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AXIONS, SN 1987A, and ONE PION EXCHANGE

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Abstract

Nucleon-nucleon, axion bremsstrahlung is the primary mechanism for axion emission from the nascent neutron star associated with SN 1987A, and the rate for this process has been calculated in the one pion exchange approximation (OPE). The axion mass limit which follows from SN 1987A, $m_a \lesssim 10^{-3}$eV, is the most stringent astrophysical bound, and has received much scrutiny. It has been suggested that by using OPE to calculate the cross section for the analog process, $pp \rightarrow pp + \pi^o$, and comparing the result to experimental data one can test the validity of this approximation, and further, that such a comparison indicates that OPE leads to a value for this cross section which is a factor of 30-40 too large. *If* true, this would suggest that the axion mass limit should be revised *upward* by a factor of ~ 6. We have carefully evaluated the cross section for $pp \rightarrow pp + \pi^o$ using OPE and find excellent agreement (to better than a factor of 2) with the experimental data.

I. Introduction

There has been a great deal of interest in axion emission from SN 1987A.[1-5] And indeed, consideration of the effect of axion emission on the neutrino burst observed by the KII[6] and IMB[7] detectors seems to provide strong evidence against the existence of an axion with mass in the range $10^{-3} - 2$ eV; for the DFS type axion this improved the existing astrophysical bound by a factor of ~ 10, while for the hadronic type axion, the improvement was more than a factor of 10^3.[8] If the axion exists, then the dominant emission process from SN 1987A should have been nucleon-nucleon, axion bremsstrahlung. The matrix element squared for this process has been computed in the OPE approximation;[2] the 4 direct and 4 exchange diagrams are shown in Fig. 1. Given that the pion-nucleon coupling is of order unity one might question the accuracy of such an approximation—of course, one should remember that the pion-nucleon coupling is derived by a comparison between an OPE treatment of pion-nucleon scattering and experimental data.[9] In addition, since the densities that existed at the core of the nascent neutron star associated with SN 1987A shortly after collapse were $\sim 2\rho_{\rm nuclear} \sim 8 \times 10^{14}$ g cm^{-3}, one should also worry about collective nuclear effects. Here, we will restrict our discussion to the validity of the OPE approximation itself.

Choi, et al[5] recently suggested a clever way of checking the accuracy of the OPE approximation in computing nucleon-nucleon, axion bremsstrahlung. Their idea is to compute the cross section for the analog process, nucleon-nucleon, pion bremsstrahlung using OPE and to compare to the body of existing experimental data. Both the axion and the pion are pseudo-Nambu-Goldstone bosons, and as such couple derivatively:

$$\mathcal{L}_{\rm int} = \ldots + i\frac{g_{aN}}{2m}\partial_\mu a \bar{N}\gamma^\mu\gamma^5 N + i\frac{\lambda}{2m}\partial_\mu \pi^o \bar{N}\gamma^\mu\gamma_5 N + \ldots \tag{1}$$

where N is the nucleon field, π^o is the neutral pion field, a is the axion field, $m \simeq 0.94$ GeV is the nucleon mass, $g_{aN} \sim m/(f_a/N_a)$ is the axion-nucleon coupling, and in the OPE approximation the pion-nucleon coupling $\lambda \simeq 2m/m_\pi \simeq m/f_\pi$, where $m_\pi \simeq 135$ MeV is the neutral pion mass and $f_\pi \simeq 95$ MeV is the pion decay constant. We should mention that the derivative coupling of the pion to the nucleon follows directly from the chiral Lagrangian, a low-energy, effective Lagrangian which describes the interactions of pions and nucleons. The similarity of the pion-nucleon and axion-nucleon couplings, as well as the subtle issue of when the pseudo-vector coupling can be expressed as a pseudo-scalar coupling, are addressed in Ref. 5. Here, we will use the unambiguously correct pseudo-vector coupling for both the axion and pion.

The axion mass, m_a, and PQ symmetry breaking scale, f_a, are related by

$$m_a = 0.62\,\mathrm{eV}\,[10^7\,\mathrm{GeV}/(f_a/N_a)]$$

where N_a is the color anomaly of the PQ symmetry. For details about the axion and its couplings to matter, see Refs. 10. Because of the similarity of the pion and axion couplings we see that by substituting $\lambda \to g_{aN}$ and $m_a \to m_\pi$, the matrix element for $NN \to NN + \pi^o$ can be obtained from that for $NN \to NN + a$.

The matrix element squared for nucleon-nucleon, axion bremsstrahlung has been calculated in Ref. 2; for the process $pp \to pp + a$ it is

$$\sum_{SPIN} |\mathcal{M}|^2_{\text{axion}} = \frac{256}{3} \frac{g_{ap}^2 m^2}{m_\pi^4}(3 - \beta) \tag{2}$$

where $|\mathcal{M}|^2_{\text{axion}}$ has been summed over *both* initial and final proton spins and averaged over the directions of the axion and the 3-momentum exchanged. The quantity β is related to the average of the cosine squared of the angle between the direction of the momentum transfer in the direct and exchange diagrams: for degenerate matter $\beta = 0$; and for non-degenerate matter $\beta \simeq 1$ (see Ref. 2). For our purposes here we take $\beta \simeq 1$. In addition, several approximations were made in calculating $|\mathcal{M}|^2_{\text{axion}}$: the nucleons were assumed to be non-relativistic, the axion mass was taken to be zero, and $3mT$ was assumed to be much larger than m_π^2 (here T = temperature) so that the pion mass in the pion propagator can be neglected—all good approximations in the core of the neutron star associated with SN 1987A. However, we must keep them in mind to understand the realm of validity for using $|\mathcal{M}|^2_{\text{axion}}$ to compute the corresponding matrix element squared for $pp \to pp + \pi^o$.

II. Naive OPE

Once $|\mathcal{M}|^2_\pi$ is at hand it is straightforward to obtain the cross section for $pp \to pp + \pi^o$ (Ref. 11):

$$d\sigma = \frac{1}{4} S \sum_{SPIN} |\mathcal{M}|^2_\pi (2\zeta_{12})^{-1} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 - a) d\Pi_3 d\Pi_4 d\Pi_a \tag{3}$$

where 1, 2 denote the incoming protons, 3, 4 denote the outgoing protons, a denotes the pion, $d\Pi_i = d^3p_i/2E_i(2\pi)^3$, $S = 1/2$ is the usual symmetry factor for identical particles in the final state, the factor of 1/4 is inserted to average over initial proton spins, and the kinematical factor (Moeller flux factor)

$$\zeta_{12} = s^{1/2}(s - 4m^2)^{1/2} = s(1 - 4m^2/s)^{1/2}$$

The quantity s is the center-of-mass (CM) energy squared, which is related to the KE of the incoming proton in the lab ($\equiv T_L$) by

$$s = 4m^2 + 2mT_L$$

The momenta of the incoming protons in the CM frame is

$$|\vec{p}_1|^2 = |\vec{p}_2|^2 = mT_L/2$$

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The threshold for pion production is

$$T_L|_{\textbf{threshold}} = 2m_\pi + m_\pi^2/2m = 280\,\mathrm{MeV} \simeq 2m_\pi$$

Because the axion mass was taken to be zero in computing $|\mathcal{M}|^2_{\mathrm{axion}}$ it is not possible to obtain $|\mathcal{M}|^2_\pi$ by the substitution: $g_{ap} \to \lambda$. However, one can, as a first, naive approximation, do so. Of course, by so doing one is implicitly assuming that the pion is ultra-relativistic, i.e., $T_L \gg T_L|_{\textbf{threshold}}$, so that the approximation $m_\pi = 0$ is a good one. In the next Section we will compute $|\mathcal{M}|^2_\pi$ without recourse to this assumption; for the moment we will use this naive approximation to compute $\sigma(pp \to pp + \pi^o)$. From the last two formulae we see that for the approximations made in obtaining $|\mathcal{M}|^2_\pi$ from $|\mathcal{M}|^2_{\mathrm{axion}}$ to be valid (relativistic pion, non-relativistic nucleons) T_L must satisfy:

$$2m_\pi \ll T_L \ll 2m \quad \text{or} \quad 0.3\,\mathrm{GeV} \ll T_L \ll 2.0\,\mathrm{GeV}$$

To actually compute the cross section it is most convenient to use the Dalitz representation for the 3-body phase space factor:[11]

$$(2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4 - a)d\Pi_3 d\Pi_4 d\Pi_a = (16s)^{-1}(2\pi)^{-3}dm_{34}^2 dm_{3a}^2 \tag{4}$$

where we have taken advantage of the fact that the matrix element squared is constant to perform most of the integrals, and the invariant mass variables are

$$m_{34}^2 = (p_3 + p_4)^2 \qquad m_{3a}^2 = (p_3 + a)^2$$

It is then straightforward to obtain $\sigma(pp \to pp + \pi^o)$:

$$\sigma(\mathrm{pp} \to \mathrm{pp} + \pi^o) = \frac{1}{2048\pi^3} \sum_{SPIN} |\mathcal{M}|^2_\pi \frac{I}{s^2(1 - 4m^2/s)^{1/2}} \tag{5a}$$

$$I = \int_{4m^2}^{(\sqrt{s}-m_\pi)^2} (x - 4m^2)^{1/2}[(s - x - m_\pi^2)^2 - 4xm_\pi^2]^{1/2} \frac{dx}{\sqrt{x}} \tag{5b}$$

where $x = m_{34}^2$. Using the matrix element squared which is obtained from $|\mathcal{M}|^2_{\mathrm{axion}}$ by substituting $g_{ap} \to m/f_\pi$ we find

$$\sigma = 2.46 \times 10^{-25}\,\mathrm{cm}^2 \, \frac{I}{s^2(1 - 4m^2/s)^{1/2}} \tag{6}$$

Note, that although approximations have been made in evaluating $|\mathcal{M}|^2_\pi$, the phase space integrals have been evaluated exactly.

In the limit that the 2 outgoing protons and pion are non-relativistic the integral I can be evaluated in closed form (see Byckling and Kajantie[11]),

$$I = \frac{\pi}{\sqrt{2}} \frac{(m_\pi/m)^{1/2}}{(1 + m_\pi/2m)^{3/2}} s^2(1 - 2m/\sqrt{s} - m_\pi/\sqrt{s})^2 \tag{7}$$

While the assumption that the outgoing protons are non-relativistic is probably reasonable, the assumption that the outgoing pion is non-relativistic is not a good one; moreover, it is contrary to the approximation made in computing $|\mathcal{M}|^2_\pi$. However, over the lab energy range, $0.3\,\mathrm{GeV} \leq T_L \leq 1.0\,\mathrm{GeV}$, this approximate formula for I is suprisingly accurate, to about 5%. Using this expression for I, we obtain

$$\sigma = \frac{1}{2048\sqrt{2}\pi^2} \frac{(m_\pi/m)^{1/2}}{(1+m_\pi/2m)^{3/2}} \sum_{\mathrm{SPIN}} |\mathcal{M}|^2_\pi \frac{(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2}{(1-4m^2/s)^{1/2}} \tag{8a}$$

$$\simeq 1.86 \times 10^{-25}\,\mathrm{cm}^2 \frac{(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2}{(1-4m^2/s)^{1/2}} \tag{8b}$$

a form similar to that of Choi, et al[5].

There exists a wealth of experimental data for the process $pp \to pp + \pi^o$ at energies $T_L \gtrsim 300$ MeV. The data is summarized in Refs. 12 and 13, and a convenient analytical fit is given in Ref. 12. Both the experimental data and the analytical fit are shown in Fig. 2. The analytical fit is given by the following expression:

$$\sigma = \frac{\pi}{2|\vec{p}_a|^2}\alpha(p_r/p_0)^\beta \frac{m_0^2\Gamma^2(q/q_0)^3}{(<M>^2-m_0^2)^2+m_0^2\Gamma^2} \tag{9}$$

where

$$p_r^2(s) = \frac{[s-(m-<M>)^2][s-(m+<M>)^2]}{4s}$$

$$q^2(<M>^2) = \frac{[<M>^2-(m-m_\pi)^2][<M>^2-(m+m_\pi)^2]}{4<M>^2}$$

$$q_0^2 = q^2(m_0^2)$$

$$p_0^2 = (m+m_0)^2/4 - m^2$$

$$<M> = M_0 + (\tan^{-1} Z_+ - \tan^{-1} Z_-)^{-1}\frac{\Gamma_0}{4}\ln\frac{1+Z_+^2}{1+Z_-^2}$$

$$Z_+ = 2(\sqrt{s}-m-M_0)/\Gamma_0 \qquad Z_- = 2(m+m_\pi-M_0)/\Gamma_0$$

$$\alpha = 3.772 \qquad \beta = 1.262$$

$$M_0 = 1.22\,\mathrm{GeV} \qquad m_0 = 1.188\,\mathrm{GeV}$$

$$\Gamma_0 = 0.12\,\mathrm{GeV} \qquad \Gamma = 0.099\,\mathrm{GeV}$$

The expression we obtained for the cross section by substituting $g_{ap} \to \lambda$, Eq.(6), along with the experimental data is shown in Fig. 3. Recall that the range of validity for our calculation above was: $0.3\,\mathrm{GeV} \ll T_L \ll 2\,\mathrm{GeV}$. In that region the experimental data agree very well with our calculation, to better than a factor of 2. As one moves outside this range our calculation begins to overestimate the cross section significantly. At small

T_L our calculation is not valid as the pion is not highly-relativistic: Because the pion couples derivatively $|\mathcal{M}|^2_\pi$ should vanish at threshold, and thus our approximation to $|\mathcal{M}|^2_\pi$, which is independent of T_L, should *overestimate* $|\mathcal{M}|^2_\pi$. (As we shall see shortly, $|\mathcal{M}|^2_\pi$ does not actually vanish at threshold, rather is suppressed by a factor of m_π/m.) Likewise, at large T_L, where the nucleons are becoming relativistic, we would expect our approximation to overestimate σ because the momentum dependence of nucleon propagators has been neglected. Thus, the deviation of the OPE treatment of the cross section from the experimental data, at both small and large T_L, is as expected.

In order to estimate the effect of pion production threshold on the OPE calculation of $\sigma(pp \to pp + \pi^0)$ we have calculated $|\mathcal{M}|^2_\pi$ at threshold ($T_L = 2m_\pi$) with $m_a = m_\pi \neq 0$. (Of course, the threshold dependence of phase space has already been properly taken into account.) We find

$$\sum_{\text{SPIN}} |\mathcal{M}|^2_\pi = 64\lambda^2 \frac{m}{m_\pi^3},$$

or a factor of $3m_\pi/8m \sim 1/20$ smaller than the value obtained taking $m_a = 0$.[14] Moreover, slightly above threshold ($T_L \gtrsim 300$ MeV), we find that the square of any given diagram is modified from the $m_a = 0$ result by a factor

$$1 - m^2 m_\pi^2/(p_3 \cdot a)^2 \propto |\vec{a}_{CM}|^2 \propto (T_L/2m_\pi - 1) \tag{10}$$

The effect of correcting the naive OPE cross section by this approximate threshold factor for the matrix element is also shown in Fig. 3: The agreement between the OPE result and the experimental data improves dramatically for small values of T_L. Of course, to properly take into account the fact that $m_\pi \neq 0$ one should compute the complete matrix element squared with $m_\pi \neq 0$ which we will do in the next section. But first let us compare to the results of Choi, et al[5].

Choi, et al[5] basically carried out the same calculation as we did to obtain the 'naive OPE' prediction: that is, they used the matrix element squared obtained from axion bremsstrahlung and substituted $g_{ap} \to \lambda$. They then compared this result to similar experimental data at energies $T_L = 400$, 500 MeV (although there seems to be a slight discrepancy between the data they use at 400 MeV, and that in Refs. 12 and 13). They found that the OPE approximation overestimated the experimental cross section by factors of 30-40. Our disagreements with the calculation of Choi, et al[5] are manifold. Firstly, the value of $S\sum_{\text{SPIN}} |\mathcal{M}|^2_\pi$ that they used is larger than ours by a factor of $12(f_\pi/m_\pi)^2 \simeq 6.6$. Of this, a factor of $4(f_\pi/m_\pi)^2 \simeq 2.0$ traces to their using $\lambda = 2m/m_\pi$ for the pion nucleon coupling vs. our using $\lambda = m/f_\pi$; the remainder, a factor of 3.3, apparently has to do with spin averaging and the symmetry factor S. With regard to the value of λ; one can make arguments for either choice, and in any case the discrepancy due to this is only a factor of 2. (The issue of the pion nucleon coupling is addressed further in the Appendix.) Secondly, they have apparently used the non-relativistic approximation to evaluate I. However, the numerical factor in their expression for I differs from ours by a factor of

$$\frac{2\sqrt{2}}{\pi}(m/m_\pi)^{1/2}(1+m_\pi/2m)^{3/2} \simeq 2.63$$

Taken together, these factors account for an overall factor of $6.58 \cdot 2.63 \simeq 17.3$, which is the entire discrepancy between our expressions for $\sigma(pp \to pp + \pi^o)$ and their expression. Further, they have only compared to the experimental data at two energies, $T_L = 400$, 500 MeV, energies which are not too far from the threshold for pion production, where the approximation made in using $|\mathcal{M}|^2_{\text{axion}}$ to obtain $|\mathcal{M}|^2_\pi$ (i.e., relativistic pion) is breaking down. As mentioned earlier, near threshold one expects this simple technique to overestimate $\sigma(pp \to pp + \pi^o)$. And as we will see shortly, when the exact matrix element is used the discrepancy at threshold disappears.

III. Exact OPE

We have calculated the full matrix element squared for the process $pp \to pp + \pi^o$ in the OPE approximation, making no other approximations: that is, all particles treated fully relativistically and the mass of the bremsstrahlung pion no longer taken to be zero (the square of the sum of the diagrams in Fig. 1 with the axion replaced by a neutral pion). The resulting expression for $|\mathcal{M}|^2_\pi$ is considerably more complicated than Eq.(2), and is given in the Appendix. The OPE cross section for $pp \to pp + \pi^o$ was then computed by Monte Carlo integration of the matrix element squared over phase space. Three-body phase space is characterized by 4 independent parameters which we take to be: p_3, p_4, the 3-momenta of the outgoing protons in the CM frame; and γ_3, γ_4, the cosine of the angles between the outgoing protons and incoming protons, again in the CM frame. The cross section is then given by:

$$\sigma = \frac{1}{512\sqrt{2}\pi^2}\frac{m^2}{m_\pi^4 f_\pi^2}\frac{(m_\pi/m)^{1/2}}{(1+m_\pi/2m)^{3/2}}\frac{(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2}{(1-4m^2/s)^{1/2}}\frac{\int|\hat{\mathcal{M}}|^2 d\mathbf{P}}{\int d\mathbf{P}} \tag{11}$$

$$d\mathbf{P} = \frac{\sqrt{s}p_3p_4}{E_3E_4E_\pi}\frac{1}{\cos\phi(1-\gamma_3^2)^{1/2}(1-\gamma_4^2)^{1/2}}dp_3dp_4d\gamma_3d\gamma_4$$

$$\int d\mathbf{P} = \frac{\pi^2}{\sqrt{2}}\frac{(m_\pi/m)^{1/2}}{(1+m_\pi/2m)^{3/2}}(1-2m/\sqrt{s}-m_\pi/\sqrt{s})^2$$

$$\sin\phi = \frac{\gamma_3\gamma_4-\gamma_{34}}{(1-\gamma_3^2)^{1/2}(1-\gamma_4^2)^{1/2}}$$

$$\gamma_{34} = \frac{s+2m^2-m_\pi^2+2E_3E_4-2\sqrt{s}(E_3+E_4)}{2p_3p_4}$$

where ϕ is the azimuthal angle of outgoing proton 4 (around the beam axis), γ_{34} is the cosine of the angle between the two outgoing protons, and $\mathcal{M} = (2m^2/f_\pi m_\pi^2)\hat{\mathcal{M}}$. In addition, the total volume of the 3-body phase space ($\equiv \int d\mathbf{P}$) has been approximated by Eq.(7) (which we recall is accurate to about 5%). The full matrix element squared $|\hat{\mathcal{M}}|^2$ is given in the Appendix. While the 4 integration variables are independent, there are complicated kinematical constraints amongst them, i.e., the boundaries of the region of integration are not easily expressed in closed form. We have implemented the constraints in the following way: the 4 variables p_3, p_4, γ_3, γ_4 are picked at random, and then checked to see if they satisfy the constraints; if they do, then the matrix element is evaluated, and the contribution to the integral computed. For each Monte Carlo integration we selected 10^4 sets of the 4 variables, of which about 3×10^3 satisfied the kinematical constraints; thus one expects the integration to have an accuracy of a few percent. Our results are shown in Fig. 4, along with the experimental data.

The agreement between the 'exact' OPE result and the data is quite impressive. The only significant deviation is in the region $T_L \simeq 500 - 900$ MeV where the OPE result underestimates σ by a factor of up to 3. The reason for the underestimation is simple to understand: The threshold for $\Delta(1232)$ production is 630 MeV, and in the region of 500-800 MeV the contribution of the 3,3 resonance ($pp \rightarrow \Delta p \rightarrow pp + \pi^o$) should be important, and is not accounted for in the OPE approximation. (In addition, there is the $N'(1440)$ 1,1 resonance whose production threshold is 1130 MeV.) In fact, with a little imagination one can see the Breit-Wigner shape in the deviation between the OPE results and the data.

As a check, we have compared the exact OPE result above, with the pion mass in the matrix element squared set to zero (but not in the phase space), with the previous naive OPE result, i.e., Eq.(8b). The comparison is shown in Fig. 5. The previous results are reproduced with good accuracy except near threshold. In fact, this can be easily understood: In using Eq.(2) for the matrix element squared we have set $\beta = 1$; exactly at threshold the 3-momentum transfer in the direct and exchange diagrams are equal and $\beta \rightarrow 3$. Had we used $\beta = 3$ in evaluating Eq.(2) the matrix element squared would vanish. Of course, when the matrix element squared is evaluated exactly, the angular average corresponding to β is automatically calculated. Further, as an additional check, we have compared the exact result (with $m_\pi \neq 0$) to the naive result at pion production threshold. As discussed earlier, in the naive approximation the matrix element squared is overestimated by a factor of ~ 20 at threshold, which is what we find.

IV. Discussion/Summary

In the supernova, where the nucleons have thermal distributions characterized by temperatures of order 20-80 MeV, the thermally averaged CM energy is given by

$$< s >=< (p_1 + p_2)^2 >\simeq< 4m^2 + |\vec{p}_1|^2 + |\vec{p}_2|^2 - 2\vec{p}_1 \cdot \vec{p}_2 >\simeq 4m^2 + 6mT$$

so that the average value of s corresponds to lab energies $T_L \simeq 3T \sim 60 - 240$ MeV. Such energies are below the threshold for pion production, and so direct comparison at the relevant energies is not possible. However, one can be very encouraged by the excellent agreement between the OPE approximation and the experimental data for $T_L = 280-1000$ MeV, and the fact that that region is not so far from the energies of interest. Moreover, there is no reason to expect a surprise at the lower energies relevant to SN 1987A. Given the circumstances, the agreement is much better than one might have expected: OPE does not take into account resonances—the threshold for $\Delta(1232)$ production is only $T_L \simeq 630$ MeV. Far more detailed treatments of pion bremsstrahlung exist in the literature[15], in which both the $N'(1440)$ and $\Delta(1232)$ resonances are taken into account; one could improve the calculation of axion bremsstrahlung by similar means. However, the energies in the supernova are well below the threshold for any baryon resonance, and the additional effort seems unjustified. Contrary to the claims of Choi, et al[5] this phenomenological comparison seems to validate the OPE approximation. Since the axion emission rate $\dot{\varepsilon}_a \propto m_a^2$, the axion mass limit derived scales as $\dot{\varepsilon}_a^{-1/2}$—a factor of 2 uncertainty translates into a factor of $\sqrt{2}$ uncertainty in the mass limit. While it appears that there should be little worry about using OPE to compute $\dot{\varepsilon}_a$, one must still worry about collective nuclear effects. Because of the high densities at the core of the supernova, we have no similar laboratory data to compare with. Modulo this important uncertainty, it appears that the rate of axion emission from the supernova has been calculated to adequate accuracy, especially given the other uncertainties. Likewise, our work seems to justify the use of OPE in calculating $\nu\bar{\nu}$ emission from hot neutron stars through the process nucleon-nucleon, neutrino pair bremsstrahlung[16].

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16. Neutrino pair emission from neutron stars due to nucleon-nucleon, $\nu\bar{\nu}$-bremsstrahlung has been calculated in the OPE approximation by B.L. Friman and O.V. Maxwell, *Astrophys. J.* **232**, 541 (1979). The neutrino pair bremsstrahlung process is very similar to axion bremsstrahlung, a fact which Iwamoto exploited to compute axion bremsstrahlung from very degenerate neutron stars (see N. Iwamoto, *Phys. Rev. Lett.* **53**, 1198 (1984)). We thank G.G. Raffelt for pointing out the applicability of our results to testing the OPE approximation in $\nu\bar{\nu}$ bremsstrahlung.

Appendix

Here we give the exact expression for $\sum_{\text{SPIN}} |\mathcal{M}|^2_\pi$ in the OPE approximation. Recalling that $\mathcal{M}_\pi = (2m^2/f_\pi m_\pi^2)\hat{\mathcal{M}}$, it is:

$$\sum_{\text{SPIN}} |\hat{\mathcal{M}}|^2 = 16(k^2 - m_\pi^2)^{-2}[(2\cdot 4) - m^2]\Big[4[(1\cdot 3) + m^2]$$

$$-4ma_{13}[(1\cdot a) + (3\cdot a)] + a_{13}^2\Big(2(1\cdot a)(3\cdot a) + m_\pi^2[m^2 - (1\cdot 3)]\Big)\Big]$$

$$+ \text{ above with } 1234 \to 2143$$

$$+ \text{ above with } 1234 \to 1243$$

$$+ \text{ above with } 1234 \to 2134$$

$$-4(k^2 - m_\pi^2)^{-1}(l^2 - m_\pi^2)^{-1}\Big[4m^2[(1\cdot 2) + (2\cdot 4) + (2\cdot 3) - (1\cdot 4) - (1\cdot 3) - (3\cdot 4)]$$

$$+4[-m^4 + (1\cdot 4)(2\cdot 3) + (1\cdot 3)(2\cdot 4) - (1\cdot 2)(3\cdot 4)]$$

$$+2m^3(a_{13} + a_{14})[(1\cdot a) + (3\cdot a) + (4\cdot a) - (2\cdot a)]$$

$$+2ma_{13}\Big((2\cdot 3)[-(1\cdot a) - (4\cdot a)] + (3\cdot 4)[(1\cdot a) + (2\cdot a)] + (2\cdot 4)[-(1\cdot a) - (3\cdot a)]$$

$$+(1\cdot 3)[(2\cdot a) - (4\cdot a)] + (1\cdot 2)[(4\cdot a) - (3\cdot a)] + (1\cdot 4)[(3\cdot a) - (2\cdot a)]\Big)$$

$$+2m_{14}\Big((2\cdot 3)[-(1\cdot a) - (4\cdot a)] + (3\cdot 4)[(1\cdot a) + (2\cdot a)] + (2\cdot 4)[-(1\cdot a) - (3\cdot a)]$$

$$+(1\cdot 3)[(4\cdot a) - (2\cdot a)] + (1\cdot 2)[(3\cdot a) - (4\cdot a)] + (1\cdot 4)[(2\cdot a) - (3\cdot a)]\Big)$$

$$+a_{13}a_{14}\Big(2m^2(1\cdot a)[(2\cdot a) - (3\cdot a) - (4\cdot a)] + 2(1\cdot a)[-(3\cdot 4)(2\cdot a) + (2\cdot 4)(3\cdot a) + (2\cdot 3)(4\cdot a)]$$

$$+m^2 m_\pi^2[-m^2 + (2\cdot 3) + (2\cdot 4) + (1\cdot 3) + (1\cdot 4) - (3\cdot 4) - (1\cdot 2)]$$

$$+m_\pi^2[(1\cdot 2)(3\cdot 4) - (1\cdot 3)(2\cdot 4) - (1\cdot 4)(2\cdot 3)]\Big)\Big]$$

$$+ \text{ above with } 1234 \to 1243$$

$$+ \text{ above with } 1234 \to 3421$$

$$+ \text{ above with } 1234 \to 3412$$

$$+ \text{ above with } 1234 \to 4312$$

$$+ \text{ above with } 1234 \to 4321$$

$$+ \text{ above with } 1234 \to 2143$$

$$+ \text{ above with } 1234 \to 2134$$

where we denote the 4-momenta of the incoming and outgoing protons by $1,2$ and $3,4$ respectively, the 4-momentum of the pion by a, and $k = 2 - 4$, $l = 2 - 3$, $a_{ij} = m/(i \cdot a) + m/(j \cdot a)$.

For consistency with the previous calculations of axion bremsstrahlung we have taken the coupling of the exchanged pion to the proton to be $2m/m_\pi$, while, in analogy to the axion proton coupling we have taken the coupling of the outgoing pion to be m/f_π. Had we taken the neutral pion proton coupling to be either $2m/m_\pi$ (or m/f_π) throughout our result for $\sigma(pp \to pp + \pi^o)$ would have been larger by a factor of about 2 (smaller by a factor of 4).

Figure Captions

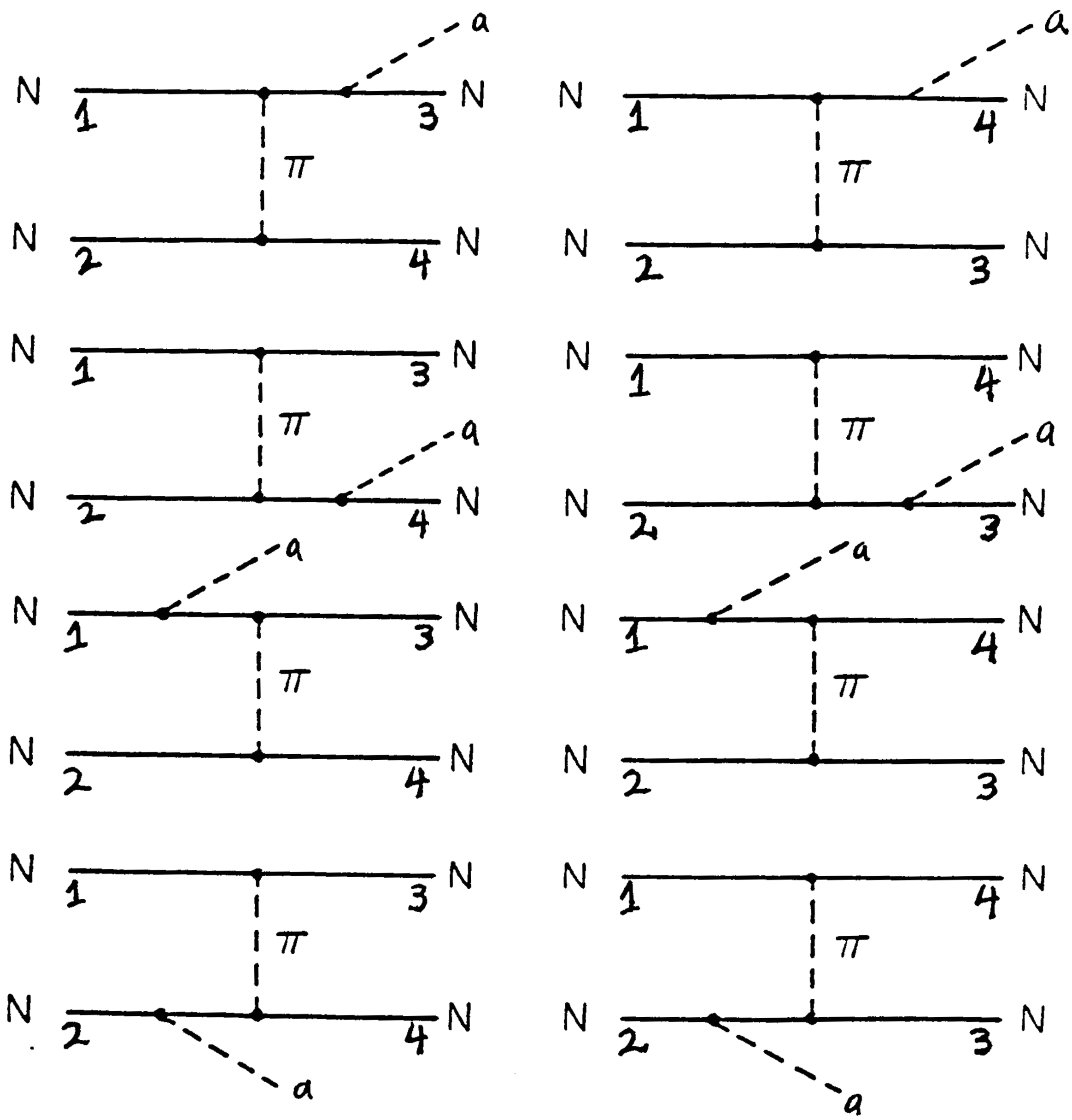
FIGURE 1—The 4 direct and 4 exchange diagrams for nucleon-nucleon, axion (or pion) bremsstrahlung, in the OPE approximation.

FIGURE 2—A summary of the experimental data for $pp \to pp + \pi^o$ and the analytical fit to the data as a function of the lab KE of the incoming proton (from Refs. 12 and 13).

FIGURE 3—The naive OPE prediction for the cross section (using the matrix element in Eq.(2), i.e., taking $m_\pi = 0$) and the experimental data. In addition, the naive OPE prediction as corrected by the approximate threshold correction to the matrix element, cf., Eq.(10), is shown.

FIGURE 4—The exact OPE prediction for the cross section (using the matrix element computed with $m_\pi \neq 0$), cf., Eq.(11), and the experimental data. The effect of neglecting the 3,3 resonance in the OPE calculation around 600 MeV is apparent.

FIGURE 5—The naive OPE cross section (as computed from the matrix element in Eq.(2)) and the exact OPE cross section with m_π set to zero in the matrix element. Except near threshold the agreement between the two is excellent; as described in the text the discrepancy at threshold is as expected.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- FIG 1 -

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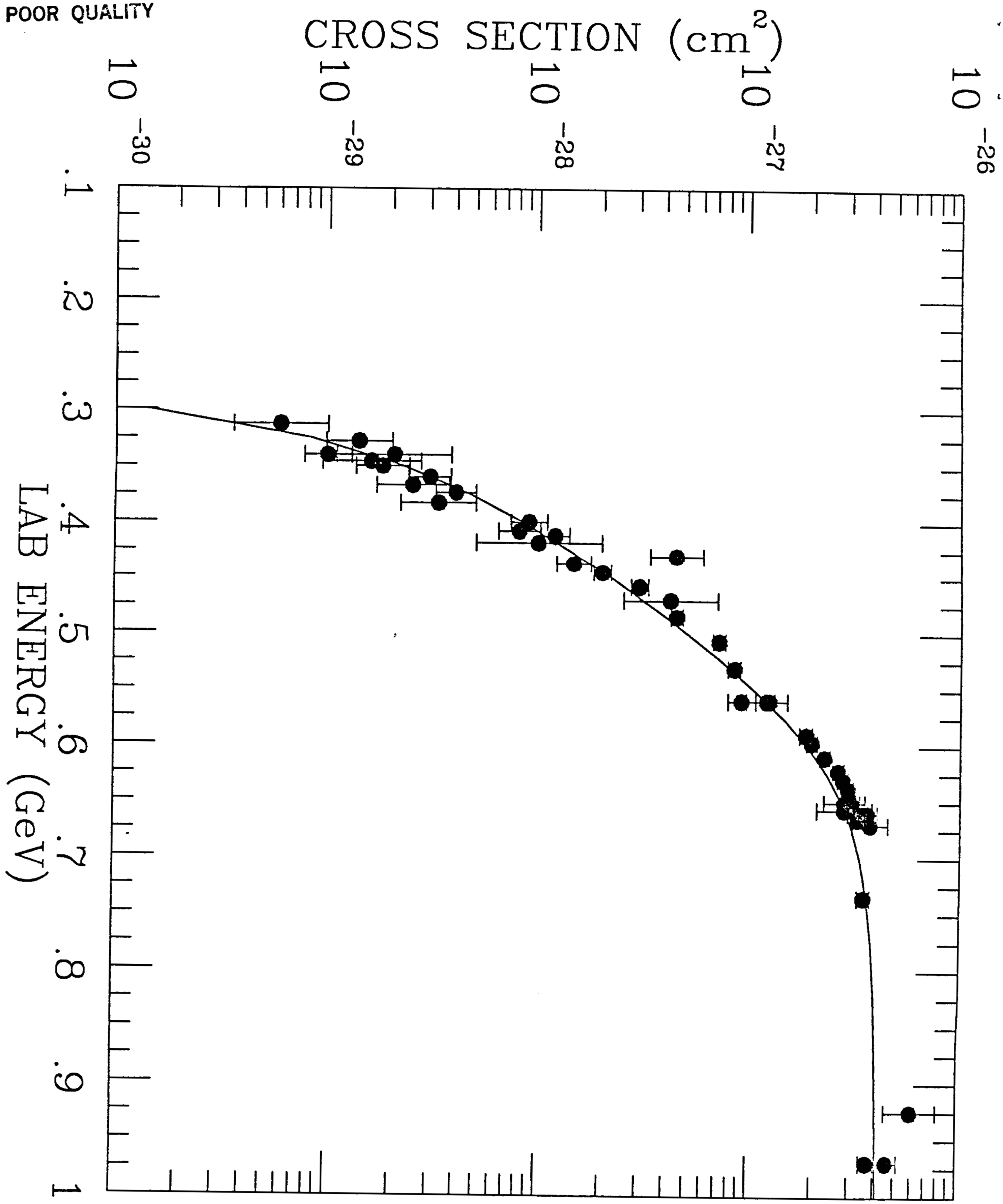
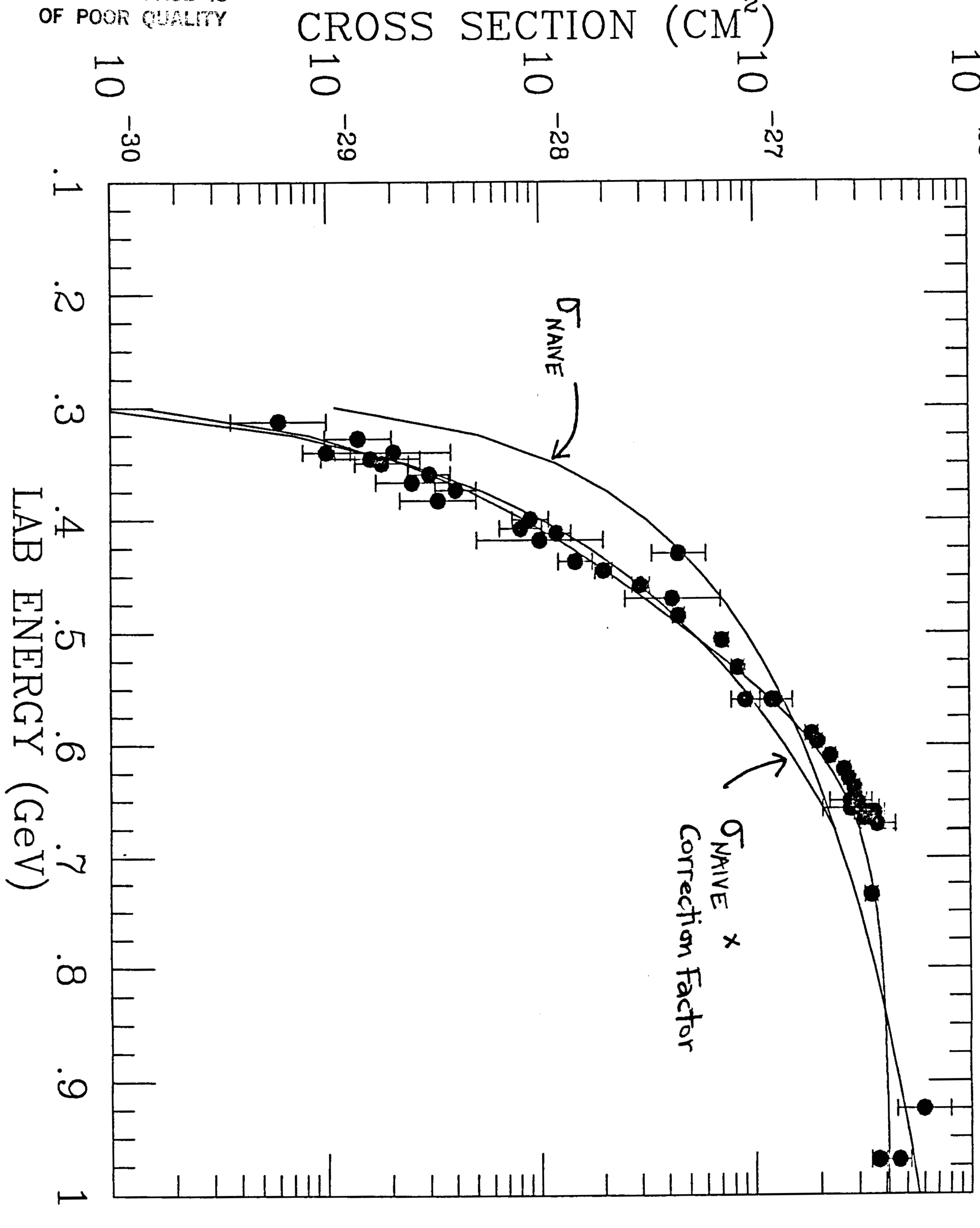
OF POOR QUALITY



—FIG 2—


CROSS SECTION (CM^2)
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- FIG 3 -

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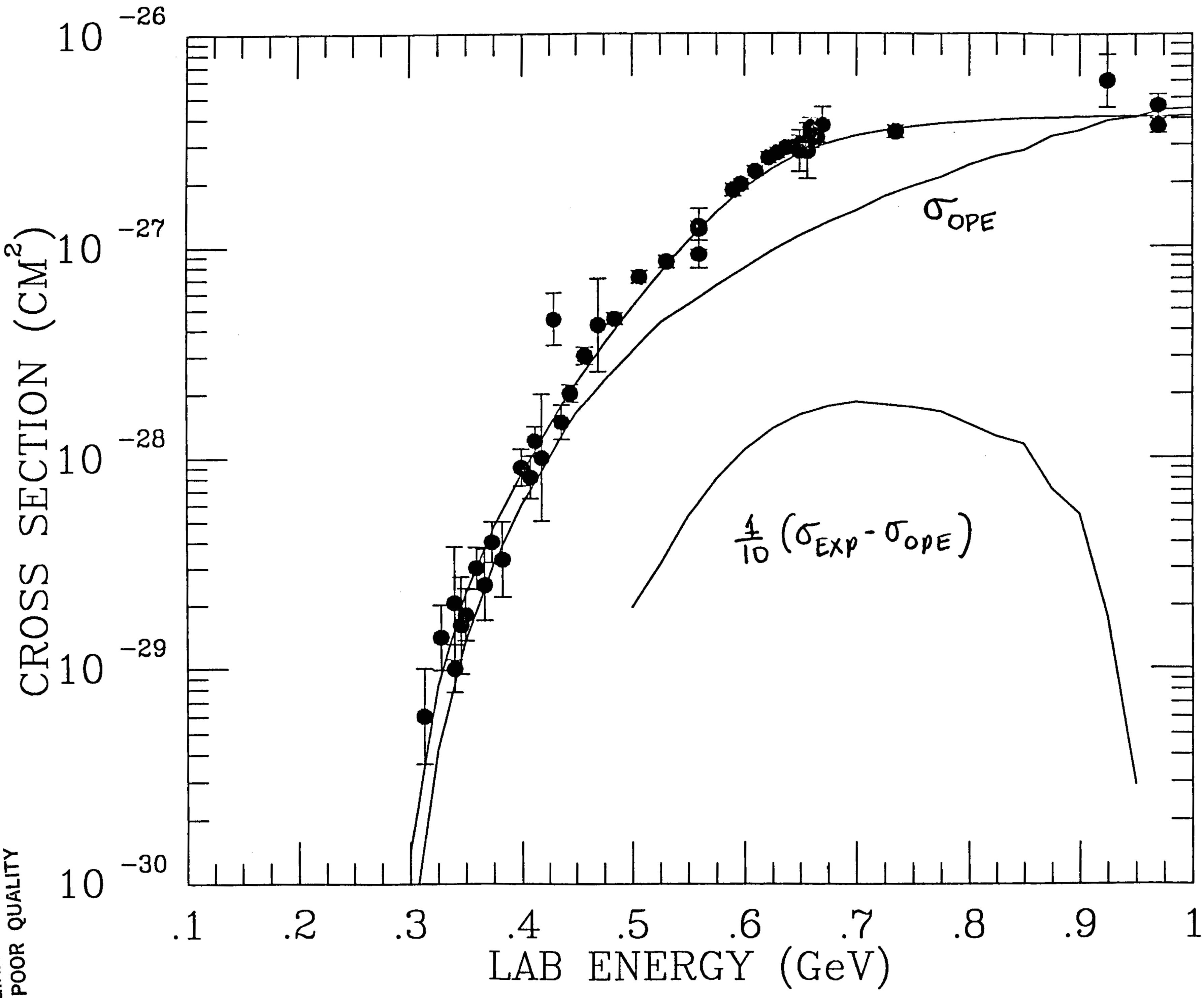
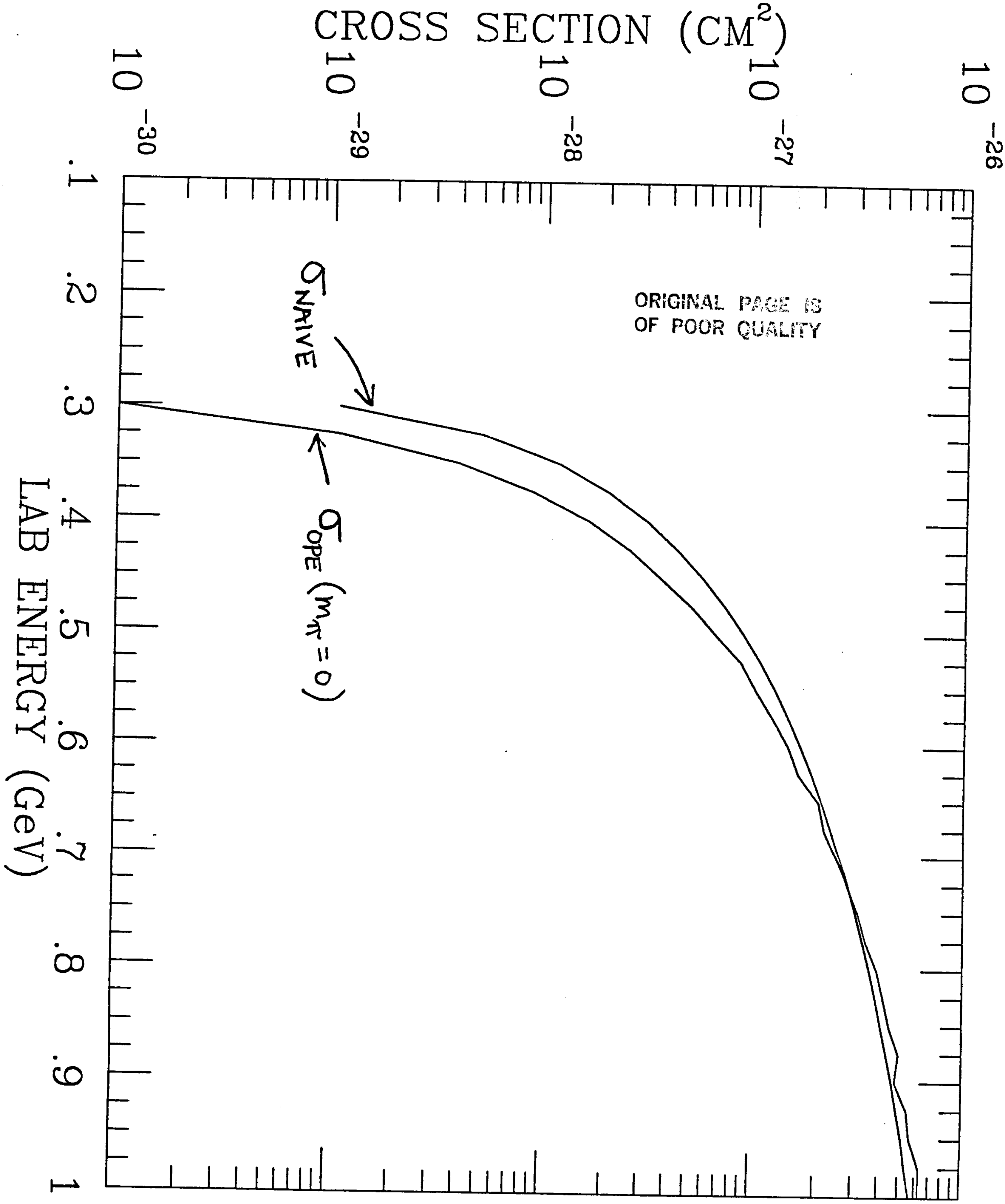


— FIG 4. —


CROSS SECTION (CM^2)
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10^{-26}
LAB ENERGY (GeV)
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.4
.5
.6
.7
.8
.9
1
σ_{NAIVE}
$\sigma_{OPE}(m_\pi = 0)$

—FIG 5—