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Computer-Aided Design of Bevel Gear Tooth Surfaces

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SUMMARY

This paper presents a computer-aided design procedure for generating bevel gears. The development is based on examining a perfectly plastic, cone-shaped gear blank rolling over a cutting tooth on a plane crown rack. The resulting impression on the plastic gear blank is the envelope of the cutting tooth. This impression and envelope thus form a conjugate tooth surface. Equations are presented for the locus of points on the tooth surface. The same procedures are then extended to simulate the generation of a spiral bevel gear. The corresponding governing equations are presented.

INTRODUCTION

Bevel gears are the principle means of motion and power transmission between intersecting shafts. Their use is widespread. The geometrical characteristics of bevel gears have long been documented by the American Gear Manufacturers' Association (AGMA, 1964) and others (Dudley, 1962; Dyson, 1969; Bonsignore, 1976; Litvin et al., 1975, 1982, 1983; Baxter, 1966; Huston and Coy, 1981, 1982a,b). Recent advances in computer-aided design present opportunities for a new look at the geometry of these gears. These computer-based procedures also provide a means for optimizing the geometry.

In a previous paper (Chang, Huston, and Coy, 1984) we have demonstrated the feasibility of this procedure to determine a spur gear tooth profile. The basic idea was to use the envelope of a family of curves to develop an involute spur gear tooth profile. The study demonstrated that the envelope of an inclined straight-line segment on a rolling gear blank is an involute of a circle. The inclined line segment in turn represented the rack tooth of a hob cutter. Such a procedure provides a means for analytically and numerically determining the tooth profile, given a cutter profile.

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In the current paper we extend these ideas to the generation of both straight and spiral bevel gears. The paper itself is divided into four sections with the section following the Symbols providing preliminary ideas useful in this sequel. The next two sections describe the formulation of string and spiral bevel gear tooth surfaces. This is followed by a discussion of applications.

SYMBOLS

The following symbols are used in the section for development of a straight bevel gear tooth.

R_m	Mean radius of crown gear in pitch plane
$[R]$	Coordinate transformation matrix; from \hat{S} to S
$[R_1]$	Coordinate transformation matrix; from S_1 to S
$[R_2]$	Coordinate transformation matrix; from S_2 to S_1
$[R_3]$	Coordinate transformation matrix; from \hat{S} to S_2
\tilde{r}	Position vector in S
\tilde{r}_g	Position vector in \hat{S}
r_{ij}	Components of $[R]$
$S(X,Y,Z)$	Cartesian coordinate system stationary with ground
$\hat{S}(\hat{X},\hat{Y},\hat{Z})$	Cartesian system stationary with generated gear
$S_1(X_1,Y_1,Z_1)$	Intermediate Cartesian coordinate system
$S_2(X_2,Y_2,Z_2)$	Intermediate Cartesian coordinate system
\hat{X}_p	Projection of \hat{X} axis on crown gear pitch plane
α	Angular position of gear referenced to system S
α_0	Initial angular position of gear
γ	Pitch angle of pinion
θ	Angle of rotation of generated gear
ϕ	Complement of pressure angle of cutter
ψ	Cutter's orientation angle on pitch plane

The following symbols are used in the section for development of a spiral bevel gear tooth.

\tilde{B}	Derivative of \tilde{T} with respect to ϕ_2
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b_i	Components of \tilde{B}
d_{ij}	$\partial e_{ij} / \partial \phi_2$
$[E]$	Coordinate transformation matrix; from S_2 to S_c
e_{ij}	Components of $[E]$
H	Horizontal machine setting of cutter
h	Addendum of cutter tooth (pitch to tip)
N_g	Number of teeth in crown gear
N_p	Number of teeth in generated gear
$[R_{c1}]$	Coordinate transformation matrix; from S_1 to S_c
$[R_{1g}]$	Coordinate transformation matrix; from S_g to S_1
$[R_{gp}]$	Coordinate transformation matrix; from S_p to S_g
$[R_{p2}]$	Coordinate transformation matrix; from S_2 to S_p
r_c	Mean radius of cutter in pitch plane
\tilde{r}_c	Position vector in S_c
\tilde{r}_g	Position vector in S_g
\tilde{r}_p	Position vector in S_p
\tilde{r}_1	Position vector in S_1
\tilde{r}_2	Position vector in S_2
$S_c(X_c, Y_c, Z_c)$	Cartesian coordinate system stationary with cutter with origin at O_c
$S_1(X_1, Y_1, Z_1)$	Cartesian coordinate system stationary with gear with origin at O_1
$S_2(X_2, Y_2, Z_2)$	Cartesian coordinate system stationary with pinion with origin at O_2
$S_g(X_g, Y_g, Z_g)$	Cartesian coordinate system stationary with ground with origin at O_g
$S_p(X_p, Y_p, Z_p)$	Cartesian coordinate system stationary with ground with origin at O_p
\tilde{T}	Coordinate translation transformation matrix; from S_2 to S_c

\tilde{T}_{lc}	Coordinate translation transformation matrix; from S_c to S_l
\tilde{T}_{gp}	Coordinate translation transformation matrix; from S_p to S_g
t_i	Components of \tilde{T}
V	Vertical machine setting of cutter
w	Angular speed ratio
Z_0	Z_c intercept of cutter surface
α	Surface coordinate of cutter
γ	Root angle of pinion
γ_0	Pitch angle of pinion
θ	Angle of rotation of system S_c reference to S_l
ϕ_1	Angle of rotation of gear
ϕ_2	Angle of rotation of gear blank
ϕ_{10}	Initial angular position of gear
ψ_0	Complement of pressure angle of cutter edge
ω_g	Rotation rate of gear
ω_p	Rotation rate of pinion

ENVELOPE OF A FAMILY OF SURFACES

Consider a family of surfaces represented by an equation of the form $F(x_1, x_2, x_3, t) = 0$ with a parameter t . Let S be a surface of the family and let it be intersected by neighboring surfaces S' . If S and S' correspond to the values t and $t + \Delta t$, the curve is represented by the simultaneous equations

$$\left. \begin{aligned} F(x_1, x_2, x_3, t) &= 0 \\ F(x_1, x_2, x_3, t + \Delta t) &= 0 \end{aligned} \right\} \quad (1)$$

It may also be represented by the equations

$$\left. \begin{aligned} F(x_1, x_2, x_3, t) &= 0 \\ \frac{F(x_1, x_2, x_3, t + \Delta t) - F(x_1, x_2, x_3, t)}{\Delta t} &= 0 \end{aligned} \right\} \quad (2)$$

The surface $[F(x_1, x_2, x_3, t + \Delta t) - F(x_1, x_2, x_3, t)]/\Delta t = 0$ goes through the curve common to the two surfaces $F(x_1, x_2, x_3, t) = 0$ and $F(x_1, x_2, x_3, t + \Delta t) = 0$. When S' approaches S as a limit (i.e., when Δt approaches zero) the intersection curve will approach a limiting curve. This curve is given by

$$\left. \begin{aligned} F(x_1, x_2, x_3, t) &= 0 \\ \frac{\partial F}{\partial t}(x_1, x_2, x_3, t) &= 0 \end{aligned} \right\} \quad (3)$$

Equation (3), when t is fixed, represents a curve on the surface of the family. The same equation, with t variable, will represent a family of curves and will generate a surface. This surface is the envelope of the given family. The result of eliminating t is the equation of the envelope (Graustein, 1935).

In the two-dimensional case the envelope of a family of curves is a curve tangent to the given family. For example, the envelope given by an inclined straight-line segment on a rolling gear blank is found to be an involute of a circle (Chang, Huston, and Coy, 1984).

In the following sections these procedures are generalized to simulate the straight bevel and spiral bevel gear tooth surface generation process. The family of surfaces created by the cutter generates an envelope in the gear blank. The envelope in the gear blank then forms a conjugate tooth surface.

DEVELOPMENT OF A STRAIGHT BEVEL GEAR TOOTH SURFACE

The cutter used for the surface generation of a straight bevel gear tooth is called the basic crown rack. Figure 1 depicts the machining model of the straight bevel gear tooth generation process. The basic crown rack R is considered to be fixed on the imaginary crown gear pitch plane C as a rack step, as depicted in Fig. 2. The generated gear blank G is a cone with the vertex at the machine center O . The gear blank is allowed to roll on the crown gear pitch plane C . The crown gear pitch plane is an imaginary fixed plane. When the gear blank rolls over the crown rack step, the envelope of the basic crown rack forms the tooth surface of the straight bevel gear.

The coordinate system used to describe the crown gear is $S(X, Y, Z)$ with origin at O . The coordinate system $\hat{S}(\hat{X}, \hat{Y}, \hat{Z})$ is fixed on the gear blank with origin at O . The gear blank is allowed to rotate through an angle θ about \hat{X} . The pitch angle of the gear blank is denoted as γ . The initial position of the gear blank is defined by angle α_0 , which is the angle between the axis X and the projection of the \hat{X} axis on the crown gear pitch plane, denoted by \hat{X}_p . During the gear blank rolling motion the angular position of the gear blank is defined by angle α , where α is measured between \hat{X}_p and X . From this geometry we obtain the relation between θ and α as

$$\alpha = \alpha_0 - \theta \tan \gamma \quad (4)$$

The coordinate system \hat{S} may be obtained by coordinate transformation of the system S in the following steps:

Step	Axis of rotation	Angle turned	Coordinate system	Rotation matrix
0	--	--	At XYZ initially	--
1	Z	α	$X_1 Y_1 Z_1$	R_1
2	Y_1	$-\gamma$	$X_2 Y_2 Z_2$	R_2
3	X_2	θ	$\hat{\hat{\hat{X}}Y\hat{Z}}$ XYZ	R_3

The indicated rotation matrices are

$$[R_1] = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$[R_2] = \begin{bmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix} \quad (6)$$

$$[R_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad (7)$$

The position vectors \tilde{r} with the components (x, y, z) and \tilde{r}_g with the components $(\hat{x}, \hat{y}, \hat{z})$ locating a typical point in S and \hat{S} are related by the expression

$$\tilde{r} = [R_1] [R_2] [R_3] \tilde{r}_g = [R] \tilde{r}_g \quad (8)$$

From Eqs. (5) to (7) the elements of $[R]$, r_{ij} ($i, j = 1, 3$), are

$$\left. \begin{aligned} r_{11} &= \cos \alpha \cos \gamma \\ r_{12} &= -\cos \alpha \sin \gamma \sin \theta - \sin \alpha \cos \theta \\ r_{13} &= -\cos \alpha \sin \gamma \cos \theta + \sin \alpha \sin \theta \\ r_{21} &= \sin \alpha \cos \gamma \\ r_{22} &= -\sin \alpha \sin \gamma \sin \theta + \cos \alpha \cos \theta \\ r_{23} &= -\sin \alpha \sin \gamma \cos \theta - \cos \alpha \sin \theta \\ r_{31} &= \sin \gamma \\ r_{32} &= \sin \theta \cos \gamma \\ r_{33} &= \cos \theta \cos \gamma \end{aligned} \right\} \quad (9)$$

EQUATION FOR CROWN RACK AND STRAIGHT BEVEL GEAR TOOTH SURFACE

The basic crown rack may be viewed as a straight cutting blade reciprocating in the radial direction. The tooth surface of the rack thus forms an inclined plane passing through the crown gear center. The pitch plane of the cutter surface is shown in Fig. 2. The normal plane view in Fig. 3 shows the rack profile. The equation of the rack surface may be expressed in terms of system S as

$$x \tan \psi - y - z \cot \phi = 0 \quad (10)$$

where ψ is the angle between the cutter face and the XZ plane, and ϕ is the complement of the pressure angle of the cutter as shown in Fig. 2. The cutter's surface may be expressed in terms of the gear blank's system \hat{S} by substituting Eqs. (8) and (9) into Eq. (10). This leads to

$$\begin{aligned} & \hat{x}[(\tan \psi \cos \alpha - \sin \alpha) \cos \gamma - \cot \phi \sin \gamma] \\ & + \hat{y}[\sin \gamma \sin \theta (\sin \alpha - \tan \psi \cos \alpha) - \cos \theta (\cos \alpha + \tan \psi \sin \alpha) \\ & - \cot \phi \cos \gamma \sin \theta] \\ & + \hat{z}[\sin \gamma \cos \theta (\sin \alpha - \tan \psi \cos \alpha) - \sin \theta (\cos \alpha + \tan \psi \sin \alpha) \\ & - \cot \phi \cos \gamma \cos \theta] = 0 = F(\hat{x}, \hat{y}, \hat{z}, \theta) \end{aligned} \quad (11)$$

Following this procedure, to determine the envelope of the cutter surface on the generated gear blank, we evaluate the partial derivative of Eq. (11) with respect to parameter θ , the rotation angle of the gear blank. This produces the relation

$$\begin{aligned} & \hat{x}[\sin \gamma (\tan \phi \sin \alpha + \cos \alpha)] \\ & + \hat{y}[\sin \theta (1 - \sin \gamma \tan \gamma) (\cos \alpha + \tan \psi \sin \alpha) \\ & + \cos \theta (\sin \gamma - \tan \gamma) (\sin \alpha - \tan \psi \cos \alpha) - \cot \phi \cos \gamma \cos \theta] \\ & + \hat{z}[\cos \theta (1 - \sin \gamma \tan \gamma) (\cos \alpha + \tan \psi \sin \alpha) \\ & - \sin \theta (\sin \gamma - \tan \gamma) (\sin \alpha - \tan \psi \cos \alpha) + \cot \phi \cos \gamma \sin \theta] = 0 \end{aligned} \quad (12)$$

Let \hat{x} be an independent variable. Then, from Eqs. (11) and (12), \hat{y} and \hat{z} may be evaluated in terms of \hat{x} . Observe that since the coefficients are functions of θ , the tooth surface has the parametric form

$$\left. \begin{aligned} \hat{x} &= \hat{x} \\ \hat{y} &= \hat{y}(\hat{x}, \theta) \\ \hat{z} &= \hat{z}(\hat{x}, \theta) \end{aligned} \right\} \quad (13)$$

where \hat{x} and θ are the surface coordinates. Equation (13) represents the envelope of the rack relative to the gear blank. This represents the tooth surface of the straight bevel gear.

DEVELOPMENT OF A SPIRAL BEVEL GEAR TOOTH SURFACE

A spiral bevel gear tooth surface is developed in a similar manner. Figure 4 depicts a circular cutter generating a spiral bevel gear. The cutter is mounted on the cradel of a generating machine. When the cutter rotates about its own axis, it forms a surface that simulates a crown gear. As the cradle, and hence the cutter, rotates about Z_g at the rate ω_g and the gear blank rotates about Z_p at the rate ω_p , the cutter will generate a spiral bevel gear. The cutting speed is independent of ω_g and ω_p . It is not related to the kinematics of tooth generation. The relation between ω_g and ω_p is simply

$$\omega_p/\omega_g = N_g/N_p \quad (14)$$

where N_g and N_p are the numbers of teeth in the crown gear and generated gear, respectively.

The coordinate system used to describe the crown gear is $S_1(X_1, Y_1, Z_1)$ with origin at O_1 . The crown gear G with frame S_1 fixed in G rotates through an angle ϕ_1 about Z_g with respect to a global coordinate system $S_g(X_g, Y_g, Z_g)$ with the origin at O_g as in Fig. 4. The position vectors \tilde{r}_1 , with the components (x_1, y_1, z_1) , and \tilde{r}_g , with the components (x_g, y_g, z_g) , locating a point in S_1 and S_g are related by the expression (see Fig. 5)

$$\tilde{r}_1 = [R_{1g}] \tilde{r}_g \quad (15)$$

where $[R_{1g}]$ is an orthogonal transformation matrix given by

$$[R_{1g}] = \begin{bmatrix} \cos \phi_1 & \sin \phi_1 & 0 \\ -\sin \phi_1 & \cos \phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

Let a coordinate system $S_c(X_c, Y_c, Z_c)$ be fixed on the cutter with origin at $O_c(H, V, 0)$. Let H and V be the horizontal and vertical machine settings (see Fig. 5). The cutter rotates through an angle θ about axis Z_c . Position vectors \tilde{r}_c , with the components (x_c, y_c, z_c) locating a point relative to S_c , and \tilde{r}_1 are related by the expression

$$\tilde{r}_c = [R_{c1}] \tilde{r}_1 + [R_{c1}] \tilde{r}_{1c} \quad (17)$$

where $[R_{c1}]$ and \tilde{T}_{1c} are

$$[R_{c1}] = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

and

$$\tilde{T}_{1c} = \begin{bmatrix} H \\ V \\ 0 \end{bmatrix} \quad (19)$$

Let the coordinate system $S_2(X_2, Y_2, Z_2)$ be fixed in the gear blank. Let S_2 rotate through an angle ϕ_2 about Z_p with respect to a second global coordinate system $S_p(X_p, Y_p, Z_p)$ (see Figs. 4, 6, and 7). The position vectors \tilde{r}_p , with the components (x_p, y_p, z_p) , and \tilde{r}_2 , with the components (x_2, y_2, z_2) , locating a point in S_p and S_2 are related by the expression

$$\tilde{r}_p = [R_{p2}] \tilde{r}_2 \quad (20)$$

where the transformation matrix $[R_{p2}]$ from S_2 to S_p is given by

$$[R_{p2}] = \begin{bmatrix} \cos \phi_2 & -\sin \phi_2 & 0 \\ \sin \phi_2 & \cos \phi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (21)$$

The two global coordinate systems S_g and S_p are related by the root angle γ of the generated gear and the addendum of the cutter tooth h (see Figs. 4, 6, and 8.) Hence, \tilde{r}_g and \tilde{r}_p are related by the expression

$$\tilde{r}_g = [R_{gp}] \tilde{r}_p + \tilde{T}_{gp} \quad (22)$$

where

$$[R_{gp}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \gamma & -\cos \gamma \\ 0 & \cos \gamma & \sin \gamma \end{bmatrix} \quad (23)$$

and

$$\tilde{T}_{gp} = \begin{bmatrix} 0 \\ 0 \\ -h \end{bmatrix} \quad (24)$$

During the cutting process the simulated crown gear rotates in such a way that the motion is conjugate with the generated gear blank. In Fig. 6 the pitch element O_2P is an instantaneous axis for these "conjugate gears." Hence, their angular velocity components on the pitch element are equal. That is,

$$\dot{\phi}_2 \sin \gamma_0 = \dot{\phi}_1 \cos(\gamma_0 - \gamma) \quad (25)$$

where γ_0 is the pitch angle of the generated gear. Integrating Eq. (25) with respect to time then leads to the relation

$$\phi_1 = \frac{\sin \gamma_0}{\cos(\gamma_0 - \gamma)} \phi_2 + \phi_{10} \quad (26)$$

where ϕ_{10} is a constant determined by initial conditions.

Let the constant parameter w be defined as

$$w = \frac{\sin \gamma_0}{\cos(\gamma_0 - \gamma)} \quad (27)$$

Combining Eqs. (14), (25), and (27) leads to the relation

$$\frac{N_g}{N_p} = \frac{w_p}{w_g} = \frac{\dot{\phi}_2}{\dot{\phi}_1} = \frac{1}{w} \quad (28)$$

CIRCULAR CUTTER SURFACE

In Fig. 9 the rotation of the cutter with a straight blade describes a conical surface of revolution with vertex angle $(\pi - 2\psi_0)$. The mean radius of the head cutter measured in the plane $Z_c = 0$ is r_c . The apex of the cone is at V with coordinates $(0,0,z_0)$ in S_c . The coordinates (x_c, y_c, z_c) of an arbitrary point C on the surface of revolution can then be expressed by the surface coordinates z_c and α as

$$\left. \begin{aligned} x_c &= (z_0 - z_c) \cot \psi_0 \cos \alpha \\ y_c &= (z_0 - z_c) \cot \psi_0 \sin \alpha \end{aligned} \right\} \quad (29)$$

Equation (29) may also be expressed in the form

$$f(x_c, y_c, z_c) = \tan^2 \psi_0 (x_c^2 + y_c^2) - (z_0 - z_c)^2 = 0 \quad (30)$$

The cutter surface may be expressed in the gear blank system S_2 by substituting from Eqs. (15), (20), and (22) into Eq. (17). This leads to

$$\tilde{r}_c = [R_{c1}][R_{1g}][R_{gp}][R_{p2}]\tilde{r}_2 + [R_{c1}][R_{1g}]\tilde{T}_{gp} + [R_{c1}]\tilde{T}_{1c} = [E]\tilde{r}_2 + \tilde{T} \quad (31)$$

where $[E]$ and \tilde{T} are defined as

$$[E] = [R_{c1}][R_{1g}][R_{gp}][R_{p2}] \quad (32)$$

and

$$\tilde{T} = [R_{c1}][R_{1g}]\tilde{T}_{gp} + [R_{c1}]\tilde{T}_{1c} \quad (33)$$

By substituting from Eqs. (16), (18), (19), (23), and (24) into Eqs. (32) and (33), the elements of $[E]$ and \tilde{T} , e_{ij} and t_i ($i, j = 1, 3$), are found to be

$$\left. \begin{aligned} e_{11} &= \cos \phi_2 \cos(\theta + \phi_1) + \sin \gamma \sin \phi_2 \sin(\theta + \phi_1) \\ e_{12} &= -\sin \phi_2 \cos(\theta + \phi_1) + \sin \gamma \cos \phi_2 \sin(\theta + \phi_1) \\ e_{13} &= -\cos \gamma \sin(\theta + \phi_1) \\ e_{21} &= -\cos \phi_2 \sin(\theta + \phi_1) + \sin \gamma \sin \phi_2 \cos(\theta + \phi_1) \\ e_{22} &= \sin \phi_2 \sin(\theta + \phi_1) + \sin \gamma \cos \phi_2 \cos(\theta + \phi_1) \\ e_{23} &= -\cos \gamma \cos(\theta + \phi_1) \\ e_{31} &= \cos \gamma \sin \phi_2 \\ e_{32} &= \cos \gamma \cos \phi_2 \\ e_{33} &= \sin \gamma \end{aligned} \right\} \quad (34)$$

and

$$\left. \begin{aligned} t_1 &= H \cos \theta + V \sin \theta \\ t_2 &= -H \sin \theta + V \cos \theta \\ t_3 &= -h \end{aligned} \right\} \quad (35)$$

Hence, the cutter surface expressed in the S_2 coordinate system may be obtained by substituting from Eqs. (31), (34), and (35) into Eq. (29). That is,

$$(e_{11} \tan \psi_0 + e_{31} \cos \alpha)x_2 + (e_{12} \tan \psi_0 + e_{32} \cos \alpha)y_2 + (e_{13} \tan \psi_0 + e_{33} \cos \alpha)z_2 = (z_0 - t_3) \cos \alpha - t_1 \tan \psi_0$$

and

$$(e_{21} \tan \psi_0 + e_{31} \sin \alpha)x_2 + (e_{12} \tan \psi_0 + e_{32} \sin \alpha)y_2 + (e_{13} \tan \psi_0 + e_{33} \sin \alpha)z_2 = (z_0 - t_3) \sin \alpha - t_2 \tan \psi_0 \quad (36)$$

SPIRAL BEVEL GEAR TOOTH SURFACE EQUATION

Equations (36) represent the generating surface seen by the gear blank. In determining the envelope of the cutter surface on the gear blank, it is useful to evaluate the partial derivative of \tilde{r}_c in Eq. (31) with respect to the parameter ϕ_2 . That is,

$$\frac{\partial \tilde{r}_c}{\partial \phi_2} = \left[\frac{\partial E}{\partial \phi_2} \right] \tilde{r}_2 + [E] \frac{\partial \tilde{r}_2}{\partial \phi_2} + \frac{\partial \tilde{T}}{\partial \phi_2} \quad (37)$$

The second term is zero since \tilde{r}_2 is fixed in S_2 . From Eq. (35) the last term is also zero because the cutting speed is independent of the gear blank rotation. Let the component of the first term be d_{ij} . Then d_{ij} can be determined from Eq. (34), and Eq. (37) may be rewritten in the form

$$\frac{\partial \tilde{r}_c}{\partial \phi_2} = [d_{ij}] \tilde{r}_2 \quad (ij = 1, 3) \quad (38)$$

where

$$\left. \begin{aligned} d_{11} &= -\sin \phi_2 \cos(\theta + \phi_1)(1 - w \sin \gamma) + (\sin \gamma - w) \cos \phi_2 \sin(\theta + \phi_1) \\ d_{12} &= -\cos \phi_2 \cos(\theta + \phi_1)(1 - w \sin \gamma) - (\sin \gamma - w) \sin \phi_2 \sin(\theta + \phi_1) \\ d_{13} &= -w \cos \gamma \cos(\theta + \phi_1) \\ d_{21} &= \sin \phi_2 \sin(\theta + \phi_1)(1 - w \sin \gamma) + (\sin \gamma - w) \cos \phi_2 \cos(\theta + \phi_1) \\ d_{22} &= \cos \phi_2 \sin(\theta + \phi_1)(1 - w \sin \gamma) - (\sin \gamma - w) \sin \phi_2 \cos(\theta + \phi_1) \\ d_{23} &= -w \cos \gamma \sin(\theta + \phi_1) \\ d_{31} &= \cos \gamma \cos \phi_2 \\ d_{32} &= -\cos \gamma \sin \phi_2 \\ d_{33} &= 0 \end{aligned} \right\} \quad (39)$$

Equation (38) represents the derivative of the equation giving transformation from the cutter system to the gear blank system. It is useful for deriving the constraint equation of the cutter's motion. The derivative of the cutter conical surface with respect to ϕ_2 , the rotation angle of the gear blank, is (from Eq. (30))

$$\frac{\partial}{\partial \phi_2} f(x_c, y_c, z_c) = 2 \tan^2 \psi_0 \left(x_c \frac{\partial x_c}{\partial \phi_2} + y_c \frac{\partial y_c}{\partial \phi_2} \right) + 2(z_0 - z_c) \frac{\partial z_c}{\partial \phi_2} = 0 \quad (40)$$

If z_c is not equal to z_0 , we may substitute from Eqs. (29) and (38) into Eq. (40). This leads to

$$\begin{aligned} & [\tan \psi_0 (d_{11} \cos \alpha + d_{21} \sin \alpha) + d_{31}]x_2 + [\tan \psi_0 (d_{21} \cos \alpha + d_{22} \sin \alpha) \\ & + d_{32}]y_2 + [\tan \psi_0 (d_{13} \cos \alpha + d_{23} \sin \alpha) + d_{33}]z_2 = 0 \end{aligned} \quad (41)$$

Equation (41) is the constraint equation of the cutter motion on the gear blank. Combining Eqs. (36) and (41) forms a set of simultaneous equations representing the envelope of the cutter relative to the gear blank. The solution of the simultaneous equations describes the tooth surface impression created by the cutter. Observe that since the coefficients are functions of (α, ϕ_1) , the solution has the parametric form

$$\left. \begin{aligned} x_2 &= x_2(\alpha, \phi_1) \\ y_2 &= y_2(\alpha, \phi_1) \\ z_2 &= z_2(\alpha, \phi_1) \end{aligned} \right\} \quad (42)$$

where α and ϕ_1 are the surface coordinates

DISCUSSION

Equations (11) to (13) and Eqs. (36), (41), and (42) determine the cutter envelopes and hence the gear tooth surface for straight and spiral bevel gears, respectively. They form the basis for a numerical and computer graphic representation of the tooth surface. In this context these equations represent an extension of the procedure described earlier (Chang, Huston, and Coy, 1984) from spur gear to bevel gear. The difference here, however, is that the equations are more detailed and extensive. Hence, numerical and computer graphic analyses are needed.

In a general sense the method simulates the kinematic relation of a cone-shaped gear blank and the cutter. With the rotation angle of the gear blank being the parameter, the envelope of the cutter profile on the gear blank describes the gear tooth profile. The straight and spiral bevel gear tooth surfaces are the forms given in Eqs. (13) and (42). The machine settings, the cutter radius, and the mean cone distance are the parameters that can be varied in a design optimization analysis. An accurate finite element mesh can also be obtained by discretizing the tooth surface.

In summary, it is believed by the authors that the analysis presented herein can form a basis for numerical design studies as well as for stress and deformation studies. Finally, the method may be readily extended to the study of nonintersecting shaft (hypoid) gears.

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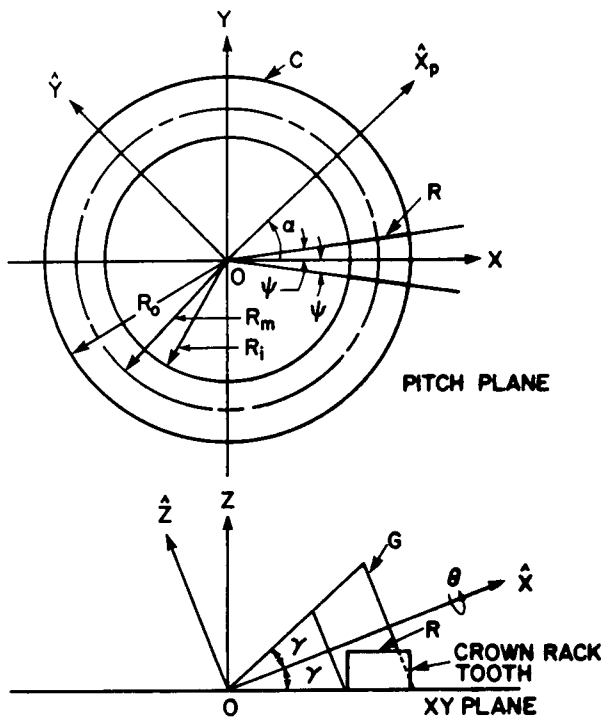


Fig. 1. Straight Bevel Gear Tooth Surface Generation

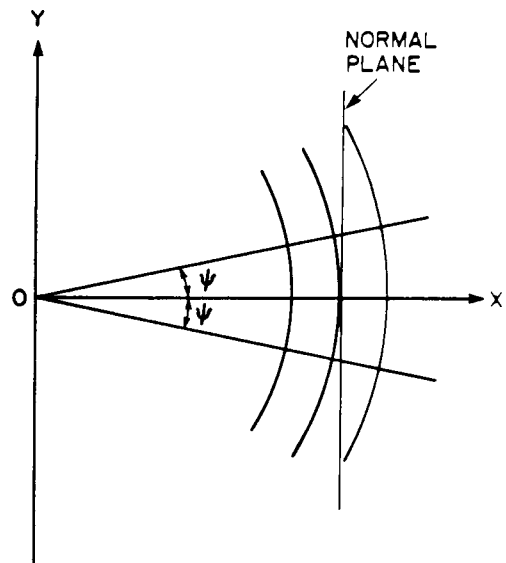


Fig. 2. The Pitch Plane

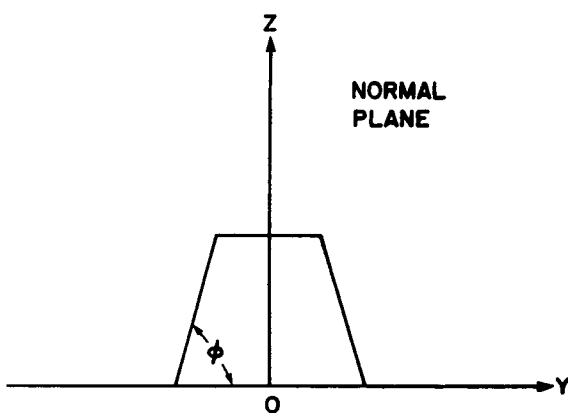


Fig. 3. The Normal Plane

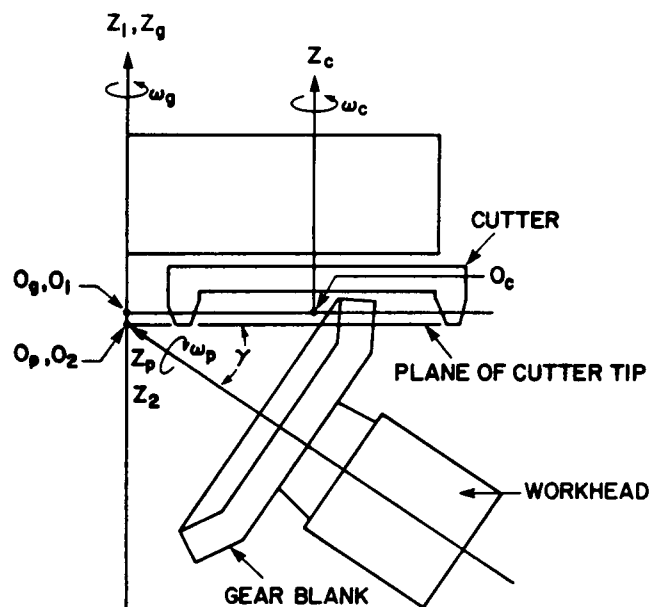


Fig. 4. Machine Setting for Spiral Bevel Gear Manufacturing

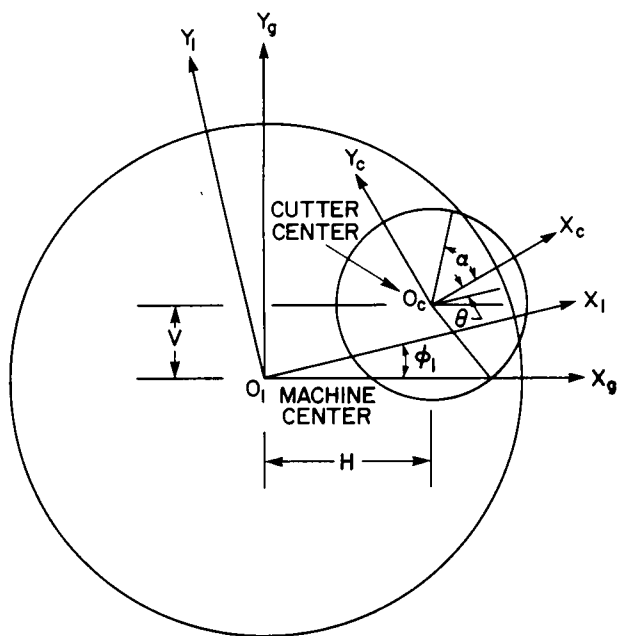


Fig. 5. Relation of Crown Gear and Cutter

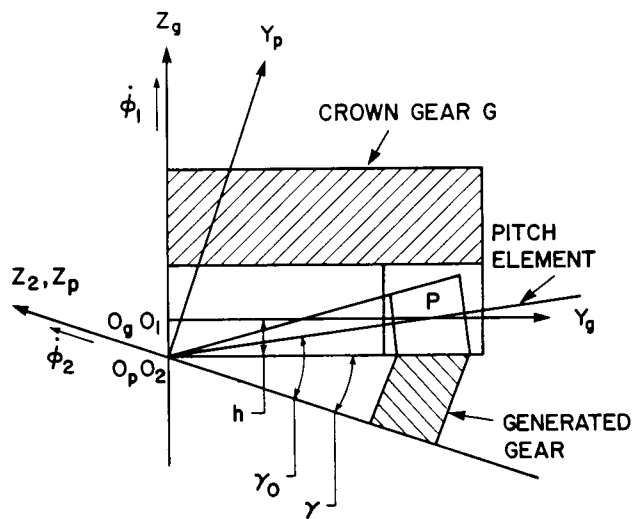


Fig. 6. Relation of Generating Gear and Generated Gear

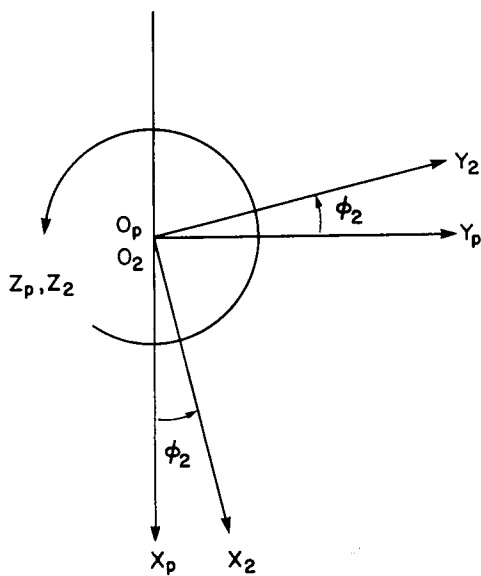


Fig. 7. Relation of Coordinate System S_p and S_2

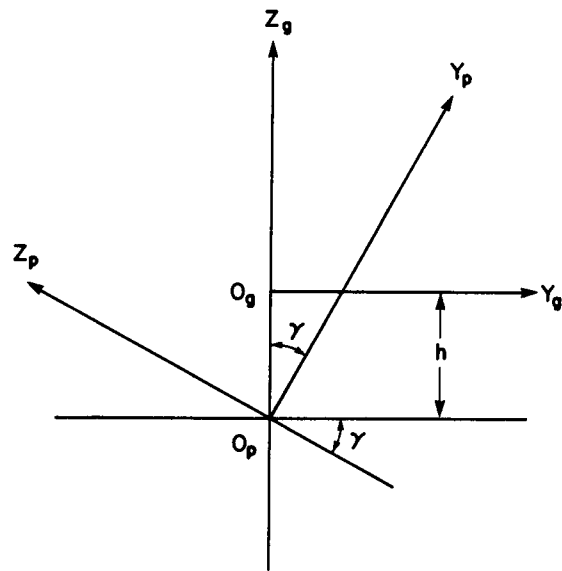


Fig. 8. Relation of Coordinate System S_g and S_p

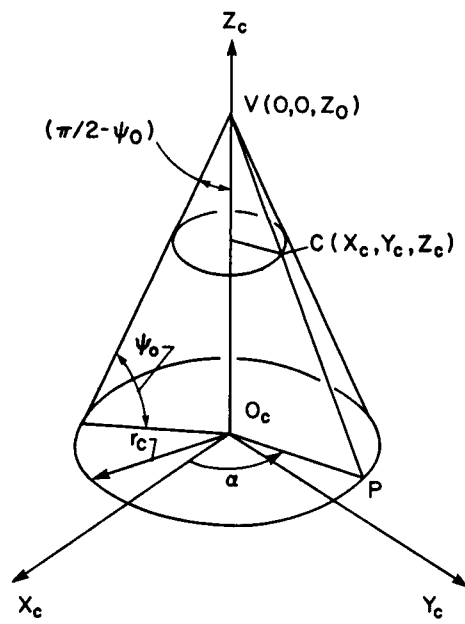


Fig. 9. Surface of Revolution of the Cutter

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16. Abstract This paper presents a computer-aided design procedure for generating bevel gears. The development is based on examining a perfectly plastic, cone-shaped gear blank rolling over a cutting tooth on a plane crown rack. The resulting impression on the plastic gear blank is the envelope of the cutting tooth. This impression and envelope thus form a conjugate tooth surface. Equations are presented for the locus of points on the tooth surface. The same procedures are then extended to simulate the generation of a spiral bevel gear. The corresponding governing equations are presented.					
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