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IMPLICATIONS OF PRESSURE DIFFUSION FOR SHOCK WAVES

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IMPLICATION OF PRESSURE DIFFUSION FOR SHOCK WAVES

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Abstract - The report deals with the possible implications of pressure diffusion for shocks in one dimensional traveling waves in an ideal gas. From this new hypothesis all aspects of such shocks can be calculated except shock thickness. Unlike conventional shock theory, the concept of entropy is not needed or used. Our analysis shows that temperature rises near a shock, which is of course an experimental fact; however, it also predicts that very close to a shock, density increases faster than pressure. In other words, a shock itself is cold.

1. Introduction

In inviscid flow theory, fluid acceleration arises from a gradient of pressure by $\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})=-\nabla p$; here we reverse this idea and regard $-\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})$ instead, as a pseudo-pressure gradient which arises from acceleration. Thus in this unconventional approach, pressure is regarded as a special type of energy density, the gradients of which contribute to fluid accelerations. As a type of energy density, pressure might diffuse (not a violation of energy conservation) and we postulate that it does so at a rate determined by the difference between ∇p and $-\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})$, that is, at a rate $\nabla\cdot[\kappa\{\nabla p+\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})\}]$, where κ is a coefficient of diffusion. For viscous fluids this diffusion term is generally non-zero. In the absence of accelerations, pressure gradients

would undergo Laplace diffusion.

The implications of the concept of pressure diffusion for fluid dynamics seem quite general. Pressure diffusion might be a prominent factor in the irreversibility of compression or expansion of an ideal gas, a fundamental phenomenon which conventional fluid dynamics is unable to explain or predict from first principles without recourse to *ad hoc* extreme increases in a poorly understood thermodynamic term, the second coefficient of viscosity [1, pp. 304-308]. Furthermore, pressure diffusion is consistent with the experimental observation that the speed of sound in CO_2, N_2O and SO_2 at 1E5 Pa and 313° K increases with frequency [2]. In fact the coefficient of pressure diffusion κ (taken, like all other thermodynamic terms in this report, as a function of density ρ and pressure p) for air near standard conditions can be estimated from such experimental data [3] to be 300 m^2/sec.

In this new approach to fluid flow modeling, the state variables of the model are density ρ, the three velocity components in the vector $\mathbf{v}$, and pressure p. To the usual Navier-Stokes mass conservation equation for $\frac{\partial \rho}{\partial t}$ and momentum conservation equation for $\frac{\partial \mathbf{v}}{\partial t}$ is added to new equation for $\frac{\partial p}{\partial t}$. The pressure equation can be derived from an energy conservation equation in which pressure diffusion is present as a term and from specification of an internal energy function for the fluid.

2. Fundamental Equations

The fundamental equations of Newtonian fluid dynamics with pressure diffusion are [1, pp. 2, 47-49]

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \tag{1}$$

and

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} + \rho^{-1}[-\nabla p + \mu \nabla^2 \mathbf{v} + (\zeta + \frac{\mu}{3})\nabla D$$

$$+ (M + M^*)\nabla \mu - \frac{2}{3} D \nabla \mu + D \nabla \zeta] + \mathbf{g} \tag{2}$$

Here μ is dynamic viscosity; ζ is the second coefficient of viscosity; M is the matrix with $\{\partial v_i / \partial x_j\}$ in row i, column j; M^* denotes the transpose of M, the matrix with $\{\partial v_j / \partial x_i\}$ in row i, column j; and g is a constant gravitational acceleration vector. Obviously we require that μ and ζ are differentiable functions of ρ and p. Dilation D is the trace of the matrix M, that is $D = tr(M) = \sum_{i=}^{3} M_{ii}$. Implicit in (2) is the assumption that a special property of the energy density p is that the gradient ∇p contributes to $\frac{\partial \mathbf{v}}{\partial t}$.

In this report a fluid is regarded as a continuum of matter which can be completely described by its density ρ velocity v and pressure p. A fluid is presumed to change with time in such a manner that matter and momentum are conserved; thus use will be made of two of the familiar Navier-Stokes equations (1) and (2). Density, velocity, and pressure dynamics can then be predicted provided an equation for $\frac{\partial p}{\partial t}$ in terms of density, velocity and pressure (including the spatial gradients thereof) can be obtained.

This is done in the next section. Indeed it is shown that

$$\frac{\partial p}{\partial t} = -\mathbf{v}\cdot\nabla p - BD + \left(\rho\frac{\partial u}{\partial p}\right)^{-1}\left[\left[\mu\,\mathrm{tr}\{M(M+M^*)\} + \left(\zeta - \frac{2}{3}\mu\right)D^2 + \nabla\cdot(\tau\nabla T) + \nabla\cdot\left[\kappa\left\{\nabla p + \rho\left(\frac{d\mathbf{v}}{dt}\right) - \mathbf{g}\right\}\right]\right]\right] \qquad (3)$$

where T is temperature (a function of ρ and p), τ is a heat diffusion coefficient, κ is a pressure diffusion coefficient, and B is the bulk modulus of elasticity (all functions of ρ and p). Throughout we assume that ρ and p always have positive values and that the parameters u, μ, ζ, τ and κ are analytic functions of ρ and p; these parameters are all non-negative and $\frac{\partial u}{\partial p}$ is assumed positive. If $\kappa=0$, then (3) reduces to a well known but seldom used energy conservation equation [4, p. 33].

3. Derivation of the Pressure Equation

The region R of flow will be assumed to be finite although possibly changing with time and for which at each time a general version of Gauss' Divergence theorem holds. The boundary ∂R of R must change smoothly with time and at each time ∂R must be a closed oriented surface with an outward pointing normal vector field **n** defined uniquely almost everywhere. It follows that for any smooth vector field **F** over R,

$$\int_R \nabla\cdot\mathbf{F}\ dV = \int_{\partial R}\mathbf{F}\cdot\mathbf{n}\ dA.$$

Let us regard R as partitioned into a large number of small cubical cells, each with edge length ℓ.

A standard expression of conservation of energy in Newtonian fluids [4, p. 33] is

$$\frac{\partial}{\partial t}(\rho u+\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v})=-\nabla\cdot[(\rho u+p+\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v})\mathbf{v}]+\nabla\cdot[(\zeta-\tfrac{2}{3}\mu)D\mathbf{v}$$
$$+\mu(M+M^{*})\mathbf{v}]+\nabla\cdot(\tau\nabla T)+\rho\mathbf{g}\cdot\mathbf{v} \qquad (4)$$

The term $-\nabla\cdot[(\rho u+p+\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v})\mathbf{v}]$ arises from the rate of energy carried by fluid flow through the walls of a fixed cell, namely $(\rho u+p+\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v})\mathbf{v}\cdot\mathbf{n}$, where $\mathbf{n}$ is the outward normal vector of the cell. The fluid carries energy as internal energy per unit volume ρu and kinetic energy density $\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v}$. The rate of work through cell walls by viscous forces results in the next term. The term $\nabla\cdot(\tau\nabla T)$ arises from the flow of energy through the cell walls by the conduction of heat, and $\rho\mathbf{g}\cdot\mathbf{v}$ is, of course, the contribution of gravitational force to the rate of change of energy in the cell.

Pressure is the result of a force per unit area. In the presence of acceleration, a momentum change can be equated with a force. Thus if a fluid accelerates through the wall of a cubic cell of edge length ℓ, the pseudo pressure generated by the momentum change (force) is $\rho\ell^3(\frac{d\mathbf{v}}{dt}-\mathbf{g})/\ell^2$, including the effect of gravity. Discounting gravity, the pseudo pressure gradient (between two such cubes which share a face) generated by the momentum change is $\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})$. Thus the net rate of pressure diffusion in general form is taken to be $\nabla\cdot[\kappa\{\nabla p+\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})\}]$, where κ is a diffusion parameter. This term is thus added to the list of power flux summands on the right-side of (4) to yield

$$\frac{\partial}{\partial t}(\rho u+\tfrac{1}{2}\rho \mathbf{v}\cdot\mathbf{v})=-\nabla\cdot[(\rho u+p+\tfrac{1}{2}\rho \mathbf{v}.\mathbf{v})\mathbf{v}]+\nabla\cdot[(\zeta-\tfrac{2}{3}\mu)D\mathbf{v}+\mu(M+M^*)\mathbf{v}]$$
$$+\nabla\cdot(\tau\nabla T)+\rho\mathbf{g}\cdot\mathbf{v}+\nabla\cdot[\kappa\{\nabla p+\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})\}] \qquad (4a)$$

combining (2) and (4a) leads to

$$\rho\frac{du}{dt}=-pD+\mu tr[M(M+M^*)]+(\zeta-\frac{2}{3}\mu)D^2$$
$$+\nabla\cdot(\tau\nabla T)+\nabla\cdot[\kappa\{\nabla p+\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})\}] \qquad (5)$$

since $\quad \frac{du}{dt}=\frac{\partial u}{\partial p}\frac{dp}{dt}-\frac{\partial u}{\partial \rho}\rho D$

and $\quad \frac{dp}{dt}=\frac{\partial p}{\partial t}+\mathbf{v}\cdot\nabla p$

we can use (5) to solve for $\frac{\partial p}{\partial t}$, that is,

$$\frac{\partial p}{\partial t}=-\mathbf{v}\cdot\nabla p+(\frac{\partial u}{\partial p})^{-1}[\rho\frac{\partial u}{\partial p}-\frac{p}{\rho}]D+(\rho\frac{\partial u}{\partial p})^{-1}[\mu tr\{M(M+M^*)\}$$
$$+(\zeta-\frac{2}{3}\mu)D^2+\nabla\cdot(\tau\nabla T)+\nabla\cdot[\kappa\{\nabla p+\rho(\frac{d\mathbf{v}}{dt}-\mathbf{g})\}]] \qquad (6)$$

If we can show that the bulk modulus of elasticity $B=\rho(\frac{\partial p}{\partial \rho})_s$, where s is entropy [4, pp. 165] can be written in terms of the internal energy u of the fluid by the relation

$$B=(\frac{\partial u}{\partial p})^{-1}[-\rho\frac{\partial u}{\partial p}+\frac{p}{\rho}]$$

then making use of this relation into equation (6), one obtains equation (3). This is shown as follows.

Generally $u=u(\rho,p)$. If specific entropy is held constant, then we may write $p=f(\rho)$ and

$$(\frac{\partial u}{\partial \rho})_s=\frac{\partial}{\partial \rho}[u(f(\rho),\rho)]=\frac{\partial u}{\partial p}(\frac{\partial p}{\partial \rho})_s+\frac{\partial u}{\partial \rho}$$

Recall that $p=\rho^2(\frac{\partial u}{\partial p})_s$ is a consequence of the Gibbs equation $du = Tds+\rho^{-2}pd\rho$. It follows that

$$\frac{p}{\rho} = \frac{\partial u}{\partial p}\rho\,(\frac{\partial p}{\partial \rho})_s + \rho\frac{\partial u}{\partial \rho}$$

and so

$$B = \rho(\frac{\partial p}{\partial \rho})_s = (\frac{\partial u}{\partial p})^{-1}[-\rho\frac{\partial u}{\partial \rho} + \frac{p}{\rho}] \qquad (7)$$

as claimed. In fact (7) can be used to define B without reference to entropy. Thus the concept of entropy is not essential in describing the dynamics of Newtonian fluids.

3. One-Dimensional Flow

For one-dimensional fluid flow, the equations (1), (2) and (3) reduce to

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v) \tag{8}$$

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial x} + \rho^{-1}[-\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\eta\frac{\partial v}{\partial x})] + g \tag{9}$$

$$\begin{aligned}\frac{\partial p}{\partial t} &= -v\frac{\partial p}{\partial x} - B\frac{\partial v}{\partial x} + (\rho\frac{\partial u}{\partial p})^{-1}[\eta(\frac{\partial v}{\partial x})^2 \\ &\quad + \frac{\partial}{\partial x}(\tau\frac{\partial T}{\partial x}) + \frac{\partial}{\partial x}[\kappa\{\frac{\partial p}{\partial x} + \rho(\frac{dv}{dt} - g)\}]] \end{aligned} \tag{10a}$$

$$\begin{aligned} &= -v\frac{\partial p}{\partial x} - B\frac{\partial v}{\partial x} + (\rho\frac{\partial u}{\partial p})^{-1}[\eta(\frac{\partial v}{\partial x})^2 \\ &\quad + \frac{\partial}{\partial x}(\tau\frac{\partial T}{\partial x}) + \frac{\partial}{\partial x}[\kappa\frac{\partial}{\partial x}(\eta\frac{\partial v}{\partial x})]] \end{aligned} \tag{10b}$$

where $\eta = \frac{4}{3}\mu + \zeta$.

For an ideal gas $u = \frac{p}{\rho(\gamma-1)}$, so $B=\gamma p$ and $\rho(\frac{\partial u}{\partial p})^{-1}=\gamma-1$, where $\gamma-1$ is a positive constant.

A traveling wave moving in x-direction with speed c is a solution of (1), (2) and (3) in which each of ρ, v and p can be written as a function of the waveform parameter $y=x-ct$. Taking $g=0$ and $\tau=0$, the conservation law equivalents of (8), (9) and (10) are

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v) \tag{11}$$

$$\frac{\partial}{\partial t}(\rho v) = -\frac{\partial}{\partial x}(\rho v^2 + p - \eta\frac{\partial v}{\partial x}) \tag{12}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\tfrac{1}{2}\rho v^2+\rho u) &= -\frac{\partial}{\partial x}[\tfrac{1}{2}\rho v^2+p+\rho u)v]+\frac{\partial}{\partial x}(v\eta\frac{\partial v}{\partial x}) \\ &\quad +\frac{\partial}{\partial x}[\kappa\frac{\partial}{\partial x}(\eta\frac{\partial v}{\partial x})] \end{aligned} \tag{13}$$

Denoting the conditions before and after shock with

subscripts 1 and 2 respectively, (11), (12), (13) yield

$$\rho_1(v_1-c) = \rho_2(v_2-c) \tag{14}$$

$$\rho_1(v_1-c)^2 + p_1 - \eta_1\frac{\partial v_1}{\partial x} = \rho_2(v_2-c)^2 + p_2 - \eta_2\frac{\partial v_2}{\partial x} \tag{15}$$

$$\{\tfrac{1}{2}\rho_1(v_1-c)^2+p_1+\rho_1 u_1\}(v_1-c)-(v_1-c)\eta_1\frac{\partial v_1}{\partial x}-\kappa\frac{\partial}{\partial x}(\eta_1\frac{\partial v_1}{\partial x}) =$$

$$\{\tfrac{1}{2}\rho_2(v_2-c)^2+p_2+\rho_2 u_2\}(v_2-c)-(v_2-c)\eta_2\frac{\partial v_2}{\partial x}-\kappa\frac{\partial}{\partial x}(\eta_2\frac{\partial v_2}{\partial x}) \tag{16}$$

As mentioned before, a traveling wave moving in x-direction with speed c is a solution of (1), (2), (3) in which each of ρ, v and p can be written as a function of the waveform parameter $y=x-ct$. The waveforms of such a wave differ from those derived from conventional theory. Hence an alternative to the Rankine-Hugoniot equation [4, p.317] may be desirable. From (14), (15), (16) one can show that

$$(u_2+\rho_2 p_2)-(u_1+\rho_1 p_1)=\tfrac{1}{2}(p_2-p_1)(\rho_2^{-1}+\rho_1^{-1})+\tfrac{1}{2}(\rho_2^{-1}-\rho_1^{-1})$$
$$\cdot(\eta_2\frac{\partial v_2}{\partial x}+\eta_1\frac{\partial v_1}{\partial x})+(\frac{\kappa}{J})[\frac{\partial}{\partial x}(\eta_2\frac{\partial v_2}{\partial x})-\frac{\partial}{\partial x}(\eta_1\frac{\partial v_1}{\partial x})] \tag{17}$$

When $\eta_2=\eta_1=0$ and $\kappa=0$, (17) reduces to the conventional equation.

4. Proposed Continuation of this Research

The assumption of pressure diffusion leads to seemingly realistic waveforms for one dimensional traveling waves with shocks. However, an accompanying mechanism for calculating waveforms through a shock remains to be devised.

If, as indicated above, a shock is very cold, metastable phase transition phenomena might be involved and a local alternative specification of internal energy $u(\rho,p)$ must be

made. A first step is the use of the Van der Waals gas model. The internal energy differential for a Van der Waals gas is

$$du = \frac{(1-b\rho)}{(\gamma-1)\rho\left[1-2\,\dfrac{\rho^2 a(1-b\rho)}{p+a\rho^2}\right]}dp - \frac{1}{\gamma-1}\left(\frac{p}{\rho^2}+a\gamma\right)d\rho \qquad (18)$$

The proposed continuation of this research involves use of the pressure diffusion theory to devise a means of calculating ρ, v, and p values through a shock (conserving mass, momentum and energy) starting with du as given by (18).

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