere. Within the atmosphere, the trajectory control is achieved by means of lift and bank angle modulations Hence, this type of flight with a combination of propulsive and nonpropulsive maneuvers, is also called synergetic maneuver or space flight[2-7].

Am AOTV baseline mission is a round trip to GEO with a maximum return weight to LEO. In a typical mission [Fig.1], the AOTV with its payload is launched from Kennedy Space Center as a single Shuttle payload into a 296.5 km circular orbit inclined at 28.5 deg. The AOTV delivers its payload propulsively to GEO at 35810 km inclined at 0 deg. On its return journey, the vehicle dips into the atmosphere to achleve the necessary velocity depletion and inclination change and finally reaches the Space Station orbit at 556 km .

In this report, we obtain fuel-optimal trajectories for orbital transfer vehicles using aeroassist technology. The maneuver involves the transfer from HEO to LEO with a prescribed plane change and at the same time minimization of the fuel consumption. It is known that the change in velocity, also called the characteristic velocity, is a convenient parameter to measure the fuel consumption. For minimum-fucl maneuver, the objective is then to minimize the total characteristic velocity for deorbit, boost, and reorbit (or circularization). The simulations are carried out using the industry standard Program to Optimize Simulated Trajectories (POST) [8]

## 2. VEHICLE CONFIGURATIONS

In general, there are three types of AOTV configurations, depending on their lift/drag ratios [9].
(i) Low L/D configuration [Fig.2]: This, also called a lifting brake or an aerobrake, consists of a payload, propulsion, and miscellaneous subsystems that are packaged in a cylindrical structure. The aerobrake looking like a large umbrella of diameter 15.25 m heat shield, is used for deceleration and inclination change by utilizing the drag and lift of the aerobrake at low angles of attack. The aerobrake is considered a low L/D concept with an experimental L/D of 0.25 at 15 deg angle of attack.
(ii) Moderate L/D configuration [Fig.3]: This has a cylindrical afterbody of 4.57 m diameter with a raked-off nose to provide the necessary aerodynamic performance. The nose was designed for stagnation heating with an ablative shield. This configuration has a moderate L/D of 0.6 at 35 deg angle of attack.
(iii) High L/D configuration [Fig.4]: This vehicle has an estimated L/D of 2.18 at 11 deg angle of attack. For high L/D capability, the liquid oxygen is stored in two separate tanks to provide a tapered nose, and inflated chins are used to continue this tapering along the body. A large deployable flap is
needed to trim the vehicle at low angle of attack for maximum $L / D$ performance.

## 3. MISSION DESCRIPTION

The mission comprises of deorblt, aeroassist (or atmospheric flight), boost and reorbit (or circularization) phases.

Initially, the spacecraft is in circular orbit of radius $R_{d}$, well outside the Earth's atmosphere, moving with a circular velocity $v_{d}=\sqrt{\mu / R_{d}}$. Deorbit is accomplished by means of an impulse $\Delta V_{d}$, to transfer the vehicle from a circular orbit to elliptic orbit with perigee low enough to intersect the dense part of the atmosphere. Since the elliptic velocity at $D$ is less than the circular velocity at $D$, the impulse $\Delta V_{d}$ is executed so as to oppose the circular velocity $V_{d}$. In other words, at point $D$, the velocity required to put the vehicle into elliptic orbit is less than the velocity required to maintain it in circular orbit. The deorbit impulse $\Delta V_{d}$ causes the vehicle to enter the atmosphere at radius $R_{a}$ with a velocity $V_{e}$ and flight path angle $\gamma_{e}$. It is known that the optimal-energy loss maneuver from the circular orbit is simply the Hohmann transfer and the impulse is parallel and opposite to the instantaneous velocity vector.

Using the principle of conservation of energy and angular momentum at the deorbit point $D$, and the atmospheric entry point $E$, we get [10],

$$
\begin{align*}
& V_{e}^{2} / 2-\mu / R_{a}=\left(V_{d}-\Delta V_{d}\right)^{2} / 2-\mu / R_{d}  \tag{1}\\
& R_{a} V_{e} \cos \left(-\gamma_{e}\right)=R_{d}\left(V_{d}-\Delta V_{d}\right) \tag{2}
\end{align*}
$$

from which solving for $\Delta V_{d}$ we get
$\Delta V_{d}=\sqrt{\mu / R_{d}}-\sqrt{2 \mu\left(1 / R_{a}-1 / R_{d}\right) /\left[\left(R_{d} / R_{d}\right)^{2} / \cos ^{2} \gamma_{e}-1\right]}$

It is easily seen that the minimum value of the deorbit impulse $\Delta V_{d m}$ obtained at $\gamma_{e}=0$, corresponds to an ideal transfer with the space vehicle grazing the atmospheric boundary. To ensure proper atmospheric entry, deorbit impulse $\Delta V_{d}$ must be higher than the minimum deorbit impulse $\Delta V_{d m}$ which is given by

$$
\begin{equation*}
\Delta V_{d m}=\sqrt{\mu / R_{d}}-\sqrt{2 \mu\left(1 / R_{a}-1 / R_{d}\right) /\left[\left(R_{d} / R_{a}\right)^{2}-1\right]} \tag{4}
\end{equation*}
$$

During the aeroassist (or atmospheric flight), the vehicle needs to be
controlled by lift and/or bank angle to achieve the necessary velocity reduction (due to atmospheric drag) and the plane change. Because of the loss of energy during a turn, a second impulse is required to boost the vehicle back to orbital altitude.

The vehicle exits the atmosphere at point $F$, with a velocity $V_{f}$ and flight path angle $\gamma_{f}$. The additional impulse $\Delta V_{b}$, required at the exit point $F$ for boosting into an elliptic orbit with apogee radius $\mathrm{R}_{\mathrm{c}}$ and the reorbit (or circularization) impulse $\Delta V_{c}$ required to insert the vehicle into a circular orbit, are obtained by using the principle of conservation of energy and angular momentum at the exit point $F$, and the reorbit or circularization point C. Thus, we have,

$$
\begin{align*}
& \left(V_{f}+\Delta V_{b}\right)^{2} / 2-\mu / R_{a}=\left(V_{c}-\Delta V_{c}\right)^{2} / 2-\mu / R_{c}  \tag{5}\\
& \left(V_{f}+\Delta V_{b}\right) R_{a} \cos \gamma_{f}=R_{c}\left(V_{c}-\Delta V_{c}\right) \tag{6}
\end{align*}
$$

Solving for $\Delta V_{b}$ and $\Delta V_{c}$ from the above equations (5) and (6),

$$
\begin{align*}
& \Delta V_{b}=\sqrt{2 \mu\left(1 / R_{a}-1 / R_{c}\right) /\left[1-\left(R_{a} / R_{c}\right)^{2} \cos ^{2} \gamma_{f}\right]}-V_{f}  \tag{7}\\
& \Delta V_{c}=\sqrt{\mu / R_{c}}-\sqrt{2 \mu\left(1 / R_{a}-1 / R_{c}\right) /\left[\left(R_{c} / R_{a}\right)^{2} / \cos ^{2} \gamma_{f}-1\right]} \tag{8}
\end{align*}
$$

## 4. TRAJECTORY SIMULATION

It is known that the change in speed, $\Delta V$, also called the characteristic velocity, is a convenient parameter to measure the fuel consumption. For minimum-fuel maneuver, the objective is then to minimize the total characteristic velocity. A convenient performance index is the sum of the characteristic velocities for deorbit, boost, and reorbit. Thus,
$J=\Delta V_{d}+\Delta V_{b}+\Delta V_{c}$
Where, $\Delta V_{d}, \Delta V_{b}$, and $\Delta V_{c}$ are the deorbit, boost, and reorbit characteristic velocities respectively, and are related as

$$
\begin{align*}
& \Delta V_{d}=\sqrt{\mu / R_{d}}-\left(R_{a} / R_{d}\right) V_{e} \cos \left(-\gamma_{e}\right)  \tag{10}\\
& \Delta V_{c}=\sqrt{\mu / R_{c}}-\left(R_{a} / R_{c}\right)\left(V_{f}+\Delta V_{b}\right) \cos \gamma_{f} \tag{11}
\end{align*}
$$

Let us note that for a given circular orbit, the impulses $\Delta V_{b}$ and $\Delta V_{c}$ are completely determined by the variables $V_{f}$ and $\gamma_{f}$ at the exit conditions of the atmospheric portion of the trajectory. The velocity $v_{e}$ and the flight path angle $\gamma_{e}$ at the entry point are dependent only on the magnitude of the deorbit impulse $\Delta V_{d}$. Therefore, the optimal trajectory problem needs to consider the segment of the trajectory within the atmosphere.

Trajectories for the AOTV are calculated using the three-dimensional version of the Program to Optimize Simulated Trajectories (POST) [13]. A 1976 US standard atmosphere is used. The results are given for the high L/D configuration only. Similar results are obtalned for the other low L/D and moderate L/D configurations and will be reported separately.

A propulsive maneuver with impulsive burn having a characteristic velocity ( $\Delta V_{b}$ ) of $1491.25 \mathrm{~m} / \mathrm{sec}$ and a specific impulse of 456 sec targets the AOTV with a transfer orbit perigee of that lies within the Earth's atmosphere. Some of the plane change to reacquire the Shuttle inclination is accomplished during this maneuver and the remaining being obtained from aeromaneuvering capability of the AOTV. Upon reaching the atmosphere interface at 120 km , the AOTV flies the aerodynamic portion of its trajectory at constant angle of attack but with variable bank angle, rolling the lift vector about the velocity vector. Rotating the lift vector modulates the drag via altitude control but also turns the vehicle through the remaining inclination change. The aeroassist phase of the mission ends as the AOTV exits through the at mospheric boundary at 120 km . Sufficient energy is dissipated during the aerobraking for the orblt apogee to be reduced to 556 km , the assumed orbit for subsequent Space Station rendezvous.

A typical set of numerical values used for simulation purposes is given below [8].
weight of the vehicle excluding the payload $=112,625 \mathrm{~N}$
aerodynamic reference area of the vehicle $=30.8$ sq. m
aerodynamic reference length of the vehicle $=15.67 \mathrm{~m}$
Using the above mentioned data, simulations are carried out for obtaining the nominal trajectories using POST. The nominal solution has the following entry and exit status.

Entry status: $H_{e}=120 \mathrm{~km} ; \quad V_{e}=10306 \mathrm{~m} / \mathrm{sec}$
$\gamma_{e}=-5.1541$ degrees; $i_{e}=0 \mathrm{deg}$
Exit status: $\quad H_{f}=120 \mathrm{~km} ; \quad \mathrm{V}_{\mathrm{f}}=8062.5 \mathrm{~m} / \mathrm{sec}$
$\gamma_{f}=1.9484 \mathrm{deg} ; i_{f}=23.039 \mathrm{deg}$
total flight time $=579.2 \mathrm{sec}$

Time histories of altitude $H$, velocity $V$, and flight path angle $\gamma$, for
total flight time of 579 seconds, are shown in Figures 5-7 respectively. The variation of bank angle and orbit inclination are depicted in Figures 8 and 9. Those for the heating rate, and dynamic pressure are shown in Figures 10 and 11.

Figure 5 shows the time history of altitude. The spacecraft enters and exits the atmosphere at an altitude of 120 km . The minlmum nltitude renched f 55.47 km . The velocity versus time is shown in Figure 6. The vehicle enters the atmosphere with a velocity of $10306 \mathrm{~m} / \mathrm{sec}$ and leaves the atmosphere with a speed of $8062.5 \mathrm{~m} / \mathrm{sec}$, thus giving a velocity reduction of $1243.5 \mathrm{~m} / \mathrm{sec}$. The profile of flight path angle with time is shown in Figure 7. The spacecraft enters the atmosphere with an inclination of -5.1541 degrees and exits with 1.9484 degrees. Figure 8 shows the variation of bank angle during the atmospheric flight. Initially the vehicle enters the atmosphere with a bank angle of -60.76 degrees to pull the vehicle into the atmosphere and decreases further to -105 deg and approaches zero degrees at the exit of the atmosphere. Fig. 9 shows the variation of orbit inclination. At the entry the inclination is assumed to be 0 degrees and at the exit the vehicle acquires an inclination of 23.04 degrees and any further inclination required may be obtained propulsively.

## 5. CONCLUDING REMARKS

In this report, we have addressed the minimization of fuel consumption during the atmospheric portion of an aeroassisted, noncoplanar, orbital transfer vehicles. The simulations are carried out for a high L/D configuration using the industry standard Program to Optimize Simulated Trajectorles (POST) [13]. The results for other moderate L/D and low L/D configurations are being investigated and will be reported separately.

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Fig. 2 Low L/D configuration


Fig. 3 Moderate L/D configuration
! $=$

Fig. 4 High L/D configuration






Fig. 10 Time history of heating rate

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