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A CONTINUUM DEFORMATION THEORY FOR METAL-MATRIX COMPOSITES AT HIGH TEMPERATURE

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Structural alloys used in high-temperature applications exhibit complex thermomechanical behavior that is time-dependent and hereditary. Recent attention is being focused on metal-matrix composite materials for aerospace applications that, at high temperature, exhibit all the complexities of conventional alloys (e.g., creep, relaxation, recovery, rate sensitivity) and, in addition, exhibit further complexities because of their strong anisotropy.

Here, a continuum theory is presented for representing the high-temperature, time-dependent, hereditary deformation behavior of metallic composites that can be idealized as pseudohomogeneous continua with locally definable directional characteristics. Homogenization of textured materials (molecular, granular, fibrous) and applicability of continuum mechanics in structural applications depends on characteristic body dimensions, the severity of gradients (stress, temperature, etc.) in the structure and on the relative size of the internal structure (cell size) of the material. Examination reveals that the appropriate conditions are met in a significantly large class of anticipated aerospace applications of metallic composites to justify research into the formulation of continuum-based theories.

The point of view taken here is that the composite is a material in its own right, with its own properties that can be measured and specified for the composite as a whole. Experiments for this purpose are outlined in detail in reference 1. This viewpoint is aimed at satisfying the structural analyst or design engineer who needs reasonably simple methods of structural analysis to predict deformation behavior in complex multiaxial situations, particularly at high temperature where material response is enormously complex. Indeed, the prediction of component lifetime depends critically on the accurate prediction of deformation behavior.

THEORETICAL DEVELOPMENT

As in references 1 and 2, the starting point here is the assumed existence of a dissipation potential function Ω for the composite material; that is,

$$\Omega = \Omega(\sigma_{ij}, \alpha_{ij}, d_i d_j, T) \tag{1}$$

in which σ_{ij} denotes the components of (Cauchy) stress, α_{ij} the components of a tensorial internal state variable (internal stress), $d_i d_j$ the components of a directional tensor, and T the temperature. The symmetric tensor $d_i d_j$ is formed by a self product of the unit vector d_i denoting the local fiber direction. As pointed out in reference 3, account can be taken of more than a single family of

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fibers inherent to the continuum element. An extension of the present work to two families of fibers has been considered by the author but is not presented here. The function Ω is taken to depend on temperature, however the present emphasis is on isothermal deformation at high homologous temperature so that the temperature dependence will not be shown explicitly hereafter. The presumption of constant high homologous temperature justifies ignoring extensive cyclic hardening. Extension to a full nonisothermal theory is in progress.

A simple thermodynamic formalism based on reasonable assumptions is shown in reference 4 to support the existence of a dissipation potential for a homogeneous (one-constituent) solid. Here, this formalism is assumed to be extendable to a two-constituent (fiber/matrix), psuedohomogeneous composite material. The thermodynamic structure leads to the generalized normality conditions,

$$\dot{\varepsilon}_{ij} = \frac{\partial \Omega}{\partial \sigma_{ij}} \tag{2}$$

and

$$\frac{\dot{\alpha}_{ij}}{h} = \frac{\partial \Omega}{\partial \alpha_{ij}} \tag{3}$$

in which $\dot{\varepsilon}_{ij}$ denotes the components of the rate of (small) strain and h is a scalar function of the internal stress α_{ij}. The orientation tensor $d_i d_j$ is taken to be constant under small deformations, otherwise an evolutionary equation would need to be specified for it as well.

As in references 1, 2, and 5, Ω is taken to depend on the state variables through the scalar functions F and G; that is,

$$\Omega = \Omega(F,G) \tag{4}$$

where

$$F(\Sigma_{ij},\ d_i d_j) \tag{5}$$

$$\Sigma_{ij} = s_{ij} - a_{ij} \tag{6}$$

and

$$G(a_{ij},\ d_i d_j) \tag{7}$$

The term Σ_{ij} is the effective stress and s_{ij} and a_{ij} are the deviatoric parts of σ_{ij} and α_{ij}, respectively.

The functions F and G each depend on two symmetric second-order tensors. Form invariance (objectivity) of F and G, and hence of Ω, requires that they depend only on certain invariants and invariant products of their respective tensorial arguments (integrity basis - ref. 6). A subset of these invariants for F (i.e., $F(I_1,I_2,I_3)$) is taken as follows:

$$I_1 = J_2 - I + \frac{1}{4} I_3 \tag{8}$$

$$I_2 = I - I_3 \tag{9}$$

$$I_3 = (I_0)^2 \tag{10}$$

where

$$J_2 = \frac{1}{2} \Sigma_{ij}\Sigma_{ji} \tag{11}$$

$$I = d_i d_j \Sigma_{jk}\Sigma_{ki} \tag{12}$$

$$I_0 = d_i d_j \Sigma_{ji} \tag{13}$$

The invariant I_1 corresponds to the square of the maximum effective shear stress on planes containing the fibers and in a direction normal to them (transverse shear), I_2 corresponds to the square of the maximum effective shear stress on planes containing the fibers but directed along the fibers (longitudinal shear) and I_3 is the square of the effective normal stress in the local fiber direction. A similar set of invariants is chosen for G, denoted by I_1', I_2', and I_3', and is obtained by replacing Σ_{ij} by a_{ij} in equations (11) to (13).

The function $F(I_1,I_2,I_3)$ is chosen to be linear in I_1, I_2, I_3 (quadratic in stress) as

$$F = \left(\frac{I_1}{K_T^2} + \frac{I_2}{K_L^2} + \frac{9}{4Y_L^2} I_3\right) - 1 \tag{14}$$

where K_T, K_L, and Y_L correspond physically to the (threshold) strengths of the composite element in transverse shear, longitudinal shear, and longitudinal tension (compression), respectively. Defining

$$\eta = \frac{K_L}{K_T} \tag{15}$$

and

$$\omega = \frac{Y_L}{Y_T} \tag{16}$$

where Y_T relates to the strength in transverse tension (compression), F becomes

$$F = \frac{1}{K_T^2}\left[I_1 + \frac{1}{\eta^2} I_2 + \frac{9}{4(4\omega^2 - 1)} I_3\right] - 1 \tag{17}$$

Experiments for determining K_T, η, and ω using thin-walled tubular specimens with varying fiber orientations are outlined in reference 1. Here, F plays the role of a Bingham-Prager threshold function with K_T (as indicated earlier) a threshold shear stress in transverse shear. If no threshold exists - that is, if inelastic

deformation occurs for applied stress however small - the function F can be taken homogeneous in stress with K_T playing the role of a "drag" stress as identified in other theoretical developments.

It is noted that with $\eta = \omega = 1$ (and with $K_T = K_L = K$) F reduces to

$$F = \frac{J_2}{K^2} - 1 \tag{18}$$

as taken in reference 5 for an isotropic solid.

Similarly, the function G is taken as

$$G = \frac{1}{K_T^2}\left[I_1' + \frac{1}{\eta^2} I_2' + \frac{9}{4(4\omega^2 - 1)} I_3'\right] \tag{19}$$

Using equations (4) to (19) in equations (1) to (3), taking h in equation (3) as $h(G)$ and

$$f(F) = \frac{\partial\Omega}{\partial F} \tag{20}$$

and

$$\gamma(G) = h(G)\frac{\partial\Omega}{\partial G} \tag{21}$$

results in a flow law

$$\dot{\varepsilon}_{ij} = f(F)\Gamma_{ij} \tag{22}$$

and an evolutionary law

$$\dot{a}_{ij} = h(G)\dot{\varepsilon}_{ij} - \gamma(G)\pi_{ij} \tag{23}$$

where

$$\Gamma_{ij} = \Sigma_{ij} - \xi[d_k d_i \Sigma_{jk} + d_j d_k \Sigma_{ki} - 2I_0 d_i d_j] - \frac{1}{2}\zeta I_0(3d_i d_j - \delta_{ij}) \tag{24}$$

and

$$\pi_{ij} = a_{ij} - \xi[d_k d_i a_{jk} + d_j d_k a_{ki} - 2I_0' d_i d_j] - \frac{1}{2}\zeta I_0'(3d_i d_j - \delta_{ij}) \tag{25}$$

where

$$\xi = \frac{\eta^2 - 1}{\eta^2} \qquad 0 \leq \xi \leq 1 \tag{26}$$

$$\zeta = \frac{4(\omega^2 - 1)}{4\omega^2 - 1} \qquad 0 \leq \zeta \leq 1 \tag{27}$$

Once again, with $\eta = \omega = 1$ ($\xi = \zeta = 0$) equations (22) and (23) reduce to the flow and evolutionary laws of reference 5 for an isotropic solid.

Equations (17), (19), (22), and (23), with the accompanying definitions, provide an anisotropic representation that accounts for observed monotonic behavioral features such as strain-rate dependent plasticity, primary creep, and secondary creep. Application to a particular composite requires specification of the parameters η and ω, characterizing the anisotropy; K_T, the strength in transverse shear; and the functions $f(F)$, $h(G)$, and $\gamma(G)$ (ref. 1).

CONSIDERATIONS FOR STRESS REVERSALS AND REDUCTIONS

Reversals and reductions of stress following inelastic deformation of metallic alloys are known to initiate micromechanistic processes that are not present, or at least not controlling, under monotonic conditions. For example, forward stressing may result in hardening through pile-ups of gliding dislocations against obstacles (for example, "forest" dislocations threading slip planes that accumulate in front of moving dislocations - ref. 7). Upon abrupt reversal of stress the immobilized dislocations become remobilized, finding fewer obstacles in their paths as they begin to move backward along their slip planes. This constitutes a relatively rapid microstructural rearrangement; that is, an abrupt change in mobile dislocation density, precipitated by a reversal of stress (dynamic recovery). Interpreted on a somewhat more macroscopic scale, this may correspond closely to the very rapid changes in the "stored energy of cold work" observed upon stress reversal by Halford (ref. 8).

Reductions in stress at elevated temperature are also known to cause microstructural rearrangements in time (thermal recovery) through diffusion-controlled mechanisms such as climb and annihilation of dislocations, even in the absence of significant inelastic strain recovery. Although, at high temperature, these mechanisms always may be present and contribute as competing mechanisms under steady-state conditions, they may become the controlling mechanisms, at least for some time, following stress reductions.

Phenomenological representations in which internal state variables (e.g., α_{ij}) serve as macroscopic measures of the current microstructure must reflect these sometimes abrupt internal changes that occur upon stress reversals and the more gradual changes that occur under reductions of stress. The theory for isotropic metals presented in reference 5 accounts for these behaviors by allowing analytically different regions of the state space: that is, regions of the space (σ_{ij}, α_{ij}) governed by different analytical forms for the evolution of the internal state (α_{ij}). Such idealizations have strong precedent in classical continuum plasticity. Details of the representation are given in reference 5. Here, it is assumed that the idealization can be extended to the anisotropic behavior of metallic composites.

Guided by reference 5, the crucial regions of the state space are bounded by

$$F \lesseqgtr 0 \quad , \quad s_{ij}\Gamma_{ji} \lesseqgtr 0 \quad \text{and} \quad s_{ij}\pi_{ji} \lesseqgtr 0 \tag{28}$$

Accordingly, the flow and evolutionary equations (22) and (23) are modified, respectively, as

$$\dot{\varepsilon}_{ij} = P(s_{ij}\,\Gamma_{ji})\, f\langle F\rangle\, \Gamma_{ij} \tag{29}$$

$$\dot{a}_{ij} = h(\hat{G})\,\dot{\varepsilon}_{ij} - \gamma(\hat{G})\pi_{ij} \tag{30}$$

where

$$\hat{G} = (G - G_0)P(s_{ij}\pi_{ji}) + G_0 \tag{31}$$

$$P(x) = 1 \qquad x > 0$$

$$P(x) = 0 \qquad x \leq 0 \tag{32}$$

and

$$f\langle F\rangle = \begin{cases} f(F) & F > 0 \\ 0 & F \leq 0 \end{cases} \tag{33}$$

A complete statement of the isothermal theory is thus given by equations (17), (19), (29), and (30), with the accompanying definitions. It is again noted that for $\eta = \omega = 1$ these equations reduce to those of reference 5 that have been applied successfully in representing the cyclic thermomechanical response of isotropic solids and structures (refs. 5 and 9 to 11).

IMPLEMENTATION IN STRUCTURAL ANALYSIS CODES

The present theory has been implemented into the commercial finite element code MARC by Dr. A.K. Arya (ref. 11). Several trial calculations have been made under uniaxial conditions using material functions and parameters that approximate a tungsten/copper composite material (ref. 3). A transversely isotropic continuum elasticity theory (ref. 12) has been used in conjunction with the present visco-plastic theory. The results of the calculations (ref. 11) show the expected responses of rate-dependent plasticity, creep, and relaxation as well as appropriate anisotropic features.

The theory has also been implemented into a research-oriented code NFAP developed by Prof. T.Y. Chang together with his colleagues and students at the University of Akron. Several of the uniaxial predictions of MARC have been successfully duplicated using NFAP and predictions of structural response - for example, composite beams, plates, and shells - are in progress.

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