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## IN-FLIGHT DETERMINATION OF SPACECRAFT MAGNETIC BIAS INDEPENDENT OF ATTITUDE

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### ABSTRACT

A simple algorithm for the in-flight determination of the magnetic bias of a spacecraft is presented. The algorithm, developed for use during the Hubble Space Telescope mission, determines this bias independently of any attitude estimates and requires no spacecraft sensor data other than that from the spacecraft magnetometer(s). Estimates of the algorithm's accuracy and results from a number of numerical studies on the use of this algorithm are also presented.

### I. INTRODUCTION

The determination of the magnetic bias of a spacecraft before launch can be quite difficult, particularly if the spacecraft is fairly large. The lack of such a bias determination can severely restrict the usefulness of onboard magnetometers in measuring the ambient magnetic field and thus also in determining spacecraft attitude. Attitudes based on magnetometer data are often the first to be determined during a spacecraft's mission. Having an algorithm that can determine the spacecraft magnetic bias at the location of each magnetometer without requiring knowledge of the spacecraft attitude is therefore desirable. We present here a simple algorithm that fulfills this need. The algorithm, developed specifically for the Hubble Space Telescope (HST) mission, requires no spacecraft sensor data other than that from the spacecraft magnetometer(s). The only other required input data are magnetic field values from a data base geomagnetic field model. Section II of this article presents the algorithm; Section III, an analytical error model; Section IV, numerical examples of the use of the algorithm; and Section V, our conclusions.

## II. MAGNETIC BIAS DETERMINATION ALGORITHM

The spacecraft magnetic bias,  $\mathbf{D}$ , at the location of an onboard magnetometer is related to the measured magnetic field,  $\mathbf{M}$ , and the local geomagnetic field,  $\mathbf{B}$ , as follows:

$$\mathbf{M} - \mathbf{D} = \mathbf{A} \mathbf{B} , \quad (1)$$

where  $\mathbf{M}$  and  $\mathbf{D}$  are defined in spacecraft coordinates and  $\mathbf{B}$  in geocentric coordinates, and  $\mathbf{A}$  is the (unknown) spacecraft attitude transformation matrix. The bias  $\mathbf{D}$  is assumed to be time independent in the spacecraft reference frame. The value of  $\mathbf{B}$  is to be obtained from a geomagnetic field model. Obtaining  $\mathbf{B}$  from the model requires both a spacecraft ephemeris and the magnetometer measurement times. The final accuracy of the determination of  $\mathbf{D}$  will depend on the accuracies of the magnetometer calibration and the geomagnetic field model (including differential errors introduced by spacecraft ephemeris errors and timing errors), as well as noise in the magnetometer readings. Taking the scalar product of each side of equation (1) with its transpose (noting that  $\mathbf{A}^T = \mathbf{A}^{-1}$ ) results in

$$\mathbf{M}^2 - 2 \mathbf{M}^T \mathbf{D} + \mathbf{D}^2 = \mathbf{B}^2 \quad (2)$$

Equation (2) is independent of spacecraft attitude because  $\mathbf{B}^2$  is invariant under rotation. We have, however, reduced a three-component vector equation to a scalar equation. Because  $\mathbf{D}$  has three components, measurements must be taken in such a way as to provide at least three linearly independent versions of equation (2). This can be done either by taking measurements at a number of positions in the orbit or by changing the spacecraft attitude. With the latter approach, it is still not necessary to know what the various attitudes are -- only that they are distinct from one another.

Equation (2) may be used as the basis for constructing a least-squares loss function for use in solving for  $\mathbf{D}$ . The loss function is constructed using an error variable,  $e_i$ , defined by

$$e_i = (\mathbf{M}_i^2 - 2 \mathbf{M}_i^T \mathbf{D} + \mathbf{D}^2) - \mathbf{B}_i^2 , \quad (3)$$

where the subscript  $i$  indicates measurement number. The corresponding least-squares loss function,  $J$ , is given by

$$J = 1/2 \sum w_i (e_i - \langle e_i \rangle)^2 \quad (4a)$$

$$= 1/2 \sum w_i [ (M_i^2 - 2 M_i^T \mathbf{D} + D^2) - B_i^2 - \langle e_i \rangle ]^2 , \quad (4b)$$

where  $w_i$  is a weight, inversely proportional to the variance of  $e_i$ , associated with the  $i$ th measurement, and  $\langle e_i \rangle$  is the expectation value of  $e_i$ . The implicit assumption made in selecting equation (4a) as the loss function is that each  $e_i$  is an independent random variable with Gaussian (or nearly Gaussian) distribution. If this assumption is true, then the value of  $\mathbf{D}$  that minimizes  $J$  is the optimal estimate for the true spacecraft bias. Specification of the values for  $w_i$  and  $\langle e_i \rangle$  is not necessarily a trivial matter. The simplest approach, of course, is to assume  $w_i = 1$  and  $\langle e_i \rangle = 0$  for all  $i$ . We present in Section III a simple analytic model from which more exact expressions for  $w_i$  and  $\langle e_i \rangle$  can be derived. It is worth bearing in mind, of course, that because of the uncertainties involved in modeling errors (both in their magnitude and the form of their distribution), the concept of an "optimal" solution for  $\mathbf{D}$  becomes a bit fuzzy. All solutions for  $\mathbf{D}$  within a small error window about the value that minimizes  $J$  will be essentially equally valid.

We now construct an algorithm to determine the value of  $\mathbf{D}$  that minimizes  $J$ . Equation (3) is quadratic in the unknowns, i.e., in the components of  $\mathbf{D}$ . This is awkward because it prevents simply applying the standard tools of linear algebra to construct a least-squares solution for the problem. We note, however, that if the quantity  $D^2$  on the right-hand side of equation (3) were a constant, then standard linear techniques would be applicable. We therefore construct a modified version,  $J_c$ , of the loss function:

$$J_c = 1/2 \sum w_i [ (M_i^2 - 2 M_i^T \mathbf{D} + D_c^2) - B_i^2 - \langle e_i \rangle ]^2 , \quad (5)$$

where the term  $D_c^2$  is to be considered an adjustable parameter rather than a function of the components of  $\mathbf{D}$ . Standard linear least-squares techniques (e.g., see Reference 1) may be used to find the value of  $\mathbf{D}$  that minimizes  $J_c$  for any specified value of  $D_c^2$ ; the solution is

$$\mathbf{D} = 1/2 (\sum w_i \mathbf{M}_i \mathbf{M}_i^T)^{-1} [ \sum w_i (M_i^2 - B_i^2 - \langle e_i \rangle) \mathbf{M}_i + D_c^2 \sum w_i \mathbf{M}_i ] . \quad (6)$$

We may now select the value of  $\mathbf{D}$  that simultaneously satisfies equation (6) and the constraint

$$D^2 = D_c^2 . \quad (7)$$

A solution for  $\mathbf{D}$  selected in this manner is not guaranteed to be a minimum for the loss function  $J$ . We do expect, however, that a vector  $\mathbf{D}_c$  chosen to satisfy equations (6) and (7) simultaneously will be both a good estimate for the solution that would minimize  $J$  and a good estimate for the true bias. In particular, we note that in the limit of noise-free data, there exists a value of  $\mathbf{D}$  (namely, the true bias) that simultaneously zeroes  $J$  and  $J_c$  while also satisfying the constraint of equation (7). Because  $J$  and  $J_c$  are by definition nonnegative, a solution that zeroes them both is a minimum for both. It follows, therefore, that  $\mathbf{D}_c$  will be the exact solution for  $\mathbf{D}$  in the limit of zero noise.

With the constraint of equation (7) applied, equation (6) can be rewritten as

$$\mathbf{D}_c = \mathbf{U} + D_c^2 \mathbf{V} , \quad (8)$$

where

$$\mathbf{U} = 1/2 (\sum w_i \mathbf{M}_i \mathbf{M}_i^T)^{-1} \sum w_i (M_i^2 - B_i^2 - \langle e_i \rangle) \mathbf{M}_i , \quad (9)$$

$$\mathbf{V} = 1/2 (\sum w_i \mathbf{M}_i \mathbf{M}_i^T)^{-1} \sum w_i \mathbf{M}_i . \quad (10)$$

Solving equation (8) for  $\mathbf{D}_c$  first requires a solution for  $D_c^2$ . This can be found as follows.

Multiplying each side of equation (8) with its transpose results in

$$D_c^2 = D_c^T D_c = U^T U + 2 U^T V D_c^2 + V^T V (D_c^2)^2 . \quad (11)$$

This can be written as a simple quadratic in  $D_c^2$  :

$$y = a (D_c^2)^2 + b D_c^2 + c = 0 , \quad (12)$$

where

$$a = V^T V , \quad (13)$$

$$b = (2 U^T V - 1) , \quad (14)$$

$$c = U^T U . \quad (15)$$

The solutions of equation (12) are

$$D_c^2 = [ -b \pm (b^2 - 4ac)^{1/2} ] / 2a . \quad (16)$$

That the negatively signed solution is the proper solution can be demonstrated in the following manner. Any real solution for  $D_c^2$  must have  $b^2 \geq 4ac$ . Parameters  $a$  and  $c$  are nonnegative because  $V$  and  $U$  are real vectors. From this it follows that  $|b| \geq (b^2 - 4ac)^{1/2}$ . In order that  $D_c^2$  be nonnegative, it follows in turn that  $b \leq 0$ . Both solutions for  $D_c^2$  are therefore nonnegative. For cases in which neither bias nor measurement error exists,  $c = 0$  and  $b = -1$ . The negatively signed solution is clearly required to give  $D_c^2 = 0$  for such cases. The parabola  $y(D_c^2)$  defined in equation (12) remains the same in form (i.e., open upwards, with two nonnegative crossings of the  $D_c^2$ -axis) for cases of nonzero bias and error. Any physically realizable measurement situation may be imagined as achieved by a continuous change through a family of situations that starts with zero bias and zero error. During this change, the function  $y$ , and therefore the two solutions for equation (16), change smoothly. The negatively and positively signed solutions of equation (16) do not merge (and therefore cannot switch places) with each other unless the family of situations

contains a "point" having  $b^2 = 4ac$ . An incremental shift in the family to a neighboring one having  $b^2 > 4ac$  everywhere is equally acceptable and removes the conceptual possibility of sign switching for the true solution. The negatively signed solution is therefore always the correct solution for equation (16). Equations (16) and (8) can thereby be solved to obtain  $\mathbf{D}_C$ .

The value of  $\mathbf{D}_C$  may be used either as an acceptable estimate of the magnetic bias or as an initial estimate in an iterative algorithm to find the value of  $\mathbf{D}$  that minimizes the loss function  $J$ . The iterative algorithm is as follows.  $\mathbf{D}$  is expressed as

$$\mathbf{D} = \mathbf{D}_0 + \mathbf{d}, \quad (17)$$

where  $\mathbf{D}_0$  is an estimate (initially set to  $\mathbf{D}_C$ ) of  $\mathbf{D}$ , and  $\mathbf{d}$  is a correction to this estimate. Equation (3) may then be written as

$$e_i = (M_i^2 + D_0^2 - 2 \mathbf{M}_i^T \mathbf{D}_0 - 2 \mathbf{M}_i^T \mathbf{d} + 2 \mathbf{D}_0^T \mathbf{d} + d^2) - B_i^2. \quad (18)$$

The loss function  $J$  becomes

$$J = 1/2 \sum w_i [ (M_i^2 - B_i^2 + D_0^2 - 2 \mathbf{M}_i^T \mathbf{D}_0 + d^2 - \langle e_i \rangle) - 2 (\mathbf{M}_i - \mathbf{D}_0)^T \mathbf{d} ]^2. \quad (19)$$

Taking the gradient of  $J$  with respect to the components of  $\mathbf{d}$  yields

$$\begin{aligned} \nabla J = \sum w_i [ & (M_i^2 - B_i^2 + D_0^2 - 2 \mathbf{M}_i^T \mathbf{D}_0 + d^2 - \langle e_i \rangle) - 2 (\mathbf{M}_i - \mathbf{D}_0)^T \mathbf{d} ] \\ & [ 2 \mathbf{d} - 2 (\mathbf{M}_i - \mathbf{D}_0) ]. \end{aligned} \quad (20)$$

Neglecting the terms in equation (20) that are quadratic or cubic in  $\mathbf{d}$  yields

$$\begin{aligned} \nabla J = 2 \sum w_i \{ & (M_i^2 - B_i^2 + D_0^2 - 2 \mathbf{M}_i^T \mathbf{D}_0 - \langle e_i \rangle) \mathbf{d} \\ & - [ (M_i^2 - B_i^2 + D_0^2 - 2 \mathbf{M}_i^T \mathbf{D}_0 - \langle e_i \rangle) - 2 (\mathbf{M}_i - \mathbf{D}_0)^T \mathbf{d} ] (\mathbf{M}_i - \mathbf{D}_0) \}, \end{aligned} \quad (21)$$

which may be rearranged to read

$$\nabla J = 2 (Y \mathbf{d} - \mathbf{Z}), \quad (22)$$

where

$$Y = \sum w_i [ 2 (M_i - D_0) (M_i - D_0)^T + (M_i^2 - B_i^2 + D_0^2 - 2 M_i^T D_0 - \langle e_i \rangle) I ], \quad (23)$$

$$Z = \sum w_i [ (M_i^2 - B_i^2 + D_0^2 - 2 M_i^T D_0 - \langle e_i \rangle) (M_i - D_0) ], \quad (24)$$

and  $I$  is the 3-by-3 identity matrix. The desired solution for  $\mathbf{d}$  is found by setting  $\nabla J$  to zero and multiplying both sides of the resulting equation by  $Y^{-1}$ :

$$\mathbf{d} = Y^{-1} \mathbf{Z}. \quad (25)$$

The loop is closed by adding  $\mathbf{d}$  to  $D_0$ , thus obtaining a new estimate for  $\mathbf{D}$ . The process continues until  $\mathbf{d}$  is made arbitrarily close to zero. As noted above, we expect that a solution for  $\mathbf{D}$  obtained using this iterative technique should not differ significantly from  $D_c$ .

The uncertainty,  $\Delta \mathbf{D}$ , associated with the estimated value of  $\mathbf{D}$  should be of order  $\sigma/N^{1/2}$ , where  $\sigma$  is the root-mean-square (rms) magnetometer measurement error, and  $N$  is the total number of measurements made. A more accurate estimate of the uncertainty of  $\mathbf{D}$  can be made as follows. If equation (25) had resulted from a "standard" least-squares problem, i.e., one in which all measurement errors were associated with the vector  $\mathbf{Z}$  while the matrix  $Y$  was composed of known constants or model parameters, then the uncertainty  $\Delta \mathbf{D}$  would be given by

$$\Delta \mathbf{D} = [ 2 J(\mathbf{D}) / (N-1) ]^{1/2} \mathbf{W}, \quad (26)$$

with  $\mathbf{W}$  defined as the vector with components equal to the square root of the diagonal components of  $Y^{-1}$ . If all measurement errors are "extracted" from  $Y^{-1}$ , it reduces to a matrix  $X$  defined as

$$X = [ \sum w_i^2 (M_i - D) (M_i - D)^T ]^{-1} \quad (27a)$$

$$= [ \sum w_i^2 (A_i B_i) (A_i B_i)^T ]^{-1} . \quad (27b)$$

We therefore take our estimate of  $\Delta D$  to be as given by equation (26), with  $W$  defined as the square root of the diagonal of  $X$ . In Section III we give an evaluation of  $\Delta D$  more directly comparable with the simple estimate  $\sigma/N^{1/2}$ .

### III. ANALYTICAL ERROR MODEL

In this section we present a simple analytical model for the magnetometer errors and use it to derive expressions for the parameters  $w_i$  and  $\langle e_i \rangle$ . We further apply this model to a simple observing scenario and calculate the corresponding uncertainty  $\Delta D$ . In principle, errors associated with both  $M_i$  and  $B_i$  exist, the latter resulting from errors both in the geomagnetic field model and the spacecraft ephemeris. For simplicity, we assume here that the total error is sufficiently dominated by the error in  $M_i$  that the error in  $B_i$  can be neglected. We define  $r_i$  as the error vector associated with  $M_i$  and assume it to be composed of three independent random variables, each having Gaussian distribution with mean zero and variance  $\sigma^2$ . The full variance of  $r_i$  is  $3\sigma^2$ .

Using this model, the variable  $e_i$  is related to  $r_i$  by the relation

$$\begin{aligned} e_i &= (M_i - D_i)^2 - B_i^2 \\ &= (B_i + r_i)^2 - B_i^2 \\ &= 2 B_i^T r_i + r_i^2 . \end{aligned} \quad (28)$$

The expectation value of  $e_i$  is given by



$$\begin{aligned}
\langle e_i \rangle &= \int G(\mathbf{r}_i) e_i d\mathbf{v}_r \\
&= \int G(\mathbf{r}_i) (2 \mathbf{B}_i^T \mathbf{r}_i + r_i^2) d\mathbf{v}_r \\
&= \int G(\mathbf{r}_i) (2 B_i r_i \cos(\theta) + r_i^2) d\mathbf{v}_r \\
&= 3\sigma^2, \tag{29}
\end{aligned}$$

where  $G(\mathbf{r}_i)$  is the Gaussian distribution function (proportional to  $\exp(-r_i^2/2\sigma^2)$ ),  $d\mathbf{v}_r$  is a volume element in  $r$ -space, and  $\theta$  is the angle between  $\mathbf{B}_i$  and  $\mathbf{r}_i$ . (The integral  $\int f \cos(\theta) d\mathbf{v}_r$  yields zero for any  $f(\mathbf{r}_i)$  that is spherically symmetric.) The variance,  $s_i^2$ , of  $e_i$  is in turn given by

$$\begin{aligned}
s_i^2 &= \int G(\mathbf{r}_i) (e_i - \langle e_i \rangle)^2 d\mathbf{v}_r \\
&= \int G(\mathbf{r}_i) [2 B_i r_i \cos(\theta) + r_i^2 - 3\sigma^2]^2 d\mathbf{v}_r \\
&= \int G(\mathbf{r}_i) [4 B_i^2 r_i^2 \cos^2(\theta) + 2 B_i r_i \cos(\theta) (r_i^2 - 3\sigma^2) \\
&\quad + r_i^4 - 2 r_i^2 (3\sigma^2) + (3\sigma^2)^2] d\mathbf{v}_r \\
&= 4/3 B_i^2 (3\sigma^2) + 5 \sigma^2 (3\sigma^2) - 2 (3\sigma^2)^2 + (3\sigma^2)^2 \\
&= 2 \sigma^2 (2 B_i^2 + 3\sigma^2) . \tag{30}
\end{aligned}$$

The weight  $w_i$  may be set equal to  $s_i^{-2}$ . We assumed for the derivation of equations (29) and (30) that the error in  $\mathbf{M}_i$  dominated that in  $\mathbf{B}_i$ . This was done for simplification and is in no way essential. Allowing an error in  $\mathbf{B}_i$  having components with Gaussian distribution of variance  $\sigma_b^2$  yields the following expressions for  $\langle e_i \rangle$  and  $s_i^2$ :

$$\langle e_i \rangle = 3\sigma^2 - 3\sigma_b^2, \tag{31}$$

$$s_i^2 = 2 \sigma^2 (2 B_i^2 + 3\sigma^2) + 2 \sigma_b^2 (2 B_i^2 + 3\sigma_b^2) + 2 (3\sigma^2 3\sigma_b^2) . \tag{32}$$

For illustration, we present here typical values of  $B$ ,  $\sigma$ , and  $\sigma_b$  for the specific case of the HST mission. The rms noise level associated with the HST magnetometers has been determined to be approximately 0.01 gauss (Reference 2). The geomagnetic field model to be used is the International Geomagnetic Reference Field (IGRF) for 1975 (Reference 3). A typical value for  $B$  at an altitude of 445 kilometers is about 0.3 gauss. Typical errors to be associated with the reference field can be estimated on the basis of the work by Trombka and Cain (Reference 4), wherein the IGRF for 1975 was compared with that for 1965. The rms error at 445 kilometers was found to be approximately 0.002 gauss. Launch date for the HST mission is currently scheduled for June 1989, 15 years after the completion of the 1975 IGRF. Assuming linear error growth with time, the error to be associated with the reference field at HST launch should be about 0.003 gauss. The error in the estimate of the local geomagnetic field value due to spacecraft position error can be estimated as  $|(\nabla \cdot \mathbf{B})| |\Delta \mathbf{R}|$ , where  $\nabla \cdot \mathbf{B}$  is the divergence of  $\mathbf{B}$ , and  $\Delta \mathbf{R}$  is the ephemeris error for the spacecraft. The definitive ephemeris for the HST is expected to have an rms accuracy of 0.2 kilometer (Reference 5). Given that the near-Earth dipole field falls off approximately as  $R^{-3}$ , the magnitude of  $\nabla \cdot \mathbf{B}$  can be estimated as  $3 B / R_0$ , where  $R_0$  is the semimajor axis of the spacecraft orbit. Together these yield a field error of approximately 0.03 milligauss. The error in  $B_i$  is dominated by the error in the field model. The ratio  $\sigma^2 / \sigma_b^2$  is approximately 10, indicating that neglecting  $\sigma_b^2$  in equations (31) and (32) would be a reasonable approximation.

We now use equations (26), (27b), and (30) to provide a simple expression for  $\Delta \mathbf{D}$  for an idealized observing scenario. We consider a situation in which the geomagnetic field is constant and the spacecraft changes attitude in such a way as to have the field aligned with each of the principal spacecraft axes for one-third of the measurements. For this scenario the variance  $s_i^2$  is the same for all  $i$ , and  $w_i$  may be set to 1. The loss function  $J(\mathbf{D})$  and matrix  $\mathbf{X}$  are then for large  $N$  given by

$$J(\mathbf{D}) = 1/2 N s^2 = N \sigma^2 (2 B^2 + 3 \sigma^2), \quad (33)$$

$$\mathbf{X} = (2/3 N B^2 \mathbf{I})^{-1}, \quad (34)$$

from which it follows that each component of  $\Delta \mathbf{D}$  is given by

$$\begin{aligned}
 \Delta D &= [ 2 J(\mathbf{D}) / N ]^{1/2} W \\
 &= [ 2 \sigma^2 ( 2 B^2 + 3 \sigma^2 ) ]^{1/2} ( 2/3 N B^2 )^{-1/2} \\
 &= \sigma / N^{1/2} 6^{1/2} [ 1 + ( 3 \sigma^2 / 2 B^2 ) ]^{1/2} .
 \end{aligned} \tag{35}$$

The estimate specified above indicates that, for situations in which the measurement errors are small compared with the field strength, the algorithm described in this article will provide an estimate for the spacecraft bias that is about a factor of  $6^{1/2}$  (i.e.,  $\sim 2.5$ ) less accurate than that which could be obtained with the same measurements if attitude information were independently available.

In situations where the use of the algorithm described in this article is intended as a preliminary calibration of the spacecraft bias, many of the details of algorithm may be neglected. As mentioned earlier, setting  $\langle e_i \rangle = 0$  and  $w_i = 1$  is the simplest approach in practice. If  $B^2$  is fairly constant, as should be roughly true (to within a factor of about 4) for a spacecraft at constant altitude, then taking the weights for all variables  $e_i$  to be equal would not be a bad approximation. It would be inappropriate, however, for spacecraft with highly eccentric orbits. Intuitively, it might be expected that neglecting  $\langle e_i \rangle$  could introduce errors in the solution for  $\mathbf{D}$  of perhaps as much as  $\sigma$ . (Our numerical examples actually show this estimate to be too high.) Errors of this order are probably not significant for preliminary estimates. In the HST mission, for example, for which final spacecraft bias calibration will be performed using high-accuracy attitude information, the algorithm described here will be used with  $\langle e_i \rangle = 0$  and  $w_i = 1$  and without iteration.

#### IV. NUMERICAL EXAMPLES

In this section we present the results of a number of numerical simulations that illustrate the use of the algorithm and verify its accuracy. After verifying that the algorithm obtains the exact

solution when no noise is added to the measurements, we ran six test scenarios using noisy data. Each scenario comprises  $K$  bias determinations for each of two magnetometers, with each determination based upon  $N$  data points. Both  $K$  and  $N$  have the value 100 for all of the scenarios. The strength of the geomagnetic field is  $\sim 0.35$  gauss in all cases, i.e., typical of that experienced by a spacecraft in near-Earth orbit, and the true values for spacecraft bias at the two magnetometer locations are

$$\mathbf{D}_1 = ( 0.005, -0.015, 0.010 )^T \text{ gauss} , \quad (36)$$

$$\mathbf{D}_2 = ( -0.170, 0.280, 0.220 )^T \text{ gauss} . \quad (37)$$

We selected these values to test the algorithm with bias values substantially below and approximately equal to the ambient field strength. For each scenario, we calculated the average error in the estimated bias (for both  $\mathbf{D}_c$  and  $\mathbf{D}$ ) and the standard deviation of these errors for each magnetometer. For calculating  $\mathbf{D}$ , we defined convergence as  $|\mathbf{d}| < 0.001 \sigma/N^{1/2}$ .

We divide the six scenarios into two groups. The scenarios in the first group (scenarios 1, 2, and 3) all use a constant geomagnetic field. We are therefore able to compare the results of these scenarios directly with the analytic calculations presented in Section III. The scenarios in the second group (scenarios 4, 5, and 6) use a geomagnetic field constructed to simulate that which could be experienced by a spacecraft in near-Earth orbit. The height of the simulated orbit is 522 kilometers; the inclination is 28.5 degrees; and the right ascension of ascending node, argument of perigee, and latitude at epoch time (i.e., time of the first data point) are all 0 degrees. The geomagnetic field experienced by a spacecraft with this orbit is reasonably simulated (at least for our purposes) using the following parameterization:

$$B_{1,i} = 0.01 + 0.17 \cos(\theta_i) \text{ gauss} , \quad (38)$$

$$B_{2,i} = -0.19 + 0.15 \sin(\theta_i) \text{ gauss} , \quad (39)$$

$$B_{3,i} = 0.20 + 0.07 \sin(\theta_i) \text{ gauss} , \quad (40)$$

where  $\theta_i = 7.2 (i - 1)$  degrees. The sample spacing corresponds roughly to one data point per

minute over a single orbit. We present details for the individual scenarios in the following paragraphs and the principal numerical results in Table 1 (for scenarios 1, 2, and 3) and Table 2 (for scenarios 4, 5, and 6) at the end of this section. The specific entries in the tables are average error, standard deviation of the errors, estimated uncertainty based on equation (26), and for scenarios 1 and 2 the "theoretical" uncertainty calculated using equation (35).

Scenario 1. This scenario is a numerical execution of the idealized case described in Section III. It is designed to demonstrate the algorithm's accuracy for situations where the magnetometer measurements are uniformly distributed. The values for the parameters  $B$  and  $\sigma$  are 0.35 gauss and 0.01 gauss, respectively. As can be seen from Table 1, the magnetic bias estimates are equally accurate for both magnetometers, and no significant difference exists between the estimates  $\mathbf{D}$  and  $\mathbf{D}_c$ . Typically, only two iterations are required to obtain the estimate  $\mathbf{D}$  after  $\mathbf{D}_c$  has been determined. The standard deviation for each of the error components is  $\sim 0.0018$  gauss, in contrast to the "theoretical" value of  $\sim 0.0025$  gauss predicted by both equations (26) and (35). This difference may be attributable to the qualitative nature of the concept of "extracting" the measurement errors from the matrix  $\mathbf{Y}^{-1}$  that was used in deriving equation (27b). The average error in the estimated bias for each component of  $\mathbf{D}$  (or  $\mathbf{D}_c$ ) is zero to within plus or minus about twice  $1.8 \sigma / (N K)^{1/2}$  (i.e.,  $\sim$  twice 0.00018 gauss), which is what would be expected given the standard deviation of the errors.

Scenario 2. This scenario is a repetition of scenario 1, except that in this case the parameter  $\langle e_i \rangle$  is set to zero. This scenario is designed to determine whether the algorithm with this simplification provides results of essentially the same accuracy as does the full algorithm. This, in fact, proves to be the case. The standard deviation of the errors is essentially the same as for scenario 1 (i.e.,  $\sim 1.8 \sigma / N^{1/2}$ ), and the average of the errors for any given component of  $\mathbf{D}$  or  $\mathbf{D}_c$  is shifted only slightly ( $\sim 0.0006$  gauss) from zero. This offset, though statistically significant, is small compared with the typical uncertainty of  $\sim 0.002$  gauss for any single determination. It appears that a user of the algorithm can apply the simplification of neglecting both  $\langle e_i \rangle$  and the steps of iterating between  $\mathbf{D}_c$  and  $\mathbf{D}$  without substantially corrupting the results of the calculation.

Scenario 3. This scenario is designed to investigate the observability problems that the algorithm encounters if an observing run with insufficient variation of the geomagnetic field is used. The field strength and noise levels are the same as for scenario 1, but the three attitudes used are all within 14 degrees of one another. Specifically, the right ascension, declination, and roll for the three attitudes are, in degrees, (0, 0, 0), (10, 0, 0), and (0, 10, 0); and the geomagnetic field is directed toward sky coordinates (0, 0). As shown in Table 1, component 1 of the standard deviation vector for the errors of  $\mathbf{D}$  is substantially smaller (by a factor of between 7 and 9) than components 2 and 3. Furthermore, the same kind of ratio between components applies for the uncertainty estimate derived using equation (26). Despite this basic agreement (i.e., proportionality) between the standard deviation vector and the uncertainty estimate, the average errors of the components of  $\mathbf{D}$  are larger in size (i.e., offset from zero) by amounts that are statistically significant when compared with their expected rms size of  $K^{-1/2}$  times the standard deviation. This is true in particular for component 1 for magnetometer 1 and components 2 and 3 for magnetometer 2. Although the offsets are statistically significant, they are all still smaller than the corresponding standard for the given component of  $\mathbf{D}$ . The calculation of  $\mathbf{D}$  remains basically good, but significantly degraded, for this scenario. In contrast to these results for  $\mathbf{D}$ , the bias estimate  $\mathbf{D}_c$  is a poor estimate of the true bias (particularly in the case of magnetometer 2, for which the true bias is large). For the scenario presented here, the averages of the errors for components 2 and 3 of  $\mathbf{D}_c$  for magnetometer 2 are nearly 10 times as large as the rms error of a single measurement. These results emphasize the need for the user to arrange for data acquisition in a manner that guarantees substantial field variation in the spacecraft's frame. They further indicate that if such uniform data acquisition is for some reason not possible, then application of the full algorithm (i.e., continuing on to calculate  $\mathbf{D}$  after  $\mathbf{D}_c$ ) is imperative.

Scenario 4. This scenario, as well as the next two, is designed to test the algorithm using the more realistic geomagnetic field model described at the beginning of this section (i.e., as specified by equations (38) through (40)). The parameters  $w_i$  and  $\langle e_i \rangle$  are set in accordance with equations (29) and (30), with  $\sigma$  again equal to 0.01 gauss. As can be seen in Table 2, the results for scenario 4 with respect to accuracy of the solution are essentially the same as those for scenario 1 (component 3 of the error for both  $\mathbf{D}$  and  $\mathbf{D}_c$  for magnetometer 2 is just slightly greater than three

times the expected rms value). These results demonstrate that the algorithm works well for realistic ambient magnetic field values, as well as for the idealized cases such as scenario 1.

Scenario 5. This scenario is a repetition of scenario 4, but with  $w_i$  set to 1 and  $\langle e_i \rangle$  set to 0. This scenario is designed with the same purpose as scenario 2, i.e., to check the accuracy of the algorithm when it is applied with the indicated simplifications. Comparing the results of this scenario and those of scenario 4 indicates (as was noted for scenario 2) that a mild degradation of results occurs when the simplified algorithm is used, but that this degradation is quite small. In actual operations, use of the simplified algorithm would almost certainly be acceptable. For the realistic error estimate being used in these scenarios (i.e.,  $\sigma \sim 0.01$  gauss), the final uncertainty in the bias estimate (using either  $\mathbf{D}$  or  $\mathbf{D}_c$ ) is of order 0.003 gauss.

Scenario 6. This scenario is a repetition of scenario 4, but with  $\sigma$  set to 0.1 gauss. The purpose is to test the robustness of the algorithm when used with very noisy data; the noise level used is about one-third of the ambient field strength. The algorithm requires approximately six iterations to converge and yields estimates for the bias with errors of order 0.04 gauss. In one case the algorithm failed to converge even after 10 iterations. As is clear in Table 2, the components of the vector of average errors are offset from zero by as much as 10 times the expected rms value. There is no indication that the estimate  $\mathbf{D}$  is any better determined than is  $\mathbf{D}_c$ .

The basic conclusions to be drawn from the six scenarios are that (1) the algorithm works very well for cases in which magnetometer noise levels are like those expected for actual spacecraft operations, (2) results obtained using the simplifications of setting  $w_i$  to 1 and  $\langle e_i \rangle$  to 0 and not iterating beyond the first estimate  $\mathbf{D}_c$  are very nearly as good as those obtained using the full algorithm, and (3) the user of the algorithm should make every effort to arrange that magnetometer data acquisition be done in a way that provides substantial variation in all components. The results of scenarios 4 and 5 indicate that the kind of magnetic variation that a spacecraft with constant attitude in near-Earth orbit automatically experiences over a single orbit is adequate to the needs of the algorithm.

Table 1. Statistical Results from Scenarios 1 through 3

Magnetometer:	1			2		
Component:	1	2	3	1	2	3
<b>Scenario 1</b>						
<b>D:</b> Average Error	0.00038	0.00038	0.00000	0.00008	0.00033	0.00027
Standard Deviation	0.00179	0.00184	0.00182	0.00166	0.00190	0.00181
Estimated Uncertainty	0.00242	0.00242	0.00239	0.00246	0.00246	0.00242
<b>D<sub>C</sub>:</b> Average Error	0.00037	0.00042	-0.00002	0.00029	-0.00001	-0.00001
Standard Deviation	0.00178	0.00183	0.00181	0.00165	0.00188	0.00182
Estimated Uncertainty	0.00242	0.00242	0.00239	0.00246	0.00246	0.00242
Theoretical Uncertainty	0.00246	0.00246	0.00243	0.00246	0.00246	0.00243
<b>Scenario 2</b>						
<b>D:</b> Average Error	0.00081	0.00081	0.00043	0.00051	0.00076	0.00070
Standard Deviation	0.00179	0.00184	0.00182	0.00166	0.00190	0.00181
Estimated Uncertainty	0.00242	0.00242	0.00238	0.00245	0.00245	0.00242
<b>D<sub>C</sub>:</b> Average Error	0.00080	0.00085	0.00041	0.00072	0.00041	0.00042
Standard Deviation	0.00178	0.00183	0.00181	0.00165	0.00188	0.00181
Estimated Uncertainty	0.00242	0.00242	0.00238	0.00245	0.00245	0.00242
Theoretical Uncertainty	0.00246	0.00246	0.00243	0.00246	0.00246	0.00243
<b>Scenario 3</b>						
<b>D:</b> Average Error	0.00093	0.00000	-0.00127	0.00018	-0.00807	-0.00955
Standard Deviation	0.00209	0.01530	0.01677	0.00149	0.01342	0.01386
Estimated Uncertainty	0.00242	0.01958	0.01943	0.00235	0.01899	0.01885
<b>D<sub>C</sub>:</b> Average Error	0.00077	0.00146	-0.00150	0.01664	-0.09821	-0.09171
Standard Deviation	0.00178	0.01265	0.01405	0.00552	0.01369	0.01398
Estimated Uncertainty	0.00242	0.01959	0.01944	0.00295	0.02386	0.02368

Note: All values are in gauss.



Table 2. Statistical Results from Scenarios 4 through 6

Magnetometer:	1			2			
Component:	1	2	3	1	2	3	
Scenario 4							
D:	Average Error	-0.00003	0.00035	0.00032	0.00047	-0.00023	0.00077
	Standard Deviation	0.00272	0.00240	0.00212	0.00285	0.00243	0.00220
	Estimated Uncertainty	0.00375	0.00317	0.00292	0.00373	0.00315	0.00290
D <sub>C</sub> :	Average Error	-0.00006	0.00045	0.00023	0.00097	-0.00050	0.00076
	Standard Deviation	0.00270	0.00239	0.00211	0.00282	0.00241	0.00219
	Estimated Uncertainty	0.00375	0.00317	0.00292	0.00373	0.00315	0.00290
Scenario 5							
D:	Average Error	-0.00008	0.00056	0.00087	0.00054	0.00003	0.00128
	Standard Deviation	0.00270	0.00243	0.00216	0.00285	0.00244	0.00220
	Estimated Uncertainty	0.00378	0.00306	0.00323	0.00376	0.00304	0.00321
D <sub>C</sub> :	Average Error	-0.00011	0.00066	0.00077	0.00102	-0.00016	0.00122
	Standard Deviation	0.00269	0.00241	0.00215	0.00283	0.00242	0.00219
	Estimated Uncertainty	0.00378	0.00306	0.00323	0.00376	0.00304	0.00321
Scenario 6							
D:	Average Error	0.00086	0.02297	0.04112	0.00856	0.01891	0.04873
	Standard Deviation	0.03783	0.03083	0.03081	0.04478	0.03685	0.04229
	Estimated Uncertainty	0.03798	0.03188	0.02992	0.03749	0.03146	0.02953
D <sub>C</sub> :	Average Error	-0.00252	0.02606	0.02527	0.03283	-0.00035	0.02891
	Standard Deviation	0.02238	0.01896	0.01700	0.02563	0.02270	0.02359
	Estimated Uncertainty	0.03817	0.03203	0.03007	0.03847	0.03228	0.03031

Note: All values are in gauss.

## V. CONCLUSIONS

We have presented a simple algorithm for the in-flight determination of the magnetic bias of a spacecraft. The algorithm, developed for use during the HST mission, is independent of any attitude estimates and requires no telemetry from spacecraft sensors other than the magnetometers at the locations where the bias is being determined. We have presented an analytical model for the errors in a typical observing scenario, as well as the results from a number of numerical studies demonstrating the general stability and accuracy of the algorithm. The algorithm works well and should be a useful tool for preliminary spacecraft magnetic bias determination for a variety of spacecraft missions.

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## REFERENCES

1. K. A. Brownlee, Statistical Theory and Methodology in Science and Engineering. New York: John Wiley & Sons, Inc., 1960
2. H. Dougherty, *Effect of Magnetometer Noise on System Performance*, Memorandum LMSC/F227860 0/64-20, B/579, Lockheed Missiles & Space Company, Inc., 1988
3. B. R. Leaton, *International Geomagnetic Reference Field 1975*, Transactions of the American American Geophysical Union, 1976, vol. 57, no. 3, p. 120
4. B. T. Trombka and J. C. Cain, Computation of the IGRF I. Spherical Expansions, NASA X-922-74-303, National Aeronautics and Space Administration, Goddard Space Flight Center, August 1974
5. E. Zavaleta, Operations Support Computing Facility (OSCF) to Space Telescope (ST) Payload Operations Control Center (POCC) Interface Control Document, ST-ICD-32, CSC/TM-82/6152, Computer Sciences Corporation, May 1983, Section 3.2.4