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The effects of particle loading on turbulence structure and modelling

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1. Introduction

Conventional wisdom implies that the presence of particles provides an additional dissipation or attenuation of turbulence. However, it is not clear how this extra dissipation may be incorporated into turbulence models or how it depends on the parameters of the problem including the particle time constant, the mass loading, and the various dimensionless parameters describing the turbulence. Simply focusing on the extra dissipation completely neglects the effect of turbulence structure on the instantaneous particle concentration field and the possibility of interactions between the particle cloud and instability mechanisms generating turbulence. For example, particles selectively concentrated in particular structures may cause rapid attenuation of that structure or trigger a new instability mechanism. Therefore, it is important to determine the effect of turbulence structure on the behavior of the particle cloud.

The objective of the present research was to extend the DNS approach to particle-laden turbulent flows using a simple model of particle/flow interaction. The program addressed the simplest type of flow, homogeneous, isotropic turbulence, and examined interactions between the particles and gas phase turbulence. The specific range of problems examined include those in which the particle is much smaller than the smallest length scales of the turbulence yet heavy enough to slip relative to the flow. The particle mass loading is large enough to have a significant impact on the turbulence, while the volume loading was small enough such that particle-particle interactions could be neglected. Therefore, these simulations are relevant to practical problems involving small, dense particles conveyed by turbulent gas flows at moderate loadings.

This report presents a sample of the results illustrating modifications of the particle concentration field caused by the turbulence structure and also illustrating attenuation of turbulence by the particle cloud.

2. Overview of the simulations

The numerical method used in the present research is based on the pseudo-spectral technique developed by Rogallo (1981) for solving the incompressible Navier-Stokes equations. A modification of Rogallo's code was used to simultaneously track a large number of particles using a vectorized, second-order interpolation scheme. This code is time advanced using second-order Runge-Kutta, and at each substep the fluid velocity at the particle position was calculated

using trilinear interpolation. Numerical experiments showed that more accurate interpolation schemes do not significantly improve the results. The new position was obtained by then integrating the particle equation of motion

$$\frac{dv_i(t)}{dt} = \frac{u_i(X_j(t),t) - v_i(t)}{\tau_p} \tag{2.1}$$

$$\frac{dX_i(t)}{dt} = v_i(t) \tag{2.2}$$

where $X_j(t)$ and $v_i(t)$ are the position and velocity of the particle, τ_p is the particle time constant, and $u_i(X_j(t),t)$ is the velocity of the fluid. Stokes' law has been used to calculate the drag on a particle, and it is, therefore, assumed that the particle is much smaller than any lengthscale of the flow (the Kolmogorov scale, η). It is also assumed that the particles are sufficiently dense such that other forces in the full equation of motion, e.g. buoyancy and added mass, are negligible compared to the Stokes drag. Finally, the particles are assumed to occupy a negligible volume fraction, and particle-particle interactions are also assumed negligible.

To calculate the effect of the particles on the turbulence, the momentum equation of the fluid was modified by a source term on the right-hand side

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_j\partial x_j} - \frac{c}{\tau_p\rho}(u_i - v_i) \tag{2.3}$$

where $c(x_i, t)$ is the particle mass per unit of volume, or particle density (repeated indices imply summation).

The problem studied is homogeneous, isotropic turbulence in a box. "Natural" isotropic turbulence decays, and the lack of stationarity complicates analysis of the results. Therefore, the low wave number modes (large scales) are artificially forced to maintain stationarity using the scheme developed by Hunt, et al (1987). Results were obtained using 373000 particles for simulations using 32^3 points and $1.\times10^6$ particles for simulations using 64^3 points. The relatively coarse grid 32^3 calculations permitted a parameter study in particle time constant and mass loading ratio. For the 32^3 simulations the values of the particle time constant divided by the eddy turnover time used were 0.14, 0.75, and 1.50. The mass-loadings used in the 32^3 simulations were 0.1, 0.5, and 1.0. All three values of the mass loading were used for each of the time constants for these simulations. For the 64^3 simulations the values of the dimensionless time constant were 0.075, 0.15, and 0.52 and the mass loadings were 0.1, 0.5, and 1.0.

For each simulation the two phases were uncoupled and time-advanced to reach a statistically stationary state and eliminate initial transients of the forcing. At this point in time, the momentum equation of the fluid was coupled to the particle momentum equation with a specified mass loading. Once the phases

were coupled, the simulations were time advanced just over 12 eddy turnover times for the 32^3 simulations and slightly over 6 eddy turnover times for the 64^3 simulations.

Results from the simulations of no coupling ($\phi = 0$ where ϕ is the mass loading) between the phases provide a baseline case to compare the effect of increased mass loading on the turbulence statistics. These simulations are also useful for examining the effect of turbulence "structure" on the particle concentration field. For these baseline simulations the Reynolds number based on the Taylor microscale was approximately 37. This relatively low value for the Reynolds number assured good resolution of the velocity field.

3. Results and discussion

Maxey (1987) has shown that particles will tend to accumulate in regions of high strain rate or low vorticity. This effect can be measured by using the second invariant of the deformation tensor, $\partial u_i / \partial x_j$. The second invariant of $\partial u_i / \partial x_j$ is given by

$$II_d = -\frac{1}{2}\left(s^2 - \frac{1}{2}\omega_j\omega_j\right) \tag{3.1}$$

where s^2 is the magnitude of the strain rate, $s_{ij}s_{ji}$, ω_j is the jth component of the vorticity vector and $\omega_j\omega_j$ is by definition the enstrophy.

Thus, from equation (3.1) highly vortical regions correspond to II_d being large and positive (where the number density is small), while regions of high strain correspond to II_d being large and negative (where the number density is large).

Contours of number density for each of the particles used in the simulations showed distinct regions where particles had collected. It was found that the peak number density is as much as 40-50 times the mean value for the lightest particles used in the simulations. By comparing these contours with the contours of the second invariant of $\partial u_i / \partial x_j$, it was found that the particles showed a tendency to accumulate in regions of low vorticity or high strain rate. As the particles were made more sluggish (i.e., larger τ_p), there is less of a tendency for them to accumulate in these regions. For the heaviest particle used in the 32^3 simulations ($\tau_p/T_e = 1.50$), the particle concentration field was found to be essentially random.

The preceding results were quantified by examining the conditional expectation of the number density given the value of the enstrophy. The conditional expectation showed that the effect of increasing particle inertia reduced the likelihood of finding the particles in regions of low vorticity (figure 1). For the 64^3 simulations, the lightest particles ($\tau_p/T_e = 0.075$) actually showed *less* of a tendency to accumulate in regions of low vorticity than did particles which were twice as heavy ($\tau_p/T_e = 0.150$). This can be explained by the fact that very light particles can follow nearly all of the turbulent motions and would, therefore, show no preference to be in regions of low vorticity.
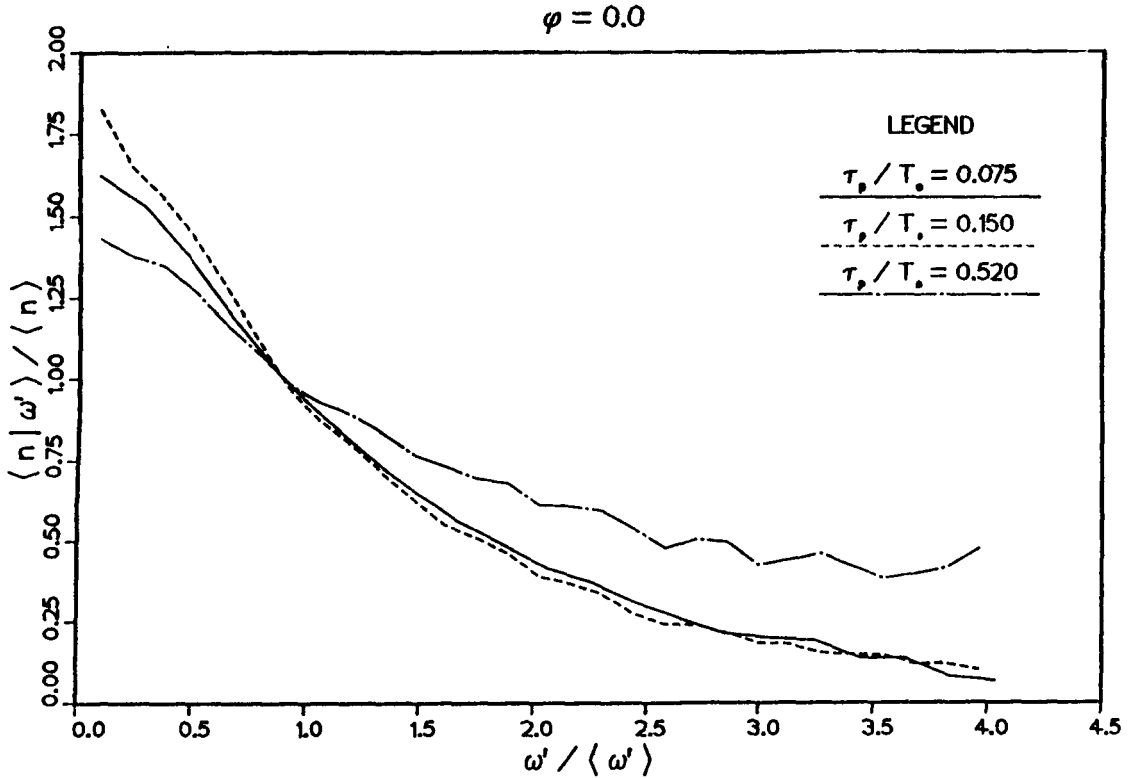
$$\varphi = 0.0$$

FIGURE 1.   Conditional expectation of number density given enstrophy for the 64³ simulations, $\phi = 0$.

For increasing values of the mass loading, the conditional expectation of number density given the enstrophy for the light particles showed that they had less of a tendency to be in regions of low vorticity. However, this was not the case for the heavier particles.

Correlations between the number density and enstrophy showed that as the loading was increased the lightest particles became less correlated with enstrophy. For the larger time constants, however, the number density and enstrophy become more correlated for increasing values of the time constant and increased loading. Correlations between number density and II_d showed that as the mass loading was increased the correlation between number density and II_d increased. This increase in the correlation was more significant for larger values of the time constant.

Once the two phases were coupled, the turbulence evolved to a new stationary state within about two eddy turnover times. For all the time constants used in the simulation, it was found that the time required to come to a new equilibrium was longer for increasing loadings, and the maximum development time for any case was just over two eddy turnover times. Time averaged statistics such as turbulence energy showed that increased mass loading decreased the turbulence
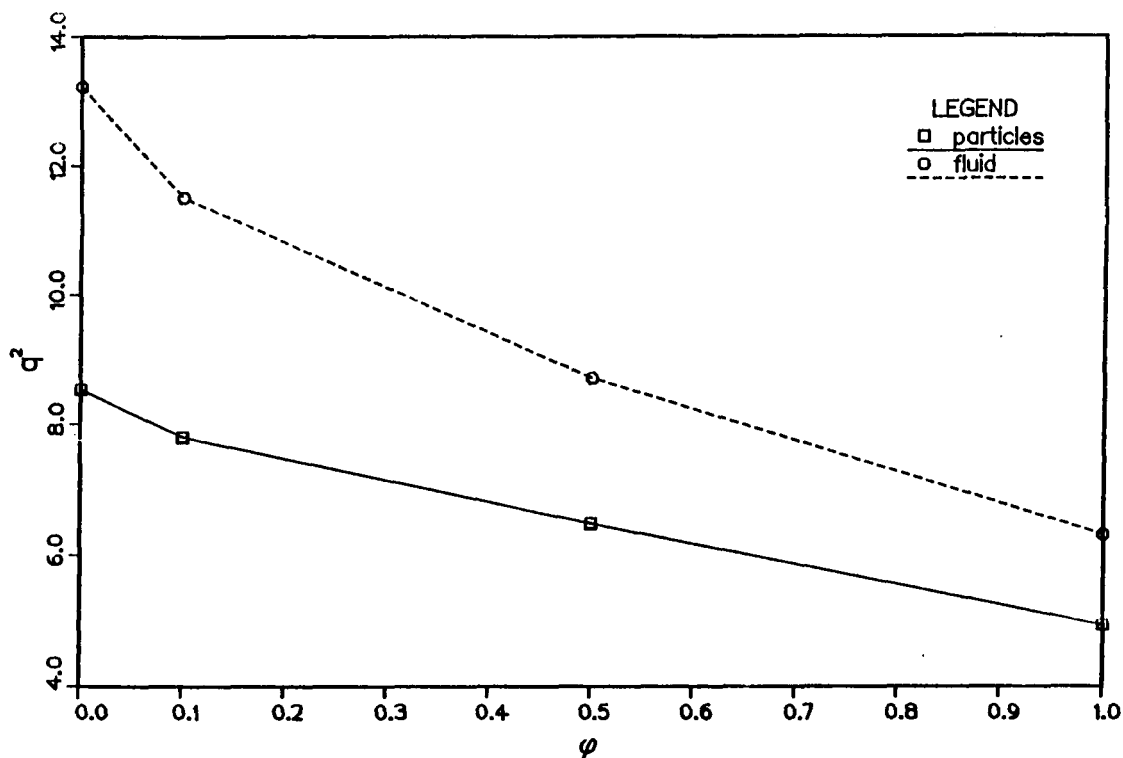
FIGURE 2. Turbulence energy versus loading from the 32^3 simulations, $\tau_p/T_e = 0.75$.

energy (figure 2). These results are consistent with the fact that the drag of the particles on the turbulence acts as an additional source of dissipation. This can be shown be deriving the transport equations for $u_i u_i$ (beginning with equation (2.3)).

Frequency spectra measured *along the particle path* were nearly identical for all simulations. The differences in the turbulence spectrum for the three cases can be attributed to the fact that the lighter particles are found more often in regions of low vorticity and high strain rate than are the heavier particles. This causes a sampling bias resulting in the differences between the spectra along the particle path. Measured values of the particle velocity spectra were found to be in excellent agreement with a theoretical prediction of Csanady (1963). It was somewhat surprising to find such close agreement with Csanady's theory in view of the fact that the particles are concentrated within particular regions of the turbulence. Examination of the power spectra of the turbulence for increasing values of the mass loading showed that energy was removed nearly equally from all frequencies. This may be due in part to the low Reynolds number of the simulations. Selective energy loss within certain frequency bands may occur at higher Reynolds numbers.

Correlations between enstrophy and pressure were decreased more by the light particles than by the heavy particles for increasing values of the mass loading. Since regions of low pressure are associated with regions of high vorticity, these results indicate that the lighter particles cause more distortion of these eddies than do the heavier particles. The fact that the lighter particles show a more pronounced tendency to accumulate in regions of lower vorticity (for $\phi = 0$) than do the heavier particles may provide some explanation for this.

Correlations between II_d and pressure showed that for the time constants and mass loadings used in the simulations the correlation between these two quantities remained reasonably constant, decreasing slightly for the lightest particles. Correlating II_d with pressure correlates regions of low pressure with vortices (large positive II_d) and regions of high pressure with straining regions (large negative II_d). Therefore, these results indicate that the correlation between the straining regions with regions of high pressure must be increasing to compensate for the loss of correlation between the vortical regions and regions of low pressure for the lighter particles.

In summary, it was found that for the case of zero loading there are significant effects of the turbulence "structure" on the resulting concentration field. These results were quantified by measuring conditional expectations and correlations. It is shown that the lighter particles show a strong tendency to be in regions of low vorticity and high strain rate.

For increasing values of the mass loading the power spectra of the turbulent fluctuations do not show any frequencies to be preferentially modified by the particles. This may be partially due to the fact that there is not a large range of scales in the simulations.

The lightest particles used in these simulations were found to modify the turbulence field differently than did the heavier particles. Since the light particles show a more pronounced tendency to accumulate in regions of low vorticity for $\phi = 0$, these regions are modified more by the light particles as the loading is increased than the heavy particles. Evidently, this selective modification by the light particles causes more of a distortion of the turbulent eddies than the more uniform modification by the heavier particles.

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