

**FINAL REPORT FOR THE UNMANNED MULTIPLE  
EXPLORATORY PROBE SYSTEM (MEPS)  
FOR MARS OBSERVATION**

**Volume II. Calculations and Derivations**

A design project by students in the Department of Aerospace Engineering at Auburn University, Auburn, Alabama, under the sponsorship of NASA/USRA Advanced Design Program.

Auburn University  
Auburn, Alabama  
June, 1988

The following seniors contributed to the MEPS design team:

Daniel E. Adams	Propulsion Systems Aerobraking System
Christopher M. Crumbly	Propulsion Systems Structural Analysis
Steve E. Delp	Guidance and Control
Michelle A. Guidry	Surface Rover Systems Surface Laboratory
Michael E. Lisano	Mars Lander Systems Sample Return Vehicle Surface Laboratory
James D. Packard	Command Information Center Trajectory Analysis Observation Satellite System Aerobraking System
Scott A. Striepe	Structural Analysis (NASTRAN)

## ABSTRACT

This volume of the final report on the unmanned Multiple Exploratory Probe System (MEPS) details all calculations, derivations, analyses, and computer programs that support the information presented in the first volume.

## TABLE OF CONTENTS

<u>Title</u>	<u>Page</u>
Abstract	ii
List of Symbols	iv
Introduction	1
Structural Mass Calculation	2
Proposed Strength Analysis	5
Analysis of Earth-Mars Trajectory	7
Propellant Analysis for the Secondary Propulsion System	12
Analysis of the Main Propulsion System	17
Aerobrake Analysis	24
Derivations for the Aerobraking Program	28
Analysis for the Observation Satellite	31
Calculation of Approximate Mass of Mars Lander	34
List of References	42
Appendix A (Computer Program for the Aerobraking Analysis)	43
Appendix B (Optimization of Propellant Mass of a Solid-Propellant Rocket)	51
Appendix C (Calculation of Stagnation Temperature on Mars Lander)	64
Appendix D (Optimization of Recovery System Mass For Mars Landers)	67

## LIST OF SYMBOLS

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
A	Area of the Aerobrake	ft <sup>2</sup>
A <sub>s</sub>	Area of the Solar Array Panel	ft <sup>2</sup>
a	Acceleration	ft/sec <sup>2</sup>
a	Length of the Semi-Major Axis	ft
a <sub>r</sub>	Length of the Semi-Major Axis of an Elliptic Transfer Orbit	ft
b	Length of the Semi-Minor Axis	ft
C <sub>d</sub>	Drag Coefficient	--
C <sub>e</sub>	Effective Exhaust Velocity	ft/sec
c	Chord of an Arc	ft
d	Diameter of Propellant Tank	ft
d	Distance from Center of Mars to x-Position of Intersection Points (Orbit and Atmosphere)	ft
e	Orbit Eccentricity	--
F	Force	lbf
F	Sum of Design Factors	--
g <sub>e</sub>	Gravity of Earth	ft/sec <sup>2</sup>
h	Length of Cylindrical Tank	ft
I <sub>sp</sub>	Specific Thrust	sec
ℓ	Length of Cylindrical tank	ft
M	Total Vehicle Mass	lbm
M <sub>i</sub>	Vehicle Mass Prior to Propulsive Burn	lbm

LIST OF SYMBOLS (continued)

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$M_v$	Vehicle Mass Prior to Propulsive Burn	lbm
$M_{LH_2}$	Mass of Liquid Hydrogen	lbm
$M_{LO_2}$	Mass of Liquid Oxygen	lbm
$M_{MMH}$	Mass of Monomethyl Hydrazine	lbm
$M_{N_2O_4}$	Mass of Nitrogen Tetroxide	lbm
$M_p$	Propellant Mass	lbm
$m$	Mass	lbm
$n$	Number of Engine Stages	lbm
O/F	Oxidizer/Fuel Ratio	---
$P_a$	Available Power	W
$p$	Semi-latus Rectum	ft
$\dot{q}_{rad}$	Radiative Heat Flux from Air to Body	$\frac{Btu}{ft^2 \cdot sec}$
$\dot{q}_{conv}$	Convective Heat Flux from Air to Body	$\frac{Btu}{ft^2 \cdot sec}$
$R$	Radius of a Sphere or Circle	ft
$r$	Radius of Orbit	ft
$r_a$	Apoapsis Length	ft
$r_c$	Circular Orbit Radius Following Transfer	ft
$r_i$	Circular Orbit Radius Prior to Transfer	ft
$r_p$	Periapsis Length	ft
$r_{\odot}$	Distance From Sun to Earth	ft

LIST OF SYMBOLS (continued)

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$r_g$	Distance From Sun to Mars	ft
S	Solar intensity at Mars	mW/in <sup>2</sup>
s	Segment Length	ft
T	Thrust	lbf
T <sub>s</sub>	Temperature at the Stagnation Point of the Body	Rankine
t	Duration of Time for Engine Burn	sec
$u_\infty$	Freestream Velocity	ft/sec
V	Orbit Velocity	ft/sec
V <sub>oo</sub>	Burn-out Velocity	ft/sec
V <sub>per</sub>	Velocity at Periapsis	ft/sec
V <sub>∞'⊕</sub>	Velocity of Spacecraft Relative to Earth	ft/sec
V <sub>∞'♂</sub>	Velocity of Spacecraft Relative to Mars	ft/sec
V <sub>∞'☉</sub>	Velocity of Spacecraft Relative to the Sun	ft/sec
V <sub>⊕</sub>	Velocity of the Earth Relative to the Sun	ft/sec
V <sub>♂</sub>	Velocity of Mars Relative to the Sun	ft/sec
V <sub>∞</sub>	Hyperbolic Excess Speed	ft/sec
ΔV	Propulsive Burn	ft/sec
ϕ	Volume	ft <sup>3</sup>
ϕ <sub>liq</sub>	Volume of Liquid Hydrogen	ft <sup>3</sup>

LIST OF SYMBOLS (continued)

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$V_{LO_2}$	Volume of Liquid Oxygen	ft <sup>3</sup>
$V_{NH_2}$	Volume of Monomethyl Hydrazine	ft <sup>3</sup>
$V_{N_2O_4}$	Volume of Nitrogen Tetroxide	ft <sup>3</sup>
$\alpha$	Angle of Attack	degrees
$\Gamma$	Angle between sun's rays and the normal to the panel	degrees
$\Delta$	Sweep Angle of Wing Leading Edge	degrees
$\delta$	Deadweight Ratio	---
$\epsilon$	Emissivity of the Fluid	---
$\eta$	Efficiency of the Solar Array Panel	percent
$\theta$	Angle between radii	degrees
$\theta_v$	Cone Half-angle	degrees
$\lambda$	Payload Ratio	---
$\mu_{\oplus}$	Gravitational Parameter for the Earth	ft <sup>3</sup> /sec <sup>2</sup>
$\mu_{\circlearrowright}$	Gravitational Parameter for Mars	ft <sup>3</sup> /sec <sup>2</sup>
$\mu_{\odot}$	Gravitational Parameter for the Sun	ft <sup>3</sup> /sec <sup>2</sup>
$\nu_0$	Argument of Periapsis	degrees
$\rho_{\infty}$	Freestream Density	slug/ft <sup>3</sup>
$\rho_{LH_2}$	Density of Liquid Hydrogen	lbm/ft <sup>3</sup>
$\rho_{LO_2}$	Density of Liquid Oxygen	lbm/ft <sup>3</sup>



LIST OF SYMBOLS (continued)

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$\sigma_{sb}$	Stefan-Boltzman Constant	$\frac{\text{BTU}}{\text{ft}^2 \cdot \text{sec} \cdot \text{R}}$
$\tau$	Period of an Orbit	hours
$\tau_{EM}$	Period of the Earth-Mars Transfer Orbit	hours

## INTRODUCTION

In Volume One of the report on the Multiple Exploratory Probe System (MEPS) the final design is presented. However, in most cases the reasoning or rationale behind many of the design decisions are not given to the reader. Volume Two alleviates this problem by presenting calculations, derivations, computer programs, and additional arguments for the final design.

Several areas are discussed in this volume. First, the structural mass calculation and the structural analysis are presented. The calculation of the propulsive burns for the Earth-Mars transfer are shown, as well as the burns required for the aerobraking and the satellite. The secondary and main propulsion systems are studied to obtain the mass of propellant (oxidizer and fuel) required for the entire trip and the size of the propellant tanks; engine analysis is also presented in more detail in this volume. The necessary equations for the aerobraking program are derived, and the lander system is analyzed for determination of mass and stagnation temperature. In addition the recovery system is optimized. Appendices present various programs and supplemental plots.

## STRUCTURAL MASS CALCULATION

The total structural mass was calculated by summing the mass of each component of the structural system. The structural mass will be divided into stringers, bulkheads, the cylindrical shells, the caps on the ends of the modules, and the connectors. Aluminum will be used for the structural material (specific weight = 173.4 lb/ft<sup>3</sup>).

### Stringers

There are 36 longitudinal stringers arranged circumferentially along the length of each module. Each stringer is assumed to have a cross-sectional area of 1 in<sup>2</sup> = 0.006944 ft<sup>2</sup>. Therefore, the total stringer mass is

$$36 \times (\text{module length}) \times (0.006944 \text{ ft}^2) \times (173.4 \text{ lbm/ft}^3)$$
$$\text{mass (lbm)} = 43.365 \times \text{length}$$

### Bulkheads

The bulkheads are I-beams located along the interior circumference. The cross-sectional area of this beam is 1.25 in<sup>2</sup> = 0.008681 ft<sup>2</sup>. For the mass of each bulkhead,

$$(2 \cdot \pi) \times (12.5 \text{ ft}) \times (0.008681 \text{ ft}^2) \times (173.4 \text{ lbm/ft}^3)$$
$$\text{mass} = 118.261 \text{ lbm (per bulkhead)}$$

The  $(2 \cdot \pi \cdot 12.5 \text{ ft})$  term is the circumference of the bulkhead, where 12.5 ft is the radius of the module.

### Cylindrical Shells

The structural mass of the cylinder obviously depends on the length of the module. Using a thickness of 1/4 in (.020833 ft),

$$(2 \cdot \pi \cdot 12.5 \text{ ft}) \times (0.020833 \text{ ft}) \times (173.4 \text{ lbm/ft}^3) \times \text{length}$$
$$\text{mass (lbm)} = 283.83 \times \text{length}$$

### End Caps on Cylinders

The end cap is a plate modelled on the end of the module, and two caps are designed for each module. Using the same thickness as the cylindrical shells (.020833 ft),

$$2[\pi \cdot (12.5 \text{ ft})^2 \times (0.020833 \text{ ft}) \times (173.4 \text{ lbm/ft}^3)]$$
$$\text{mass} = 3547.98 \text{ lbm}$$

### Connectors

Each connector is one inch thick and are five feet in length (with the exception of the aerobrake connector, which is 10 feet long). The design of the MEPS vehicle requires a total of four 5-foot long and one 10-foot long connectors; only two 5-foot long connectors and the 10-foot long connector will remain on the vehicle during aerobraking. For the total mass of the connectors,

$$(2 \cdot \pi \cdot 12.5 \text{ ft}) \times (0.08333 \text{ ft}) \times (\text{length}) \times (173.4 \text{ lbm/ft}^3)$$
$$\text{mass for Earth configuration (lbm)} = 1135 \times 30 \text{ ft} = 34050$$
$$\text{mass for Mars configuration (lbm)} = 1135 \times 20 \text{ ft} = 22700$$

## Total Vehicle Mass

Using the individual calculations given above, the total mass of the MEPS vehicle can be determined:

### Equatorial Lander:

stringers	(43.37 lbm/ft · 30 ft)	=	1305 lbm
bulkhead	(4 · 118.26 lbm)	=	475 lbm
cylinder	(283.83 lbm/ft · 30 ft)	=	8515 lbm
caps		=	3550 lbm
total mass		=	13845 lbm

### Satellite/CIC:

stringers	(43.37 lbm/ft · 25 ft)	=	1085 lbm
bulkhead	(3 · 118.26 lbm)	=	355 lbm
cylinder	(283.83 lbm/ft · 25 ft)	=	7100 lbm
caps		=	3550 lbm
total mass		=	12100 lbm

### Secondary Propulsion:

stringers	(43.37 lbm/ft · 10 ft)	=	435 lbm
bulkhead	(2 · 118.26 lbm)	=	240 lbm
cylinder	(283.83 lbm/ft · 10 ft)	=	2840 lbm
caps		=	3550 lbm
total mass		=	7065 lbm

### Main Propulsion:

stringers	(43.37 lbm/ft · 70 ft)	=	3040 lbm
bulkhead	(5 · 118.26 lbm)	=	595 lbm
cylinder	(283.83 lbm/ft · 70 ft)	=	19870 lbm
caps		=	3550 lbm
total mass		=	27055 lbm

### Polar Lander:

stringers	(43.37 lbm/ft · 50 ft)	=	2170 lbm
bulkhead	(6 · 118.26 lbm)	=	710 lbm
cylinder	(283.83 lbm/ft · 50 ft)	=	14195 lbm
caps		=	3550 lbm
total mass		=	20625 lbm

### Total Vehicle (Structural Mass):

Earth departure	=	111,500 lbm
Mars arrival	=	67,100 lbm

## PROPOSED STRENGTH ANALYSIS

The following analysis approach will be used to find (a) the proper material for the stringers, module skin, and bulkheads; (b) the correct material and thickness of the connectors; and (c) the material composition and overall number of pins for module connections.

A complete finite element model for the MEPS vehicle is being placed on MSC/pal. This model includes the proper lengths of the modules and connectors (using aluminum as the initial material for all components) and the payload mass inside each module. The force cases for dynamic analysis of the system are being obtained from the propulsion and orbital insertion analyses. These cases will include thrust from the initial departure burn, acceleration and thrust from the deceleration burn to insert into Martian orbit, and the acceleration and drag from the maximum-force aerobraking pass. The model and forces will be translated into a NASTRAN input file using a program available on MSC/pal2.

NASTRAN will be executed using the input cases outlined above and the output will be evaluated; to utilize this evaluation, the following procedure should be applied. If the evaluation shows that the structural integrity of the connector is in doubt, two cases should be run. First, use aluminum for the module and a metal matrix composite (MMC) for the connector;

second, use aluminum for both components but increase the thickness of the connector. If the evaluation shows question in the module's strength, the following runs will be conducted sequentially until a proper solution is reached: (1) increase the stiffener and bulkhead sizes (areas); (2) change the material of the module to MMC and use the original stiffener and bulkhead sizes alone; (3) use aluminum for all components but increase the thickness of the module; and (4) increase the sizes of all components while still using aluminum.

Once the proper sizes for a minimum stress on the entire vehicle have been determined, an analysis technique outlined by R. E. Peterson in Stress Concentration Design Factors (Wiley Press, 1974) will be used to determine the stress in a pin hole of a connector. From this information, strength or failure of the connection can be determined. If the pin shows failure, then the material of the pins must be changed; however, if the pin hole indicates failure, then the thickness around the hole will be increased. Also from this stress information, and some assistance from Dr. W. A. Foster, Jr. of Auburn University, the minimum amount of pin connections can be determined.

## ANALYSIS OF EARTH-MARS TRAJECTORY

After MEPS has been moved to the ecliptic plane, a propulsive burn will be conducted to start the vehicle on the journey to Mars. A Hohmann (minimum energy) transfer will be employed to save fuel, although the time of flight will be extended. This section will introduce the analysis of the Earth-Mars trajectory, including the magnitudes of the required propulsive burns and the determination of the time of flight.

### Earth Departure

The semi-major axis of the transfer is defined as

$$a_r = \frac{1}{2} \cdot (r_{\oplus} + r_{\odot}) = \frac{1}{2} \cdot (4.908 \text{ E11} + 7.477 \text{ E11}) \text{ ft}$$

$$a_r = 6.193 \text{ E11 ft}$$

To determine the propulsive burns for the start of the transfer, the approach to the problem must be considered. The transfer between Earth and Mars will require determination of relative velocities, as the situation is not a simple transfer between two orbits about the same body. Now two bodies must be taken into account--the Earth and the sun.

The velocity of the vehicle relative to the sun can be expressed in terms of velocities about earth:

$$V_{\infty/\odot} = V_{\oplus} + V_{\infty/\oplus} = V_{\oplus} + V_{\infty}$$

where  $V_{\infty}$  is the hyperbolic excess speed. This speed can be expressed as

$$V_{\infty} = V_{\infty/\odot} - V_{\oplus}$$



A fundamental equation used in astrodynamics is the Vis-Viva equation. This equation allows the calculation of a velocity at a point in an orbit if the parameters of the orbit are known:

$$v = \sqrt{\mu \cdot \left( \frac{2}{r} - \frac{1}{a} \right)}$$

where  $\mu$  is the gravitational parameter of the body (planet) which influences the vehicle,  $r$  is the distance from the body where the propulsive burn is applied, and  $a$  is the semi-major axis of the transfer ellipse.

Applying the Vis-Viva equation to find the velocity of the vehicle relative to the sun, the required inputs are

$$\mu_{\odot} = 4.687 \text{ E}21 \text{ ft}^3/\text{sec}^2$$

$$r_{\oplus} = 4.908 \text{ E}11 \text{ ft}$$

$$a = 6.193 \text{ E}11 \text{ ft}$$

The resulting velocity is

$$V_{\infty \odot} = 107,383.46 \text{ ft/sec}$$

The velocity of the Earth is calculated by assuming the Earth is in a circular orbit about the sun. Using the equation for velocity in a circular orbit,

$$V_{\oplus} = \sqrt{\frac{\mu_{\odot}}{r_{\oplus}}}$$

and the appropriate values given above,

$$V_{\oplus} = 97,722.64 \text{ ft/sec}$$

The hyperbolic excess speed can now be determined:

$$V_{\infty} = V_{\infty \odot} - V_{\oplus}$$

$$V_{\infty} = (107,383.46 - 97,722.64) \text{ ft/sec}$$

$$V_{\infty} = 9660.82 \text{ ft/sec}$$

The burnout velocity is expressed as

$$V_{b.} = \sqrt{v_{\infty}^2 + \frac{2\mu_{\oplus}}{r}}$$

At an radius of 22,567,193.75 ft (3714.153 n mi) from the center of the Earth, with

$$\mu_{\oplus} = 1.408 \text{ E16 ft}^2/\text{sec}^2$$

the burnout velocity has the value of

$$V_{b.} = 36621.86 \text{ ft/sec}$$

The velocity of the vehicle relative to the Earth is given by the expression for circular velocity, using the gravitational parameter for the Earth and the radius of 22,567,193.75 ft:

$$V_{\infty, \oplus} = 24,978.28 \text{ ft/sec}$$

With this value and the value for the burnout velocity the required propulsive burn can be calculated:

$$\Delta V = V_{b.} - V_{\infty, \oplus}$$

$$\Delta V = 11643.58 \text{ ft/sec}$$

### Mars Capture

The equations for analyzing the propulsive burn to allow capture by Mars are similar to those used for the Earth departure analysis.

The hyperbolic excess speed is expressed as

$$|V_{\infty}| = |V_{\infty, \odot} + V_{\delta}|$$

The velocity of the spacecraft relative to the sun is determined by the Vis-Viva equation, with the following inputs:

$$\mu_{\odot} = 4.687 \text{ E21 ft}^2/\text{sec}^2$$

$$r_{\delta} = 7.477 \text{ E11 ft}$$

$$a_r = 6.193 \text{ E11 ft}$$

Substitution into the Vis-Viva equation yields the value of this velocity:

$$V_{\infty, \odot} = 70,489.39 \text{ ft/sec}$$

The velocity of Mars about the sun is calculated under the assumption that Mars is moving in a circular orbit about the sun:

$$V_{\odot} = \sqrt{\frac{\mu_{\odot}}{r_{\odot}}}$$

$$V_{\odot} = 79174.22 \text{ ft/sec}$$

Thus the hyperbolic excess speed has the value of

$$V_{\infty} = 8684.83 \text{ ft/sec}$$

The magnitude of the propulsive burn required for Mars capture is expressed as

$$\Delta V = V_{\infty, \oplus} - V_{\infty, \odot}$$

where  $V_{\infty, \odot}$  is the velocity of the spacecraft at the point of closest approach to Mars. Because the vehicle is on a hyperbolic approach to Mars the velocity of the vehicle relative to Mars is expressed with the same equation as the burnout velocity used for the Earth departure:

$$V_{\infty, \oplus} = \sqrt{V_{\infty}^2 + \frac{2\mu_{\oplus}}{r_p}}$$

$$\mu_{\oplus} = 1.5066 \text{ E15 ft}^2/\text{sec}^2$$

$$r_p = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$V_{\infty, \oplus} = 17,643.74 \text{ ft/sec}$$

The elliptic orbit of the vehicle after Mars capture is defined as 1,640,500 ft (270 n mi) x 108,151,603 ft (17800 n mi).

Given the radius of Mars,

$$r_{\oplus} = 11,134,073.5 \text{ ft}$$

the semi-major axis of the ellipse can be calculated using the following:

$$a = \frac{1}{2} \cdot [(1,640,500 \text{ ft} + r) + (108,151,603 \text{ ft} + r)]$$

$$a = 66,030,125 \text{ ft} \quad (10,867.37 \text{ n mi})$$

At periapsis the velocity of the vehicle is determined (using the Vis-Viva equation) to have the following value:

$$V_{p.e.} = 14,596.52 \text{ ft/sec}$$

Using the expression for the propulsive burn required for Mars capture,

$$\Delta V = V_{p.e.} - V_{p.m.}$$

$$\Delta V = 3047.22 \text{ ft/sec}$$

#### Time of Flight

The time required for the transfer from Earth to Mars can be calculated from the period of the elliptical orbit. The orbit period is defined as

$$\tau = \sqrt{\frac{2\pi}{\mu_{\odot}}} a^{3/2}$$

Substitution of the appropriate values yields

$$\mu_{\odot} = 4.687 \text{ E}21 \text{ ft}^3/\text{sec}^2$$

$$a_{\tau} = 6.193 \text{ E}11 \text{ ft}$$

$$\tau = 44,728,466.79 \text{ seconds} = 517.7 \text{ days}$$

Since this period is the time of flight for an entire elliptic orbit, the required time to reach Mars is one-half the period, or

$$\tau_{EM} = 258.85 \text{ days}$$

## PROPELLANT ANALYSIS FOR THE SECONDARY PROPULSION SYSTEM

The secondary propulsion system will be employed upon approach to Mars. The purpose of this engine system is to slow down MEPS to obtain an elliptic orbit about Mars and to help in the final stages of orbit circularization. The analysis presented in this section concerns the calculation of the propellant mass (oxidizer and fuel) for each  $\Delta V$  burn and the required volume of the fuel tanks.

The masses of each MEPS module which will be placed into orbit about Mars are presented below. These values do not include propellant.

Aerobrake	12,000 lbm
Equatorial Lander/Rover	65,000 lbm
Satellite System	3,500 lbm
CIC	3,000 lbm
Structure	67,100 lbm

The total mass of MEPS excluding propellant is 140,000 lbm.

Four  $\Delta V$  burns will be required during the circularization process at Mars. The first burn will place the MEPS vehicle system into a highly elliptic orbit about Mars. During the appropriate orbit a second  $\Delta V$  burn will be performed at the orbit apoapsis to lower the periapsis into the Martian atmosphere. Following aerobraking, a third burn moves the periapsis out of

the atmosphere, and the fourth  $\Delta V$  burn will provide final orbit circularization by adjusting the apoapsis. The calculated values of these burns will now be presented.

Burn 1 (orbit capture)	3047.90 ft/sec
Burn 2 (lower periapsis)	75.1214 ft/sec
Burn 3 (raise periapsis)	301.6568 ft/sec
Burn 4 (adjust apoapsis)	12.1129 ft/sec

The secondary propulsion system will use three engines similar to the Space Shuttle Orbiting Maneuvering System (OMS). These engines have a specific thrust of 280 seconds. With the help of an equation relating the burn to the specific thrust and initial and final masses, the propellant mass (initial mass before burn) required for each burn can be calculated.

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln(M_i / M_f)$$

$$M_i = M_f \cdot e^{(\Delta V / I_{sp} \cdot g_0)}$$

Use of this equation will begin the propellant analysis required for the secondary propulsion system; for each burn the mass of the propellant (oxidizer and fuel) must be determined. The final vehicle mass is 140,000 lbm. Substitution of this mass and the value for the  $\Delta V$  burn (12.1129 ft/sec) results in the total vehicle mass prior to apoapsis adjustment (or, following periapsis raising):

$$M_{i,4} = M_{f,3} = (140,000 \text{ lbm}) \cdot e^{(12.1129 / (280 \cdot 32.174))}$$

$$M_{f,3} = 140,188.36 \text{ lbm}$$

The propellant mass required for apoapsis adjustment is determined simply by subtracting the total mass following adjustment from the mass prior to the maneuver:

$$M_{p, \Delta} = (140,188.36 - 140,000) \text{ lbm}$$

$$M_{p, \Delta} = 188.36 \text{ lbm}$$

Following this procedure, the propellant mass breakdown is given in the accompanying table.

#### Propellant Mass Required For $\Delta V$ Burns

<u>Burn Number</u>	<u><math>\Delta V</math> (ft/sec)</u>	<u>Total Vehicle Mass (lbm)</u>	<u>Propellant Mass (lbm)</u>
1	3047.90	205,026.35	58,850.44
2	75.12	146,175.91	1213.86
3	301.66	144,962.05	4773.69
4	12.11	140,188.36	188.36

Note that the total vehicle mass on approach to Mars is determined to be 205,026.35 lbm. The total mass of the propellant used during the  $\Delta V$  burns is 65,026.35 lbm.

To calculate the mass of oxidizer ( $N_2O_4$ ) and fuel (MMH) the oxidizer/fuel ratio (1.65) will be used. Every 1.65 parts of oxidizer is accompanied by 1 part of fuel, for a total of 2.65 parts of propellant. From the ratio,

$$\text{mass of } N_2O_4 = (1.65/2.65) \cdot M_p$$

$$\text{mass of MMH} = (1.0/2.65) \cdot M_p$$

For the total mass of propellant given above (65,030 lbm),

$$\text{mass of } \text{N}_2\text{O}_4 = 40,491 \text{ lbm}$$

$$\text{mass of MMH} = 24,540 \text{ lbm}$$

The volumes of the nitrogen tetroxide and mono-methyl hydrazine are calculated with the equation for density:

$$\text{Volume} = \text{mass/density}$$

where

$$\rho_{\text{N}_2\text{O}_4} = 85.50 \text{ lbm/ft}^3$$

$$\rho_{\text{MMH}} = 53.83 \text{ lbm/ft}^3$$

Using the oxidizer and fuel masses given above, the respective volumes are

$$V_{\text{N}_2\text{O}_4} = 473.57 \text{ ft}^3$$

$$V_{\text{MMH}} = 455.88 \text{ ft}^3$$

The shape of the tanks can now be determined. If cylindrical tanks (25 ft. diameter) are used, the length may be calculated using

$$l = V/(\pi \cdot R^2)$$

For the nitrogen tetroxide,

$$l_{\text{N}_2\text{O}_4} = 0.965 \text{ foot}$$

and for the mono-methyl hydrazine,

$$l_{\text{MMH}} = 0.929 \text{ foot}$$

Note that the required length is only one foot, which is very impractical.

Spherical tanks will now be considered. For the radius of the tank,

$$R = \left( \frac{3 \cdot V}{4 \cdot \pi} \right)^{1/3}$$



Calculation of the sphere radii for the oxidizer and fuel tanks yields, respectively,

$$R_{N_2O_4} = 4.835 \text{ feet}$$

$$R_{MMH} = 4.774 \text{ feet}$$

From this analysis the spherical tank is the optimum design. The tanks can be contained side-by-side within the cylindrical compartment of the MEPS vehicle; the radii of the spheres may be increased to five feet for ease of construction, thus providing for a compartment 10 feet in length with sufficient room for the 3 engines.

## ANALYSIS OF THE MAIN PROPULSION SYSTEM

The following section contains the analysis of the engines considered for MEPS. Four engines were compared on the basis of thrust, specific thrust, weight, and burntime; the Space Transportation Main Engine (STME) was chosen for the main propulsion system. The necessary data (propellant mass and volume, module length/tank size) is presented for the STME, and a staging analysis is shown.

### Engine Comparisons

Four engines were compared in the analysis of the main propulsion system--the J-2, RL-10-A-1, Space Shuttle Main Engine (SSME), and the Space Transportation Main Engine (STME). The J-2 was used for the third stage of the Saturn rockets. The original RL-10 engine was used for the early Saturn rockets, and has seen use on the Titan; the RL-10-A-1 is more of an engine design as this engine has not been produced. The SSME is currently in operation on the Space Shuttle orbiters, while the STME is a second generation SSME-based engine which also has not gone into production.

By using Newton's Second Law, and assuming the initial mass (engine and propellant) to be equal, the four engine candidates can be compared:

$$\Sigma F = T = m \cdot a$$

$$T = \frac{M \cdot dV}{dt}$$

Rewriting the latter equation as an expression for time,

$$dt = \frac{M \cdot dV}{T}$$

$$\Delta t = \frac{M \cdot \Delta V}{T}$$

This final equation is used for calculation of the burn time of each engine. Note that the comparisons were made on the basis of thrust levels of approximately equal magnitudes; for approximately 450,000 lb of thrust the appropriate number of engines must be considered.

<u>J-2</u>	Thrust	200,000 lbf x 2 engines
	Isp	418 seconds
	Mass	3480 (6960) lbm
	Burn time	635.53 seconds

$$t = \frac{(702588.2 \text{ lbm}) \cdot (11641.26 \text{ ft/sec})}{2 \cdot (200000 \text{ lbf}) \cdot (32.174 \text{ lbm-ft/lbf-sec}^2)}$$

total mass leaving orbit = 702588.2 lbm  
 delta-V burn required = 11641.26 ft/sec

<u>RL-10-A-1</u>	Thrust	15,000 lbf x 29 engines
	Isp	433 seconds
	Mass	298 (8642) lbm
	Burn time	584.40 seconds

<u>SSME</u>	Thrust	470,000 lbf x 1 engine
	Isp	433 seconds
	Mass	6700 lbm
	Burn time	540.90 seconds

<u>STME</u>	Thrust	435,000 lbf x 1 engine
	Isp	449 seconds
	Mass	7455 lbm
	Burn time	584.40 seconds

A short duration burn time is desirable because of the decreased risk of course deviation during the burn (ref 3). The shortest burn time is achieved by the engine with the greatest

thrust; by the above data this engine is the SSME. The next lowest burn time occurs with the RL-10-A-1 engines and the STME. The RL-10-A-1 was found to be unfeasible since 29 units are required to obtain a comparable thrust level. Although the STME has a longer burn time than the SSME, the STME is designed to be more reliable and less expensive than the SSME (ref 2); thus the Space Transportation Main Engine is selected over the Space Shuttle Main Engine.

Comparison of the STME to the J-2 engine is based on thrust, weight, burn time, and design. The two J-2 engines produce 400,000 lbf of thrust and weigh 6960 lbm; the STME weighs slightly more but produces greater thrust (435,000 lbf). The burn time of the STME is considerably less than that of the J-2. In addition, the STME is being designed specifically for reusability and space applications (one design of the STME nozzle expands the flow at the exit to the optimum pressure for operation in the vacuum of space).

#### STME Engine Information

The engine data required for the analysis of the MEPS mission will now be presented. Some of the engine particulars have been previously stated.

Isp = 449 seconds  
Thrust = 435,000 lbf  
Mass = 7455 lbm  
Oxidizer/Fuel Ratio = 6.0  
Area Ratio = 55/141

The initial mass of the MEPS vehicle prior to leaving Earth may be calculated using the following analysis:

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln(M_i / M_r)$$

$$M_i = M_r \cdot e^{(\Delta V / (I_{sp} \cdot g_0))}$$

The vehicle mass before the engines and the polar lander system are released is 381,505 pounds (see previous section). For a required delta-V burn of 11641.26 ft/sec to begin the Earth-Mars transfer,

$$M_i = (381,505 \text{ lb}) \cdot e^{(11641.26 / ((449) \cdot 32.174))}$$

$$M_i = 854,020 \text{ lb}$$

The mass of the propellant is the difference between the initial mass (vehicle plus propellant) and the final mass (vehicle only). For the given conditions,

$$M_p = M_i - M_r$$

$$M_p = 854,019.4 \text{ lb} - 381,501.4 \text{ lb}$$

$$M_p = 472,518 \text{ lb}$$

The oxidizer/fuel ratio for the engine is given as 6.0. Every six parts of liquid oxygen must be accompanied by one part of liquid hydrogen; thus a total of seven parts of oxidizer and fuel will be available. Using this development, the masses of the liquid oxygen and liquid hydrogen can be determined.

$$M_{L_{O_2}} = 6/7 \cdot M_p = 6/7 \cdot 472,518 \text{ lb} = 405,015.43 \text{ lb}$$

$$M_{L_{H_2}} = 1/7 \cdot M_p = 67,502.57 \text{ lb}$$

Using the densities of the oxidizer and fuel, and the relationship between density and volume, the volumes of the liquid

oxygen and liquid hydrogen can be calculated. Knowing these volumes will allow the sizing of the fuel and oxidizer tanks.

$$\rho_{L_{O_2}} = 71.07 \text{ lb/ft}^3 \text{ at } -297^\circ \text{ F}$$

$$\rho_{L_{H_2}} = 4.42 \text{ lb/ft}^3 \text{ at } -423^\circ \text{ F} \quad (5:4-23)$$

The density is defined as the mass per unit volume. Therefore,

$$V_{L_{O_2}} = 405,015 \text{ lb} \cdot (1 \text{ ft}^3 / 71.07 \text{ lb}) = 5699 \text{ ft}^3$$

$$V_{L_{O_2}} = 42,727.7 \text{ gallons}$$

$$V_{L_{H_2}} = 67,503 \text{ lb} \cdot (1 \text{ ft}^3 / 4.42 \text{ lb}) = 15,271.95 \text{ ft}^3$$

$$V_{L_{H_2}} = 114,499.8 \text{ gallons}$$

A pressure vessel is normally spherical, or cylindrical with hemispherical ends. The diameter of the MEPS vehicle must be considered to determine which type tank will hold the liquid oxygen and hydrogen. For the diameter of 25 feet, the volume of a spherical tank is

$$V = 4/3 \cdot \pi \cdot (12.5 \text{ ft})^3 = 8181.23 \text{ ft}^3$$

This volume falls between the required volumes for the oxidizer and fuel. Thus, a spherical tank will be employed for the liquid oxygen, and the cylindrical/hemispherical tank will be used for the liquid hydrogen.

For the calculated volume of liquid oxygen the corresponding tank size is determined to be

$$\text{volume} = 5699 \text{ ft}^3 = 4/3 \cdot \pi \cdot R^3$$

$$R = 11.08 \text{ ft}$$

If boil-off of the liquid oxygen (12.0 lb/hr) is considered, the actual size of the tank must be increased to account for the

expendable oxidizer. Based on a thirty day transport and construction period, the volume of the LO<sub>2</sub> lost to boil-off is

$$\text{mass} = (720 \text{ hrs}) \cdot (12.0 \text{ lb/hr}) = 8640 \text{ lb}$$

$$\text{volume} = (8640 \text{ lb}) \cdot (1 \text{ ft}^3 / 71.07 \text{ lb}) = 121.57 \text{ ft}^3$$

This additional volume yields an increase in the diameter of the spherical tank to 22.5 feet.

The volume of liquid hydrogen is much larger than that of the liquid oxygen and, as mentioned previously, a cylindrical tank with hemispherical endcaps will be required. For the tank to fit snugly inside the MEPS vehicle (diameter of 25 feet), the length of the tank can be calculated:

$$\text{volume} = 4/3 \cdot \pi \cdot (12.5)^3 + \pi \cdot (12.5)^2 \cdot h = 15,271.95 \text{ ft}^3$$

$$\text{length} = h = 14.4 \text{ ft}$$

The tank size will increase under consideration of boil-off. The rate for LH<sub>2</sub> is 18.0 lb/hr, and for the same thirty day period used earlier,

$$\text{mass} = (720 \text{ hrs}) \cdot (18.0 \text{ lb/hr}) = 12,960 \text{ lb}$$

$$\text{volume} = (12,960 \text{ lb}) \cdot (1 \text{ ft}^3 / 4.42 \text{ lb}) = 2932.13 \text{ ft}^3$$

The change in the length of the tank is now determined:

$$\text{length} = h = (15,271.95 + 2932.12 - 8181.23) / 490.87$$

$$h = 20.45 \text{ ft}$$

Since the endcaps have a radius of 12.5 feet, the total length of the LH<sub>2</sub> tank is

$$25 \text{ ft} + 20.45 \text{ ft} = 45.45 \text{ ft}$$

If the two tanks are mounted bulkhead to bulkhead, the total length of the main propulsion module is

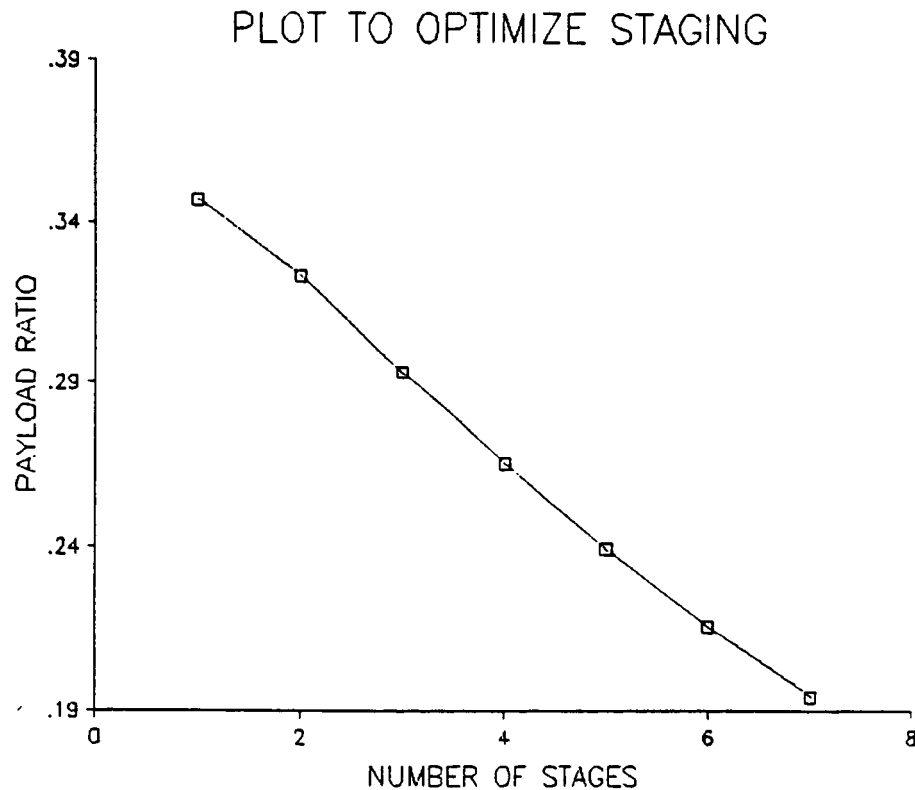
$$45.45 \text{ ft} + 22.5 \text{ ft} = 67.95 \text{ ft}$$

### Staging Analysis

In order to determine the optimum number of stages for Earth departure, the following equation is used:

$$\lambda = [e^{-\Delta v / c} - \delta]^n$$

The results of the analysis performed on the STME are plotted below. From this plot, one stage is shown to be the optimum configuration for the engines.





## AEROBRAKE ANALYSIS

The aerobrake will be used for orbit circularization about Mars. This section will present the determination of the weight of the aerobrake and the calculation of the propulsive burns required for the circularization analysis.

### Weight Determination

Using information provided by Bill Willcockson, OTV Program Manager at Martin Marietta Aerospace (Denver), the mass of an aerobrake can be sized with the aerobrake area. From values presented in reference 7,

$$\text{Area} = 142 \text{ ft}^2$$

$$\text{mass of rigid surface insulation (RSI)} = 401 \text{ lbm}$$

$$\text{mass of flexible surface insulation (FSI)} = 8890 \text{ lbm}$$

$$\text{structure weight} = 11032 \text{ lbm}$$

The weight of the RSI will remain 401 lbm since a diameter of 25 feet is used for the Martin Marietta brake as well as the proposed brake. The weight of the FSI will require a calculation. For the Martin aerobrake,

$$A = \frac{\pi}{4} [(142 \text{ ft})^2 - (25 \text{ ft})^2] = 15345.895 \text{ ft}^2$$

Obtaining a weight to area ratio,

$$\frac{\text{weight}}{\text{area}} = \frac{8890 \text{ lbm}}{15345.895 \text{ ft}^2} = .57931 \text{ lbm/ft}^2$$

For the MEPS aerobrake,

$$A = \frac{\pi}{4} [(95 \text{ ft})^2 - (25 \text{ ft})^2] = 6597.345 \text{ ft}^2$$

The weight of the FSI is calculated using the weight/area ratio previously determined:

$$W_{FSI} = (.57931 \text{ lbm/ft}^2) \cdot (6597.345 \text{ ft}^2)$$

$$W_{FSI} = 3821.895 \text{ lbm}$$

To determine the weight of the aerobrake structure, size the weights using a weight/area ratio:

$$\frac{W_{aerobrake}}{\frac{1}{4} \cdot (95 \text{ ft})^2} = \frac{11032 \text{ lbm}}{\frac{1}{4} \cdot (142 \text{ ft})^2}$$

$$W_{aerobrake} = 4937.701 \text{ lbm}$$

#### Propulsive Burns Used For Mars Orbit Circularization

Although aerobraking will be applied during the MEPS mission, complete orbit circularization will require propulsive burns using the secondary propulsion system. These burns must be considered for three different maneuvers: lowering the periapsis prior to aerobraking, and raising the periapsis and adjusting the apoapsis after braking. Calculation of each  $V$  will be made by applying the Vis-Viva equation:

$$V = \sqrt{\mu_g \cdot (2/r - 1/a)}$$

where  $r$  is the length of either the periapsis or apoapsis, measured from the center of Mars, and  $a$  is the semi-major axis of the elliptic orbit.

To decrease the periapsis, the burn will be applied at the apoapsis of the initial elliptic orbit about Mars. The desired periapsis altitude has been determined to be 314,976 ft (51.84 nautical miles). The lengths of the initial periapsis and apoapsis are 120,598,076.5 ft (19848.27 n mi) and 12,774,573.5 ft (2102.46 n mi), respectively, measured from the center of Mars.

The lengths of the semi-major axes of the two different elliptical orbits (same initial apoapsis, two different periapses) are determined:

$$r_a = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$a_{r_1} = \frac{1}{2}(r_a + r_p) = 66,686,325 \text{ ft (10,975.37 n mi)}$$

$$r_p = 11,449,049.5 \text{ ft (1884.31 n mi)}$$

$$a_{r_2} = 66,023,563 \text{ ft (10,866.29 n mi)}$$

Applying the Vis-Viva equation, the velocities at the apoapsis for each elliptical orbit are calculated:

$$V_1 = \left[ (1.5066 \text{ E15 } \frac{\text{ft}}{\text{sec}}) \cdot \left( \frac{2}{120598076.5 \text{ ft}} - \frac{1}{66686325 \text{ ft}} \right) \right]^{1/2}$$

$$V_1 = 1546.98 \text{ ft/sec}$$

$$V_2 = \left[ (1.5066 \text{ E15 } \frac{\text{ft}}{\text{sec}}) \cdot \left( \frac{2}{120598076.5 \text{ ft}} - \frac{1}{66023563 \text{ ft}} \right) \right]^{1/2}$$

$$V_2 = 1471.85 \text{ ft/sec}$$

The propulsive burn required to lower the periapsis is determined by taking the difference between the apoapsis velocities given above:

$$\Delta V = V_2 - V_1 = -75.1214 \text{ ft/sec}$$

The minus sign indicates the burn will be applied in the direction opposite that of the MEPS vehicle (retrofire).

The same analysis is performed for the burns to raise the periapsis and apoapsis. The necessary inputs and output are presented:

to raise the periapsis to 1,640,500 ft (270 n mi):

$$r_p = 12,717,736.74 \text{ ft (2093.11 n mi) -- from program}$$

$$r_p = 11,449,049.5 \text{ ft (1884.31 n mi)}$$

$$a_r = 12,083,391.5 \text{ ft (1988.71 n mi)}$$

$$r_p = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$a_r = 12,746,155.2 \text{ ft (2097.79 n mi)}$$

$$\Delta V = 301.6568 \text{ ft/sec}$$

to raise the apoapsis to 12,774,573.5 ft (2102.46 n mi) from  
the center of Mars:

$$r_p = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$r_a = 12,717,736.7 \text{ ft (2093.11 n mi)}$$

$$a_r = 12,746,155.2 \text{ ft (2097.79 n mi)}$$

$$r_a = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$a_r = 12,774,573.5 \text{ ft (2102.46 n mi)}$$

$$\Delta V = 12.1129 \text{ ft/sec}$$

## DERIVATIONS FOR THE AEROBRAKING PROGRAM

The program included in Appendix A is used to execute the iterations for the aerobraking process. With inputs concerning the orbit of a vehicle about Mars, and parameters of the aerobrake, the complete aerobraking passage can be analyzed. The output presents the time for aerobraking, the drag forces that act on the aerobrake, and the parameters of the final orbit.

The program requires several derivations--the location of the intersection of a circle (Mars atmosphere) and an ellipse (vehicle orbit); the length of segment between the intersection points (total distance travelled within the atmosphere); and the drag coefficient of the aerobrake.

### Intersection Points

The equations of an ellipse and a circle are given, respectively, as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$(x - ae)^2 + y^2 = r^2$$

where  $a$  is the semi-major axis and  $e$  is the eccentricity of the orbit; and  $a \cdot e$  is the location of the center of the circle representing the Martian atmosphere (i.e., the center of Mars). In addition, the trajectory equation, which gives the location of any point on the ellipse, is defined as

$$r = \frac{p}{1 + e \cdot \cos \gamma_0}$$

In the above equation all of the variables (semi-latus rectum, eccentricity, and argument of periapsis) are given parameters of an elliptic orbit.

Solving for  $y^2$  in the two equations,

$$y^2 = b^2 - \frac{b^2 x^2}{a^2}$$

$$y^2 = r^2 - (x - ae)^2$$

Equating the  $y^2$  terms,

$$b^2 - \frac{b^2 x^2}{a^2} = r^2 - (x - ae)^2$$

$$b^2 - \frac{b^2}{a^2} \cdot x^2 = r^2 - x^2 + 2aex - a^2 e^2$$

$$\left(1 - \frac{b^2}{a^2}\right) \cdot x^2 - 2aex + (b^2 - r^2 + a^2 e^2) = 0$$

Using the quadratic equation to solve for  $x$ ,

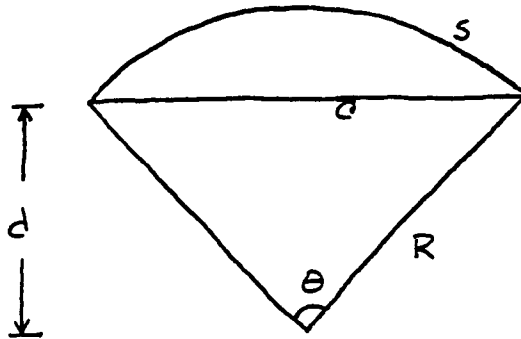
$$x = \frac{2ae \pm \sqrt{4a^2 e^2 - 4 \cdot (b^2 - r^2 + a^2 e^2) \cdot (1 - b^2/a^2)}}{2 \cdot (1 - b^2/a^2)}$$

Choosing only the negative value of the square root (due to the geometry of the problem), the  $x$ -location of the intersection points is known. Substitution of  $x$  back into an expression for  $y$  will yield the complete location of the points.

### Segment Length

The values of  $x$  and  $y$  obtained as the intersection points will be used in this derivation. The angle created between radii from the center of Mars is denoted as  $\theta$ ,  $s$  is the segment length,  $R$  is the radius,  $c$  is the chord of the arc, and  $d$  is the distance from the center of Mars to the  $x$ -position of the intersection points.

Using the geometry of an arc,



$$c = 2y$$

$$d = (x - ae)$$

$$R = (d^2 + y^2)^{1/2}$$

$$\theta = 2 \cdot \tan^{-1} \left( \frac{c}{2d} \right) = 2 \cdot \tan^{-1} \left( \frac{c/2}{d} \right) = 2 \cdot \tan^{-1} \left( \frac{y}{x-ae} \right)$$

Finally,

$$s = R\theta$$

Thus the segment is known, and this value can be used to determine the change in velocity due to drag forces during aerobraking.

#### Determination of the Drag Coefficient

From hypersonic equations for a cone, the drag coefficient is given as

$$C_D = 2 \sin^2 \theta_v + (1 - 3 \sin^2 \theta_v) \sin^2 \alpha$$

where

$$\theta_v = \text{cone half-angle}$$

$$\alpha = \text{angle of attack}$$

For the MEPS mission the design for zero angle of attack (using momentum wheels and the cone's inherent stability) allows cancellation of the second term. Therefore (1:681),

$$C_D = 2 \sin^2 \theta_v$$

## ANALYSIS FOR THE OBSERVATION SATELLITE

This section will contain the calculations of the propulsive burns required for orbital transfer of the satellite. In addition, the determination of the solar array panel (applicable to the CIC as well) will also be presented.

### Propulsive Burns

A transfer between a 1,640,500 ft (270 n mi) orbit and a 2,313,105 ft (327.64 n mi) orbit will be required to put the satellite into the observation orbit. For these calculations a Hohmann (minimum energy) transfer will be assumed.

Calculation of the altitudes is the first step:

$$r_g = 11,134,073.5 \text{ ft (1832.47 n mi)}$$

$$\begin{aligned} r_i &= (11,134,073.5 + 1,640,500) \text{ ft} \\ &= 12,774,573.5 \text{ ft (2102.46 n mi)} \end{aligned}$$

$$\begin{aligned} r_o &= (11,134,073.5 + 2,313,105) \text{ ft} \\ &= 13,447,178.5 \text{ ft (2005.25 n mi)} \end{aligned}$$

The gravitational parameter of Mars is given as

$$\mu_g = 1.5066 \text{ E15 ft}^3/\text{sec}^2$$

For a Hohmann transfer, first calculate the circular velocities of the two orbits:

$$v_c = \sqrt{\frac{\mu_g}{r}}$$

$$v_i = \sqrt{\frac{1.5066 \text{ E15 ft}^3/\text{sec}^2}{12774573.5 \text{ ft}}} = 10,859.7819 \text{ ft/sec}$$

$$v_o = \sqrt{\frac{1.5066 \text{ E15 ft}^3/\text{sec}^2}{13447178.5 \text{ ft}}} = 10,584.8341 \text{ ft/sec}$$



Now determine the semi-major axis of the transfer ellipse:

$$a_r = \frac{1}{2}(r_i + r_f) = 13,110,876 \text{ ft (2157.82 n mi)}$$

To obtain the propulsive burn required to leave the initial orbit, the Vis-Viva equation of astrodynamics (see trajectory analysis section) will be used. The result is

$$V_{r_1} = 10,998.2401 \text{ ft/sec}$$

The burn is found by subtracting the circular velocity from the velocity at the periapsis:

$$\Delta V_1 = V_{r_1} - V_i = 138.4 \text{ ft/sec}$$

The speed at the apoapsis and the propulsive burn required to achieve the final circular orbit are calculated in a similar manner:

$$V_{r_2} = 10,448.0164 \text{ ft/sec}$$

$$\Delta V_2 = 136.64 \text{ ft/sec}$$

Using these burns the mass of propellant required for the transfer can be calculated (the proper equation may be found in the section on the secondary propulsion system). First obtain the mass ratio for each  $\Delta V$ :

$$\left(\frac{M_i}{M_r}\right)_1 = 1.0155$$

$$\left(\frac{M_i}{M_r}\right)_2 = 1.0153$$

The final mass of the satellite in the observation is approximately 3500 lbm. Backing out the mass required by the second  $\Delta V$ ,

$$M_i = M_r + M_p$$

$$1.0153 \cdot M_r = M_r + M_p$$

$$0.0153 \cdot (3500 \text{ lbm}) = M_p$$

$$M_p = 53.55 \text{ lbm}$$

Obtaining the propellant mass used in the first  $\Delta V$  with the same calculations,

$$1.0155 \cdot M_p = M_r + M_p$$

$$0.0155 \cdot M_r = M_p$$

$$0.0155 \cdot (3500 \text{ lbm} + 53.55 \text{ lbm}) = M_p$$

$$M_p = 55.08 \text{ lbm}$$

The total mass of propellant is

$$M_{p_T} = M_p + M_p$$

$$M_{p_T} = 108.93 \text{ lbm}$$

#### Area for the Solar Array

The area of a solar array panel is calculated using

$$A_a = \frac{P_a}{S \cdot \eta \cdot F \cdot \cos \Gamma} \quad (\text{ref 5})$$

where

$P_a$  = available power (1500 watts)

$S$  = solar intensity at Mars (54.14 mW/ft<sup>2</sup>) (5:410)

$\eta$  =  $\frac{\text{power output of array}}{\text{power input of sun}}$  (9.6%)

$F$  = sum-total of array design and degradation factors

misc. assembly and degradation 0.95

radiation (for silicon cells) 0.74

configuration (flat plate array) 1.00

$F = 2.69$  (5:123-125)

$\Gamma$  = angle between sun's rays and the normal to the panel

$\Gamma = 0$

$\cos \Gamma = 1$

Calculation of the area yields

$$A_a = \frac{1500 \text{ W}}{(54.14 \text{ W/ft}^2) \cdot (0.096) \cdot (2.69) \cdot 1} = 108 \text{ ft}^2$$

## CALCULATION OF APPROXIMATE MASS OF MARS LANDER

To calculate the approximate mass of a Mars lander the masses of the major components of the lander must be estimated. These major components are (1) the Sample Return Vehicle (SRV); (2) the rovers; (3) the automated laboratory; (4) an upper and lower aeroshell; (5) a platform for the SRV to sit upon; (6) landing gear; and (7) a recovery system consisting of a solid rocket motor and three parachutes.

### Mass of the Sample Return Vehicle (SRV)

The mass of the solid rocket booster that will propel the SRV can be obtained from a program written to simulate the launch of a solid rocket booster; booster specifications include the fuel, payload, and planet of launch. This program, titled "Stages", can be used to study the effect of changing propellant mass on the final altitude and velocity achieved by the rocket.

To use "Stages" (listed in Appendix A), the following parameters for a launch must be known or assumed:

1. the combustion temperature of the propellant ( $^{\circ}\text{R}$ )
2. the density of the propellant ( $\text{lbm}/\text{in}^3$ )
3. the propellant cross-sectional diameter (feet)
4. the propellant burn rate ( $\text{in}/\text{sec}$ )
5. the specific heat ratio and perfect gas constant for the burning propellant

ORIGINAL PAGE IS  
OF POOR QUALITY

6. the radius of the planet (n.mi.), the latitude of launch site (degrees), the angular velocity of the planet surface, and the gravitational acceleration on the planet surface
7. the desired altitude after launch
8. the mass of the final payload to be put into orbit.

The first seven parameters were designated for a launch from a pole of Mars of a rocket propelled by the propellant DB/AP-HMX/AL (Double Base/Aluminum Perchlorate-Cyclotetramethylene Tetranitramine/Aluminum), selected for its high combustion temperature (6700 degrees Rankine) and burn rate (.55 in/sec). The rocket booster was designed to have a propellant cross-sectional area of 2.91667 feet and a deadweight ratio (ratio of booster non-propellant mass to total booster mass) of 0.12. The desired orbit was specified to be circular at an altitude of 270.0 nautical miles.

The eighth parameter (payload mass) was designed to be a lightweight vessel that would carry up to 100 lbm of Martian soil and air samples in a refrigerated chamber; on board the ship would be a small reaction control system and an aeroshell. The mass of this vehicle is estimated to be 1000.0 lbm (200 lbm for the refrigeration chamber, 500 lbm for the reaction control system, 100 lbm of samples, 50 lbm for the aeroshell, and 150 lbm for onboard guidance and control computers).

Once these parameters for the SRV launch have been specified, a "target burn time" (which is equal to the mass of propellant divided by the mass consumption rate) is entered into the "Stages" subroutine named "Launch". This subroutine is a numeri-

ORIGINAL PAGE IS  
OF POOR QUALITY

cal integration of the equation of motion for a single-stage rocket being launched in a gravity field, and will fire for the entire "target burn time" unless the propellant is completely consumed or the target altitude is reached. As the launch proceeds, the spacecraft is rotated through a "pitch program", arbitrarily selected to vary the direction of the rocket's weight vector as its altitude increases.

To optimize the propellant mass, and thus the initial mass of the SRV, five plots of data from "Stages" are constructed (see Appendix B). These plots show:

1. variation of final altitude with "target burn time" (Figure B.1)
2. variation of final velocity with "target burn time" (Figure B.2)
3. variation of payload ratio (payload mass/initial mass) with "target burn time" (Figure B.3)
4. variation of "excess mass" (mass excluding payload mass after launch) with "target burn time" (Figure B.4)
5. variation of final acceleration with "target burn time" (Figure B.5)

The first plot is used to determine a minimum value for "target burn time" (TBT) by observing that below a particular value for TBT the desired altitude is not reached due to insufficient propellant mass. The second plot is then used to find the range of values of TBT above the minimum altitude value for which the final velocity is at least sufficient to achieve a circular orbit at the design altitude.

The third, fourth, and fifth plots are used to find the optimum value of TBT from the range of TBT values determined with the first two plots. An optimum payload ratio can be selected from the third plot; an optimum "excess mass" can be selected from the fourth plot; and an optimum final acceleration can be selected from the fifth plot.

For the SRV the payload ratio was optimized because the ratio will yield a shorter and less massive booster than the booster for which the final acceleration is minimum. From Figure B.1 the desired altitude is reached for TBT greater than or equal to 350 seconds. From the second plot the required orbital velocity is achieved only for TBTs ranging from 100 to 460 seconds; therefore the optimum range of TBTs is between 350 and 460 seconds. From the third plot the TBT for the highest payload ratio is found to be 350 seconds. From the fourth plot excess mass is seen to be a minimum for the optimum range at TBT of 350 seconds. From Figure B.5 final acceleration is seen to be a maximum for the optimum range at TBT equal to 350 seconds.

The value used for the payload ratio of the SRV, based upon optimization by use of "Stages", is 0.068120. This ratio yields an initial SRV mass of approximately 14,700 lbm.

#### Mass of the Upper and Lower Aeroshell

The masses of the upper and lower aeroshell can be estimated by determining the approximate geometry of the aeroshells and selecting a material with which the aeroshells will be made. The material that is selected for the shell must be strong enough to

withstand large aerodynamic forces and aerothermodynamic heating incurred upon descent through the Martian atmosphere. The maximum temperature that will occur on the lander during descent will be located at the stagnation point of the vessel, which is located at the center of the lower aeroshell. The stagnation point temperature that the lander encounters at an altitude of 100 nautical miles (calculated as 1630 degrees Rankine in Appendix C) is used to determine the type of material to be used for the upper and lower aeroshell (as a first approximation).

An "outer blanket" of carbon-carbon heat-transfer resistant tiles, or a one-piece carbon-carbon sheet, will cover the bottom of the lower aeroshell. The inner part of the lower aeroshell and the upper aeroshell will be composed of CLAD 2014 aluminum alloy (density of .101 lbm/in<sup>3</sup>). Modeling the upper aeroshell as a conic frustrum 23 feet high, with a base diameter of 25 feet, a top diameter of seven feet, and a thickness of .30 inches, an approximate upper aeroshell volume of 233,989.7 cubic inches and a mass of approximately 23,633.0 lbm are determined. Modeling the lower aeroshell as a segment of a sphere with a base diameter of 25 feet, a segment height of five feet, and a thickness of .30 inches yields an approximate lower aeroshell volume of 20,722 cubic inches and a mass of approximately 2093.0 lbm.

#### Mass of Platform and Landing Gear

A metal disk twenty-five feet in diameter and one-half inch thickness is used to model the platform which the SRV, rovers, and autonomous laboratory sit upon; this platform has a volume

of 35,342 cubic inches. A strong material that can withstand the effects of the exhaust plume of the launching SRV is needed to comprise the platform; AM-355 stainless steel is chosen for its favorable resistance to high temperature and corrosion. AM-355 stainless steel has a density of .282 lbm/in<sup>3</sup>, so the mass of the platform is approximately 9966 lbm.

Each strut of the landing gear was modeled as a quarter-inch thick AM-355 stainless steel pipe, one foot in outer diameter and five feet long, fastened to a square AM-355 stainless steel pipe with sides four feet in length and a thickness of one-half inch. The total volume of each strut is 1857 cubic inches and the total mass of each strut is approximately 525 lbm. The landing gear system will consist of four struts so the total landing gear mass is 2100 lbm.

#### Mass of the Rovers and Automated Laboratory

Each rovers is to be no more massive than 2500 lbm (5000 lbm for the two rovers on each lander). The mass of the automated laboratory will not exceed 1000 lbm.

#### Mass of the Lander Recovery System

The mass of the recovery system for the lander was determined by use of a program written by D. Bell. The program calculates the optimum recovery system mass, consisting of one to six parachutes and a solid fuel retrorocket, for a generic ve-



hicle of specifiabile mass (minus recovery system mass) that is landing on the surface of Earth or Mars. The program inputs are

1. the mass of the vehicle without recovery system
2. the desired terminal velocity for the main parachutes
3. the number of parachutes desired
4. the specific impulse, thrust, and mass fraction of the solid rocket motor used for descent
5. the required velocity upon impact with the planet surface
6. the desired height above the ground at which a constant-velocity descent of the vehicle begins (the rocket is fired such that the thrust equals the weight of the vehicle -- "constant velocity falling height")

A set of plots can be obtained by making a series of runs of this program (see Figures D.1, D.2, D.3). These plots are used to optimize the main chute terminal velocity, impact velocity, constant velocity falling height, the mass of the parachute system, and the mass of the solid rocket motor required to land the vehicle.

The total (approximate) mass of a lander is found to be 58,500 lbm, excluding the mass of the recovery system. A set of runs of the optimizing program were made using this value of the vehicle mass; the results can be seen in the figures of Appendix D. The optimum main chute terminal velocity for this vehicle is determined to be 75 feet per second. The optimum impact velocity for the vehicle is 10 feet per second (assuming that the terminal velocity under consideration for the vehicle is the optimum value). The optimum constant velocity falling height is

five feet (assuming that the terminal velocity and impact velocity are optimum values. The optimum recovery system mass for the lander (using the aforementioned variables) is determined to be as follows:

parachute system mass = 1610 lbm  
solid rocket motor mass = 2639 lbm  
total recovery system mass = 4249 lbm

A more detailed breakdown of the mass of the recovery system is shown in Appendix D. This breakdown is the final output of the optimizing program.

Statement of Approximate Total Mass of Mars Lander

As stated at the beginning of this section, the approximate total mass of the Mars lander is the sum of the masses of its major components:

COMPONENT	MASS (lbm)
SRV	14,700
Upper Aeroshell	23,633
Lower Aeroshell	2,093
Platform	9,966
Landing Gear	2,100
Rover Systems	5,000
Laboratory	1,000
Parachute System	1,610
Solid Rocket Motor	2,639
TOTAL APPROXIMATE LANDER MASS	62,749

## LIST OF REFERENCES

1. Grimminger G., E.P. Williams, and G.B.W. Yong. "Lift on Inclined Bodies of Revolution in Hypersonic Flow". Journal of the Aeronautical Sciences, Volume 17, Number 11, November 1950.
2. McNeely, Ralph. Professional Engineer, Preliminary Design, Propulsion Group. Marshall Space Flight Center, Huntsville, Alabama. Personal communication.
3. Nichols, J.O., Ph.D. Professor, Aerospace Engineering Department. Auburn University, Auburn, Alabama. Personal communication.
4. Nicolai, Leland M. Fundamentals of Aircraft Design. E.P. Domicone Printing Services: Fairborn, Ohio, 1975.
5. Raushenbach, H.S. Solar Cell Array Design Handbook. Van Nostrand Reinhold Co.: New York, 1980.
6. Space Shuttle News Reference. National Aeronautics and Space Administration.
7. Willcockson, W.H. Orbital Transfer Vehicle Concept Definition and System Analysis Study: Volume X-- Aerocapture for Manned Mars Missions. Martin Marietta Aerospace, Denver, Colorado, 1987.

**APPENDIX A**

**Computer Program for the Aerobraking Analysis**

THIS PROGRAM RUNS THROUGH AN AEROBRAKE ANALYSIS OF MARS  
INPUTS ARE MADE IN SI UNITS, AND OUTPUT IS WRITTEN IN  
ENGLISH UNITS

REAL MU, MASS, MASSE  
OPEN(UNIT=7, FILE='AEROBRKE.DAT', STATUS='OLD')

PRINT\*, 'INPUT THE PERIAPSIS ALTITUDE IN km '  
READ(6, \*) PERIAP

DEFINE THE PERIAPSIS FROM THE CENTER OF MARS  
PERIAP = PERIAP + 3393.5

PRINT\*, 'INPUT THE INITIAL SEMI-MAJOR AXIS IN km '  
READ(6, \*) AINIT

PRINT\*, 'INPUT THE DESIRED APOAPSIS DISTANCE IN km '  
READ(6, \*) APOAPF

CALCULATE THE TIME FOR THE FIRST 1.5 ORBITS BEFORE MAKING  
PERIAPSIS CHANGE (AT APOAPSIS) FOR AEROBRAKING PROCESS.

THE ELLIPTIC ORBIT FOR THIS PERIOD IS (500 km X 33363 km)

SMAJ1 = .5\*((500.+3393.5)+(33363.+3393.5))

THE GRAVITATIONAL PARAMETER MU (kg<sup>3</sup>/sec<sup>2</sup>)

MU = 42656.

PI = 3.141592654

PERD1 = 2\*PI\*SQRT(SMAJ1\*\*3/MU)

TIME1 = 1.5\*PERD1

CALCULATE THE PARAMETERS OF THE ELLIPTIC ORBIT ABOUT MARS  
USING THE PERIAPSIS FROM INPUT. THIS ORBIT IS ACTUALLY ONLY  
HALF AN ORBIT, MAKING THE JOURNEY FROM APOAPSIS TO AEROBRAKING  
PERIAPSIS

PERIOD = 0.0

APOAP = 0.0

CALL PARAMS(PERIAP, APOAP, AINIT, PERIOD)

PERDHR = PERIOD/3600.

DETERMINE THE TIME FOR THE HALF-ORBIT FROM THE APOAPSIS TO  
THE PERIAPSIS OF AEROBRAKING

TIME2 = .5\*PERIOD

OBTAIN THE INPUTS FOR THE AEROBRAKING PROCESS

PRINT\*, 'INPUT THE ATMOSPHERIC DENSITY FOR THE ALTITUDE (kg/m<sup>3</sup>) '

READ(6, \*) RHO

PRINT\*, 'INPUT THE MASS OF THE SPACE VEHICLE (kg) '

READ(6, \*) MASS

CONVERT THE MASS TO ENGLISH UNITS

MASSE = MASS\*32.174/14.57

PRINT\*, 'INPUT THE HALF-ANGLE OF THE CONICAL AEROBRAKE (deg) '

READ(6, \*) THETA

PRINT\*, 'INPUT THE DIAMETER OF THE AEROBRAKE (m) '

READ(6, \*) DIAM

DIAM = DIAM/1000.

DETERMINE THE AREA OF THE CONICAL AEROBRAKE

THETAR = THETA\*PI/180.

PART = 1./(TAN(THETAR)\*\*2)

AREA = PI\*(DIAM/2. )\*\*2\*SQRT(1.+PART)

DETERMINE THE DRAG COEFFICIENT OF THE AEROBRAKE  
BASED ON NEWTONIAN METHODS

CD = 2.\*SIN(THETAR)\*\*2

CALCULATE THE ENGLISH-UNIT COUNTERPARTS OF THE ABOVE VALUES

PERAPE = PERIAP\*3280.839895

AINITE = AINIT\*3280.839895

DIAME = DIAM\*3280.839895

AREAE = AREA\*10763910.42

RHOE = RHO\*0.0019435035  
THE PERIAPSIS, APOAPSIS, SEMI-MAJOR AXIS, AND AEROBRAKE DIAMETER  
ARE CONVERTED FROM km TO ft. THE AREA IS CHANGED FROM km^2 TO ft^2  
AND THE DENSITY FROM kg/m^3 TO slugs/ft^3.

TIMTTL = TIME1+TIME2

TIMTLH = TIMTTL/3600.

WRITE(7,\*)' AEROBRAKE ANALYSIS '

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' HALF-ANGLE FOR CONICAL AEROBRAKE (deg): ',THETA

WRITE(7,\*)' DIAMETER OF THE AEROBRAKE (ft): ',DIAME

WRITE(7,\*)' SURFACE AREA OF THE AEROBRAKE (ft^2): ',AREAE

WRITE(7,\*)' MASS OF SPACE VEHICLE (lb): ',MASSE

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' ATMOSPHERIC CONDITIONS: '

WRITE(7,\*)' PERIAPSIS FROM CENTER OF MARS (ft): ',PERAPE

WRITE(7,\*)' DENSITY (slug/ft^3): ',RHOE

WRITE(7,\*)' '

WRITE(7,\*)' INITIAL ORBITAL PARAMETERS: '

WRITE(7,\*)' '

WRITE(7,130)

WRITE(7,\*)' '

WRITE(7,140) PERAPE, APOAPE, AINITE, PERDHR

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' APPROX TIME (hr) PRIOR TO AEROBRAKING: ',TIMTLH

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' '

WRITE(7,\*)' AEROBRAKE PROCEDURE: '

WRITE(7,\*)' '

WRITE(7,100)

WRITE(7,110)

WRITE(7,\*)' '

SET INITIAL CONDITIONS FOR VARIABLES PRIOR TO DO-LOOP

SMAJ = AINIT

X = 0.0

Y = 0.0

SEG = 0.0

PHI = 0.0

ASECTR = 0.0

TIMTTL = 0.0

DO 50 I = 1, 500

ECCNTY = (APOAP-PERIAP)/(APOAP+PERIAP)

SMIN = SQRT(SMAJ\*\*2\*(1.-ECCNTY\*\*2))

CALCULATE THE INTERSECTION POINTS OF THE ELLIPTIC ORBIT  
AND THE MARTIAN ATMOSPHERE

CALL NTRSEC(SMAJ, SMIN, ECCNTY, X, Y)

CALCULATE THE LENGTH OF SEGMENT OF THE ELLIPTIC ORBIT  
ENCLOSED BY THE MARTIAN ATMOSPHERE

CALL SEGMNT(X, Y, SMAJ, ECCNTY, SEG, PHI, ASECTR)

DETERMINE THE VELOCITY OF THE SPACECRAFT AT PERIAPSIS

VELCTY = SQRT(MU\*(2./PERIAP-1./SMAJ))

THE SEMI-MAJOR AXIS IS THE "OLD" SEMI-MAJOR AXIS

CALCULATE THE DRAG ON THE VEHICLE DURING THE AEROBRAKING PROCESS  
UNITS ARE (KG\*KM/SEC^2) AND (LB)

DRAG = .5\*CD\*(RHO\*1.E9)\*VELCTY\*\*2\*AREA  
DRAGE = .5\*CD\*RHOE\*VLCTYE\*\*2\*AREAE

DETERMINE THE TIME (IN MINUTES) OF THE AEROBRAKE PASSAGE

TIME = 2.\*ASECTR\*SQRT(SMAJ/MU)/SMIN  
TIME = TIME/60.

DETERMINE THE NEW SEMI-MAJOR AXIS

ENRGY1 = -MU/(2.\*SMAJ)  
SMAJ = -MU/(-2.\*DRAG\*SEG/MASS+2.\*ENRGY1)

DETERMINE THE PARAMETERS OF THE NEW ELLIPTIC ORBIT

CALL PARANS(PERIAP, APOAP, SMAJ, PERIOD)

PERIOD OF THE ORBIT IS IN HOURS

PERDHR = PERIOD/3600.

CONVERT SI UNITS TO ENGLISH UNITS

SMAJE = SMAJ\*3280.839895  
APOAPE = APOAP\*3280.839895

CHECK IF THE APOAPSIS IS LESS THAN THE RADIUS OF THE  
MARTIAN ATMOSPHERE

IF(APOAP .LE. 3643.5) GO TO 80

45 WRITE(7, 120) I, PERAPE, APOAPE, SMAJE, PERDHR, DRAGE, TIME

IF(APOAP .LE. APOAPF) GO TO 60

TIMTTL = TINTTL+PERIOD

50 CONTINUE

IF(APOAP .GT. APOAPF) GO TO 85

DETERMINE THE TIME TO TRAVEL THE HALF ORBIT FROM THE AEROBRAKING  
PERIAPSIS TO THE APOAPSIS.

60 TIMEPA = .5\*PERIOD

A DELTA-V BURN WILL BE PERFORMED AT THE APOAPSIS TO RAISE THE  
PERIAPSIS TO 500 km. DETERMINE THE PERIOD OF THE NEW ORBIT, AND  
THE TIME TO TRAVEL FROM THE APOAPSIS TO THE PERIARSIS.

SMAJAP = .5\*((500.+3393.5)+APOAP)

PERDAP = 2.\*PI\*SQRT(SMAJAP\*\*3/MU)

TIMEAP = .5\*PERDAP

IF(APOAP .EQ. APOAPF) THEN

PERDF = PERDAP

ELSE

GO TO 70

END IF

GO TO 75

BECAUSE THE FINAL APOAPSIS FROM AEROBRAKING IS LESS THAN THE DESIRED  
APOAPSIS, A DELTA-V BURN WILL HAVE TO BE APPLIED AT THE PERIAPSIS  
TO RAISE THE APOAPSIS SO THAT THE FINAL CIRCULAR ORBIT IS OBTAINED  
THE PERIOD OF THIS ORBIT, AND THE TIME TO COMPLETE ONE ORBIT (THUS  
FINALIZING THE CIRCULARIZATION OF THE ORBIT ABOUT MARS) IS DETERMINED

70 SMAJF = 500.+3393.5

PERDF = 2.\*PI\*SQRT(SMAJF\*\*3/MU)

75 TIMTL = TIMTTL+TIMEPA+TIMEAP+PERDF

TIMHR = (TIMTL+TIME1+TIME2)/3600.

```

WRITE(7,*)'TIME BREAKDOWN (hrs): '
WRITE(7,*)'  TIME TO INITIALIZE ORBIT:           ',TIME1/3600.
WRITE(7,*)'  TIME TO TRAVEL FROM APOAP TO PERIAP: ',TIME2/3600.
WRITE(7,*)'  TIME FOR AEROBRAKING PASSAGE:       ',TIMTTL/3600.
WRITE(7,*)'  TIME TO TRAVEL FROM PERIAP TO APOAP: ',TIMEPA/3600.
WRITE(7,*)'  TIME TO TRAVEL FROM APOAP TO PERIAP: ',TIMEAP/3600.
WRITE(7,*)'  TIME FOR 1 ORBIT AFTER CIRCULARIZE: ',PERDF/3600.
WRITE(7,*)'  '
WRITE(7,*)'  TOTAL TIME FOR AEROBRAKING PROCESS: ',TIMHR

```

```

PRINT*,I
PRINT*, 'AEROBRAKING TIME= ', (TIMTL+TIME1+TIME2)/3600.
PRINT*, 'PER TO APO= ', TIMEPA/3600.
PRINT*, 'APO TO PER= ', TIMEAP/3600.
PRINT*, 'ORBIT AFTER CIRCULARIZATION= ', PERDF/3600.
GO TO 90

```

```

80 WRITE(7,*)'AEROBRAKING IS NOT POSSIBLE FOR THIS PERIAPSIS'
GO TO 90

```

```

85 WRITE(7,*)'FINAL APOAPSIS HAS NOT BEEN REACHED'

```

```

130 FORMAT(3X, 'PERIAPSIS (ft)', 6X, 'APOAPSIS (ft)', 6X,
1 'SEMI-MAJOR AXIS', 5X, 'PERIOD (hrs)')
140 FORMAT(2X, F15. 5, 5X, F15. 5, 5X, F15. 5, 5X, F11. 5)

```

```

100 FORMAT(4X, 'PASS', 5X, 'PERIAPSIS', 7X, 'APOAPSIS', 7X,
1 'SEMI-MAJOR', 4X, 'PERIOD', 7X, 'DRAG', 8X, 'PASSAGE')
110 FORMAT(3X, 'NUMBER', 7X, '(ft)', 11X, '(ft)', 11X, 'AXIS (ft)',
1 5X, '(hrs)', 7X, '(lb)', 7X, 'TIME (min)')
120 FORMAT(4X, I3, 5X, F13. 3, 3X, F13. 3, 3X, F13. 3, 3X, F7. 3, 3X, F10. 3, 4X, F8. 3)

```

```

90 CLOSE (UNIT = 7)
STOP
END

```

```

SUBROUTINE PARAMS(RP, RA, A, PERD)

```

```

C THIS SUBROUTINE CALCULATES THE APOAPSIS AND PERIOD OF THE
ELLIPTIC ORBIT, USING THE VALUES OF THE PERIAPSIS AND SEMI-
MAJOR AXIS FROM THE MAIN PROGRAM

```

```

THE PERIAPSIS, APOAPSIS, AND SEMI-MAJOR AXIS ARE IN km
THE PERIOD IS IN sec, THE GRAVITATIONAL PARAMETER IS (km^3/sec^2)

```

```

REAL MU
RA = 2. *A-RP
PI = 3.141592654
MU = 42656.0
PERD = 2. *PI*SQRT(A**3/MU)
RETURN
END

```

```

SUBROUTINE NTRSEC(A, B, E, X, Y)

```

```

C THIS SUBROUTINE CALCULATES THE POINTS OF INTERSECTION OF THE
SPACE VEHICLE'S ELLIPTIC ORBIT AND THE ATMOSPHERE'S CIRCULAR
ORBIT, USING THE SEMI-MAJOR AXIS AND THE ECCENTRICITY FROM THE
MAIN PROGRAM, AND THE SEMI-MINOR AXIS FROM SUBROUTINE PARAMS

```



RADIUS = 250. +3393.5

X1 = 2. \*A+E

X2A = 4. \*A\*\*2+E\*\*2

X2B = 4. \*(B\*\*2-RADIUS\*\*2+A\*\*2+E\*\*2)\*(1. -B\*\*2/A\*\*2)

X2 = SQRT(X2A-X2B)

X3 = 2\*(1. -B\*\*2/A\*\*2)

THE INTERSECTION POINTS OF THE ELLIPTIC ORBIT ARE X AND Y

X = (X1-X2)/X3

Y = SQRT(RADIUS\*\*2-(X-A+E)\*\*2)

RETURN

END

SUBROUTINE SEGMNT(X, Y, A, E, SEG, PHI, AREA)

THIS SUBROUTINE CALCULATES THE LENGTH OF THE SEGMENT (km) OF THE SPACE VEHICLE'S ELLIPTIC ORBIT BOUNDED BY THE MARTIAN ATMOSPHERE USING THE INTERSECTION POINTS, SEMI-MAJOR AXIS, AND ECCENTRICITY FROM THE MAIN PROGRAM

C = 2. \*Y

D = X-A+E

R = SQRT(D\*\*2+Y\*\*2)

PHI IS THE ANGLE OF THE BOUNDED SEGMENT, AND AREA IS THE AREA OF THE BOUNDED PORTION OF THE ORBIT

PHI = 2. \*ATAN(C/(2. \*D))

SEG = R\*PHI

AREA = .5\*R\*SEG

RETURN

END

**AEROBRAKE ANALYSIS**

ORIGINAL PAGE IS  
OF POOR QUALITY

HALF-ANGLE FOR CONICAL AEROBRAKE (deg): 70.00000000  
 DIAMETER OF THE AEROBRAKE (ft): 95.00000000  
 SURFACE AREA OF THE AEROBRAKE (ft^2): 7543.12402000  
 MASS OF SPACE VEHICLE (lb): 200000.00000000

**ATMOSPHERIC CONDITIONS:**

PERIAPSIS FROM CENTER OF MARS (ft): 1.14484910E+07  
 DENSITY (slug/ft^3): 2.42937948E-10

**INITIAL ORBITAL PARAMETERS:**

PERIAPSIS (ft)	APOAPSIS (ft)	SEMI-MAJOR AXIS	PERIOD (hrs)
11448491.00000	120592192.00000	66020340.00000	24.12273

APPROX TIME (hr) PRIOR TO AEROBRAKING: 48.79166030

**AEROBRAKE PROCEDURE:**

PASS NUMBER	PERIAPSIS (ft)	APOAPSIS (ft)	SEMI-MAJOR AXIS (ft)	PERIOD (hrs)	DRAG (lb)	PASSAGE TIME (min)
1	11448491.000	113458000.000	62453248.000	22.194	388.908	11.691
2	11448491.000	107067800.000	59258144.000	20.513	386.799	11.762
3	11448491.000	101310824.000	56379656.000	19.037	384.695	11.835
4	11448491.000	96097224.000	53772856.000	17.732	382.594	11.909
5	11448491.000	91353344.000	51400916.000	16.572	380.499	11.983
6	11448491.000	87018344.000	49233416.000	15.535	378.407	12.060
7	11448491.000	83041360.000	47244924.000	14.603	376.319	12.137
8	11448491.000	79379664.000	45414076.000	13.762	374.235	12.216
9	11448491.000	75997080.000	43722784.000	13.001	372.155	12.296
10	11448491.000	72862736.000	42155612.000	12.308	370.079	12.378
11	11448491.000	69950144.000	40699320.000	11.676	368.006	12.461
12	11448491.000	67236472.000	39342480.000	11.097	365.937	12.546
13	11448491.000	64701880.000	38075184.000	10.565	363.872	12.633
14	11448491.000	62329112.000	36888804.000	10.075	361.810	12.721
15	11448491.000	60103032.000	35775760.000	9.623	359.751	12.811
16	11448491.000	58010348.000	34729420.000	9.204	357.695	12.902
17	11448491.000	56039316.000	33743904.000	8.815	355.642	12.996
18	11448491.000	54179508.000	32813998.000	8.453	353.592	13.091
19	11448491.000	52421676.000	31935084.000	8.115	351.545	13.188
20	11448491.000	50757564.000	31103028.000	7.800	349.501	13.288
21	11448491.000	49179764.000	30314128.000	7.505	347.459	13.389
22	11448491.000	47681636.000	29565064.000	7.229	345.419	13.493
23	11448491.000	46257200.000	28852846.000	6.969	343.382	13.599
24	11448491.000	44901056.000	28174772.000	6.725	341.347	13.708
25	11448491.000	43608300.000	27528396.000	6.495	339.314	13.819
26	11448491.000	42374508.000	26911498.000	6.278	337.282	13.933
27	11448491.000	41195628.000	26322060.000	6.073	335.252	14.050
28	11448491.000	40067988.000	25758238.000	5.879	333.224	14.169
29	11448491.000	38988220.000	25218354.000	5.695	331.197	14.292

31	11448491.000	36960228.000	24204360.000	5.355	327.146	14.546
32	11448491.000	36006576.000	23727534.000	5.197	325.122	14.679
33	11448491.000	35089896.000	23269194.000	5.048	323.098	14.815
34	11448491.000	34207976.000	22828234.000	4.905	321.075	14.955
35	11448491.000	33358782.000	22403636.000	4.769	319.051	15.099
36	11448491.000	32540420.000	21994456.000	4.639	317.027	15.247
37	11448491.000	31751138.000	21599814.000	4.514	315.003	15.400
38	11448491.000	30989304.000	21218898.000	4.395	312.978	15.557
39	11448491.000	30253406.000	20850948.000	4.282	310.953	15.720
40	11448491.000	29542030.000	20495260.000	4.172	308.925	15.887
41	11448491.000	28853862.000	20151176.000	4.068	306.897	16.061
42	11448491.000	28187662.000	19818076.000	3.967	304.866	16.240
43	11448491.000	27542282.000	19495386.000	3.871	302.833	16.426
44	11448491.000	26916638.000	19182564.000	3.778	300.797	16.619
45	11448491.000	26309712.000	18879102.000	3.689	298.758	16.818
46	11448491.000	25720544.000	18584518.000	3.603	296.715	17.026
47	11448491.000	25148234.000	18298362.000	3.520	294.669	17.242
48	11448491.000	24591922.000	18020206.000	3.440	292.618	17.467
49	11448491.000	24050806.000	17749648.000	3.363	290.561	17.701
50	11448491.000	23524116.000	17486302.000	3.288	288.500	17.946
51	11448491.000	23011118.000	17229804.000	3.216	286.431	18.202
52	11448491.000	22511124.000	16979808.000	3.146	284.356	18.470
53	11448491.000	22023460.000	16735976.000	3.079	282.273	18.752
54	11448491.000	21547492.000	16497992.000	3.013	280.182	19.048
55	11448491.000	21082604.000	16265547.000	2.950	278.081	19.360
56	11448491.000	20628198.000	16038344.000	2.888	275.969	19.690
57	11448491.000	20183694.000	15816093.000	2.829	273.846	20.039
58	11448491.000	19748528.000	15598510.000	2.770	271.711	20.409
59	11448491.000	19322144.000	15385317.000	2.714	269.561	20.804
60	11448491.000	18903988.000	15176239.000	2.659	267.395	21.225
61	11448491.000	18493504.000	14970998.000	2.605	265.213	21.677
62	11448491.000	18090140.000	14769316.000	2.552	263.011	22.164
63	11448491.000	17693322.000	14570907.000	2.501	260.788	22.690
64	11448491.000	17302464.000	14375477.000	2.451	258.540	23.261
65	11448491.000	16916940.000	14182715.000	2.402	256.266	23.886
66	11448491.000	16536080.000	13992285.000	2.354	253.961	24.574
67	11448491.000	16159161.000	13803826.000	2.306	251.622	25.337
68	11448491.000	15785364.000	13616927.000	2.260	249.244	26.190
69	11448491.000	15413744.000	13431118.000	2.213	246.820	27.155
70	11448491.000	15043186.000	13245838.000	2.168	244.344	28.260
71	11448491.000	14672308.000	13060399.000	2.122	241.805	29.547
72	11448491.000	14299343.000	12873917.000	2.077	239.192	31.074
73	11448491.000	13921898.000	12685194.000	2.032	236.489	32.934
74	11448491.000	13536531.000	12492511.000	1.986	233.672	35.280
75	11448491.000	13137854.000	12293172.000	1.938	230.708	38.391
76	11448491.000	12716375.000	12082433.000	1.889	227.544	42.842

AEROBRAKING COMPLETE

TIME BREAKDOWN (hrs):

TIME TO INITIALIZE ORBIT:	36.73029330
TIME TO TRAVEL FROM APOAP TO PERIAP:	12.06136420
TIME FOR AEROBRAKING PASSAGE:	472.14114400
TIME TO TRAVEL FROM PERIAP TO APOAP:	0.94430298
TIME TO TRAVEL FROM APOAP TO PERIAP:	1.02305233
TIME FOR 1 ORBIT AFTER CIRCULARIZE:	2.05304074

TOTAL TIME FOR AEROBRAKING PROCESS: 524.95318600

**APPENDIX B**

**Optimization of Propellant Mass of a  
Solid-Propellant Rocket**

\*\*\*\*\*  
MIKE LISANO  
AUEURN UNIVERSITY AEROSPACE ENGINEERING SENIOR DESIGN PROJECT  
UNMANNED MARS MISSION  
-----

"STAGES"

PROGRAM TO OPTIMIZE THE PROPELLANT MASS OF A SOLID-PROPELLANT  
ROCKET LAUNCHING IN AN ARBITRARY GRAVITY FIELD FOR A GIVEN  
DESIRED ORBIT (PITCH PROGRAM INCLUDED)

\*\*\*\*\*

COMMON/ROCK/TCCMB,DF,DIA,BR,GAMMA,RGAS,GC  
COMMON/PLAN/GRAV,R,A,OMEGA,RAD,ALAT  
COMMON/LAUN/V(610),G(610),H(610),AM(610),DELV(610),ACC(610)

UNIT CONVERSION FACTOR GC (LBM-FT/LBF-SEC\*\*2)  
GC=32.174

\*\*\*\*\*

SPECIFICATIONS OF ROCKET:  
(CCOMBUSTION TEMPERATURE OF PROPELLANT, DEGREES RANKINE)  
TCCMB=6700.  
(DENSITY OF FUEL, LBM/IN\*\*3)  
DF=.065  
(PROPELLANT CROSS-SECTION DIAMETER, FT)  
FDIA=2.91667  
DIA=FDIA\*12.0  
(PROPELLANT BURN RATE, IN/SEC)  
BR=0.55  
(SPECIFIC HEAT RATIO OF BURNING PROPELLANT (DEFAULT:AIR))  
GAMMA=1.4  
(PERFECT GAS CONSTANT OF BURNING PROPELLANT (DEFAULT:AIR),  
FT-LBF/LBM-R)  
RGAS=53.3  
(DEADWEIGHT RATIO OF BOOSTER)  
DWRAT=.12  
(MASS OF PAYLOAD TO BE CARRIED INTO ORBIT, LBM)  
AMF=1000.0

\*\*\*\*\*

DATA FOR LAUNCH:  
(RADIUS OF PLANET, N MI)  
RAD=1841.0  
(LATITUDE OF LAUNCH SITE, DEGREES)  
DLAT=0.0  
ALAT=DLAT/53.295773  
(ANGULAR VELOCITY OF PLANET, RAD/SEC)  
OMEGA=.00067  
(DESIRED ALTITUDE, N MI)  
ALT=270.0  
HF=ALT\*6080.

```

(TOTAL HEIGHT, FT)
R=(RAD*6080.)+HF
(SEMI-MAJOR AXIS OF DESIRED ORBIT, N MI (DEFAULTS TO A=R))
SMA=RAD+ALT
A=SMA*6080.
(GRAVITATIONAL ACCELERATION ON THE SURFACE OF THE PLANET,
 FT/SEC**2)
GSLRF=12.332
(GRAVITATIONAL PARAMETER OF PLANET IN FT**3/SEC**2)
GRAV=1.5065E15
*****

```

```

10 WRITE(6,11U)
   FORMAT(///,3X,'OPTIMIZATION OUTPUT:')

```

```

-----
SUBROUTINE "ROCKET" CALCULATES THE EXHAUST VELOCITY (C) AND TIME
RATE OF CHANGE OF MASS (DMDT) OF THE ROCKET:

```

```

11 CALL ROCKET(C,DMDT)
   WRITE(6,111)C,DMDT
   FORMAT(//,3X,'EXHAUST VELOCITY OF ROCKET =',F12.3,1X,'FT/SEC',
*//,3X,'PROPELLANT CONSUMPTION RATE =',F12.6,1X,'LBM/SEC')

```

```

-----
SUBROUTINE "SPEEDS" CALCULATES THE SPEED OF THE ROCKET DUE
TO PLANETARY SPIN BEFORE TAKEOFF, AND THE SPEED OF THE
ROCKET IN THE DESIRED ORBIT:

```

```

12 CALL SPEEDS(VSURF,VORB)
   WRITE(6,112)VSURF,ALT,VORB
   FORMAT(//,3X,'VELOCITY OF PLANET SURFACE =',F12.3,1X,'FT/SEC',
*//,3X,'DESIRED ORBIT ALTITUDE =',F12.3,1X,'N MI',//,3X,
*'VELOCITY OF ROCKET IN DESIRED ORBIT =',F12.3,1X,'FT/SEC')

```

```

-----
SUBROUTINE "LAUNCH" CALCULATES THE TOTAL CHANGE IN SPEED AND
CHANGE IN ALTITUDE OF A SINGLE STAGE ROCKET BEING LAUNCHED IN
THE GRAVITY FIELD OF A GIVEN PLANET. AERODYNAMIC FORCES ON THE
ROCKET HAVE BEEN NEGLECTED. THE INITIAL MASS OF THE ROCKET IS
INCREMENTED FROM THE MASS REQUIRED FOR A TEN MINUTE BURN TO THAT
REQUIRED TO REACH THE DESIRED ALTITUDE (HF) AND VELOCITY (VORB).

```

```

CALL LAUNCH(DWRAT,VSURF,GSURF,AMF,DMDT,C,HF,RAD,VORB)

```

```

STCP
END

```

```

*****
SUBROUTINE ROCKET(C,DMDT)
COMMON/ROCK/TCOMB,DF,DIA,BR,GAMMA,RGAS,GC
A=(3.1415927*(DIA**2))/4.
DMDT=A*DF*BR
C=SQRT((2.*GAMMA*RGAS/(GAMMA-1.))*GC*TCOMB)

```

RETURN  
END

\*\*\*\*\*

SUBROUTINE SPEEDS(VSURF,VORB)  
COMMON/PLAN/GRAV,R,A,OMEGA,RAD,ALAT  
VSURF=CMEGA\*RAD\*6080.\*COS(ALAT)  
VORB=SGRT(GRAV\*((2./R)-(1./A)))

RETURN  
END

\*\*\*\*\*

SUBROUTINE LAUNCH(DWRAT,VSURF,GSURF,AMF,DMDT,C,HF,RAD,VORB)  
COMMON/LAUN/V(610),G(610),H(610),AM(610),DELV(610),ACC(610)

TARGET BURN TIME (SEC). . . DETERMINES INITIAL MASS AM(1)

TBURN=350.0

TIME INCREMENTS (SEC)

DT=1.0

T=C.0

AM(1)=((TBURN\*DMDT)/(1.0-DWRAT))+AMF

AMFF=(AM(1)-AMF)\*DWRAT+AMF

V(1)=VSURF

DELV(1)=0.0

G(1)=GSURF

H(1)=0.0

DTHET=0.0

I=1

I=I+1

T=T+DT

THET=DTHET/57.29578

AM(I)=AM(I-1)-(DMDT\*DT)

IF(AM(I).LT.AMFF)GOTO 1900

DELV(I)=C\*LOG(AM(I-1)/AM(I))-(G(I-1)\*COS(THET)\*DT)

V(I)=V(I-1)+DELV(I)

H(I)=H(I-1)+(((V(I)+V(I-1))/2.)\*DT)\*SIN(THET)

G(I)=GSURF\*((RAD\*6080.)\*\*2)/(((RAD\*6080.)+H(I))\*\*2)

ACC(I)=DELV(I)/(DT\*32.174)

PITCH PROGRAM (ARBITRARY, FIVE STEP, INITIAL THETA = 0 DEG.,  
FINAL THETA = 90 DEG.)

IF(H(I).GE.HF)GOTO 1900

IF(H(I).GT.(HF\*.4))GOTO 1800

IF(H(I).GT.(HF\*.15))GOTO 1700

IF(H(I).GT.(HF\*.05))GOTO 1600

IF(H(I).GT.(HF\*.01))GOTO 1500

1500 DTHET=45.0

GOTO 100

1600 DTHET=65.0

GOTO 100

1700 DTHET=80.0

GOTO 100

1800 DTHET=85.0

GOTO 100

1900 T=T-DT

```
VEL=V(I-1)
HT=H(I-1)/6080.
AC=ACC(I-1)
AMM=AM(I-1)
PR=AMF/AM(1)
TPR=AM(I-1)/AM(1)
EXC=AM(I-1)-AMF
```

12

```
WRITE(6,212)TBURN,AM(1),T,VEL,HT,AC,AMM,PR,TPR,EXC
FORMAT(/,3X,'DATA FOR SINGLE-STAGE ROCKET LAUNCH:',
*//,3X,'THE TARGET TIME OF BURN IS ',F8.3,1X,'SEC',
*//,3X,'THE INITIAL MASS OF THE ROCKET IS ',F12.3,1X,'LBM',
*//,3X,'THE TOTAL TIME OF BURN IS ',F8.3,1X,'SEC',
*//,3X,'THE FINAL VELOCITY IS ',F12.3,1X,'FT/SEC',
*//,3X,'THE FINAL ALTITUDE IS ',F12.4,1X,'N MI',
*//,3X,'THE FINAL ACCELERATION IS ',F6.2,1X,'GS',
*//,3X,'THE FINAL MASS OF THE ROCKET IS ',F12.3,1X,'LBM',
*//,3X,'THE DESIGN PAYLOAD RATIO OF THE ROCKET IS ',F9.6,
*//,3X,'ACTUAL FINAL MASS/INITIAL MASS IS ',F9.6,
*//,3X,'THE EXCESS MASS AFTER FIRING IS ',F12.3,1X,'LBM')
```

```
RETURN
END
```

```
*****
```



OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC

PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC

VELOCITY OF PLANET SURFACE = 783.551 FT/SEC

DESIRED ORBIT ALTITUDE = 270.000 N MI

VELOCITY OF ROCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:

THE TARGET TIME OF BURN IS 250.000 SEC

THE INITIAL MASS OF THE ROCKET IS 10771.480 LBM

THE TOTAL TIME OF BURN IS 250.000 SEC

THE FINAL VELOCITY IS 14047.491 FT/SEC

THE FINAL ALTITUDE IS 210.3703 N MI

THE FINAL ACCELERATION IS 4.35 GS

THE FINAL MASS OF THE ROCKET IS 2172.578 LBM

THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.092838

ACTUAL FINAL MASS/INITIAL MASS IS 0.201697

THE EXCESS MASS AFTER FIRING IS 1172.578 LBM

OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC

PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC

VELOCITY OF PLANET SURFACE = 783.551 FT/SEC

DESIRED ORBIT ALTITUDE = 270.000 N MI

VELOCITY OF ROCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:

THE TARGET TIME OF BURN IS 350.000 SEC

THE INITIAL MASS OF THE ROCKET IS 14680.072 LBM

THE TOTAL TIME OF BURN IS 336.000 SEC

THE FINAL VELOCITY IS 13328.603 FT/SEC

THE FINAL ALTITUDE IS 268.7500 N MI

THE FINAL ACCELERATION IS 3.03 GS

THE FINAL MASS OF THE ROCKET IS 3123.147 LBM

THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.068120

ACTUAL FINAL MASS/INITIAL MASS IS 0.212747

THE EXCESS MASS AFTER FIRING IS 2123.147 LBM

OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC

PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC

VELOCITY OF PLANET SURFACE = 783.551 FT/SEC

DESIRED ORBIT ALTITUDE = 270.000 N MI

VELOCITY OF RCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:

THE TARGET TIME OF BURN IS 450.000 SEC

THE INITIAL MASS OF THE ROCKET IS 18588.664 LBM

THE TCTAL TIME OF BURN IS 394.000 SEC

THE FINAL VELOCITY IS 10955.684 FT/SEC

THE FINAL ALTITUDE IS 268.4356 N MI

THE FINAL ACCELERATION IS 1.37 GS

THE FINAL MASS OF THE ROCKET IS 5036.794 LBM

THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.053796

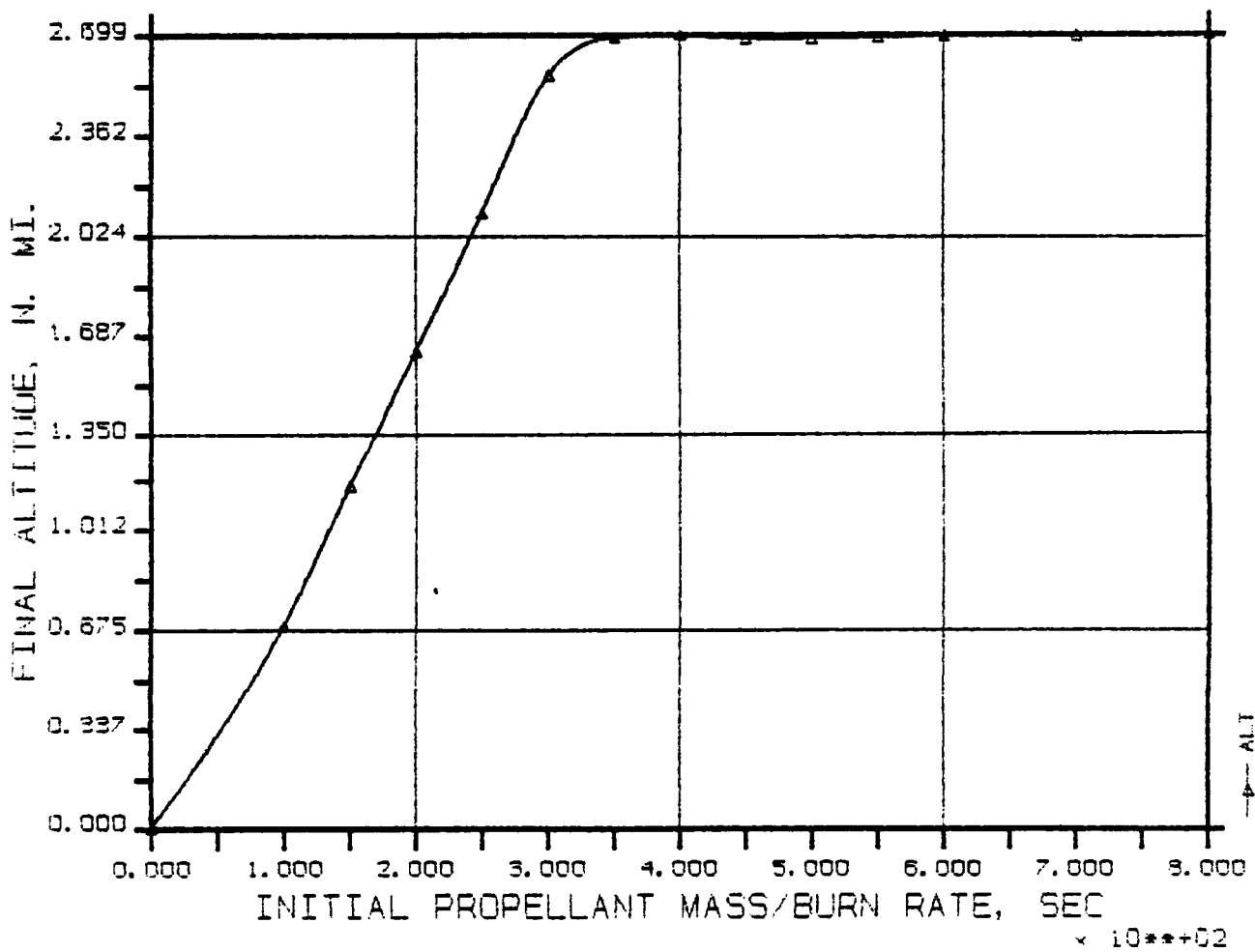
ACTUAL FINAL MASS/INITIAL MASS IS 0.270961

THE EXCESS MASS AFTER FIRING IS 4036.794 LBM

ORIGINAL PAGE IS  
OF POOR QUALITY

FIGURE B.1

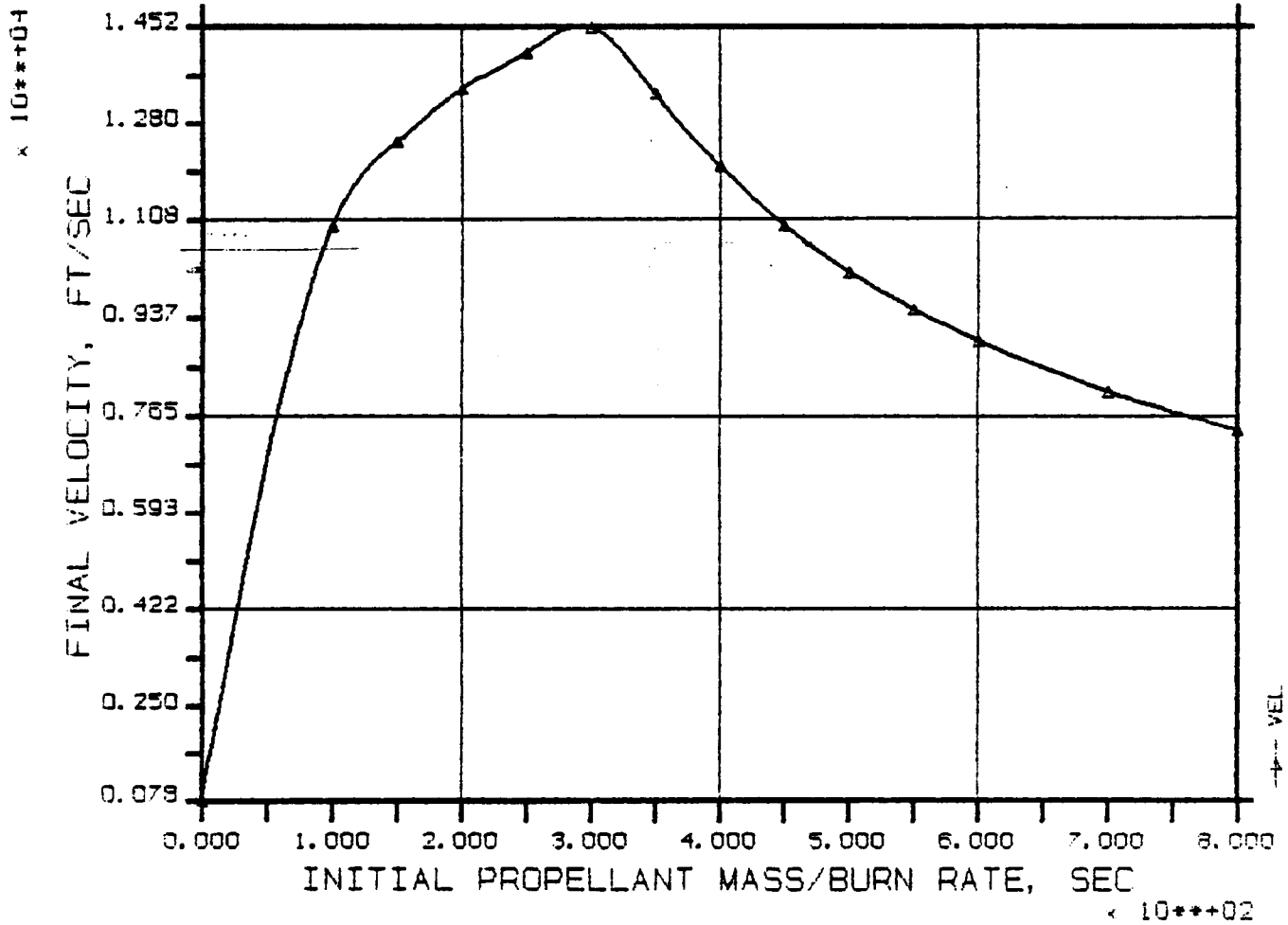
FINAL ALTITUDE VS. "DESIGN BURN TIME"



ORIGINAL PAGE IS  
OF POOR QUALITY

FIGURE B.2

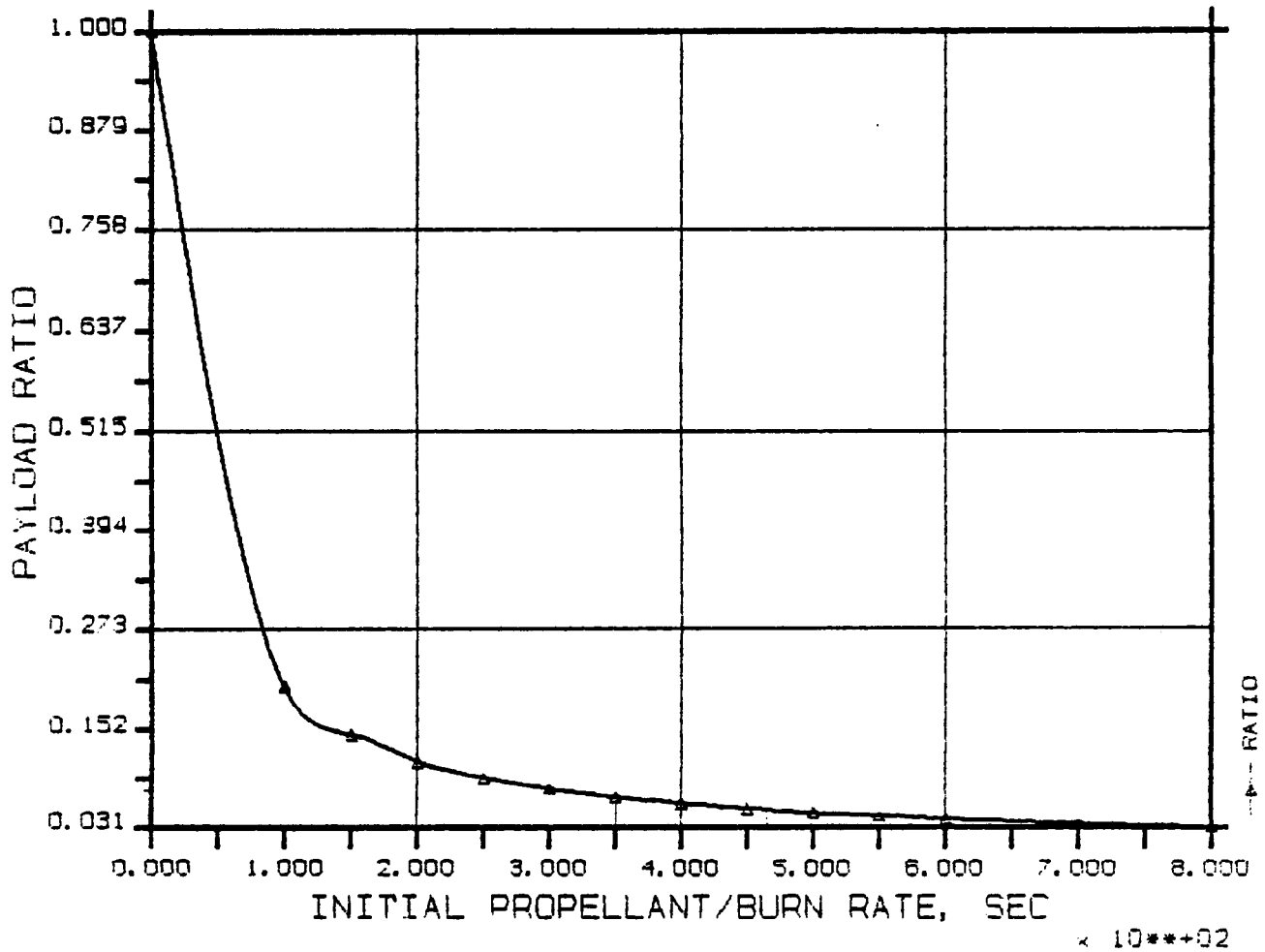
FINAL VELOCITY VS. "DESIGN BURN TIME"



ORIGINAL PAGE IS  
OF POOR QUALITY

FIGURE B.3

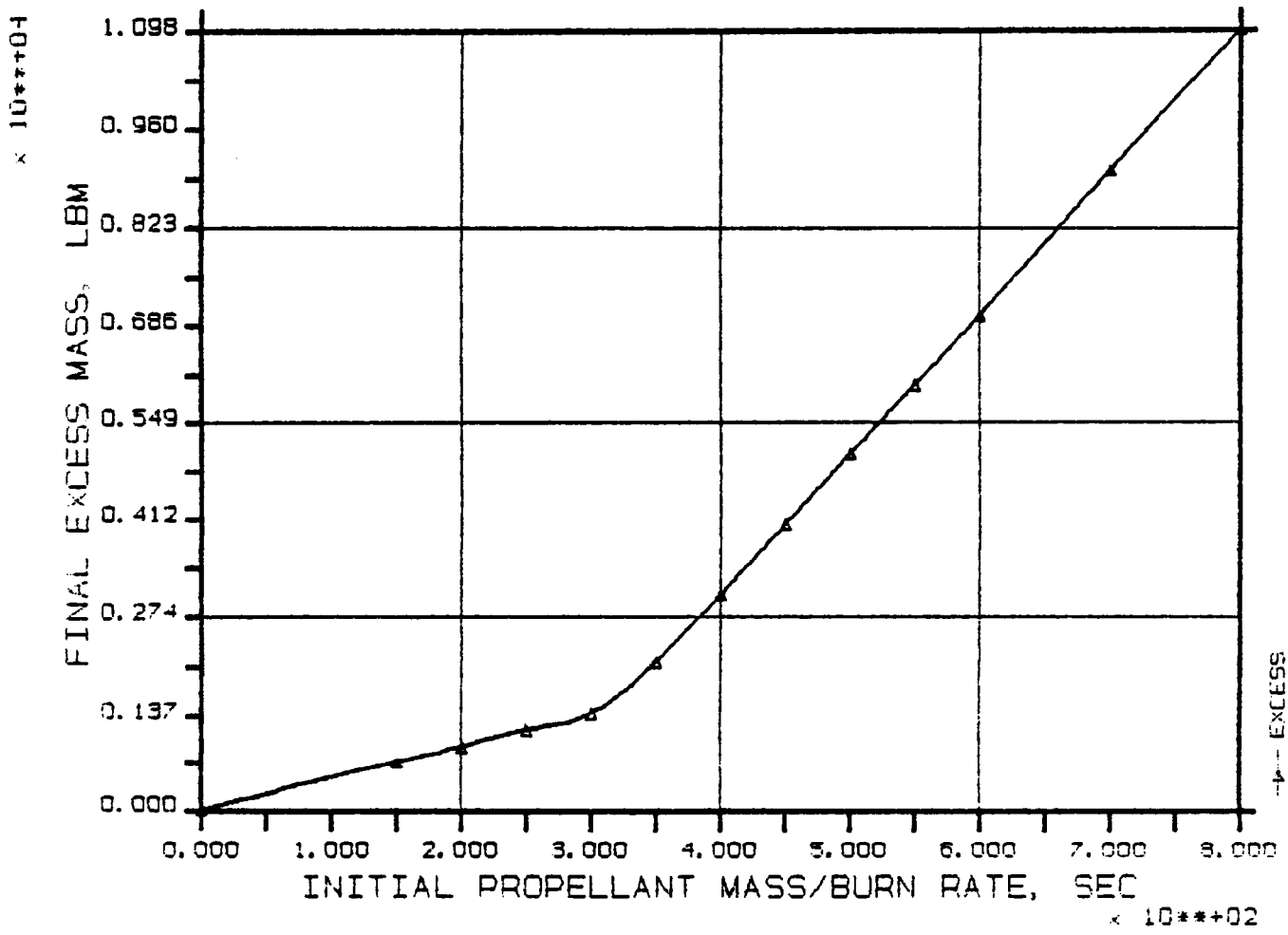
PAYLOAD RATIO VS. "DESIGN BURN TIME"



ORIGINAL PAGE IS  
OF POOR QUALITY

FIGURE B.4

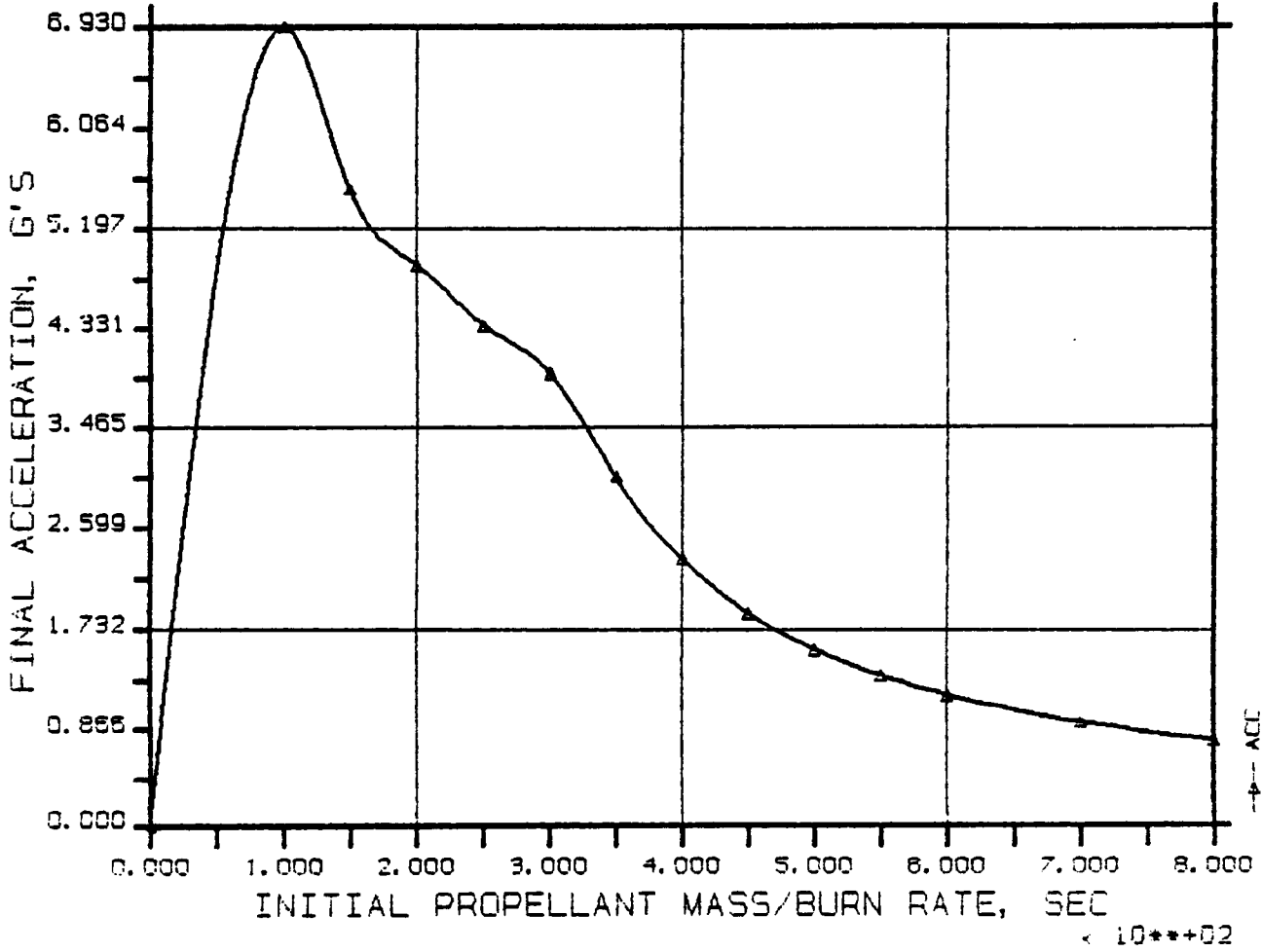
EXCESS MASS VS. "DESIGN BURN TIME"



ORIGINAL PAGE IS  
OF POOR QUALITY

FIGURE B.5

FINAL ACCELERATION VS. "DESIGN BURN TIME"





**APPENDIX C**

**Calculation of Stagnation Temperature on Mars Lander**

Apply the method of Nicolai (5:4.6):

From Stefan's Law,

$$T_s = \left( \frac{\dot{q}_{s,air}}{\epsilon \cdot \sigma_{sb}} \right)^{.25}$$

where  $T_s$  is the temperature at the stagnation point of the body (degrees Rankine)

$\dot{q}_{s,air}$  is the radiative heat flux from air to body (Btu/ft<sup>2</sup>/sec)

$\epsilon$  is the emissivity of the fluid

$\sigma_{sb}$  is the Stefan-Boltzman constant (.481 E-12 Btu/ft<sup>2</sup>/sec/°R)

For the atmosphere of Mars the emissivity is assumed to be approximately the same value as the emissivity of the Earth's atmosphere, or

$$\epsilon = 0.8$$

For the flow-body system to be in equilibrium, the heat radiated to the body must equal the heat convected from the body to the flow:

$$\dot{q}_{s,air} = \dot{q}_{c,conv}$$

An empirical formula from Nicolai allows the calculation of the convective heat flux:

$$q_{c,conv} = 15 \cdot \left( \frac{\rho_{\infty}}{R} \right)^{.5} \cdot \left( \frac{u_{\infty}}{1000} \right)^3 \cdot \cos \Delta$$

where  $\rho_{\infty}$  = freestream density (slugs/ft<sup>3</sup>)

$u$  = freestream velocity (ft/sec)

$R$  = radius of curvature of the nose (ft)

$\Delta$  = sweep angle of wing leading edge (zero degrees)

At an altitude of 100 nautical miles the density of the Martian atmosphere is determined:

$$\rho_{100 \text{ n.mi.}} = \frac{P_{100 \text{ n.mi.}}}{R_{\text{CO}_2} \cdot T_{100 \text{ n.mi.}}}$$

where  $R_{\text{CO}_2} = 35.10 \text{ ft}\cdot\text{lb}_f/(\text{lb}_m\cdot^\circ\text{R})$

$$T_{100 \text{ n.mi.}} = 324.6 \text{ }^\circ\text{R}$$

$$P_{100 \text{ n.mi.}} = 0.0042837 \text{ lb}_f/\text{ft}^2$$

The values of T and P were obtained from Viking data.

The velocity at 100 nautical miles was estimated by assuming that the specific total mechanical energy of the lander at 100 nautical miles is the same as the specific total mechanical energy at 270 nautical miles. This assumption implies that no work is done by drag forces, which would decrease the total energy of the lander. Thus the velocity estimate is high:

$$u_{\infty, 100} = 11,744 \text{ ft/sec}$$

This estimate leads to a convective heat flux of

$$\dot{q}_{\text{conv}} = 2.7199 \text{ Btu/ft}^2/\text{sec} = \dot{q}_{\text{rad}}$$

and a stagnation temperature of

$$T_s = 1630 \text{ }^\circ\text{R}$$

**APPENDIX D**

**Optimization of Recovery System Mass  
For Mars Landers**

-----  
 RECOVERY SYSTEM SUMMARY  
 -----

-----  
 Weight Statement  
 -----

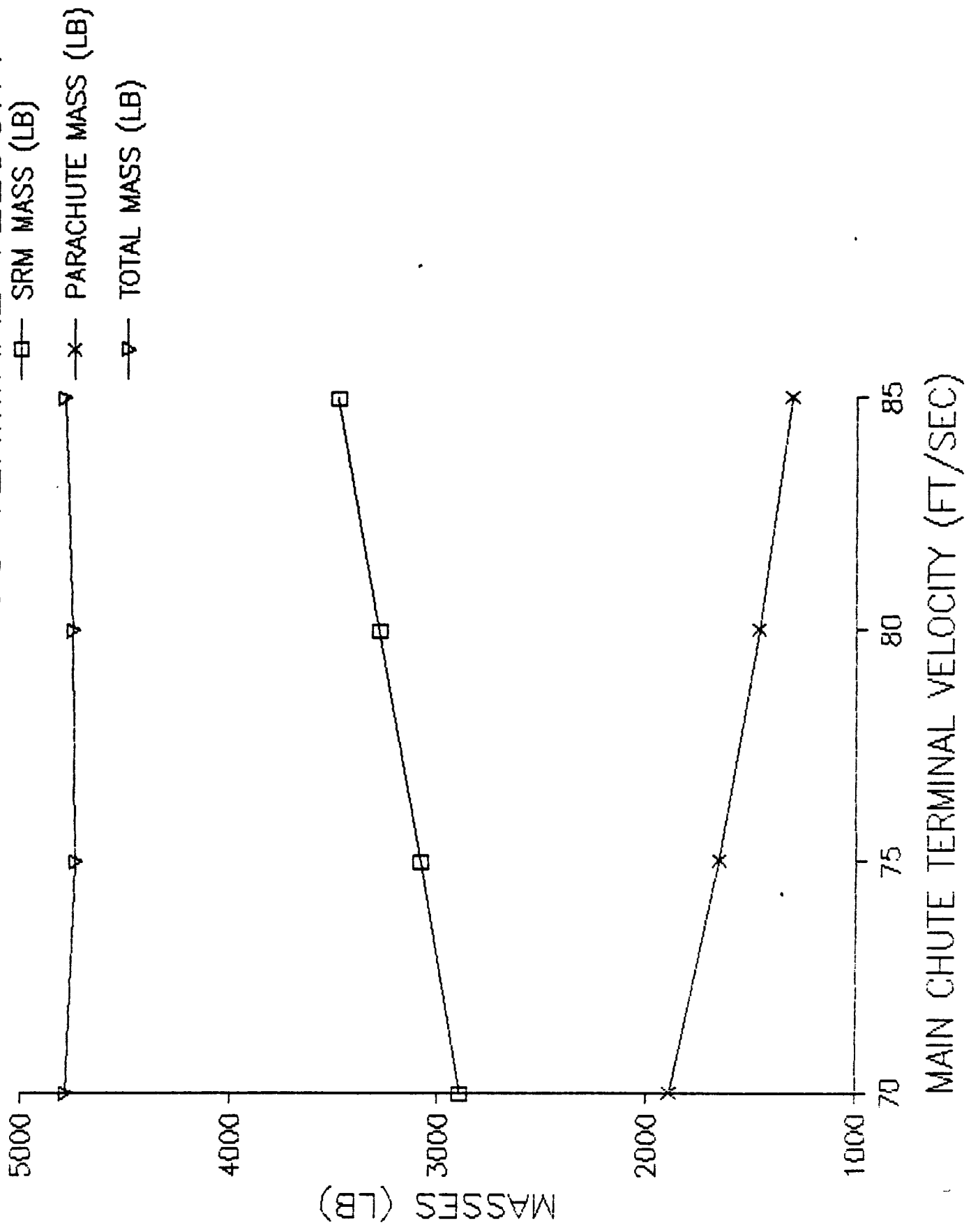
Parachute System Weight.....		1610
Pilot Chute.....	8	
Drogue Chute.....	224	
Misc. Straps.....	69	
Main Parachute		
Support Structure.....	76	
Main Parachute.....	391 X 3	
Fittings and Flotation..	59	
SRM Weight.....	+	2639
Propellant.....	2243	
Case Weight.....	396	
		-----
Recovery System Total Weight.....		4249
Basic Vehicle Weight.....	+	58500
		-----
Total Vehicle Weight (Reentry).....		62749

-----  
 Performance Characteristics  
 -----

Parachute System	SRM	
1 pilot @ 11.7 ft. diameter	ISP .....	260 sec
1 drogue @ 54.4 ft. diameter	mass fraction .....	0.850
3 mains @ 129.7 ft. diameter	thrust .....	100000 lbf
75 fps terminal velocity	burn time .....	6.04 sec
	impact velocity .....	9.92 fps
	constant velocity	
	falling height.....	5.1 ft
	total distance fallen.	210.2 ft
	maximum deceleration..	1.6 g's

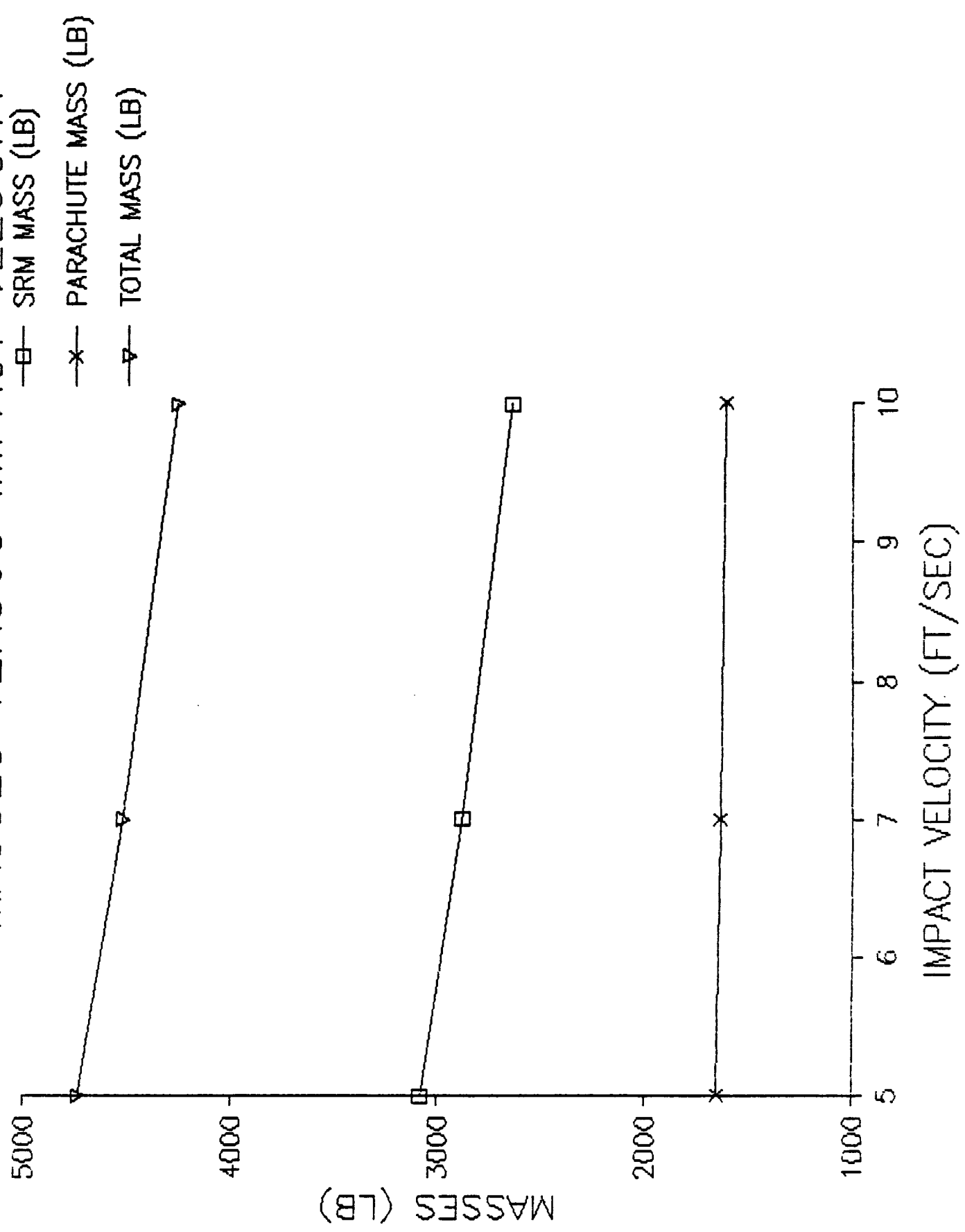
# MASSES VERSUS TERMINAL VELOCITY

FIGURE D.1



# MASSES VERSUS IMPACT VELOCITY

FIGURE D.2



# MASSES VERSUS FALLING HEIGHT

FIGURE D.3

