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REMOTE CHANCE OF RECONTACT?

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ABSTRACT

The ejection of appendages with uncertain drag characteristics presents a concern for eventual recontact. Recontact shortly after release can be prevented by avoiding ejection in a plane perpendicular to the velocity. For ejection tangential to the orbit, the likelihood of recontact within a year is high in the absence of drag and oblateness. The optimum direction of ejection of the thermal shield cable and an overestimate of the recontact probability are determined for the Cosmic Background Explorer (COBE) mission when drag, oblateness, and solar/lunar perturbations are present. The probability is small but possibly significant.

1. INTRODUCTION

The Cosmic Background Explorer (COBE) thermal shield cable is scheduled to be ejected before the Delta second-stage separation, 101 seconds (sec) after the second-stage cutoff (3699 sec after lift-off), near the first ascending node. Determining the optimum direction of ejection to prevent recontact and estimating the risk of recontact with the cable (within the constraints specified by the COBE Project Office) are the aims of this study.

Background information is presented in Section 2, the methods used are briefly described in Section 3, and the analysis is given in Section 4. Ejection for a spherical Earth without drag will be studied first to provide insight (Section 4.1). The optimum direction of ejection in this case will be determined, and the probabilities of recontact for optimum and tangential ejections will be calculated. A trajectory simulation having a near recontact with drag and oblateness will be presented and examined, and the probability of recontact will then be estimated (Section 4.2). An extremely large cross-sectional area was chosen to provide high drag so that the recontact could be obtained easily. This choice does not affect the generality of the result. The conclusions are summarized in Section 5.

In this paper, computations have a 60-sec integration step. This limitation was imposed to conserve computer time. Sixty seconds represents a relatively low level of resolution because, for example, a separation of over 1/3 kilometer (km) is possible in this time for an ejection speed of 20 feet per second (ft/sec).

The calculated probability is expected to be an overestimate. Radial separation due to drag can be greater than the minimum separation required for recontact. An otherwise possible recontact can thereby be prevented, or a new recontact can become possible. The former is much more likely than the latter because the separation required to break a recontact is small. This effect should be minor, since drag is small, and it is not included in the analysis.

2. BACKGROUND

In accordance with guidelines set by the COBE Project Office, the following constraints on the ejection are assumed. The ejection speed is between 8 and 20 ft/sec. The direction of the ejection is determined by the position of the cable cutter, which can be located at any point around the body of the spacecraft. This direction is known to within ±20 degrees (deg) in the plane perpendicular to the nadir and is assumed to have no radial component.

2.1 SPHERICAL EARTH, NO DRAG

Ejection in the direction of the positive or negative orbit normal (or in a radial direction) leaves the tangential component of the cable's velocity unchanged. Since the ejection speed is small compared to the orbital velocity, the cable's orbital velocity is nearly identical to the spacecraft's. The orbital periods are therefore almost equal, and recontact occurs within the first revolution.

For example, an 8-ft/sec ejection speed in either the radial or orbit normal direction results in a separation of only 7.5 meters (m) in one revolution. Recontact can occur at the ejection point because the radius is unchanged by the ejection. Recontact occurs in half an orbit for ejection along the orbit normal.

To prevent this form of recontact, a tangential component is required. The tangential component increases as the ejection direction moves from the orbit normal toward the tangential direction or as the ejection speed increases. The larger the tangential component, the greater the separation after one revolution. For ejection along the positive spacecraft velocity direction, the cable orbit period is greater than that of the spacecraft, and the cable will follow the spacecraft after one revolution. Ejection along the negative velocity direction produces the converse effect. After sufficiently many revolutions, an opportunity for recontact will occur because the cable makes exactly one revolution more or less than the spacecraft. Several opportunities for recontact may occur in a year. The smaller the tangential component of the ejection velocity, the fewer the number of possible recontacts within a year.

2.2 EFFECT OF DRAG

The drag on the cable cannot be determined accurately because of the cable's extremely long and thin shape. Concern arises because there is doubt about whether the drag on the cable will be greater than or less than that on the spacecraft. If the ejection is along the positive orbital velocity vector, recontact can occur only if drag on the cable is greater than on the spacecraft. For ejection along the negative velocity, drag on the cable must be less than on the spacecraft for recontact to occur.

Drag changes the time of recontact because the semimajor axis of each orbit changes. The difference in drag is small. The along-track separation produced by drag can be canceled by an appropriate change in ejection speed. As mentioned in the introduction, the radial separation could either prevent an otherwise possible recontact or create a new one. An overestimate of the probability results from neglecting this effect. There are still several opportunities for recontact in the year.

2.3 EFFECT OF OBLATENESS

Oblateness causes precession of the line of nodes and rotation of the line of apsides. The ejection gives the cable orbit a semimajor axis and inclination that may differ from those of the spacecraft, resulting in a nodal precession rate different from the spacecraft's. The COBE orbit is nearly polar (99-deg inclination). The only chance of recontact after the first orbit is therefore at the northernmost or southernmost points of the orbit (except in one rare case). For any given angle of ejection, the difference in node upon recontact is independent of the ejection speed because the increasing nodal rate is canceled by the decreasing time to recontact as ejection speed increases.

The difference in rate of change of the argument of perigee of the spacecraft and cable orbits is negligible. The rate of change of the argument of perigee is 2.75 deg/day. This rate places the apsis initially at the ascending node at the southernmost point in the orbit in 32 days, and at the northernmost point in 98 days. The worst case occurs in 65 days, when this apsis is at the descending node, as happens for a tangential ejection at 8.9 ft/sec. The distance between orbits at the northernmost or southernmost points is then 5.3 km. Variations in semimajor axis of ± 9 km are typical, but it is expected that the spacecraft and cable orbits will vary in the same fashion. Recontact is therefore not possible when the line of apsides is along the line of nodes. Recontact is only possible when the appropriate apsis is near the northernmost or southernmost point. The fraction of time spent in an orientation in which the distance between orbits at these points is within the required minimum separation distance for recontact will be determined and used to estimate the probability of recontact.

3. METHOD AND SOFTWARE USED

3.1 ANALYTICAL CALCULATIONS

The analytical calculations shown here are strictly valid in the absence of drag, oblateness, and solar/lunar perturbations. Standard equations for two-body circular and elliptical orbits are used. These equations permit separation distances to be calculated by determining the orbital periods from knowledge of the ejection velocity.

3.2 SOFTWARE USED

The EPHGEN program was used to generate the ephemeris of COBE with given epoch elements and that of the ejected cable with epoch elements calculated when the magnitude and direction of the velocity of ejection relative to COBE are known. The EPHCMP and GTDS-COMPARE programs were run to compare the COBE and the cable ephemerides. The output contains the spacecraft separation at specified time intervals from epoch. An 8-by-8 model of the geopotential zonal and tesseral harmonics and the Goddard Trajection Determination System (GTDS) Atmospheric Density Model No. 3 were used. A 60-sec integration step was used to provide efficiency with adequate accuracy.

3.3 METHODS FOR CALCULATING PROBABILITY

An empirical method for determining the probability was devised. It involves counting all the ways in which recontact can occur and determining the extent of the dispersions in ejection velocity for each, such that a maximum separation distance is not exceeded. A close encounter will be said to occur in one synodic period.

3.3.1 NO DRAG OR OBLATENESS

When drag and oblateness are not considered, the method is as follows. Values of ejection velocity that give a recontact on the first close encounter are obtained. Dispersions about these values are found that produce the maximum tolerance in separation distance, conservatively assumed to be 20 m. An average value of the dispersions is determined over the specified velocity range. The average dispersion is divided by the magnitude of the range of velocities and multiplied by the number of possible revolutions consistent with the range of velocities to obtain the probability. An equivalent method, which can also be used with drag and oblateness, equates the probability of recontact for a given ejection velocity with the fraction of a revolution in which the minimum separation occurs. The different ways in which recontact can occur in a year on subsequent close encounters are enumerated so that the probability can be determined for the year. The probability of recontact is the same for all close encounters for a given ejection velocity.

3.3.2 DRAG AND OBLATENESS

When drag and oblateness are considered, the method is as follows. The COBE orbit is nearly polar (99-deg inclination). Precession of the nodes therefore prevents recontact except at the northernmost or southernmost points of the orbit (except in one rare case). The probability for recontact is estimated as above. An additional factor related to the orientation of the cable orbit is needed. This is the only place where radial separation is taken into account. The result is an overestimate, as explained in the introduction, since radial separation due to drag is ignored.

4. CALCULATIONS AND RESULTS

4.1 ANALYTICAL CALCULATIONS

In this section, ejection will be considered in the absence of drag and oblateness. Recontact must occur at or near the ejection point. The orbits are coplanar throughout. The probability of recontact for the first close encounter and for the first year will be calculated. The optimum angle of ejection will be determined, and the associated probability of recontact will be calculated.

The COBE and cable orbits are characterized by the following parameters:

Radius of circular COBE orbit	7278.14 km
Period of circular COBE orbit	102.98887 minutes (min)
Velocity of COBE	7.40046 km/sec
Assumed maximum cable ejection speed	20 ft/sec
Assumed minimum cable ejection speed	8 ft/sec
Uncertainty in the angle of ejection	±20 deg
Cable length and diameter	25 ft x 3/32 inches (in.)
Area/mass COBE (ft^2/kilogram (kg))	160/2260 = 0.07
Nominal area/mass cable* (ft^2/kg)	0.154/1 = 0.15
Rate of change of COBE semimajor axis	0.5 km/year (yr)
Rate of change of cable semimajor axis	1.0 km/yr

The control parameters, in order of increasing importance for recontact, are the angle of ejection Θ (Figure 1), the speed of ejection, and the coefficient of drag/area-to-mass ratio. Only the first can be influenced by design. It is assumed that the ejection produces no radial component. The ejection speed and direction are assumed to be random within the specified limits.

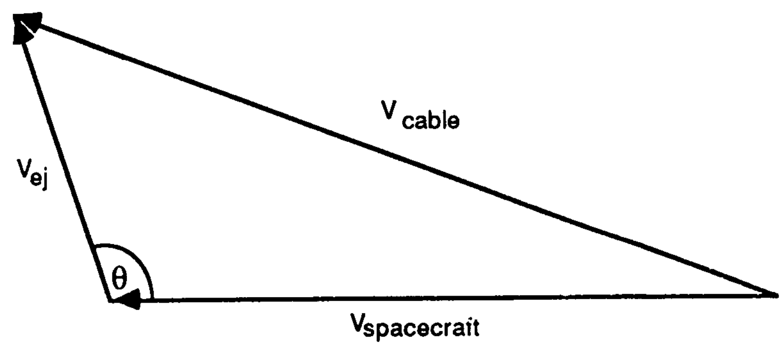

Figure 1. Definition of Angle of Ejection (Θ)

*Based on an effective length of 20 ft calculated by assuming a random orientation over a solid angle of 4π.

4.1.1 TANGENTIAL EJECTION

Spacecraft and cable orbits are shown in Figure 2 for ejection in the direction of the velocity. Drag and oblateness are not considered here. The numbers shown in the figure refer to the order in which the close encounters occur. The meeting point is at the point of ejection. Recontact occurs for a close encounter at or near the meeting point.

Let the first close encounter occur in x_1 cable orbits. A convenient approximation from which cable velocity can be determined with satisfactory accuracy is

$$x_1 \, t_{cable} = (x_1 + 1) \, t_{s/c}$$

where t is the orbital period.

Therefore,

$$x_1 = t_{s/c}/(t_{cable} - t_{s/c})$$

The nth close encounter is given by

$$x_n = n \, x_1$$

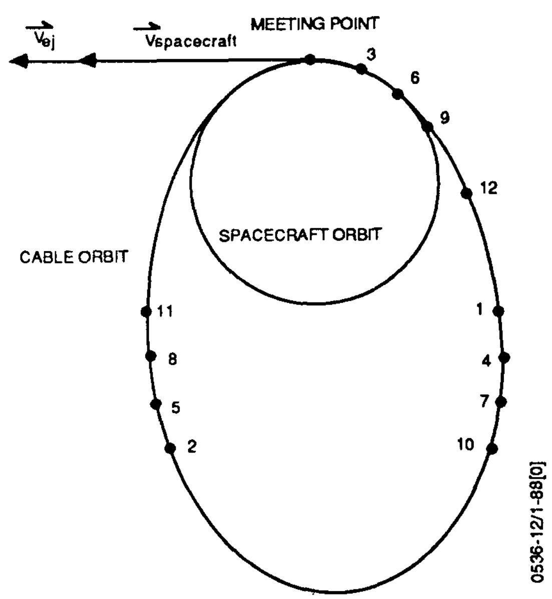

Figure 2. Close Encounters for Ejection Along the Spacecraft Velocity

Table 1 shows values of x_1 for four cases: ejection at 20 or 8 ft/sec in the direction of the plus or minus spacecraft velocity vector.

Table 1. x_1 for Various Cases

V_{ej} (ft/sec)	+ OR -	V_{CABLE} (km/sec)	τ_{CABLE} (min)	$\tau_{CABLE} - \tau_{S/C}$ (sec)	PER-ORBIT SEPARATION (km)	x_1
20	+	7.406556023	103.2440093	15.31	113.3	403.667
20	-	7.394364023	102.7349948	15.23	112.7	404.662
8	+	7.402898423	103.0907762	6.11	45.2	1010.655
8	-	7.398021623	102.8871713	6.10	45.1	1011.655

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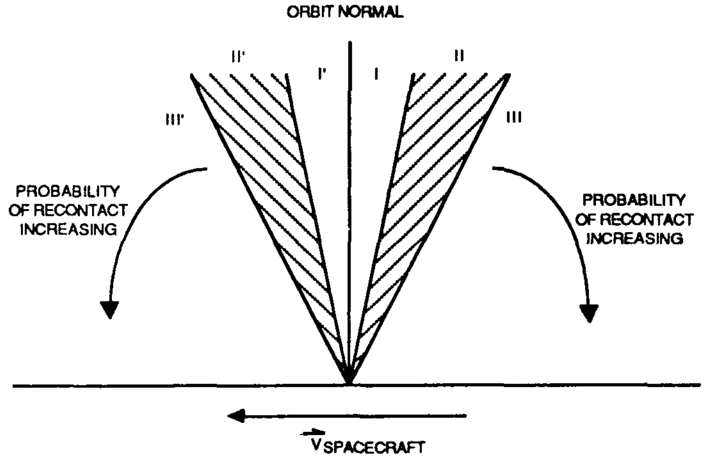
The first close encounter will occur at the meeting point when $x_1 = 404$, if $V_{ej} = 19.983$ ft/sec. Recontract takes place about 29 days after ejection. All subsequent close encounters will also occur at the meeting point. For $x_1 = 405$, $V_{ej} = 19.934$ ft/sec. Clearly, numerous close encounters occur between 8 and 20 ft/sec.

4.1.2 WINDOW OF NO RECONTACT

Recontact is possible for either ejection along the orbit normal or for a tangential ejection. An intermediate region of ejection directions exists for which no recontact is possible within a year.

Approximately 5100 spacecraft orbits occur per year. The circumference of the orbit is 45,730 km. A per-orbit separation of greater than 20 m or less than 8.95 km will guarantee that no recontact will occur in the first year. For $V_{ej} = 8$ ft/sec, a separation of 20 m occurs in half an orbit for $\Theta = 89.96$ deg or $\Theta = 90.11$ deg. A per-orbit separation of 8.95 km occurs for tangential ejection at 1.59 ft/sec. For $V_{ej} = 20$ ft/sec, the 8.95-km per-orbit separation therefore occurs for $\Theta = 94.52$ deg or $\Theta = 85.43$ deg.

These limits on Θ define regions II and II' in Figure 3. Hence, if Θ is in region II or II', no recontact is possible within a year because no close encounters can occur. Unfortunately, Θ cannot be placed in regions II and II' because the uncertainty in the angle of ejection then extends to the orbit normal direction. In regions I and I', recontact occurs in half an orbit. In regions III and III', at least one close encounter occurs in the year. The number of possible close encounters in a year increases as Θ approaches 0 or 180 deg.



	REGION	ANGULAR WIDTH (DEG)	PROBABILITY OF RECONTACT
RECONTACT IN 1 ORBIT	I I'	0.04 0.11	100
NO RECONTACT	II II'	4.53 4.41	0
RECONTACT POSSIBLE	III III'	85.43 85.48	NONZERO

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Figure 3. Regions of Recontact; No Drag or Nodal Precession

4.1.3 TANGENTIAL EJECTION, NO DRAG AND OBLATENESS

Consider ejection along the negative spacecraft velocity vector. Defining recontact to be a close approach of 20 m and assuming it occurs in about 404 spacecraft revolutions (V_{ej} = 20 ft/sec), the approximate positions of the spacecraft and cable for which a minimum separation of 20 m occurs (i.e., r = 7278.12) are shown in Figure 4 and determined by the angle γ. For the cable orbit, a = 7266.15458722, e = 0.0016495, and c = ae = 11.9855. At point p, r = 7278.12 and γ is 3.3009 deg. This is $\pm$0.0091692 revolution. In 1011 spacecraft revolutions (V_{ej} = 8 ft/sec), γ is 5.225 deg; in 2023 revolutions (V_{ej} = 4 ft/sec), γ is 7.394 deg.

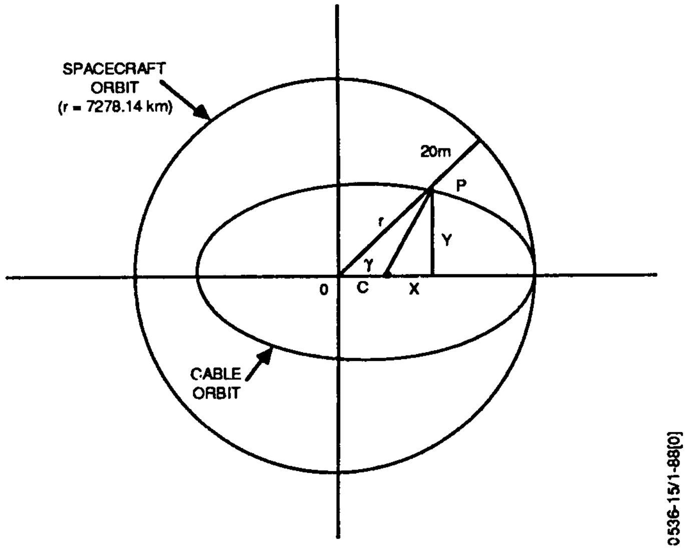

Figure 4. Cable and Spacecraft Orbits

Selected values relating x_1, which is here the number of spacecraft orbits until the first close encounter, to the ejection velocity are shown in Table 2. Some variations in ejection speed that still result in recontact (as given by γ) can be

Table 2. V_{ej}* for Various Values of x_1

x_1	V_{ej}
403	20.0824947
404	20.0327856
404.6622675	20
404.9908308	19.9837743
405	19.9833218
405.0091692	19.9828694
406	19.9341018
1011	8.0051852
1011.6552847	8
1012	7.9972750

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* V_{ej} = 8 AND 20 ft/sec, APPROXIMATELY.

seen in this table. They correspond to x_1 = integer $\pm$0.0091692 (V_{ej} = 20 ft/sec). A few values of V_{ej} are shown graphically in Figure 5. In the darkened intervals, recontact (i.e., a separation of 20 m or less) occurs.

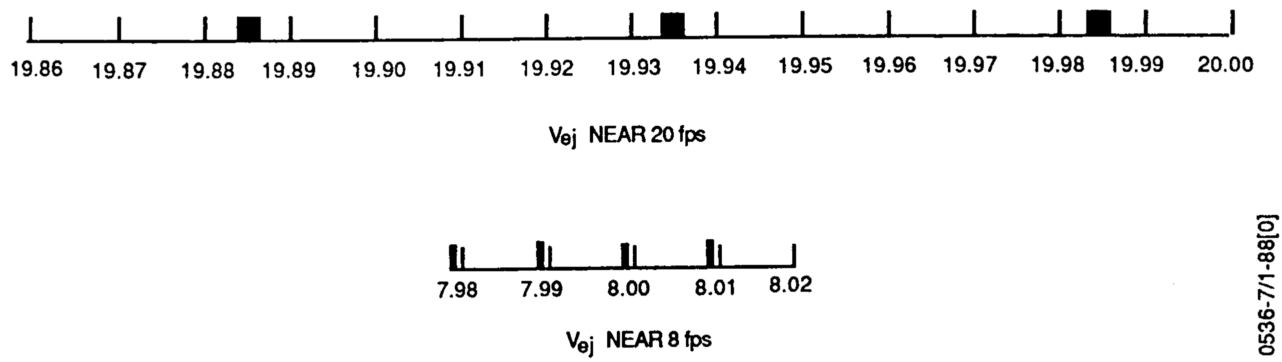

Figure 5. Some Values of V_{ej} Resulting in Recontact in 1 Year

The number of intervals is 1012 - 405 = 607. The width of the interval for x_1 = 405 is $\pm$0.0004524, and for x_1 = 1012 it is $\pm$0.000115. The average width is (0.0009048 + 0.00023)/2 = 0.00057. The separation between intervals is 0.049 and 0.008, respectively. The width of the total domain is (20 - 8) = 12 ft/sec. Therefore, the probability (P) of recontact on the first close encounter is

$$P = \frac{(607)(0.00057)}{12} = 2.88\%$$

This remarkably large probability is obtained because the spacecraft and cable orbits are coplanar upon recontact.

The probability of recontact for any close encounter in the absence of drag will now be calculated. In 1 year, as many as 12 close encounters can occur. The various values for the fractional part of x_1 giving recontact on a specified close encounter are shown in Table 3. The distribution of values is approximately uniform, except for a gap from 0 to 1/12. Referring to Table 3, a close encounter can occur in 10 ways at the meeting point when there are 5 close encounters in the year ($V_{ej} \approx$ 8 ft/sec and $x_1 \approx$ 1011), and a close encounter can occur in 46 ways when there are 12 close encounters in the year ($V_{ej} \approx$ 20 ft/sec and $x_1 \approx$ 405).

Table 3. Fractional Part of x_1 Giving Recontact on Various Close Encounters

CLOSE ENCOUNTER NUMBER											
1	2	3	4	5	6	7	8	9	10	11	12
0	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
		2/3	3/4	2/5	5/6	2/7	3/8	2/9	3/10	2/11	5/12
				3/5		3/7	5/8	4/9	7/10	3/11	7/12
				4/5		4/7	7/8	5/9	9/10	4/11	11/12
						5/7		7/9		5/11	
						6/7		8/9		6/11	
										7/11	
										8/11	
										9/11	
										10/11	

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The probability of recontact (P) in the vicinity of any value of x_1 is now

$$P = \frac{\text{(number of ways) x (width of recontact interval)}}{\Delta V}$$

where ΔV is the difference in ejection velocity between possible recontacts on successive orbits.

For $x_1 = 405$ ($V_{ej} \approx 20$ ft/sec), separation within 20 m occurs when the width of the recontact interval is 0.0009 (as shown above) and $\Delta V = 0.049$. Therefore,

$$P(V_{ej} \approx 20) = 46(0.0009)/0.049 = 84\%$$

For x_1 = 1012 ($V_{ej} \approx$ 8 ft/sec), separation within 20 m occurs when the width of the recontact interval is 0.00015 and ΔV = 0.008. Therefore,

$$P(V_{ej} \approx 8) = 10(0.00023)/0.008 = 29\%$$

The average value for x_1 = 405 to 1012 is 56 percent. The possibility of more than one recontact in a year is ignored. It can also be shown that

$$P(V_{ej} \approx 4 \text{ ft/sec}) = 4\%$$

4.2 NUMERICAL COMPUTATIONS

The preceding results apply to the case without drag and oblateness. When these effects are considered for ejection at the ascending node, the circumstances of the problem change. The knowledge gained in the idealized problem will be used to obtain an estimate of the probability of recontact in the actual problem.

To demonstrate the possibility of recontact in the presence of drag, oblateness, and solar/lunar perturbations, the case was considered in which drag on the cable is very much larger than that on COBE, so that recontact could be obtained easily. A tangential ejection of about 20 ft/sec in the positive velocity direction was assumed. The cross-track, radial, and along-track separations needed to be minimized simultaneously. Variation of the speed of ejection and coefficient of drag enabled this to be done, albeit tediously.

A value of the area that gave near-zero values of the cross-track and radial separations at the time of the closest approach of the cable and spacecraft was found iteratively to be 4.2 x 10^{-7} km^2. Drag on the cable was then 60 times larger than on the spacecraft. The best value of the ejection velocity giving the minimum separation for this area was then found iteratively.

4.2.1 RECONTACT WITH EXTERNAL PERTURBATIONS

The minimum separation was found to decrease linearly with decreasing ejection speed until a minimum value of 0.294 km was obtained at 19.79433 ft/sec (Figure 6). The minimum occurred as expected at the southernmost point. Figure 7 shows the

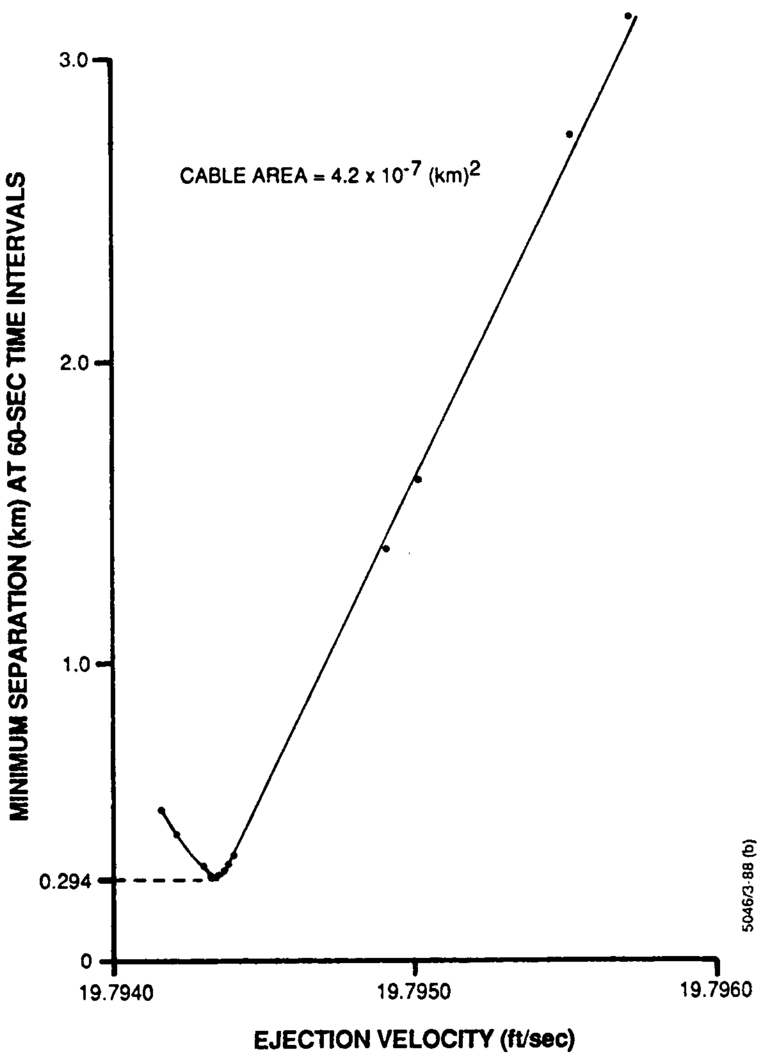

Figure 6. COBE/Cable Separation: Variation of Minimum Separation With Cable Ejection Velocity

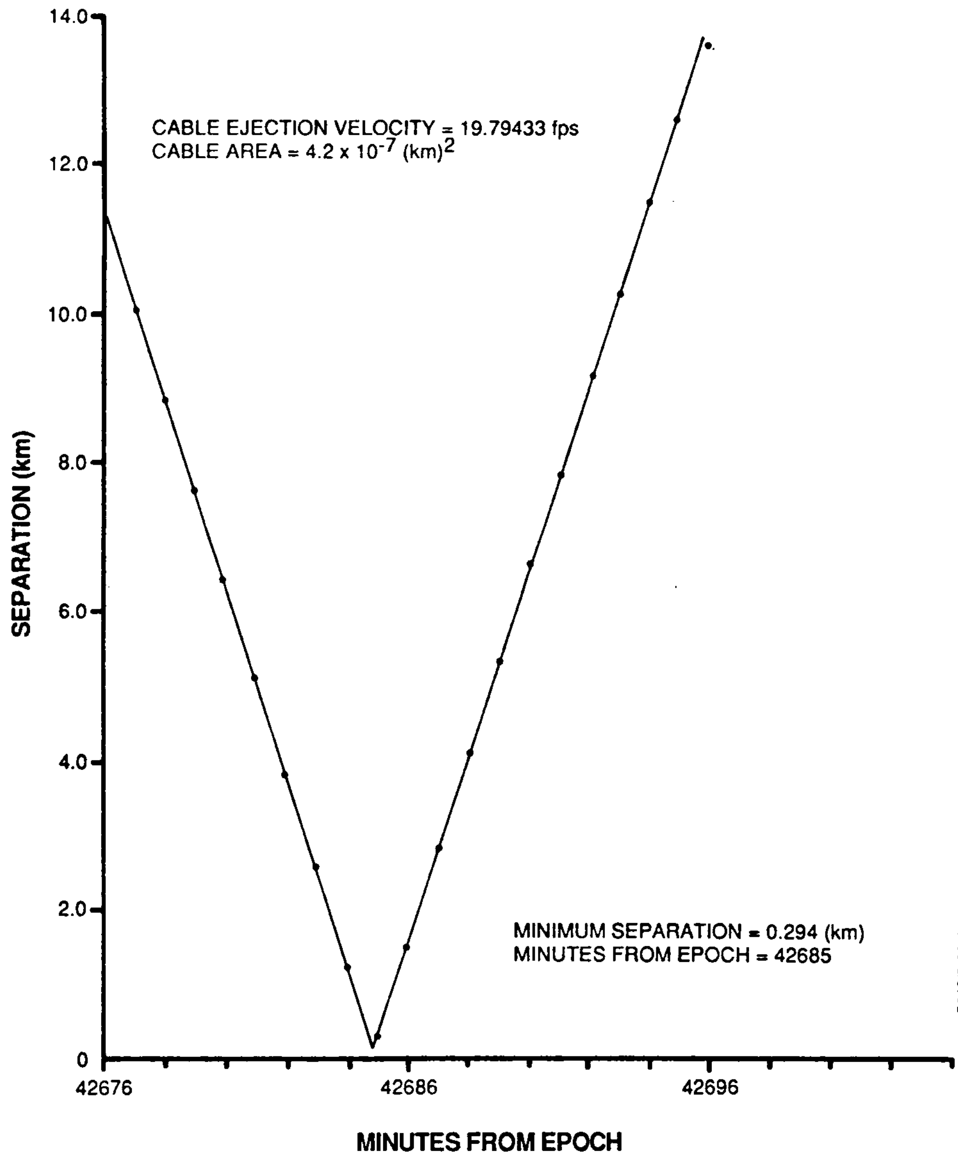

Figure 7. COBE/Cable Separation: Variation of Separation With Time for Case of Closest Approach With Realistic Drag and Geopotential Models

change in the separation with time when the ejection speed is 19.79433 ft/sec and the cable area is 4.2×10^{-7} km^2. For an ejection speed of 19.79435 ft/sec, the variation of the minimum separation with area is shown in Figure 8.

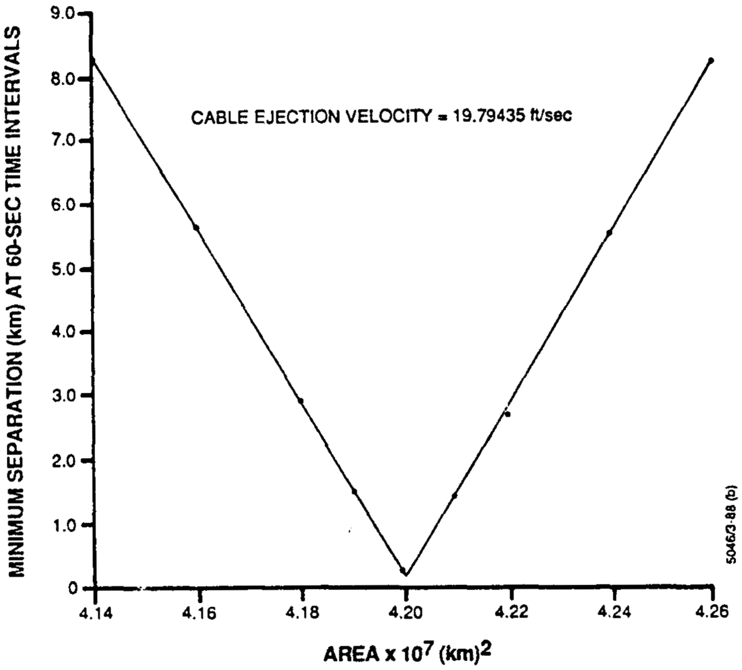

Figure 8. COBE/Cable Separation: Variation of Minimum Separation With Area of Cable

4.2.2 ESTIMATE OF PROBABILITY FOR RECONTACT WITHIN 1 YEAR WITH ALL EXTERNAL PERTURBATIONS - TANGENTIAL EJECTION

The minimum separation criterion for the numerical simulation of a recontact was 1/3 km. A minimum separation distance of 0.294 km was obtained for an ejection along the spacecraft velocity vector with

$$V_{ej} = 19.79433 \text{ ft/sec}$$

$$\text{area} = 4.2 \times 10^{-7} \text{ km}^2$$

A smaller minimum separation could be obtained for a smaller integration step. The recontact occurred about 30 days after epoch, when perigee was at the southernmost point. The variations in ejection speed and area separately that produce an increase of 1/3 km in the minimum separation distance are

$$\Delta V_{ej} = \pm\ 0.00027 \text{ ft/sec}$$

$$\Delta \text{area} = \pm\ 0.00273 \times 10^{-7} \text{ km}^2$$

These values are interpolated from the data (not shown) used to generate Figures 6 and 8.

The variation in ejection speed that produces an increase of 20 m in the along-track separation is given by interpolation (data not shown):

$$\Delta V_{ej} = \pm\ 0.0000085 \text{ ft/sec}$$

Using this value for ΔV_{ej}, the probability of recontact is obtained by dividing by 0.049 ft/sec (the velocity difference for recontact on orbit numbers 404 and 405). The probability is 0.035 percent. An analytical result will be given in Section 4.3.2 for comparison.

4.3 CALCULATING THE PROBABILITY OF RECONTACT

4.3.1 TANGENTIAL EJECTION

The angle between the orbit planes upon recontact is 0.165 deg (see below). The minimum separation occurs very near the meeting point at this angle. Using the equations in Section 4.1.1, an along-track separation of 20 m for recontact on orbit 404 can be shown to result from the value of

$$\Delta V_{ej} = \pm\ 0.000008761 \text{ ft/sec}$$

This is in agreement with the numerical result. The probability of recontact is 0.036 percent.

4.3.2 OPTIMUM DIRECTION OF EJECTION

Tangential ejection was considered above. It remains to consider a nontangential ejection. The maximum difference in inclination of the cable orbit from the spacecraft orbit is 0.044 deg, which occurs for a 20-ft/sec ejection at ±70 deg. This produces a 5.6-km displacement of the northernmost points of the orbits, or the southernmost points. Although recontact can no longer occur exactly at the northernmost or southernmost points, the displacement is small and recontact remains possible at some nearby point. The dihedral angle between the orbit planes is therefore largely composed of the difference in the right ascensions of the ascending nodes. It will be assumed that the difference in inclination is negligible and that the dihedral angle between the orbit planes is equal to the difference in node between the two orbits.

An ejection in the negative spacecraft velocity direction will be considered since the area-to-mass ratio of the cable is nominally greater than that of the spacecraft. In Table 4, the dihedral angle is calculated for ejection angles between -70 and 70 deg from the negative spacecraft velocity vector. For these limits, the ejection is at least 20 deg away from the orbit normal. The probability of recontact for a 20-m minimum separation criterion is calculated only for angles near 47.9 deg. This is a special case. The calculation uses the dihedral angle to determine the angle γ required for a 20-m cross-track separation. Minimum separation occurs away from the meeting point. Below 44.4 deg and above 51.0 deg, minimum separation occurs in the vicinity of the meeting point. The probability in this case is calculated below. A dihedral angle of 0.164 deg was derived from the numerical results for a 180-deg angle of ejection. This agrees with the result of 0 deg shown in Table 4.

A positive dihedral angle indicates that the cable orbit leads the spacecraft orbit in the nodal precession, while a negative dihedral angle indicates that the cable orbit lags the spacecraft orbit. There is a singular point at 47.9 deg for which the probability is relatively high. At this angle, the rates of precession of the spacecraft and cable orbits are equal and the orbits are coplanar upon recontact. Calculation of the probability is analogous to that in Section 4.1.3. If the small difference in inclination is not neglected, the orbits are never perfectly coplanar. The location of a possible recontact is no longer at the northernmost or southernmost points, but the probability remains high, though not as high as for the coplanar case. The angle of 47.9 deg should be avoided.

When the angle of ejection is between 44.4 and 51.0 deg, the magnitude of the dihedral angle is less than or equal to 0.019 deg. The minimum separation is then the cross-track separation, which may be several kilometers away from the meeting point. However, when the magnitude of the dihedral angle is less than or equal to 0.0027 deg, the orbits are virtually coplanar. The minimum separation becomes the radial separation, which is greater than the cross-track separation. The probability of recontact then becomes large, as shown in Section 4.1.3. Above 51.0 deg, the uncertainty of ±20 deg in the angle of ejection will make avoidance of both the orbit normal and the angle of 47.9 deg almost impossible to guarantee.

Table 4. Dihedral Angle Between Orbit Planes for Various Angles of Ejection

ANGLE OF EJECTION (deg)	DIHEDRAL ANGLE (deg)	PROBABILITY OF RECONTACT (%)
70	-0.243	–
60	-0.093	–
51.0	-0.019	0.264
47.9	-0.0001	6.140*
44.4	0.019	0.264
35	0.060	–
15	0.124	–
0	0.165	–
-15	0.204	–
-35	0.268	–
-60	0.422	–
-70	0.571	–

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* AVERAGED OVER EJECTION SPEEDS OF 8 TO 20 ft/sec. A FACTOR OF 2 IS INCLUDED BECAUSE DRAG CREATES A SECOND MEETING POINT. THE CALCULATION IS ANALOGOUS TO THAT IN SECTION 4.1.3 FOR COPLANAR ORBITS.

The probability of recontact will now be determined for ejection angles outside the above range (Table 5). The case of a tangential ejection was discussed above. The calculation required finding the magnitude of ΔV_{ej} that produced a 20-m along-track separation at the meeting point. Minimum separation does not occur exactly at the meeting point. For ejection angles between -70 and 20 deg, the dihedral angle is greater than 0.1 deg, and the minimum separation occurs sufficiently close to the meeting point to make a negligible difference in this calculation. The

Table 5. Probability of Recontact in 1 Year for Various Ejection Angles for a 20-Meter Minimum Separation Criterion

Θ (deg)	V_{ej} (ft/sec)	V_{ej} TANGENTIAL COMPONENT (ft/sec)	X_i	P (%)	NUMBER OF CLOSE ENCOUNTERS	P_1 YEAR	P_{av} (%)	FRACTION OF TIME*	P_{final} (%)
+20	20	18.79	431	0.038	42	1.596	1.080	0.05	0.054
+20	8	7.52	1076	0.094	6	0.564			
0	20	20.00	404	0.036	46	1.656	1.273	0.05	0.064
0	8	8.00	1012	0.089	10	0.890			
-35	20	16.38	494	0.043	32	1.376	1.012	0.06	0.060
-35	8	6.55	1235	0.108	6	0.648			
-70	20	6.84	1183	0.104	6	0.624	0.442	0.07	0.031
-70	8	2.74	2958	0.259	1	0.259			

* REFERS TO THE FRACTION OF TIME THE ORBITS ARE SEPARATED BY 20 METERS OR LESS AT THE NORTHERNMOST OR SOUTHERNMOST POINTS. THE VALUE IS $2\gamma/\pi$.

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change in ejection speed needed to shift the recontact by one revolution is also required. This is divided into ΔV_{ej} to obtain the probability of recontact on any close encounter (P). Ejection speeds of 20 ft/sec and 8 ft/sec are considered. The number of close encounters possible is enumerated by reference to Table 3, and the probability for the year is determined by multiplication. A simple average is then calculated. To obtain an estimate of the probability of recontact (P_{final}) from this, a factor is needed that takes into account the orientation of the cable orbit with respect to the northernmost or southernmost points. The value of this factor is available from calculating the angle γ (Figure 4). The fraction of time that the relevant apsis spends within a domain of $\pm\gamma$ from the northernmost or the southernmost point is then $2\gamma/\pi$. This fraction is typically less than 0.1. The probability of recontact can be seen to be a decreasing function of $|\Theta|$. Notice for $\Theta = 0$ deg, $V_{ej} = 20$ ft/sec, that P = 0.036 percent, in agreement with the numerical result.

A minimum separation criterion of 20 m was used. For a more realistic criterion of 10 m, P is half as large, but γ is also smaller. For example, the probability of recontact within 10 m for a tangential ejection is 0.022 percent.

5. CONCLUSIONS

Recontact of the COBE thermal shield cable is inevitable for ejection in the positive or negative orbit normal directions. A relatively high probability of recontact exists for ejection at or near 47.9 deg, because the cable and spacecraft orbits would be nearly coplanar upon recontact. It is then possible for recontact to occur at a node. Ejection in the vicinity of the above directions should be avoided.

Recontact in the presence of drag and oblateness is possible only at the northernmost or southernmost points of the orbits, provided that ejection does not take place along the orbit normal or near 47.9 deg. The probability depends upon the dihedral angle subtended between the orbit planes upon recontact, for angles of ejection between 44.4 and 51.0 deg. For all other angles, the probability depends upon the along-track separation at the meeting point for dispersions about a recontact velocity.

A numerical simulation provided an example of a near recontact for a tangential ejection. An analytical determination of the probability of recontact agreed with the numerical results.

The optimum direction of ejection is the one that minimizes the number of close encounters within a year, maximizes the dihedral angle between orbit planes upon recontact (i.e., avoids 47.9 deg), but guarantees that ejection will not take place along the orbit normal. For an uncertainty of $\pm$20 deg in ejection angle, the recommended angle is therefore -69.96 deg. The direction of this ejection is such as to increase the inclination. The probability of having a separation of 20 m or less at this value was calculated to be 0.031 percent. The probability of recontact for a tangential ejection was found to be 0.064 percent. These values are overestimates.

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