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Chapter 1

PROBLEMS INHERENT TO HYPERSONIC PROPULSION

1.1 Introduction

An engineer designing an air-breathing propulsion system capable of reaching near orbital velocity is faced with a task significantly different in many basic ways from that faced in the design of current military and civilian engines. As flight speeds increase, the accompanying rise in stagnation temperature makes the design and operation of the propulsion system more difficult. Some of these problems are directly attributable to the material properties of the structure and components parts of the engine. Other difficulties, which in certain respects are even more challenging, involve the orderly combustion of the chosen fuel. In order to avoid flame out in the combustion chamber, the customary method of stabilizing the flame involves slowing the flow to about 100 feet per second. At high flight Mach numbers, nearly stagnating the flow produces undesireably high temperatures. The temperature after combustion can become so high that much of the fuel's energy is absorbed by internal degrees of freedom (such as vibrational energy of the molecules) and dissociation of the molecules into fragmented parts or atoms. In order to produce thrust, this stored energy must be reclaimed. The chemical kinetic rates of the necessary reactions make this difficult.

Currently, a number of possible ways of overcoming these problems associated with hypersonic propulsion are being investigated. One of these is the addition of heat to the flow while it still possesses appreciable kinetic energy so that the static temperature remains low enough during the energy release that energy is not absorbed in the internal degrees of freedom. A difficulty encountered in this case is arranging the combustion process so that it is complete in a length which can be accommodated in a reasonable engine design. Consequently, much effort has been spent exploring means to increase flame speeds, such as enhancing turbulence or by the choice of fuel. The use of standing detonation waves in the combustion chamber as well as in the external flow has been suggested and examined. In order to present an overview of the challenges posed by air-breathing, hypersonic engines and to explain the motivation for this research, some problems which require special attention are briefly discussed below.

1.2 Capture Area

In any supersonic engine inlet design the capture area becomes the actual physical inlet area of the engine. As the flight speed increases, the loads on the structure (thermal and pressure) force operation at very high altitudes where the density is low. The low density demands an ever increasing inlet area to maintain the same airflow through the engine at any given speed. To house the larger inlet area in the aircraft structure can result in a design which presents an unacceptable drag penalty due to wave thickness drag. A successful design for the craft thus requires the engine and airframe designs to be integrated. The "fuselage" must sufficiently compress the flow so that the subsequent engine inlet can capture enough air to provide the thrust needed to accelerate the aircraft. Since the acceleration depends on the thrust being greater than the drag, any increase in airflow through the propulsion system to increase the thrust nust not be accompanied by a greater increase in drag. The compression waves generated by the aircraft structure most likely result in shock waves; their attendant irreversibilties compromise the details of the rest of the engine design.

1.3 Mixing

Once the air is onboard, the next task is to arrange a thorough mixing of the fuel and air so that combustion can occur. If combustion is carried out at supersonic speeds (SCRAMJET) and the air speed is too high, the required mixing time necessitates a physical length which is difficult to accomodate in the airframe design.

1.4 Burning

Once the fuel and air are mixed so that combustion will occur, the designer must arrange the flow so that the flame front is stabilized in the combustion chamber. If the flow speed is too high, the flame can blow out through the downstream end of the combustor; if it is too low, the flame can flashback upstream of the flameholder. In either case the result can be flameout and the immediate loss of thrust. Should this occur in supersonic or hypersonic flight, the flame must be quickly relit or the particular mission aborted. Additionally, burning the fuel at other than a low subsonic Mach number entails an attendant loss in stagnation pressure which complicates the engine design.

1.5 Chemical Kinetic Effects

Once a fuel is chosen, the stoichiometric air-fuel ratio is fixed and thus the chemical energy released per unit mass is fixed. Consequently, as the flight speed increases, the heat addition become smaller in comparison to the stagnation enthalpy of the flow. If the static temperature of the flow is high enough, the fuel energy is sufficient to cause the molecules to dissociate. When this occurs, the process of recovering the dissociation energy is made difficult by the fact that recombination only can occur if appropriate three-body collisions occur. These collisions are rare when the density is low. Thus, to recover this energy, the jet nozzle must be made impractically long so that the enthalpy of the flow is converted into kinetic energy and thrust is produced. Or, care must be taken so that the temperature is kept low enough that appreciable dissociation does not occur. Hence, one is led to the addition of the fuel energy at low temperature (and thus a high flow speed). As the flight speed increases, the temperatures increase, and one is faced with trying to stabilize the flame front at ever increasing flow speeds to keep the temperatures low.

1.6 Jet Nozzle

The kinetic effects mentioned above are also important in the design of the jet nozzle so that thrust is produced. To find the kinetic energy leaving the nozzle requires a consideration of the chemical kinetics of the combustion products. Care must also be taken in designing the jet nozzle just as in the inlet duct to avoid having excessive area ratios that an adverse impact on the wave drag.

1.7 Unsteady Flow Engine

One engine concept which shows particular promise involves the use of unsteady waves to process the high Mach number flow without it reaching extremely high temperatures. In this engine scheme, the large kinetic energy of a hypersonic flow is stored temporarily in a "wave tube," leaving a "cooled" flow to be processed in a more or less conventional engine in a conventional fashion. The flow is then accelerated and compressed with a compression wave for expansion in a thrust nozzle. The formidable problems discussed above that arise at high speeds are thus avoided. Exploiting these principles, however, requires a great deal of development work, as there is limited engineering experience with such devices.

The physical configuration of such an engine consists of a rotating bank of "wave tubes" (the wave tubes contain wave processes which very closely resemble those which occur in shock tubes). This propulsion system resembles the comprex or the wave engine of the Brown Bovari company, which is currently being used as a "wave supercharger" for truck and automotive applications. Flow which has been diffused but is still supersonic enters one of the "wave tubes." The tube rotates past the opening and the end is thus closed. The momentum of the flow causes it too "stretch" and cool, i.e., an isentropic expansion wave is generated at the closed end and travels down the tube cooling the flow and bringing it to rest. This is to be contrasted with a steady flow process where slowing a flow always increases the temperature.

The cooled flow might then routed to a conventional turbojet, or to a ramjet. After the fuel has been added and burned, the flow is re-introduced into the wave tube, providing the gas that is compressed or shocked by the incoming diffused flow. This shocked gas is then made to exit the wave tube and enter the thrust nozzle.

Advantages of this engine design include that the wave processes are isentropic and also cool the flow. Furthermore, present engines might be used and their operating range extended into the high supersonic and hypersonic range. The operating conditions in the engine proper do not vary with increase of flight Mach number because the bank of wave tubes provides the necessary adjustment. Also, only the wave tubes are exposed to the stagnation temperature of the flow, and they are alternately exposed to hot and cold flows. Thus, they are, in effect, aerodynamically cooled.[1]

1.8 Summary

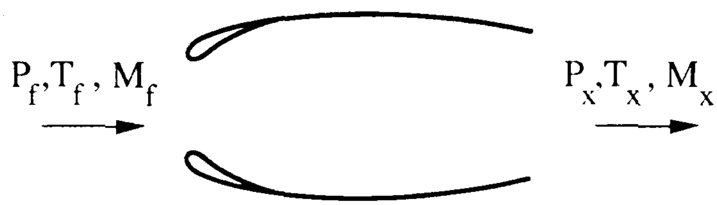
There are many serious problems which need to be solved before a successful hypersonic engine can be built. In the chapters which follow, criteria will be developed which allow the comparison of different propulsion systems. Furthermore, these criteria serve as guidelines for the designer who is exploring solutions to the difficulties of hypersonic propulsion systems.

Chapter 2

GENERAL CONSIDERATIONS OF HYPERSONIC PROPULSION

The basic tenet of what follows is that, given the inlet (flight) conditions and the exit stagnation sound speed and exit stagnation pressure of an air-breathing engine, very useful relationships that define limits imposed on a propulsion system may be derived. Since an engine essentially does no more than change the stagnation temperature and pressure of the air which passes through it, it may be treated as a "black box." Thus, a comparison of the performance of different engine designs requires little or no information about what is going on inside the engines themselves.

2.1 Basic Equations



where: M_f = flight Mach number = $\frac{u_f}{\sqrt{\gamma R T_f}}$, u_f = flight speed

P_f, T_f =flight (ambient) static pressure and temperature; they are fixed by the altitude of flight

M_x, P_x, T_x = exit conditions

2.1.1 Temperature and Pressure

In general, the stagnation temperature is the temperature attained by a moving fluid when brought to rest adiabatically. A definition for the stagnation temperature may be taken from the integrated energy equation for continuous, one-dimensional flow of an inviscid, non-heat-conducting perfect gas:

$$c_p T_o = c_p T + \frac{u^2}{2} \qquad (2.1)$$

A perfect gas obeys the thermal equation of state $p = \rho RT$. For constant specific heats and from the definition of Mach number,

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad \left(= 1 + \frac{1}{5} M^2, \ \text{for } \gamma = 1.4\right) \qquad (2.2)$$

If the stagnation process is reversible as well as adiabatic, the isentropic stagnation pressure (p_o) is reached:

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \quad (= \left(1 + \frac{1}{5}M^2\right)^{3.5} \qquad (2.3)$$

2.1.2 Thrust

From conservation of momentum along a streamline through the engine,

$$\text{Thrust} = \dot{m}(u_x - u_f) + p_x A_x - p_f A_f \qquad (2.4)$$

where: $\dot{m}$ = mass flow (taken here to be constant through the engine)

u_x, u_f = exit and flight velocities, respectively, in "engine fixed" coordinates

p_x, p_f = exit and flight static pressures (absolute), respectively

A_x, A_f = area of engine exit and inlet, respectively

The pressure terms are often dropped because their contribution to the thrust is small when the gas is completely expanded (or nearly so) in the nozzle. Defining F as the thrust per unit mass flow and dropping the pressure terms:

$$F \equiv \frac{\text{Thrust}}{\dot{m}} = u_x - u_f \qquad (2.5)$$

2.1.3 Some Useful Relations

From the definition of Mach number and sound speed (perfect gas):

$$\frac{u}{a_o} = \frac{u/a}{a_o/a} = \frac{M}{\sqrt{\gamma R T_o / \gamma R T}} = \frac{M}{\sqrt{T_o/T}}$$

and from equation (2.2),

$$\frac{u}{a_o} = \frac{M}{\sqrt{1 + \frac{\gamma - 1}{2} M^2}} \qquad (2.6)$$

2.1.3.1 Energy Limitation of Thrust

As seen in the prior section, for a given flight speed, thrust increases with exit speed, u_x. From equation (2.6), above, taken at the exit condition ($\gamma = 1.4$):

$$\frac{u_x}{a_{o_x}} = \frac{M_x}{\sqrt{1 + \frac{1}{5} M_x^2}} \qquad (2.7)$$

The exit stagnation sound speed, $a_{o_x} = \sqrt{\gamma R T_{o_x}}$, is limited by the amount of heat added during combustion. The exit Mach number, and therefore the exit speed, is determined by the extent to which the gas is expanded in the nozzle. It is possible to find a relation for the maximum exit speed.

Starting with the energy equation (2.1), and replacing the temperature with the perfect gas sound speed relation, $a = \sqrt{\gamma RT}$, produces:

$$5a_x^2 + u_x^2 = 5a_{o_x}^2 \qquad (2.8)$$

For an isentropic expansion, as $p_x \rightarrow 0$, $a_x \rightarrow 0$. Thus, the maximum exit speed possible is

$$u_{x_{max}} = \sqrt{5}\, a_{o_x} \qquad (2.9)$$

Combining equations (2.7) and (2.9) yields a relation for the percentage of complete expansion achieved in the nozzle:

$$PCE \equiv \frac{\dfrac{u_x}{a_{o_x}}}{\left(\dfrac{u_x}{a_{o_x}}\right)_{max}} = \frac{1}{\sqrt{5}} \frac{M_x}{\sqrt{1 + \frac{1}{5}M_x^2}} \qquad (2.10)$$

As seen in Figure 1, the percentage of complete expansion is over 90% by $M_x = 5$, and over 95% by $M_x = 7$. Consequently, expansion to higher Mach numbers will produce little additional thrust and the construction of nozzles for very high exit Mach numbers is not warranted.

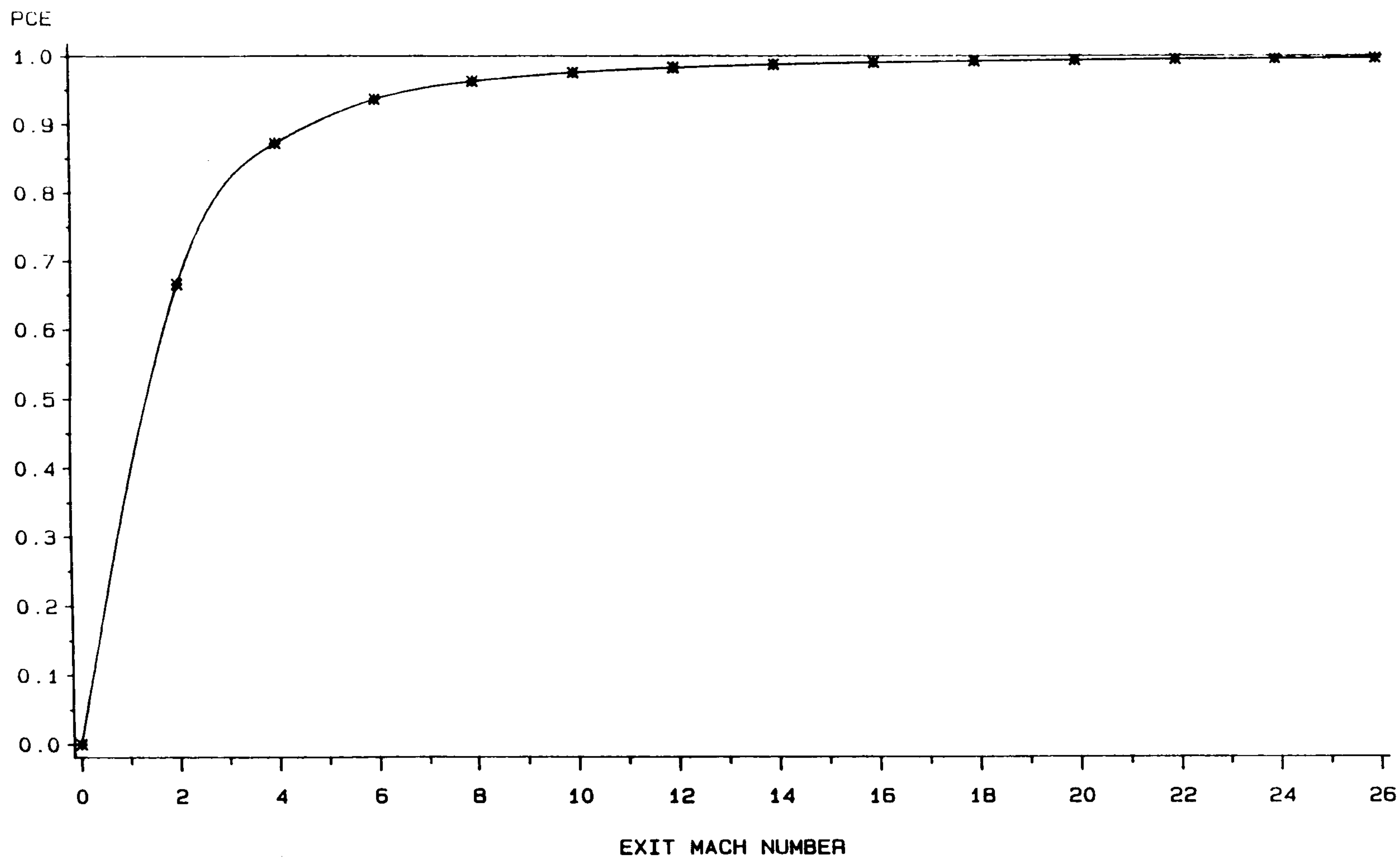


Figure 1: Percentage of complete expansion

2.1.3.2 Engine Pressure Ratio

Depending on the engine components, the exit and flight stagnation pressures are related; here, define ß so that:

$$p_{o_x} = ß\, p_{o_f} \tag{2.11}$$

where: ß = 1 for an ideal ramjet (no pressure losses in the diffuser, nozzle, or combustor, implying $M_x = M_f$)

ß < 1 for a real ramjet (pressure losses due to shocks, friction, etc.)

ß > 1 for a turbojet (see section below for a discussion of the significance of and the limitations on this statement.)

i.e., ß and T_{o_x} characterize the net effect of the "black box" (the engine). Dividing equation (2.11) by $p_f = p_x$, thereby assuming complete expansion in the nozzle, results in:

$$\frac{p_{o_x}}{p_x} = ß\,\frac{p_{o_f}}{p_f} \tag{2.12}$$

Thus from equation (2.3), taking $\gamma = 1.4$,

$$\left(1 + \frac{1}{5}M_x^2\right)^{3.5} = ß\left(1 + \frac{1}{5}M_f^2\right)^{3.5}$$

$$1 + \frac{1}{5}M_x^2 = \beta^{\frac{1}{3.5}}\left(1 + \frac{1}{5}M_f^2\right) \tag{2.13}$$

$$M_x = \sqrt{5\left[\beta^{\frac{1}{3.5}}\left(1 + \frac{1}{5}M_f^2\right) - 1\right]} \tag{2.14}$$

Thus, M_x is known given the flight Mach number and the stagnation pressure ratio across the engine.

2.2 Zero-Thrust Criterion for a Ramjet

2.2.1 Zero-Thrust Temperature Ratio

Using equation (2.5), thrust (F) is reduced to zero when:

$$u_x = u_f \tag{2.15}$$

Dividing by the flight stagnation sound speed,

$$\frac{1}{a_{of}}(u_x) = \frac{u_f}{a_{of}}$$

Now, using equation (2.6) twice,

$$\frac{1}{a_{o_f}}\left(\frac{a_{o_x} M_x}{\sqrt{1 + \frac{1}{5}M_x^2}}\right) = \frac{M_f}{\sqrt{1 + \frac{1}{5}M_f^2}}$$

Squaring both sides and then using the relation (perfect gas):

$$\left(\frac{a_{o_x}}{a_{o_f}}\right)^2 = \frac{T_{o_x}}{T_{o_f}}$$

yields the following:

$$\frac{T_{o_x}}{T_{o_f}} \cdot \frac{M_x^2}{1 + \frac{1}{5}M_x^2} = \frac{M_f^2}{1 + \frac{1}{5}M_f^2} \tag{2.16}$$

Now, using equations (2.13) and (2.14) on the left hand side,

$$\frac{T_{o_x}}{T_{o_f}} \cdot \frac{5\left[\frac{1}{\beta^{3.5}}\left(1 + \frac{1}{5}M_f^2\right) - 1\right]}{\frac{1}{\beta^{3.5}}\left(1 + \frac{1}{5}M_f^2\right)} = \frac{M_f^2}{1 + \frac{1}{5}M_f^2}$$

and then solving for the temperature ratio,

$$\left.\frac{T_{o_x}}{T_{o_f}}\right)_{\substack{zero\\thrust}} = \frac{M_f^2\, \beta^{\frac{1}{3.5}}}{5\left[\frac{1}{\beta^{3.5}}\left(1 + \frac{1}{5}M_f^2\right) - 1\right]} \tag{2.17}$$

The stagnation temperature ratio given by equation (2.17) is the ratio for zero thrust ($u_x = u_f$). At this condition all of the energy added during combustion is used to recoup stagnation pressure losses in the engine. Energy addition in excess of the amount specified by equation (2.17) yields a net engine thrust. It is shown below why the above relation only applies to engines where $\beta \leq 1$.

2.2.2 Combustor Temperature Rise

When computing the temperature after combustion, it is necessary to account for the energy required to raise the fuel temperature to the combustor exit temperature. Therfore, the heat addition due to combustion, Q, will be defined here on a per unit mass flow of air-fuel mixture basis. An energy balance across the combustion chamber yields (taking the specific heats to be constants):

$$
\begin{aligned}
(\dot{m}_{air} + \dot{m}_{fuel})\, Q &= \Delta\dot{H}_{o_{air}} + \Delta\dot{H}_{o_{fuel}} \\
&= c_{p_{air}}\, \dot{m}_{air}\, (T_{o_q} - T_{o_c}) + c_{p_{fuel}}\, \dot{m}_{fuel}\, (T_{o_q} - T_{o_{fuel}})
\end{aligned}
$$

where: $\dot{H}_o$ = stagnation enthalpy

subscripted q $\Rightarrow$ condition at the combustor exit (air + fuel)

subscripted c $\Rightarrow$ condition at the compressor exit (air)

Neglecting the difference between T_{o_c} and $T_{o_{fuel}}$ as well as the difference in the specific heats of the fuel and air results in:

$$(\dot{m}_{air} + \dot{m}_{fuel})\, Q = c_p\, (\dot{m}_{air} + \dot{m}_{fuel})\, (T_{o_q} - T_{o_c})$$

Cancelling like terms,

$$Q = c_p\, (T_{o_q} - T_{o_c}) = c_p\, (\Delta T_o)_{combustion} \qquad (2.18)$$

For a turbojet, the compressor work is equal to the turbine work, i.e.,

$$W_c = W_t$$

or

$$c_p\, (T_{o_c} - T_{o_f}) = c_p\, (T_{o_q} - T_{o_x}) \qquad (2.19)$$

Rearranging and using equation (2.18),

$$Q = c_p\, (T_{o_q} - T_{o_c}) = c_p\, (T_{o_x} - T_{o_f}) \qquad (2.20)$$

Note that, if considering a ramjet, there is no compressor/turbine combination, and thus equation (2.20) applies because $T_{o_c} = T_{o_f}$ and $T_{o_q} = T_{o_x}$. Equation (2.20) is also valid if there are shock waves present since the stagnation temperature is constant across adiabatic shocks.

2.2.3 Ideal Ramjet

For the case of the ideal ramjet (no losses) one finds that $\beta = 1$ and equation (2.17) reduces to:

$$\left.\frac{T_{o_x}}{T_{o_f}}\right)_{\substack{zero\\thrust}} = 1 \qquad (2.21)$$

Thus any heat addition gives positive thrust. This is true because, if no heat were added, the air would simply decelerate from the flight velocity in the diffuser, then accelerate back to u_f in the nozzle. Obviously, this would produce no thrust.

2.2.4 Actual Ramjet

For the case of the actual ramjet, $\beta < 1$ (but $\beta > 0$, of course), i.e. the stagnation pressure at the exit is less than at the inlet. This loss may be due to shocks, friction, etc. Consequently, when $\beta < 1$, some of the fuel energy which is added to the air is used just to get back up to the zero thrust point. Equation (2.17) determines this amount of energy.

If no heat where added in this case, the loss of pressure would cause the exit velocity reached after expansion in the nozzle to be less than the flight velocity. Mathematically, the engine would then be producing negative thrust (drag). In reality, however, the gases would not exit the rear of the engine and the assumption of steady flow at the chosen conditions would be invalid.

As an example, consider a "real" ramjet at 100,000 feet, flying at $M_f = 10$, and a pessimistic $\beta = 0.003$ (the stagnation pressure loss from a normal shock at Mach 10); then:

$$T_{o_f} = 8400° \text{ R}$$

Equation (2.17) gives:

$$\left.\frac{T_{o_x}}{T_{o_f}}\right)_{\substack{zero\\thrust}} = 1.27 \quad \Rightarrow \quad (\Delta T_o)_{\substack{zero\\thrust}} = 2272° \text{ R}$$

The temperature rise for ideal, stoichiometric combustion of a hydrocarbon fuel with a heat content of 18,000 BTU/lb_m is about 4700° R. Thus, recouping the pressure loss due to a normal shock in the inlet would use up a substantial portion of this energy, greatly reducing the energy available to produce thrust. Normal shocks can be avoided by designing for oblique shocks; taking a ß of 0.01 yields a zero-thrust temperature rise of 1326° R. However, as seen in Figure 2, by Mach 17 normal shock losses use all of the available fuel energy, making it impossible to generate thrust.

2.2.5 Zero-Thrust Pressure Ratio

The analysis of Section 2.2.1 will now be reversed, and a relation for the zero-thrust stagnation pressure ratio (ß) will be found. Taking equation (2.13) and rearranging:

$$\frac{1}{\beta^{3.5}} = \frac{1 + \frac{1}{5}M_x^2}{1 + \frac{1}{5}M_f^2} \tag{2.22}$$

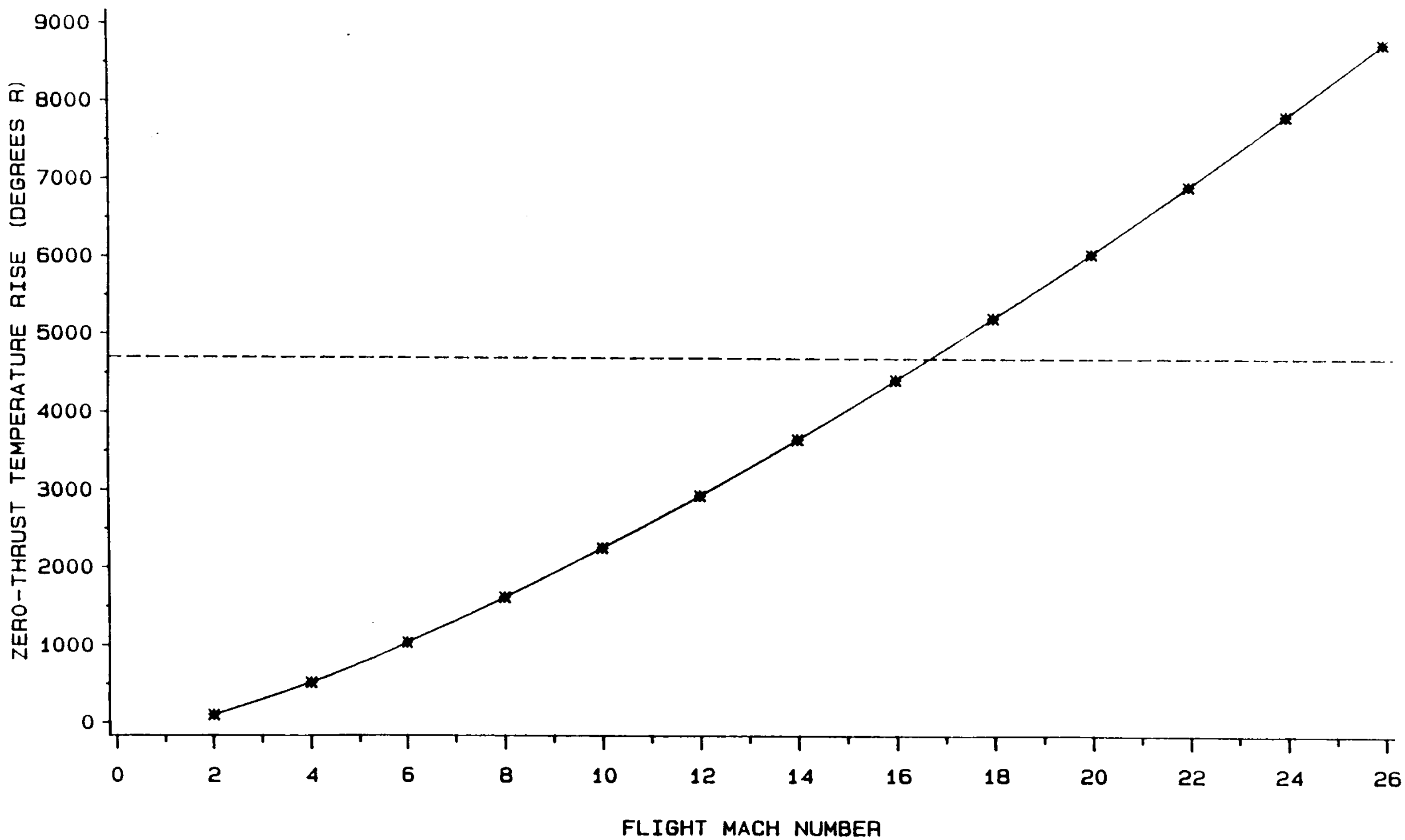


FIGURE 2: Temperature rise to recoup losses due to a normal shock

Substitution of equation (2.22) into equation (2.16) (which is derived in Section 2.2.1 for zero thrust) yields:

$$\beta^{\frac{1}{3.5}} = \frac{T_{o_x}}{T_{o_f}} \cdot \frac{M_x^2}{M_f^2} \qquad (2.23)$$

Remembering from Section 2.2.2 that Q, the heat addition, is defined per unit mass flow of air-fuel mixture, for a fuel with heat content per mass Δh_{fuel}, an energy balance across the combustion chamber shows that

$$c_p(\dot{m}_{air} + \dot{m}_{fuel})(T_{o_q} - T_{o_c}) = (\dot{m}_{air} + \dot{m}_{fuel})Q = \dot{m}_{fuel}\,\Delta h_{fuel}$$

From equation (2.20) then, defining the fuel-air ratio, $\lambda \equiv \dfrac{\dot{m}_{fuel}}{\dot{m}_{air}}$

$$T_{o_q} - T_{o_c} = \frac{Q}{c_p} = T_{o_x} - T_{o_f} = \frac{\Delta h_{fuel}}{c_p\left(\frac{1}{\lambda} + 1\right)} \qquad (2.24)$$

Rearranging,

$$\frac{T_{o_x}}{T_{o_f}} = 1 + \frac{\Delta h_{fuel}}{c_p T_{o_f}\left(\frac{1}{\lambda} + 1\right)} \qquad (2.25)$$

Putting (2.25) into (2.23):

$$\frac{1}{\beta^{3.5}} = \frac{M_x^2}{M_f^2}\left[1 + \frac{\Delta h_{fuel}}{c_p T_{of}\left(\frac{1}{\lambda} + 1\right)}\right] \qquad (2.26)$$

Squaring equation (2.14),

$$M_x^2 = 5\left[\frac{1}{\beta^{3.5}}\left(1 + \frac{1}{5}M_f^2\right) - 1\right] \qquad (2.27)$$

and substituting it and then (2.2) into equation (2.26) produces, after some rearrangement and simplification:

$$\left.\frac{p_{o_x}}{p_{o_f}}\right)_{\substack{zero\\thrust}} = \beta_{\substack{zero\\thrust}} = \left[\frac{1 + \dfrac{\Delta h_{fuel}}{c_p T_f\left(\frac{1}{\lambda} + 1\right)\left(1 + \frac{1}{5}M_f^2\right)}}{1 + \dfrac{\Delta h_{fuel}}{c_p T_f\left(\frac{1}{\lambda} + 1\right)}}\right]^{3.5} \qquad (2.28)$$

In Figure 3 the zero-thrust pressure ratio (from equation (2.28)) and the pressure ratio across a normal shock are plotted as functions of flight Mach number (altitude = 100,000 ft, $T_f = 400°$ R, $c_p = 0.24$ BTU/lb$_m$°R, $\Delta h_{fuel} = 18{,}000$ BTU/lb$_m$, $\lambda = 1/15$). By Mach 17, the normal shock pressure loss is so great that there is not enough fuel energy to produce thrust. Figure 3 makes it clear that efficient diffusion is critical to any hypersonic propulsion system: integration of the engine with the airframe is essential.

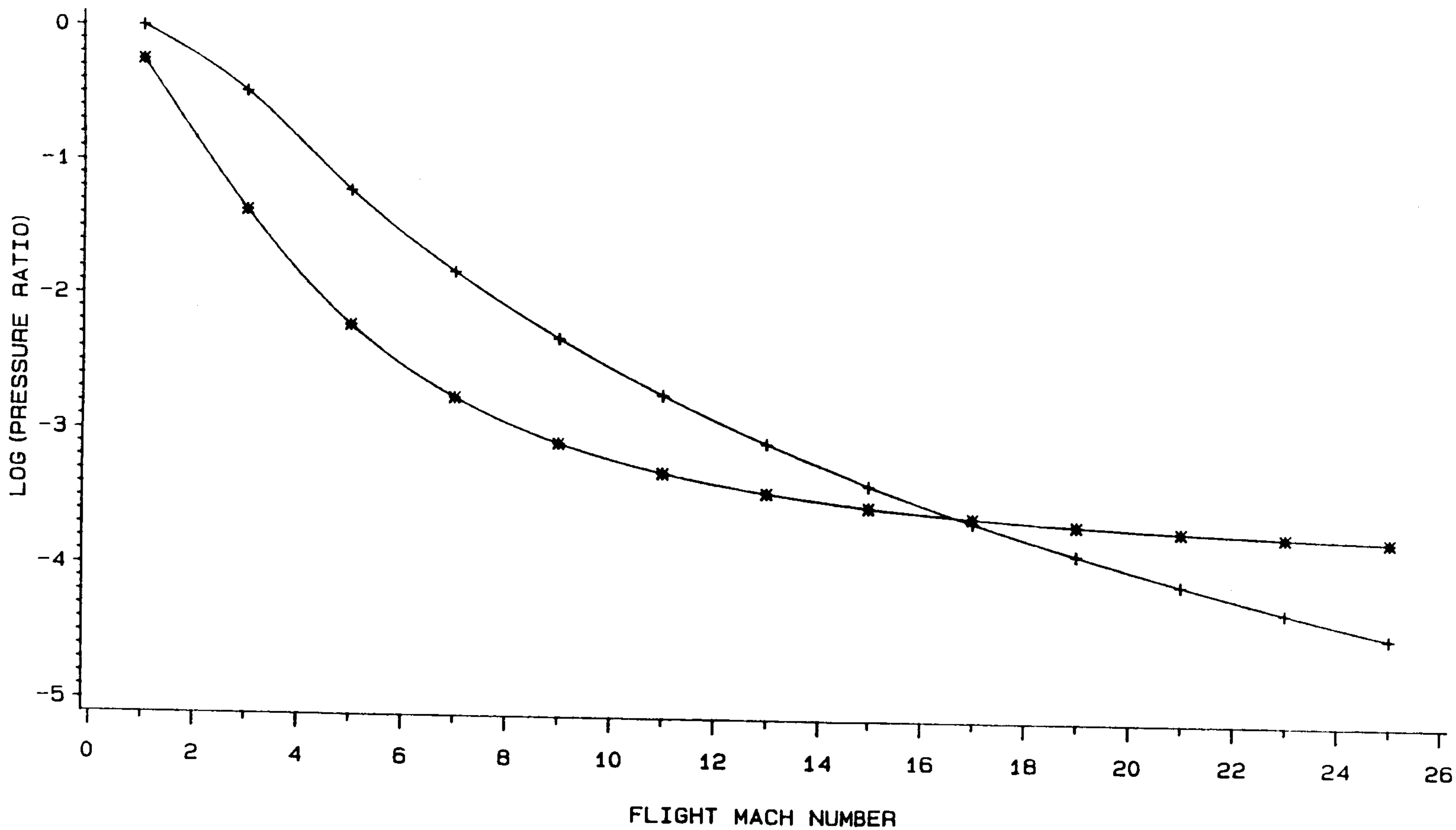


Figure 3: Pressure ratios for zero thrust and normal shock

* = zero thrust
+ = normal shock

2.3 Turbojet

The case of the turbojet is discussed below, including why equation (2.17) does not apply. As described above in Section 2.2.2, the compressor work is equal to the turbine work, and thus:

$$Q = c_p (T_{o_q} - T_{o_c}) = c_p (T_{o_x} - T_{o_f}) \tag{2.20}$$

Since T_{o_f} is known given the flight conditions, equation (2.17) gives the minimum T_{o_x} required to produce positive thrust, and thus $c_p(T_{o_x} - T_{o_f})^{zero}_{thrust}$ is known as a function of ß. Equation (2.20), therefore, shows that if

$$Q = c_p (T_{o_q} - T_{o_c}) > c_p (T_{o_x} - T_{o_f})^{zero}_{thrust}$$

then positive thrust is generated. In summary, equation (2.17) sets the minimum amount of heat that must be added to overcome engine inefficiencies and produce positive thrust, and it only requires knowledge of M_f and ß.

It was stated above (equation (2.11)) that, for the turbojet, ß > 1. The reason for this lies in an understanding of the significance of stagnation pressure. In general, the overall efficiency (η) of air-breathing engines increases with p_{o_x}. For the ideal ramjet $p_{o_x} = p_{o_f}$; thus ramjets tend to become more efficient as M_f increases (this is why ramjets are not used at subsonic speeds). At subsonic and low supersonic speeds, where the stagnation pressures are low, a compressor/turbine combination is employed to raise p_{o_x} in order to increase the engine's efficiency, η. This occurs in the following manner:

$$W_c = W_t$$

$$T_{o_c} - T_{o_f} = T_{o_q} - T_{o_x} \qquad (2.19)$$

The change in stagnation temperature is the same across the compressor and turbine, and since heat is added during combustion, the turbine inlet temperature is higher than the compressor exit temperature. Symbolically,

$$(\Delta T_o)_c = (\Delta T_o)_t = \Delta T_o$$

and

$$T_{o_q} > T_{o_c} \qquad (2.29)$$

It is known that

$$T_{o_x} = T_{o_q} - \Delta T_o \quad \text{and}$$

$$T_{o_f} = T_{o_c} - \Delta T_o$$

thus

$$\frac{T_{o_x}}{T_{o_q}} = 1 - \frac{\Delta T_o}{T_{o_q}} \quad \text{and} \qquad (2.30)$$

$$\frac{T_{o_f}}{T_{o_c}} = 1 - \frac{\Delta T_o}{T_{o_c}} \qquad (2.31)$$

Since $T_{o_q} > T_{o_c}$, an examination of equations (2.30) and (2.31) reveals that:

$$\frac{T_{o_x}}{T_{o_q}} > \frac{T_{o_f}}{T_{o_c}}$$

Or, inverting

$$\frac{T_{o_c}}{T_{o_f}} > \frac{T_{o_q}}{T_{o_x}} \qquad (2.32)$$

The pressure ratio varies with the temperature ratio (in compressors and turbines); thus, the pressure ratio across the compressor is greater than the pressure ratio across the turbine. The result is that the compressor/turbine combination yields a ß greater than unity.

An additional constraint on the temperature and pressure ratios can be derived from thermodynamic considerations. However, it is instructive to first examine a numeric example in order to display the need for this addition constraint:

At an altitude of 100,000 feet, $T_f = 400°$ R. Choosing $M_f = 2$, and $ß = 1.1$, one finds from equation (2.2):

$$T_{o_f} = 720° \text{ R}$$

and from equation (2.17):

$$\left.\frac{T_{o_x}}{T_{o_f}}\right)_{\substack{\text{zero}\\ \text{thrust}}} = 0.967$$

which is less than unity. This is impossible since it would mean that a net stagnation temperature loss across the engine could be suffered (i.e., energy withdrawn, not added) while the engine still generated positive thrust. Consequently, equation (2.17) is only valid for $ß \leq 1$, i.e., only for ramjets. The additional constraint mentioned above explains why this is true.

It is stated above that $ß = \frac{p_{o_x}}{p_{o_f}} > 1$ for the turbojet. From the Second Law of Thermodynamics, the entropy of an isolated system must increase or remain the same. In other words, it requires work input or heat removal to reduce the entropy of a system. If a system is isolated, i.e. it has no shafts, etc. attached to it, then no work can be done on it. If a control volume around an engine is taken to include the inlet and exit air streams as well as the fuel, then the engine is an isolated adiabatic system and the following condition applies:

$$\Delta s \geq 0$$

Thus for a perfect gas travelling from inlet to exit:

$$s_x - s_f = \Delta s = c_p \ln\frac{T_{o_x}}{T_{o_f}} - R \ln\frac{p_{o_x}}{p_{o_f}} \geq 0 \qquad (2.33)$$

Substituting the definition of ß and solving for the temperature ratio,

$$\frac{T_{o_x}}{T_{o_f}} \geq ß^{\frac{\gamma-1}{\gamma}} \qquad (2.34)$$

From equation (2.20),

$$Q = c_p T_{of}\left(\frac{T_{ox}}{T_{of}} - 1\right) \qquad (2.35)$$

Examination of equations (2.34) and (2.35) reveals that if ß > 1, the stagnation temperature ratio is greater than unity and Q must be positive. Thus, the Second Law requires that, in the case of engines described here, heat be added in order to achieve a ß greater than unity.

2.4 Ramjet at Very High Speed

The amount of heat that is added to a unit mass of air is determined by the fuel-air ratio, regardless of flight speed. The amount of oxygen in a unit mass of air limits the thrust per unit mass (u_x - u_f).

For an ideal ramjet ß = 1 and thus $p_{o_x} = p_{o_f}$. Assuming complete expansion in the nozzle, $p_x = p_f$, and therefore from equation (2.3), $M_x = M_f$. From this the following can be written:

$$\frac{p_{o_x}}{p_x} = \frac{p_{o_f}}{p_f}$$

and thus,

$$\left(\frac{T_{o_x}}{T_x}\right)^{3.5} = \left(\frac{T_{o_f}}{T_f}\right)^{3.5}$$

Simplifying and rearranging,

$$\frac{T_{o_x}}{T_{o_f}} = \frac{T_x}{T_f} \tag{2.36}$$

Using $M_x = M_f$ along with equation (2.6) yields:

$$\frac{u_x}{a_{o_x}} = \frac{u_f}{a_{o_f}} \tag{2.37}$$

For ideal inlet and nozzle sections there is no change in the stagnation temperature; thus an energy balance across the entire ramjet gives:

$$c_p T_{o_f} + Q = c_p T_{o_x} \tag{2.38}$$

Rearranging and employing equation (2.36):

$$1 + \frac{Q}{c_p T_{o_f}} = \frac{T_{o_x}}{T_{o_f}} = \frac{T_x}{T_f} \tag{2.39}$$

Combining equations (2.37) and (2.39):

$$\frac{u_x}{u_f} = \frac{a_{o_x}}{a_{o_f}} = \sqrt{\frac{T_{o_x}}{T_{o_f}}} = \sqrt{1 + \frac{Q}{c_p T_{o_f}}} \tag{2.40}$$

As M_f increases, T_{o_f} becomes very large. Since Q is fixed by the fuel-air ratio, equation (2.40) shows that

$$u_x \rightarrow u_f \tag{2.41}$$

and thus the thrust decreases with flight speed. Since $\frac{Q}{c_p\, T_{of}} \geq 0$ always, however,

$$u_x \geq u_f \tag{2.42}$$

and therefore, no matter how fast an ideal ramjet is travelling, any heat addition will result in positive thrust. This is also shown in Section 2.2.3. For real ramjets with shock losses, etc., equation (2.17) gives the amount of heat addition required to produce positive thrust.

Expanding equation (2.38) via equation (2.1):

$$c_pT_f + \frac{u_f^2}{2} + Q = c_pT_x + \frac{u_x^2}{2} \tag{2.43}$$

Multiplying equation (2.39) by c_pT_f,

$$c_pT_f + \frac{T_f}{T_{of}}Q = c_pT_x \tag{2.44}$$

Putting this into (2.43) and simplifying:

$$Q\left(1 - \frac{T_f}{T_{of}}\right) = \frac{1}{2}(u_x^2 - u_f^2) \tag{2.45}$$

$$= \frac{1}{2}(u_x - u_f)(u_x + u_f)$$

Using the definition of thrust per mass flow (equation (2.5)) and rearranging:

$$Q\left(1 - \frac{T_f}{T_{of}}\right) = F\,u_f \frac{\left(\frac{u_x}{u_f} + 1\right)}{2} = \frac{F\,u_f}{\eta_p} \qquad (2.46)$$

Note that, whereas many sources simply define it, here the propulsive efficiency (η_p) has been derived from the conservation equations (energy and momentum).

Inspection of equation (2.46) reveals that as the flight Mach number becomes very large, $\frac{T_f}{T_{of}}$ becomes very small. This means that the left-hand side of (2.46) is approximately Q, the heat added:

$$Q \cong \frac{F\,u_f}{\eta_p} = \frac{1}{2}(u_x - u_f)(u_x + u_f) \qquad (2.47)$$

Remember that Q is limited by the maximum (stoichiometric) fuel-air ratio. As shown above, the exit velocity approaches the inlet velocity as flight speed becomes very large. Thus, the exit-to-inlet velocity ratio approaches unity, and the propulsive efficiency approaches 100%. Consequently, equation (2.46) reduces to

$$Q \approx F\,u_f \qquad (2.48)$$

which says that all of the fuel's energy is utilized by doing work on the vehicle. This is as good as is possible.

The importance of this last point should not be overlooked. Equation (2.46) suggests the definition of an overall engine efficiency:

$$\eta \equiv \frac{Fu_f}{Q} = \eta_p\left(1 - \frac{T_f}{T_{o_f}}\right) \tag{2.49}$$

This engine efficiency is the ratio of the work actually done on the vehicle to the available fuel energy. As shown above, even though the thrust is decreasing, the overall efficiency is approaching 100% as the flight speed increases. Hence, a ramjet should be flown up to as high a speed as possible; switching to a rocket is necessary only when there is not enough oxygen available to burn the fuel needed to obtain the desired thrust.

2.5 Summary

For any propulsion system, its performance follows the proper application of just two equations, namely ($\gamma = 1.4$):

$$\frac{u_x}{a_{o_x}} = \frac{M_x}{\sqrt{1 + \frac{1}{5}M_x^2}} \tag{2.7}$$

and

$$\frac{p_{o_x}}{p_x} = \left(1 + \frac{1}{5}M_x^2\right)^{3.5} \tag{2.3}$$

All propulsion systems can be compared on the basis of the use of the fuel energy $\eta \equiv \frac{Fu_f}{Q}$ (equation (2.49)).

Chapter 3

ENTROPY CONSIDERATIONS

3.1 General Equations

From the definition of entropy, for an ideal gas with constant specific heats, the entropy change across the engine as a whole is given by:

$$s_x - s_f = \Delta s_{total} = c_p \ln\frac{T_x}{T_f} - R \ln\frac{p_x}{p_f} \tag{3.1}$$

Although the stagnation conditions through an engine might change across each component, overall, the change in the stagnation temperature of the flow through the engine is the energy added to the flow by the combustion process. To use this fact write the steady flow energy equation at the inlet and exit as follows:

$$c_p T_f + \frac{u_f^2}{2} = c_p T_{o_f} = h_{o_f} \tag{3.2}$$

$$c_p T_x + \frac{u_x^2}{2} = c_p T_{o_x} = h_{o_x} \tag{3.3}$$

Subtracting (3.2) from (3.3) results in:

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = c_p\,(T_{o_x} - T_{o_f}) - c_p\,(T_x - T_f)$$

Since from equation (2.20), $Q = c_p\,(T_{o_x} - T_{o_f})$,

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q - c_p\,(T_x - T_f) \qquad (3.4)$$

Again assuming complete expansion in the nozzle, $p_x = p_f$, and thus equation (3.1) becomes:

$$(\Delta s)_{total} = c_p \ln\frac{T_x}{T_f}$$

or

$$\frac{T_x}{T_f} = \exp\left(\frac{(\Delta s)_{total}}{c_p}\right) \qquad (3.5)$$

Putting (3.5) into (3.4):

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = \frac{F\,u_f}{\eta_p} = Q - c_pT_f\left[\exp\left(\frac{(\Delta s)_{total}}{c_p}\right) - 1\right] \qquad (3.6)$$

Equation (3.6) is valid for engines of all types; the assumptions made were adiabatic flow, ideal gas with constant specific heat, and complete expansion in the nozzle. Equation (3.6) gives the net kinetic energy change through the engine (and thereby the thrust per unit mass flow, $F = u_x - u_f$, since u_f is known) in terms of the heat added and the total entropy change across the engine.

At this point, a relation for $(\Delta s)_{total}$ is needed. In general,

$$(\Delta s)_{total} = \sum_{\substack{\text{all} \\ \text{components}}} (\Delta s)_{component} \tag{3.7}$$

The great virtue of this analysis is that it allows the inclusion of individual component parameters and efficiencies and displays their influence.

3.2 Turbojet Case

The case of the turbojet will be examined; then it will be shown that the turbojet relations reduce to the ramjet case (equation (2.45)) when the compressor and turbine are removed. For the turbojet,

$$(\Delta s)_{total} = (\Delta s)_d + (\Delta s)_c + (\Delta s)_q + (\Delta s)_t + (\Delta s)_n \tag{3.8}$$

where the subscripts d, c, q, t, n, refer to the diffuser, compressor, combustor, turbine, and nozzle, respectively.

3.2.1 Assumptions

For simplicity, consideration will only be given here to perfect diffusers and nozzles, i.e. where the air is decelerated and accelerated isentropically:

$$(\Delta s)_d = (\Delta s)_n = 0 \tag{3.9}$$

However, an asset of this analysis is that it easily allows the later inclusion of the entropy rise due to shocks in the diffuser or nozzle or friction effects.

3.2.2 Compressor

Applying equation (3.1) to the compressor:

$$(\Delta s)_c = c_p \ln\frac{T_{o_c}}{T_{o_f}} - R \ln\frac{p_{o_c}}{p_{o_f}} \tag{3.10}$$

In order to include real compressors into the analysis, compressor efficiency is defined as the ratio of the change of stagnation enthalpy for an isentropic compression to the change of stagnation enthalpy for the real compression, where both compressions are across equal pressure ratios:

$$\eta_c \equiv \frac{(\Delta h_{o_c})_s}{(\Delta h_{o_c})_a} = \frac{(T_{o_c})_s - T_{o_f}}{(T_{o_c})_a - T_{o_f}} \tag{3.11}$$

where: subscripted s $\Rightarrow$ isentropic process

subscripted a $\Rightarrow$ actual process

Dividing the numerator and the denominator by T_{o_f} and using the isentropic relation

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

yields

$$\eta_c = \frac{\left(\frac{p_{o_c}}{p_{o_f}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{(T_{o_c})_a}{T_{o_f}} - 1} \tag{3.12}$$

Note that the subscript "s" has been dropped in the numerator because, according to the definition of η_c, the pressure ratios of "s" and "a" are the same. The changes of stagnation enthalpies are different: $(\Delta h_{o_c})_s <$ $(\Delta h_{o_c})_a$. Solving for the pressure ratio while dropping the subscript "a":

$$\frac{p_{o_c}}{p_{o_f}} = \left[1 + \eta_c\left(\frac{T_{o_c}}{T_{o_f}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}} \tag{3.13}$$

Putting this into equation (3.10) and using the ideal gas relation $c_p = \frac{\gamma}{\gamma-1}R$ produces the relation for the entropy change across the compressor,

$$\frac{(\Delta s)_c}{c_p} = \ln\left[\frac{\frac{T_{o_c}}{T_{o_f}}}{1 + \eta_c\left(\frac{T_{o_c}}{T_{o_f}} - 1\right)}\right] \tag{3.14}$$

3.2.3 Combustor

Employing the usual assumption of constant pressure combustion at low speed ($p_{o_q} = p_{o_c}$), equation (3.1) for the combustor is:

$$\frac{(\Delta s)_q}{c_p} = \ln\left(\frac{T_{o_q}}{T_{o_c}}\right) \tag{3.15}$$

3.2.4 Turbine

From (3.1), the entropy change through the turbine is:

$$(\Delta s)_t = c_p \ln\frac{T_{o_x}}{T_{o_q}} - R \ln\frac{p_{o_x}}{p_{o_q}} \tag{3.16}$$

Analogous to the compressor, the turbine efficiency is defined as:

$$\eta_t \equiv \frac{(\Delta h_{o_t})_a}{(\Delta h_{o_t})_s} = \frac{T_{o_q} - (T_{o_x})_a}{T_{o_q} - (T_{o_x})_s} = \frac{1 - \dfrac{(T_{o_x})_a}{T_{o_q}}}{1 - \dfrac{(T_{o_x})_s}{T_{o_q}}} \tag{3.17}$$

Solving for the isentropic stagnation temperature ratio,

$$\frac{(T_{o_x})_s}{T_{o_q}} = 1 - \frac{1 - \dfrac{(T_{o_x})_a}{T_{o_q}}}{\eta_t}$$

and then using the same isentropic relation as for the compressor analysis:

$$\frac{p_{o_x}}{p_{o_q}} = \left[\frac{(T_{o_x})_s}{T_{o_q}}\right]^{\frac{\gamma}{\gamma-1}} = \left[1 - \frac{1 - \frac{(T_{o_x})_a}{T_{o_q}}}{\eta_t}\right]^{\frac{\gamma}{\gamma-1}} \tag{3.18}$$

where once again the "s" has been dropped. Substituting equation (3.18) into equation (3.16), again using $c_p = \frac{\gamma}{\gamma-1}R$, and dropping the subscript "a" produces the relation for the entropy change across the turbine:

$$\frac{(\Delta s)_t}{c_p} = \ln\left[\frac{\eta_t \frac{T_{o_x}}{T_{o_q}}}{\eta_t - 1 + \frac{T_{o_x}}{T_{o_q}}}\right] \tag{3.19}$$

3.2.5 Synthesis

Recalling equation (3.6) and substituting in equations (3.8) and (3,9),

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = \frac{F\,u_f}{\eta_p} = Q - c_pT_f\left[\exp\left(\frac{(\Delta s)_c + (\Delta s)_q + (\Delta s)_t}{c_p}\right) - 1\right]$$

$$= Q - c_pT_f\left[\exp\left(\frac{(\Delta s)_c}{c_p}\right)\cdot\exp\left(\frac{(\Delta s)_q}{c_p}\right)\cdot\exp\left(\frac{(\Delta s)_t}{c_p}\right) - 1\right] \tag{3.20}$$

Now, when the entropy rise relation for each component (equations (3.14), (3.15), and (3.19)) is included, one finds after simplification:

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q - c_pT_f\left[\frac{\eta_t \dfrac{T_{o_x}}{T_{o_f}}}{\left[1 + \eta_c\left(\dfrac{T_{o_c}}{T_{o_f}} - 1\right)\right]\left[\eta_t - 1 + \dfrac{T_{o_x}}{T_{o_q}}\right]} - 1\right] \tag{3.21}$$

T_{o_c} can be eliminated from the above equation by using the fact that the compressor work equals the turbine work (equation (2.19)). Dividing this equation by T_{o_f}:

$$\frac{T_{o_c}}{T_{o_f}} - 1 = \frac{T_{o_q} - T_{o_x}}{T_{o_f}} \tag{3.22}$$

Substituting (3.22) into the denominator of equation (3.21) produces:

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q - c_pT_f\left[\frac{\eta_t \dfrac{T_{o_x}}{T_{o_f}}}{\left[1 + \eta_c\left(\dfrac{T_{o_q} - T_{o_x}}{T_{o_f}}\right)\right]\left[\eta_t - 1 + \dfrac{T_{o_x}}{T_{o_q}}\right]} - 1\right] \tag{3.23}$$

Equation 3.21 or considerations leading to an equation of this type for a particular application might be used to design experiments to measure component efficiencies when the components are contained in an actual engine or operating environment. This procedure might be useful to explore the validity of matching schemes used to predict engine performance from separate tests made on the individual components. If one uses the equations relating temperatures and pressures to the efficiencies of the components to find a relation between easily measured pressures in the engine and not so easily measured temperatures an engine test scheme can be devised that is more conveniently implemented in practice such as testing under flight conditions.

3.2.6 Reduction to ramjet equation

It is asserted in the derivation of equation (3.6) that the relation is valid for any type of engine. If the compressor/turbine combination is "removed" from a turbojet, the resulting engine is a ramjet. This must be reflected in the equations. Analytically, "removal" consists of setting the stagnation temperature changes across the compressor and turbine equal to zero and their efficiencies (η_c, η_t) equal to unity in either equation (3.21) or (3.23):

$$T_{o_c} = T_{o_r},\ T_{o_x} = T_{o_q},\ \eta_c = \eta_t = 1$$

Inclusion of the above in either equation (3.21) or (3.23) leaves:

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q - c_pT_f\left[\frac{T_{o_x}}{T_{o_f}} - 1\right] \qquad (3.24)$$

$$= Q - c_pT_f\left[\frac{T_{o_x}}{T_{o_f}} - 1\right]\left(\frac{T_{o_f}}{T_{o_f}}\right)$$

$$= Q - c_p\,(T_{o_x} - T_{o_f})\frac{T_f}{T_{o_f}}$$

After using $Q = c_p(T_{o_x} - T_{o_f})$ one finds

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q\left(1 - \frac{T_f}{T_{o_f}}\right) \qquad (3.25)$$

which is the same as equation (2.45).

3.2.7 Maximum Output Turbojet

For the case where there are no losses in either the compressor or turbine,

$$\eta_c = \eta_t = 1 \qquad (3.26)$$

which, from equation (3.21), gives after simplification:

$$\frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = Q - c_pT_f\left(\frac{T_{o_a}}{T_{o_c}} - 1\right)$$

Substituting in equation (2.20) and multiplying the last term by $\frac{T_{o_c}}{T_{o_c}}$ gives

$$\Delta\left(\frac{KE}{mass}\right) = \frac{1}{2}u_x^2 - \frac{1}{2}u_f^2 = c_p(T_{o_q} - T_{o_c})\left(1 - \frac{T_f}{T_{o_c}}\right) \quad (3.27)$$

Noting that T_f and T_{o_q} are known (from flight conditions and material limitations, respectively), it is possible to find a condition on T_{o_c} which will maximize the air's kinetic energy change through the engine. Taking a derivative of equation (3.27) with respect to T_{o_c} and setting it equal to zero:

$$\frac{d}{dT_{o_c}}\Delta\left(\frac{KE}{mass}\right) = 0 = c_p\left(\frac{T_{o_q}T_f}{(T_{o_c})^2} - 1\right)$$

Solving for T_{o_c} yields,

$$T_{o_c} = \sqrt{T_{o_q}T_f} \quad (3.28)$$

Equation (3.28) is the relation for a maximum-output, ideal turbojet.

For an actual turbojet, the kinetic energy change can be plotted as a function of compressor temperature to find the maximum-output operating condition. This is accomplished by eliminating T_{o_x} from equation (3.21):

$$W_c = W_t$$

$$T_{o_c} - T_{o_f} = T_{o_q} - T_{o_x} \quad (2.19)$$

$$T_{o_x} = T_{o_q} - T_{o_c} + T_{o_f} \quad (3.29)$$

and thus, defining DKE $\equiv \frac{1}{2}(u_x^2 - u_f^2)$:

$$DKE \equiv Q - c_p T_f \left[\frac{\eta_t \left(\frac{T_{o_q} - T_{o_c}}{T_{o_f}} + 1 \right)}{\left[1 + \eta_c \left(\frac{T_{o_c}}{T_{o_f}} - 1 \right) \right] \left[\eta_t - \frac{T_{o_c} - T_{o_f}}{T_{o_q}} \right]} - 1 \right] \quad (3.30)$$

DKE is plotted in Figure 4 for the case:

$$T_{o_q} = 3000° \text{ R}, \; \eta_c = 0.6, \; \eta_t = 0.9$$

For each Mach number, only compressor stagnation temperatures greater than the flight stagnation temperature are plotted.

An examination of Figure 4 reveals that, as the flight Mach number increases, less compression is needed to reach the optimal condition. By Mach 3, a compressor is no longer necessary to reach the optimal stagnation temperature. This is due to the increase in stagnation temperatures with Mach number: since the heat addition is limited by the acceptable turbine temperature (T_{o_q}), as the compressor temperature (T_{o_c}) goes up, less heat can be added. If Q is smaller, then the kinetic energy change (and the thrust) given by equation (3.6) must be smaller. In other words, simply increasing the compression ratio is only advantageous up to a certain point, if maximum power is required. To summarize, equation (3.30) can serve as a criterion for deciding when to switch from a turbojet to a ramjet.

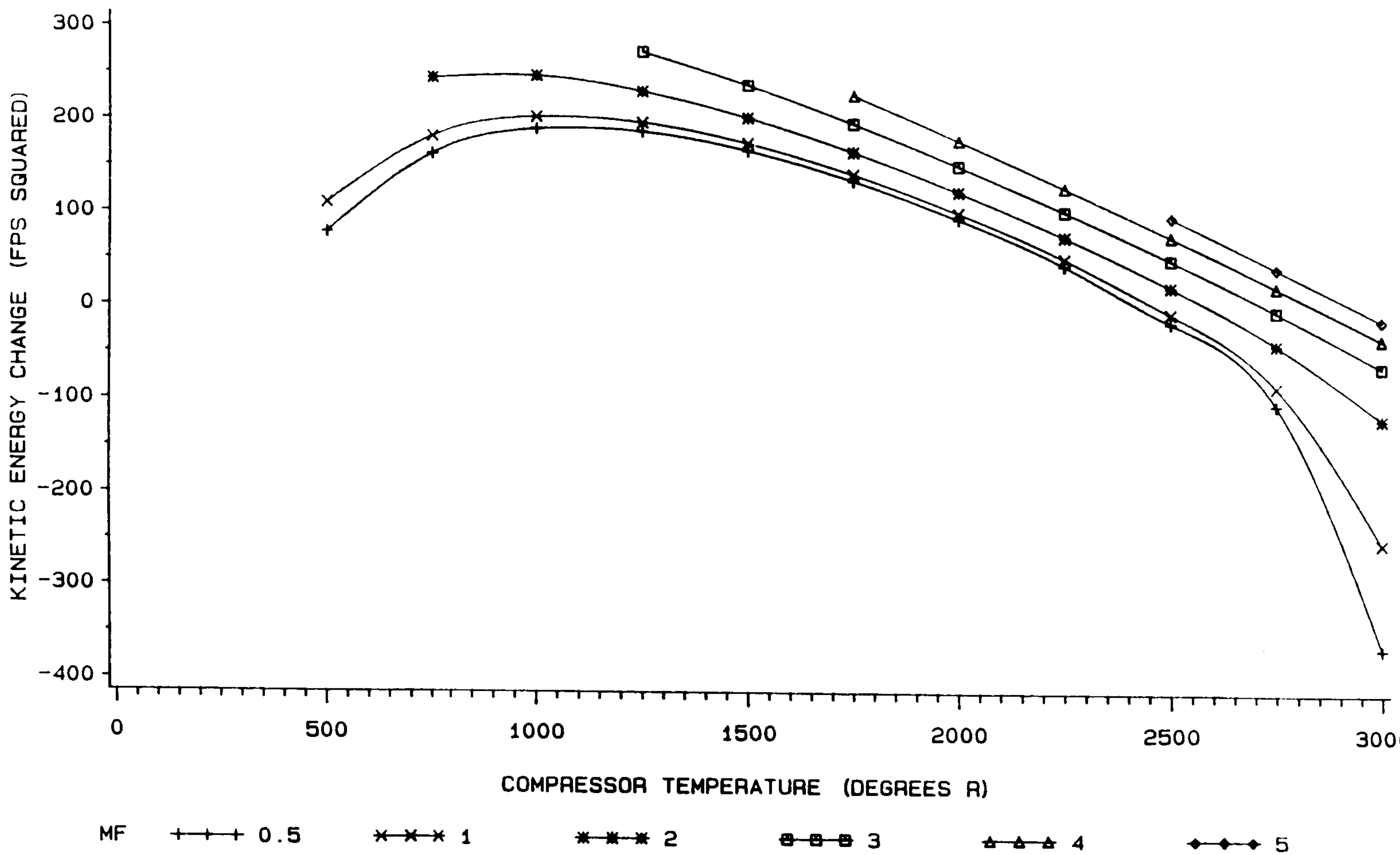


Figure 4: Kinetic energy change per mass of a "real" turbojet as a function of of compressor temp., Mach 0.5 to 5

3.3 Summary

All propulsion systems can be compared with the following equation:

$$\eta = \frac{F\,u_f}{Q} = \frac{2}{u_x + u_f}\left[1 - \frac{c_p T_f}{Q}\left[\exp\left(\frac{(\Delta s)_{total}}{c_p}\right) - 1\right]\right] \quad (3.31)$$

$(\Delta s)_{total}$ is the total irreversibilty caused by the various engine components. This equation is useful in assessing the relative importance of the various losses encountered by the flow through the engine.

Chapter 4

THRUST PRESSURE RATIO

4.1 Derivation of the Thrust Pressure

It is asserted here that hypersonic engines are not stagnation pressure sensitive (as low speed engines are); rather, as shown in Section 2.1.3.1 at high Mach number flight an engine is energy limted. In equation form this means that the maximum thrust possible is achieved when all of the fuel's energy appears as thrust work done on the vehicle:

$$F_m u_f = Q \qquad (4.1)$$

where: F_m = maximum thrust (per unit mass flow of air)

u_f = flight speed

Q = fuel energy per unit mass flow of air-fuel mixture

In general, one has for the thrust force:

$$T = \dot{m}\,[(1 + \lambda)u_x - u_f] + p_x A_x - p_f A_f \qquad (4.2)$$

where: $\dot{m}$ = mass flow rate of mixture (taken to be constant)

λ = fuel-air ratio

p_f, p_x = gage pressures at inlet and exit, respectively

A_f, A_x = engine cross-sectional area at inlet and exit, respectively

For complete expansion (or nearly so), taking $A_x \approx A_f$, and $\lambda << 1$, equation (4.2) reduces to:

$$T = \dot{m}\,(u_x - u_f) = \rho_f u_f A_f (u_x - u_f) \tag{4.3}$$

or, for the thrust per unit mass flow of air:

$$F = \frac{T}{\dot{m}} = u_x - u_f \tag{4.4}$$

Combining equations (4.3) and (4.4),

$$T = \rho_f u_f A_f F \tag{4.5}$$

Now, the maximum thrust force is found from equations (4.1) and (4.5)

$$T_m = \rho_f A_f Q \tag{4.6}$$

Defining the "thrust pressure," p_{th}, as the thrust divided by the inlet area, A_f, one finds:

$$p_{th_m} = \frac{T_m}{A_f} = \rho_f Q \tag{4.7}$$

Eliminating the density via the perfect gas equation of state, and multiplying by $1 = \gamma/\gamma$ yields:

$$p_{th_m} = \frac{\gamma Q}{a_f^2} p_f \tag{4.8}$$

4.2 Numerical Example

Operation at stoichiometric mixture would be $1/\lambda = 15$. For this analysis the fine details of the combustion process will be ignored. To simplify matters, it will be assumed that the fuel energy is added to the air, whose mass is that of the air-fuel mixture; i.e. $1/\lambda + 1 = 16$. In other words, although the general equation for the combustion process is

$$(\dot{m}_{fuel} + \dot{m}_{air})Q = \Delta H_{fuel} + \Delta H_{air} = c_{p_{fuel}}\dot{m}_{fuel}(T_{out} - T_{fuel}) + c_{p_{air}}\dot{m}_{air}(T_{out} - T_{air})$$

the above assumptions reduce it to:

$$(\dot{m}_{fuel} + \dot{m}_{air})Q = \dot{m}_{fuel}\,\Delta h_{fuel} = c_p(\dot{m}_{fuel} + \dot{m}_{air})\,\Delta T_{air} \tag{4.9}$$

where Δh_{fuel} is fuel's heat content per mass. Solving for Q and using the definition of the fuel-air ratio:

$$Q = \frac{\Delta h_{fuel}}{\frac{1}{\lambda} + 1} \quad (4.10)$$

Thus, for a fuel with a heat content of 18,000 $\frac{BTU}{lb_{m_{fuel}}}$, one finds that:

$$Q = 2.82 \times 10^7 \frac{ft^2}{s^2}$$

Additionally, taking $\gamma = 1.4$, and choosing an altitude of 100,000 ft. gives $T_f = 400°$ R and $a_f = 980$ fps and consequently the maximum thrust pressure is:

$$p_{th_m} = 41.0\, p_f \quad (4.11)$$

This result indicates the maximum thrust pressure possible for a given fuel and flight altitude. This result comes from the conservation of energy law and it is impossible to generate thrust in excess of that given by equation (4.8). It is, of course, possible to do worse: any losses, say from friction, shocks, or from compressor, turbine, or propulsive inefficiencies, will reduce the thrust (pressure). See the discussion of zero-thrust criteria for analysis of the fuel energy used to recoup pressure losses.

4.3 Thrust Pressure Ratio

A thrust pressure ratio, TPR $\equiv \frac{p_{th}}{p_{th_m}}$, can be defined. Writing the thrust pressure from equation (4.3):

$$p_{th} = \frac{T}{A_f} = \frac{\dot{m}(u_x - u_f)}{A_f} = \frac{\rho_f u_f A_f (u_x - u_f)}{A_f}$$

and rearranging,

$$p_{th} = \rho_f u_f^2 \left(\frac{u_x}{u_f} - 1 \right) \tag{4.12}$$

Substituting for ρ_f using the ideal gas equation of state, multiplying by $1 = \gamma/\gamma$ and employing the relation for the sound speed of an ideal gas, $a^2 = \gamma RT$ and the definition of the Mach number results in:

$$p_{th} = \gamma p_f M_f^2 \left(\frac{u_x}{u_f} - 1 \right) \tag{4.13}$$

Thus, the thrust ratio is, combining equation (4.13) with equation (4.8):

$$\frac{p_{th}}{p_{th_m}} = \frac{u_f^2}{\dot{Q}} \left(\frac{u_x}{u_f} - 1 \right) \tag{4.14}$$

An important question concerning the thrust pressure ratio which has been defined here is: what are its limits? This is resolved by writing the steady energy equation for the engine:

$$c_pT_f + \frac{u_f^2}{2} + Q = c_pT_x + \frac{u_x^2}{2} \qquad (4.15)$$

First, looking at the case $u_f \to \infty$, compared to the other terms in equation (4.15), c_pT_f and c_pT_x are negligible, and thus:

$$\frac{1}{2}(u_x^2 - u_f^2) = Q \qquad (4.16)$$

$$\frac{1}{2}(u_x + u_f)(u_x - u_f) = Q$$

At high speeds the approximation $\frac{1}{2}(u_x + u_f) \approx u_f$ is valid, thus

$$u_f(u_x - u_f) = Q$$

and

$$\left(\frac{u_x}{u_f} - 1\right) = \frac{Q}{u_f^2}, \qquad \text{as } u_f \to \infty \qquad (4.17)$$

Putting (4.17) back into the thrust pressure ratio equation (4.14) gives,

$$\frac{p_{th}}{p_{th_m}} \to 1, \qquad \text{as } u_f \to \infty \qquad (4.18)$$

Conversely, as u_f tends toward zero in equation (4.14)

$$\frac{p_{th}}{p_{th_m}} = \frac{u_f}{Q}(u_x - u_f) \to 0, \quad \text{as } u_f \to 0 \qquad (4.19)$$

Thus, as the flight speed goes from zero to infinity, the thrust pressure ratio goes from zero to one.

The thrust pressure ratio is a criterion by which different engines may be compared. Many of the performance parameters currently in use for subsonic and low supersonic engines are inappropriate or insufficient for hypersonic flight. The thrust pressure ratio, however, is always significant. At all speeds, it directly relates how effectively the fuel energy is being used to generate thrust. For example, it is incorrect just to say that the thrust is going to zero as flight speed increases and that this means that a rocket is better. As shown in equation (4.18), and also in Section 2.4, as flight speed increases virtually all of the fuel's energy is used to do work on the vehicle. To switch to a rocket at this point makes no sense, so long as the necessary thrust is being produced.

4.4 Summary

At any altitude, the maximum thrust for any air-breathing propulsion system is limited by the oxygen available and captured by the engine. In terms of the engine inlet area, A_f, the maximum pressure thrust is:

$$p_{th_m} = 41.0\, p_f \qquad (4.11)$$

Chapter 5

TRAJECTORY PATHS

5.1 Ramjet (Horizontal Flight Path)

From equation (2.46):

$$Q\left(1 - \frac{T_f}{T_{of}}\right) = \frac{F u_f}{\eta_p} = \frac{\text{Thrust}}{\dot{m}_{air}}\frac{u_f}{\eta_p} = \frac{\text{Thrust}}{\rho u_f A_i}\frac{u_f}{\eta_p}$$

and thus:

$$T = \text{Thrust} = \frac{\dot{m}_{air}}{u_f}\eta_p\left(1 - \frac{T_f}{T_{of}}\right)Q = \frac{\dot{m}_{air}}{u_f}\eta\, Q \qquad (5.1)$$

where: η = overall effeciency $\equiv \dfrac{F u_f}{Q} = \eta_p\left(1 - \dfrac{T_f}{T_{of}}\right)$

From the definition of the fuel-air ratio one finds:

$$\dot{m}_{air} = \frac{\dot{m}_{fuel}}{\lambda}$$

and thus:

$$T = \frac{\dot{m}_{fuel}}{\lambda u_f}\left(1 - \frac{T_f}{T_{of}}\right)\eta_p Q = \frac{\dot{m}_{fuel}}{\lambda u_f}\eta Q = \frac{\eta Q}{\lambda u_f}\left(\frac{-dm}{dt}\right) \quad (5.2)$$

where: m = instantaneous mass of the vehicle

Now, assuming level flight (where lift equals the product of the vehicle mass and gravity) and a constant lift-drag ratio, the equation of motion for the vehicle may be written:

$$m\frac{du_f}{dt} = T - D = T - \frac{mg}{(L/D)} \quad (5.3)$$

Substituting for the thrust from equation (5.2),

$$m\frac{du_f}{dt} = \frac{\eta Q}{\lambda u_f}\left(\frac{-dm}{dt}\right) - \frac{mg}{(L/D)} \quad (5.4)$$

and rearranging:

$$u_f\, du_f = -\frac{\eta Q}{\lambda}\left(\frac{dm}{m}\right) - \frac{g u_f dt}{(L/D)} \quad (5.5)$$

In order to find an expression for the range use:

$$dR = u_f dt \quad (5.6)$$

Equation (5.5) may now be integrated. The result is:

$$\frac{1}{2}u_f^2 = -\frac{\eta Q}{\lambda}\ln(m) - \frac{g}{(L/D)}R + \text{constant} \qquad (5.7)$$

Choosing the following initial and final conditions:

$$\text{at } t = 0: \quad R = 0, \quad u_f = U_i, \quad m = m_v + m_p \qquad (5.8)$$

$$\text{at } t = t_b: \quad u_f = U_b, \quad m = m_v \qquad (5.9)$$

where: U_i, U_b = initial and "burnout" velocities, respectively

m_v, m_p = vehicle and propellant masses, respectively

The constant is now found from the chosen initial condition:

$$\text{constant} = \frac{1}{2}U_i^2 + \frac{\eta Q}{\lambda}\ln(m_v + m_p) \qquad (5.10)$$

Imposing the final condition and substituting in equation (5.10) gives:

$$\frac{1}{2}(U_b^2 - U_i^2) + \frac{\eta Q}{\lambda}\ln\left(\frac{m_v}{m_v + m_p}\right) = -\frac{g}{(L/D)}R \qquad (5.11)$$

The three terms represent the increment of the kinetic energy per mass, the vehicle mass fraction, and the range, respectively. One of these parameters must be eliminated: an equation for the range can be derived once a trajectory path is chosen.

5.1.1 Vehicle Mass Fraction for Constant Acceleration

The choice of lift equal to weight implies acceleration in level flight. For a constant rate of acceleration:

$$a = \frac{du_f}{dt} = \alpha g , \qquad \alpha = \text{constant} \qquad (5.12)$$

and thus

$$U_b = \alpha g\, t_b + U_i \qquad (5.13)$$

Putting (5.12) into (5.4) produces:

$$m\alpha g = -\frac{\eta Q}{\lambda u_f}\left(\frac{dm}{dt}\right) - \frac{mg}{(L/D)}$$

Rearranging, using the defintion of the range (5.6), and then integrating,

$$g\left(\alpha + \frac{1}{(L/D)}\right)R = -\frac{\eta Q}{\lambda}\ln(m) + \text{constant}$$

Imposing the initial condition, equation (5.8),

$$\text{constant} = \frac{\eta Q}{\lambda}\ln(m_v + m_p) \qquad (5.14)$$

and then imposing the burnout condition, equation (5.9), and solving for the range:

$$R = \frac{\eta Q}{\lambda g} \frac{\ln\left(\frac{m_v + m_p}{m_v}\right)}{\left(\alpha + \frac{1}{(L/D)}\right)} \tag{5.15}$$

Note that $\alpha = 0$ represents the "cruise" condition. Putting (5.15) into (5.11) and solving for the vehicle mass fraction:

$$\ln\left(\frac{m_v}{m_v + m_p}\right) = -\frac{((L/D)\alpha + 1)}{(L/D)\alpha} \frac{\lambda}{\eta Q} \frac{(U_b^2 - U_i^2)}{2} \tag{5.16}$$

For example, taking

$$\eta = 1, \ L/D = 5, \ \alpha = 1, \ \lambda = 1/15,$$

$$U_b = 26{,}000 \text{ fps}, \ U_i = 0,$$

and finding the heat addition ($Q = 1125$ BTU/$lb_{m_{mixture}}$) from equation (4.10) for a fuel with a heat content of 18,000 BTU/$lb_{m_{fuel}}$ yields,

$$\frac{m_v}{m_v + m_p} = 0.38$$

This means that the vehicle mass would be 62% fuel at the beginning of the burn.

5.2 Vertical Rocket Comparison

The thrust of a rocket is given by:

$$T = \dot{m}_p u_p \tag{5.17}$$

where: $\dot{m}_p$ = propellant flow rate (constant) $= \frac{-dm}{dt}$

u_p = propellant exhaust speed (constant)

Now, neglecting the drag on the rocket, the equation of motion for the vertical rocket may be written:

$$m\frac{du_r}{dt} = T - mg = \dot{m}_p u_p - mg = -\frac{dm}{dt}u_p - mg \tag{5.18}$$

where: u_r = rocket flight speed

Rearranging,

$$\frac{dm}{m} = \frac{(gdt + du_r)}{u_p}$$

Integrating and choosing the initial and final conditions

$$\text{at } t = 0: \quad u_r = U_i, \quad m = m_v + m_p \tag{5.19}$$

$$\text{at } t = t_b: \quad u_r = U_b, \; m = m_v \tag{5.20}$$

yields:

$$\ln\left(\frac{m_v}{m_v + m_p}\right) = -\frac{(U_b - U_i) + gt_b}{u_p} \tag{5.21}$$

A relation for the burnout time is found by selecting an appropriate trajectory. For a constant acceleration trajectory:

$$\frac{du_r}{dt} = \alpha g \tag{5.22}$$

Integrating equation (5.22) across the initial and final conditions:

$$t_b = \frac{U_b - U_i}{\alpha g} \tag{5.23}$$

Substituting (5.23) into equation (5.21) produces the relation for the vehicle mass fraction of a constant acceleration vertical rocket:

$$\ln\left(\frac{m_v}{m_v + m_p}\right) = -\frac{1 + \alpha}{\alpha}\frac{(U_b - U_i)}{u_p} \tag{5.24}$$

For example, taking:

$$\alpha = 1, \ u_p = 10{,}800 \text{ fps} \ (H_2, O_2 \text{ rocket})$$

$$U_b = 26{,}000 \text{ fps}, \ U_i = 0$$

yields

$$\frac{m_v}{m_v + m_p} = 0.008$$

Quite obviously, a single-stage-to-orbit rocket has an unacceptable payload fraction. However, even with multi-staging, the air-breathing engine is far superior.

5.3 Summary

Considering the amount of fuel required to accelerate a given payload mass to orbital velocity, an air-breathing engine is far superior to a rocket. This is due the fact that a rocket must carry onboard both the fuel and its oxidizer. The ramjet, however, need only carry its fuel: it gets oxygen from the captured air.

This trajectory analysis suggests another difficulty which any hypersonic vehicle must overcome. Once a rate of acceleration (α) is chosen, the time of flight to reach orbital velocity is fixed. It is the same for both a ramjet and a rocket. During this time, the structure is subjected to extreme thermal and pressure stresses. As an example, consider a one-gravity acceleration from 10,000 fps (about Mach 10 at 100,000 ft.) to an orbital velocity of 26,000 fps: the time of flight (t_b) is 8.3 minutes, during which the vehicle travels 1700 miles. Thus, a vehicle is exposed to very high stagnation temperatures and pressures for a considerable length of time. For a winged vehicle in particular, this problem poses a serious challenge to the airframe designer.

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16. Abstract

This report presents criteria to assess the relative merits of different propulsion systems. Previous references focus mainly on subsonic or low supersonic flight speeds. The main focus here is on a higher range, from low supersonic to orbital velocities. Air breathing propulsion systems for hypersonic flight present the engine designer with circumstances that differ in important fundamental ways from those encountered in engines designed for operation at subsonic or low supersonic speeds. This analysis highlights the importance of various features of hypersonic engine design. Since the performance of hypersonic engines are energy limited, unlike low speed engines which are stagnation pressure limited, the efficient use of the energy of the fuel used is critical to minimize the take-off fuel mass fraction of the vehicle. Furthermore, since the required energy increase of a vehicle per incremental speed change increases with speed, the engine must be designed to operate efficiently at high speed. An analysis of engine performance in terms of entropy changes of the flow passing through the engine allows comparison of various engine designs as well as a convenient method to determine the effect of individual engine component efficiencies on overall engine performance.

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