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Topology of Modified Helical Gears and Tooth Contact Analysis (TCA) Program

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Abstract

The contents of this report covers: (i) development of optimal geometry for crowned helical gears; (ii) method for their generation; (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned helical gears and (iv) modelling and simulation of gear shaft deflection.

The developed method for synthesis is used for determination of optimal geometry for crowned helical pinion surface and is directed to localize the bearing contact and guarantee the favorable shape and low level of the transmission errors.

Two new methods for generation of the crowned helical pinion surface have been proposed. One is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. Other is based on crowning pinion tooth surface with predesigned transmission errors. The pinion tooth surface can be generated by a computer controlled automatic grinding machine.

The TCA program simulates the meshing and bearing contact of the misaligned gears. The transmission errors are also determined.

The gear shaft deformation has been modelled and investigated. It has been found the deflection of gear shafts has the same effects as those of gear misalignment.

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Table of Contents

List of Symbols

a^*	half the length of major axis of contact ellipse
a	coefficient of parabolic function
b	addendum of cutting tool or coefficient of linear function
b^*	half the length of minor axis of contact ellipse
$[b_{ij}]$	3x3 auxiliary matrix
C	Operating center distance
$\underset{\sim}{C}$	operating center distance vector
$C^o = \frac{N_1}{2P} + \frac{N_2}{2P}$	nominal center distance
$\underset{\sim}{C}^o$	nominal center distance vector
d	height of generating cone or level of transmission error.
$e_I^{(i)}, e_{II}^{(i)}$	unit vectors along principal direction of surface Σ_i if principal directions of two surface coincide, i is neglected.
$f_i(X_1, X_2 \ldots)$	function expression with respect to variable $X_1, X_2 \ldots$
$F_1(X_1, X_2 \ldots)$	function expression with respect to variable X_1, X_2
F	magnitude of $\underset{\sim}{F}$
$\underset{\sim}{F}$	acting force between gear tooth surfaces
$\underset{\sim}{F}_t$	acting force in transverse section
$\underset{\sim}{F}_z$	acting force in axial direction
$g(\phi_1)$	auxiliary function of ϕ_1
i_t, j_t, k_t	unit direction vectors of coordinate system t
L_{ij}	line of tangency of surfaces Σ_i and Σ_j
$[L_{ij}]$	projection transformation matrix (from S_j to S_i)
$m_{1p} = \frac{d\phi_p}{ds}$	kinematic ratio
$m'_{1p} = \frac{dm_{1p}}{ds}$	derivative of kinematic ratio of gear and rack cutter

$m_{21} = \frac{d\phi_2}{d\phi_1} = \frac{\omega_2}{\omega_1}$	kinematic ratio of gears
$m'_{21} = \frac{dm_{21}}{d\phi_1}$	derivative of kinematic ratio of gears
$[M_{ij}]$	coordinate transformation matrix (from S_j to S_i)
$\dot{\tilde{n}}_r$	velocity of end point of unit normal corresponding a point moving over a surface
$\tilde{n}_i^{(j)}$	unit normal vector of surface Σ_j in coordinate system S_i [sometimes (i) is omitted if it is unnecessary to specify coordinate system]
N_1	number of pinion teeth
N_2	number of gear teeth
O_i	origin of coordinate system i
P_n	diametral pitch
r_1	radius of pitch circle for pinion
r_2	radius of pitch circle for gear
R	radius of arc of generating surface
$r_i^{(j)}(u,\theta)$	position vector describing surface Σ_j with surface coordinate (u,θ) in coordinate system S_i [sometimes (j) is omitted]
S	parameter of cutting motion
$S_i(X_i,Y_i,Z_i)$	coordinate system i
t_p, t_G	surface coordinates of pinion and gear
u_i	generating surface coordinate
u_p^*	auxiliary variable
$\tilde{V}_r$	velocity of a point moving over a surface
$\tilde{V}_j^{(i)}$	velocity of surface Σ_i in coordinate system j (j is omitted sometimes)
$\tilde{V}_f^{(ij)} = \tilde{V}_f^{(i)} - \tilde{V}_f^{(j)}$	relative velocity between surface Σ_i and Σ_j at contact point in coordinate system f (f is omitted sometimes).
$V_I^{(ij)}$, $V_{II}^{(ij)}$	projection of $V^{(ij)}$ in the principal direction of surface
$X_i^{(j)}$, $Y_i^{(j)}$, $Z_i^{(j)}$	coordinates of vector in S_i [sometimes (j) is omitted]

α	half of cone angle or coordinate of generatin surface
β	generating surface coordinate
β_p	helical angle
$\Delta\gamma$	crossing angle between axes of pinion and gea
$\Delta\delta$	intersecting angle between axes of pinion an gear
$\Delta\phi_2$	kinematic error (transmission error)
$\Delta\phi_2'$	derivative of kinematic error with respec to ϕ_1
ε	elastic deformation of the contacting point
θ_i	generating surface coordinate
$\kappa_I^{(i)}, \kappa_{II}^{(i)}$	principal curvatures of surface Σ_i
$\kappa_\varepsilon^{(i)} = \kappa_I^{(i)} + \kappa_{II}^{(i)}$	auxiliary function
λ_i	rotation of certain section of gear shaft
ρ	radius of genetrix arc for revolute surface
$\sigma^{(ij)}$	angle between $e_I^{(i)}$ and $e_I^{(j)}$
Σ_i	pinion tooth surface
Σ_2	gear tooth surface (or shape)
Σ_G	gear generating surface
Σ_p	pinion generating surface
ϕ_p	angle of rotation for pinion being generated
ϕ_G	angle of rotation for gear being generated
ϕ_1	angle of pinion rotation in meshing with gear
ϕ_2	angle of gear rotation in meshing with pinion
ϕ_{20}	theoretical angle of gear rotation in meshin with pinion
ψ_1, ψ_2	new coordinates to describe parabolic functio
ψ_c, ψ_n	pressure angles in transverse section an normal section
$\underset{\sim}{\omega}_1$	angular velocity of pinion
$\underset{\sim}{\omega}_2$	angular velocity of gear
$\underset{\sim}{\omega}_p$	angular velocity of pinion being generated
$\underset{\sim}{\omega}_G$	angular velocity of gear being generated

$\underset{\sim}{\omega}^{(12)}=\underset{\sim}{\omega}^{(1)}-\underset{\sim}{\omega}^{(2)}$	relative angular velocity
μ	angle for installment of pinion cutting tool
ν_i	deflection of certain point of gear shaft

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SUMMARY

The topology of several types of modified surfaces for helical gers are proposed. The modified surfaces allow absorption of linear or almost linear function of transmission errors caused by gear misalignment and deflection of shaft. These surfaces result in the improved contact of gear tooth surfaces. The principles and corresponding programs for computer aided simulation of meshing and contact of gears have been developed. The results of this investigation are illustrated with numerical examples.

1. INTRODUCTION

Traditional methods for generation of involute helical gears with parallel axes provide developed ruled tooth surfaces for the gear teeth (Fig. 1.1). The tooth surfaces contact each other at every instant along a line, L, that is the tangent to the helix on the base cylinder. The surface normals along L do not change their orientation. The disadvantage of regular helical gears is that they are very sensitive to misalignments such as the crossing or intersection of gear axes. The misaligned gears transform rotation with a linear function of transmission errors (a main source of noise) and the bearing contact is shifted to the edge of the teeth. The frequency of transmission errors coincides with the frequency of tooth meshing. The actual contact ratio (the average number of teeth being in mesh at every instant) is close to one and is far from the expected value.

These are the reasons why we have to reconsider the canonical ideas on involute helical gears and modify their tooth surfaces. Crowning of the gear surfaces is needed to negate the effects of transmission errors and the shift of contact between the gear tooth surfaces. Deviations of screw involute gear tooth surfaces to provide a new topology that can reduce the gear sensitivity to misalignment will be developed. Theoretically, the modified tooth surfaces will be in contact at every instant at a point instead of a line. Actually, due to the load applied between meshing teeth, the contact will be spread over an elliptical area whose dimensions may be controlled. Methods for gear tooth surface generation that provide the desirable surface deviation are proposed. For economical reasons only the pinion tooth surface is modified while the gear surface is kept as a regular screw involute surface.

2. BASIC CONCEPTS AND CONSIDERATIONS

2.1 Simulation of Meshing

The investigation of influence of gear misalignment requires a numerical solution for the simulation of meshing and contact of gear tooth surfaces. The basic ideas of this method (Litvin, 1968) are as follows:

(1) The meshing of gear tooth surfaces is considered in a fixed coordinate system, S_f. Usually, the generated gear tooth surfaces may be represented in a three parametric form with an additional relation between these parameters - Gaussian

coordinates. Such a parametric form is the result of representation of a gear tooth surface as an envelope of the family of the tool surface (the generating surface) and two from the three Gaussian coordinates are inherited from the tool surface.

The continuous tangency of gear tooth surfaces is represented by the following equations

$$\underset{\sim}{r}^{(1)}(u_1, \theta_1, \psi_1, \phi_1) = \underset{\sim}{r}^{(2)}(u_2, \theta_2, \psi_2, \phi_2) \tag{2.1.1}$$

$$\underset{\sim}{n}^{(1)}(u_1, \theta_1, \psi_1, \phi_1) = \underset{\sim}{n}^{(2)}(u_2, \theta_2, \psi_2, \phi_2), \quad \left|\underset{\sim}{n}^{(1)}\right| = \left|\underset{\sim}{n}^{(2)}\right| \tag{2.1.2}$$

$$f_6(u_1, \theta_1, \psi_1) = 0 \tag{2.1.3}$$

$$f_7(u_2, \theta_2, \psi_2) = 0 \tag{2.1.4}$$

Here: u_i and θ_i are the tool surface curvilinear coordinates, ψ_i is the parameter of motion in the process of generation of the gear tooth surface, ϕ_i is the angle of rotation of the gear being in mesh with the mating gear.

Equations (2.1.1) to (2.1.4) provide that the position vectors $\underset{\sim}{r}^{(1)}$ and $\underset{\sim}{r}^{(2)}$ and surface unit normals $\underset{\sim}{n}^{(1)}$ and $\underset{\sim}{n}^{(2)}$ are equal for the gear tooth surfaces in contact (Fig. 2.1). Vector equations (2.1.1) and (2.1.2) yield five independent equations and the total equation system is

$$f_i(u_1,\theta_1,\phi_1,u_2,\theta_2,\phi_2,\psi_1,\psi_2) = 0, \text{ i.e. } [1,5],$$

$$f_6(u_1,\theta_1,\psi_1) = 0, \; f_7(u_2,\theta_2,\psi_2) = 0 \qquad (2.1.5)$$

An instantaneous point contact instead of a line contact is guaranteed if the Jacobian differs from zero, i.e. if

$$\frac{D(f_1,f_2,f_3,f_4,f_5,f_6,f_7)}{D(u_1,\theta_1,\psi_1,u_2,\theta_2,\psi_2,\phi_2)} \neq 0 \qquad (2.1.6)$$

If the inequality equation (2.1.6) is observed, then the system of equations (2.1.5) may be solved in the neighborhood of the contact point by functions

$$u_1(\phi_1),u_2(\phi_1),\psi_1(\phi_1),\ldots\ldots,\phi_2(\phi_1) \qquad (2.1.7)$$

These functions are of class C^1 (at least they have continuous derivatives of the first order). Functions (2.1.7) and equations (2.1.5) enable calculation of the transmission errors (deviation of $\phi_2(\phi_1)$ from the prescribed linear function) and the path of the contact point over the gear tooth surface.

For the case when the gear tooth surface is a regular screw involute surface, it may be directly represented in a two-parametric form and the number of equations in system (2.1.5) may be reduced to six.

2.2 Simulation of Contact

Due to the elastic approach of the gear tooth surfaces their contact is spread over an elliptical area. It is assumed that the magnitude of the elastic approach is known from experiments or may be predicted. Knowing in addition the principal curvatures and directions for two contacting surfaces at their point of contact we may determine the dimensions and orientation of the contact ellipse (Litvin, 1968).

The determination of principal curvature and directions for a surface represented in a three-parametric form is a complicated computational problem. A substantial simplification of this problem may be achieved using the relations between principal curvatures and directions, and the parameters of motion for two surfaces being in contact at a line. One of the contacting surfaces is the tool surface and the other is the generated surface.

Helical gears with modified gear tooth surfaces will be designed as surfaces being in point contact at every instant. The point of contact traces out on the surface a spatial curve (the path of contact) whose location must be controlled. The tangent to the path of contact and the derivative of the gear ratio $\frac{d}{d\phi_1}(m_{21}(\phi_1))$ may be controlled by using the relationship between principal curvatures and directions for two surfaces that are in point contact (ref. 2). Here:

$$m_{21} = \frac{\omega_2}{\omega_1} = f(\phi_1)$$

is the gear ratio.

2.3 Partial Compensation of Transmission Errors

Aligned gears transform rotation with a constant gear ratio m_{21} and

$$\phi_{20}(\phi_1) = \frac{N_1}{N_2}\phi_1 \tag{2.3.1}$$

is a linear function. Here: N_1 and N_2 are the numbers of gear teeth. An investigation of the effect of helical gear rotational axis intersection or crossing indicates that $\phi_2(\phi_1)$ becomes a piece-wise function which is nearly linear for each cycle of meshing (Fig. 2.2(a)). The transmission errors are determined by

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \phi_1\frac{N_1}{N_2} \tag{2.3.2}$$

and they are also represented by a piece-wise linear function (Fig. 2.2(b)). Transmission errors of this type cause a discontinuity of the gear angular velocity at transfer points and vibration becomes inevitable. The new topology of gear tooth surfaces proposed in this report allows the absorption of a linear function of transmission errors that results in a reduced level of vibration. This is based on the possibility to absorb a linear function by a parabolic function.

Consider the interaction of a parabolic function given by

$$\Delta\phi_2^{(1)} = -a\phi_1^2 \tag{2.3.3}$$

with a linear function represented by

$$\Delta\phi_2^{(2)} = b\phi_1 \tag{2.3.4}$$

The resulting function

$$\Delta\phi_2 = b\phi_1 - a\phi_1^2 \tag{2.3.5}$$

may be represented in a new coordinate system (Fig. 2.3):

$$\psi_2 = -a\psi_1^2 \tag{2.3.6}$$

where

$$\psi_2 = \Delta\phi_2 - \frac{b^2}{4a^2}, \quad \psi_1 = \phi_1 - \frac{b}{2a} \tag{2.3.7}$$

We consider that $\Delta\phi_2^{(1)} = -a\phi_1^2$ is a predesigned function that exists even if misalignments do not appear. The absorption of function $\Delta\phi_2^{(2)} = b\phi_1$ by the parabolic function $\Delta\phi_2^{(1)} = -a\phi_1^2$ means that gear misalignment does not change the predesigned parabolic function of transmission errors. Thus the resulting function of transmission errors $\Delta\phi_2 = \Delta\phi_2^{(1)} + \Delta\phi_2^{(2)}$ will keep its shape as a parabolic function although the gears are misaligned. The resulting function of transmission errors $\phi_2(\phi_1)$ may be obtained by translation of the parabolic function $\Delta\phi_2^{(1)}$.

The absorption of a linear function of transmission errors by a parabolic function is accompanied by the change of transfer points. The transfer points determine the positions of the gears where one pair of teeth is rotating out of mesh and the next pair is coming into mesh. The change of transfer points is determined with $\Delta\phi_1 = \left|\frac{b}{2a}\right|$ and $\Delta\phi_2 = \frac{b^2}{4a^2}$, the cycle of meshing of one pair of teeth is $\phi_1 = \frac{2\pi}{N_1}$ i ε 1,2. It may happen that the absorption of a linear function by a parabolic function is accompanied with a change that is too large. If this occurs the transfer points and the resulting parabolic function of transmission errors, $\psi_2(\psi_1)$, will be represented as a discontinuous function for one cycle of meshing (Fig. 2.4). To avoid this, it is necessary to limit the tolerances for gear misalignment.

2.4 Misalignment of Regular Helical Gears

Regular helical pinion and gear can be represented by their surface position vectors and normal vectors in coordinate system S_1 and S_2 as:

$$[r_1] = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\dfrac{\cos^2\psi_n}{\cos\psi_c}\cos(\phi_p - \psi_c)[t_p\sin\beta_p + r_1\phi_p] + r_1\sin\phi_p \\ \dfrac{\cos^2\psi_n}{\cos\psi_c}\sin(\phi_p - \psi_c)[t_p\sin\beta_p + r_1\phi_p] + r_1\cos\phi_p \\ t_p(\cos\beta_p + \sin^2\psi_n\dfrac{\sin^2\beta_p}{\cos\beta_p}) + r_1\phi_p\sin^2\psi_n t_g\beta_p \\ 1 \end{bmatrix} \tag{2.4.1}$$

$$[n_1] = \begin{bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{bmatrix} = \begin{bmatrix} \cos\psi_n\cos\beta_p\cos\phi_p + \sin\psi_n\sin\phi_p \\ -\cos\psi_n\cos\beta_p\sin\phi_p + \sin\psi_p\cos\phi_p \\ + \cos\psi_n\sin\beta_p \end{bmatrix} \tag{2.4.2}$$

$$[r_2] = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} -\dfrac{\cos^2\psi_n}{\cos\psi_c}\sin(\phi_G - \psi_c)[-t_G\sin\beta_p + r_2\phi_G] + r_2\sin\phi_G \\ \dfrac{\cos^2\psi_n}{\cos\psi_c}\sin(\phi_G - \psi_c)[-t_G\sin\beta_p + r_2\phi_G] + r_2\cos\phi_G \\ t_G(\cos\beta_p + \sin^2\psi_n\dfrac{\sin^2\beta_p}{\cos\beta_p}) - r_2\phi_G\sin^2\psi_n tg\beta_p \\ 1 \end{bmatrix} \tag{2.4.3}$$

$$[n_2] = \begin{bmatrix} n_{2x} \\ n_{2y} \\ n_{2z} \end{bmatrix} = \begin{bmatrix} \cos\psi_n\cos\beta_p\cos\phi_G + \sin\psi_n\sin\phi_G \\ -\cos\psi_n\cos\beta_p\sin\phi_G + \sin\psi_n\cos\phi_G \\ -\cos\psi_n\sin\beta_p \end{bmatrix} \tag{2.4.4}$$

where ψ_n and ψ_c are the pressure angles in gear tooth normal section and transverse section respectively; β_p is the helical angle at the pitch cylinder of the pinion and the gear; r_1 and r_2 are the radii pitch cylinder of the pinion and the gear respectively; ϕ_p, and t_p are the surface parameters of the pinion tooth surface; ϕ_G and t_G are the surface parameters of the gear tooth surface.

When the helical pinion and gear are in mesh, with their axes misaligned, their position vectors and normal vectors can be transformed to the fixed coordinate system S_f. The basic ideas have already been discussed in the Section 2.1. And the real approach of transformation and the matrices to describe the misalignment and gear rotation will be given in Section 3.8.

It is found that when the misalignment occurs the regular pinion and gear surface cannot contact in tangency. That is, in S_f, their normals can not be equal in all circumstances. In this case, only the gear tooth edge contacts pinion tooth surface in tangency. Therefore, in the fixed coordinate system, there are four equations to describe the contact, that is, the equalities of three position vectors describing gear tooth edge and pinion tooth surface as well as the zero product of gear tooth edge tangency and pinion tooth normal.

The computer aided simulation of meshing of misaligned helical gears with regular tooth surfaces shows that the transformation of rotation is accompanied with large transmission errors. There are two sub-cycles of meshing during the complete meshing cycle for one pair of teeth. These sub-cycles correspond to the meshing of (1) a curve with a surface, and (2) a point with the surface. The curve is the involute curve at the edge of the tooth of the gear and the point is the tip of the gear tooth edge. The transmission errors for the period of a cycle are represented by two linear functions (Fig. 2.5). The transformation of rotation will be accompanied with a jump of the angular velocity of the driven gear and therefore vibrations are inevitable.

The results of computation are presented for the following case. Given: number of teeth $N_1 = 20$, $N_2 = 40$, diametral pitch in normal section $P_n = 10 \text{ in}^{-1}$, gear tooth length $L = 10/P_n$ the helical angle $\beta_p = 15°$, the normal pressure angle $\psi_n = 20°$. The gear axes are crossed and form an angle $\Delta\gamma = 5$ arc minutes. The computed transmission errors are represented in table 2.4.1.

TABLE 2.4.1 - TRANSMISSION ERRORS OF REGULAR HELICAL GEARS WITH CROSSED AXES

ϕ_1, deg	-8	-5	-2	1	4	7	10
$\Delta\phi_2$, sec	4.90	3.06	1.22	-0.61	-2.45	-4.29	-6.12

2.5 Surface Deviation by the Change of Pinion Lead

Helical gears in this case are designed as helical gears with crossed axes. The crossing angle is chosen with respect to the expected tolerances of the gear misalignment ($\Delta\gamma$ is in the range of 10 to 15 arc minutes). The gear ratio for helical gears with crossed axes may be represented (Litvin, 1968) as

$$M_{12} = \frac{\omega_1}{\omega_2} = \frac{r_{b2} \sin \lambda_{b2}}{r_{b1} \sin \lambda_{b1}} \tag{2.5.1}$$

where r_{bi} and λ_{bi} are the radius of the base cylinder and the lead angle on this cylinder, $i \varepsilon 1,2$. $\left|\lambda_{p2} - \lambda_{p1}\right| = \Delta\gamma$. Here: λ_{pi} is the lead angle on the pitch cylinder. The advantage of application of crossed helical gears is that the gear ratio is not changed by the misalignment (by the change of $\Delta\gamma$). The tooth surfaces contact each other at a point during meshing. The disadvantage of this type of surface deviation is that location of the bearing contact of the gears is very sensitive to gear misalignment. A slight change of the crossing angle causes shifting of the contact to the edge of the tooth (Fig. 2.6).

The discussed type of surface deviation is reasonable to apply for manufacturing of expensive reducers of large dimensions when the lead of the pinion can be adjusted by regrinding. While changing by regrinding the parameters r_{b1} and γ_{b1}, the requirement that the product $r_{b1} \sin\lambda_{b1}$ must be kept constant. Then, the gear ratio M_{21} will be of the prescribed value and transmission errors caused by the crossing of axes will be zero.

Theoretically, transmission errors are inevitable if the axes of crossed helical gears become intersected. Actually, if gear misalignment is of the range of 5 to 10 arc minutes, the transmission errors are very small and may be neglected. The main problem for this type of misalignment is again the shift of the bearing contact to the edge (Fig. 2.6).

3. GENERATION OF PINION TOOTH SURFACE BY A SURFACE OF REVOLUTION

3.1 Basic Consideration

The purpose of this method for deviation of the pinion tooth surface is to reduce the sensitivity of the gears to misalignment. Also the transmission error must be kept to a low level and stabilize the bearing contact. This investigation shows that this goal may be achieved by the proposed method of crowning but the bearing contact cannot cover the whole surface. The reason for this is that the instantaneous contact ellipse moves across but not along the surface (Fig. 3.1).

The proposed method for generation is based on the following considerations. It is well known that the generation of a helical gear may be performed by an imaginary rack-cutter with skew teeth whose normal section represents a regular rack-cutter for spur gears (Fig. 3.2(a)). We may imagine that two generating surfaces, Σ_G and Σ_p, are applied to generate the gear tooth surface and the pinion tooth surface, respectively (Fig. 3.2(b)). Surface Σ_G is a plane (a regular rack-cutter surface), and Σ_p is a cone surface. Surfaces Σ_G and Σ_p are rigidly connected and perform translational motion, while the pinion and the gear rotate about their axes (Fig. 3.3). The generated pinion and gear will be in point contact and transform rotation with the prescribed linear function $\phi_2(\phi_1)$. However, due to gear misalignment, function $\phi_2(\phi_1)$ becomes a piecewise linear function (Fig. 2.2(a)) that is not acceptable. To absorb a linear function of transmission errors (Fig. 2.2(b)), a predesigned parabolic function of transmission errors is used. For this reason a surface of revolution that slightly deviates from the cone surface is proposed (Fig. 3.2(c)). The radius of the surface of revolution in its axial section determines the level of the predesigned parabolic function. The pinion crowning process may be accomplished by grinding, shaving or lapping.

3.2 Principle of Generation and Used Coordinate Systems

Consider two rigid connecting surfaces Σ_G and Σ_p. The generating surface Σ_G is a plane and generates the helical gear tooth surface that is an involute screw surface. Surface Σ_p is a surface of revolution. Initially, we consider that Σ_p is a cone surface and Σ_p and Σ_G contact each other along a straight line that is the generatrix of the cone.

Fig. 3.4 shows the generating surfaces Σ_G and Σ_p. Fig. 3.5 illustrates the process for generation. While the rigidly connected generating surfaces perform a translational motion, the pinion and gear rotate about their axes O_1 and O_2, respectively. The parameter of motion of the cutter, S, and the angles of rotation of the pinion and the gear, ϕ_1 and ϕ_2, are related as follows:

$$S = r_1\phi_p = r_2\phi_G \tag{3.2.1}$$

where r_1 and r_2 are the great centrodes radii, the cutter centrode is the straight line that is tangent to the gear centrodes. Point I is the instantaneous center of rotation. Coordinate systems S_1 and S_2 are rigidly connected to the helical pinion and the helical gear, whereas coordinate systems S_c and S_f are rigidly connected to the tool surface and fixed frame. The generating surface Σ_G (a plane) and Σ_p are covered with a set of contact lines L_{G2} and L_{p1} respectively---the instantaneous lines of tangency of surfaces Σ_G and Σ_2 and Σ_p and Σ_1 (Fig. 2.3, a,b). The location of these lines depends on the value of parametric ϕ_G and ϕ_p(ϕ_p and ϕ_G are related). Line L_{Gp} is the line of tangency of generating surfaces Σ_G and Σ_p. When Σ_1 and Σ_2 are generated, at any moment, one point M_i on line L_{pG} are generating corresponding point M_{pi} and M_{Gi} on the helical gear surface Σ_1 and Σ_2. When the Σ_1 and Σ_2 are meshing without misalignment M_{pi} and M_{Gi} contact each other in turn. Now it is clear why L_{pG} is not parallel with the edge of Σ_G. The reason is that in this way, the contact ratio of crowned helical pinion and regular helical gear will be higher.

3.3 Tool Surface

The pinion generating surface is a cone and may be represented in an auxiliary coordinate system S_d (Fig. 3.7) as follows:

$$\underset{\sim}{r}_d = \begin{bmatrix} x_d \\ y_d \\ z_d \\ 1 \end{bmatrix} = \begin{bmatrix} u_p \cos\theta \sin\alpha \\ d - u_p \cos\alpha \\ -u_p \sin\theta_p \sin\alpha \\ 1 \end{bmatrix} \qquad (3.3.1)$$

where $0 < u < \frac{d}{\cos\alpha}$, $0 < \theta < 2\pi$. The surface normal is represented by

$$\underset{\sim}{N}_d = \frac{\partial \underset{\sim}{r}_d}{\partial \theta_p} \times \frac{\partial \underset{\sim}{r}_d}{\partial U_p} = u_p \sin\alpha \begin{bmatrix} \cos\alpha \cos\theta_p \\ \sin\alpha \\ \cos\alpha \sin\theta_p \end{bmatrix} \qquad (3.3.2)$$

The unit surface normal is (provided $u_p \sin\alpha \neq 0$)

$$\underset{\sim}{n}_d = \begin{bmatrix} \cos\alpha \cos\theta_p \\ \sin\alpha \\ \cos\alpha \sin\theta_p \end{bmatrix} \qquad (3.3.3)$$

Figure 3.8 illustrates the installment of the conic tool in coordinate system S_c step by step as follows:

(i) From coordinate system S_d to S_b', the cone is tangent to the plane y_b', o_b', z_b' where the tangent line L_{pG} (see Section3.2) is coincident with y_b' axis. Here, we have (Fig. 3.8a).

$$[M_{b'd}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & -d\sin\alpha \\ -\sin\alpha & \cos\alpha & 0 & dtg\alpha\sin\alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.3.4)$$

where the d is the height of cone.

(ii) From coordinate system S_b' to S_b' the tangent line of cone and plane $y_b o_b z_b$ is declined with an angle μ (see Fig. 3.4), also the origin o_b' and o_b are not coincident. Here we have (Fig. 3.8b).

$$[M_{bb'}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\mu & -\sin\mu & -\dfrac{b}{\cos\psi_n} \\ 0 & \sin\mu & \cos\mu & \dfrac{b}{\cos\psi_n}\, tg\mu \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.3.5)$$

where b is the addendum height of the tool.

(iii) From coordinate system S_b to S_a, the tool is declined with a pressure angle ψ_n. For the helical gear, the ψ_n is measured at the normal cross section. Here, we have (Fig. 3.8c).

$$[M_{ab}] = \begin{bmatrix} \cos\psi_n & -\sin\psi_n & 0 & 0 \\ \sin\psi_n & \cos\psi_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.3.6)$$

(iv) From coordinate system S_a to S_c, the helix angle β_p is considered. Here we have (Fig. 3.8d)

$$[M_{ca}] = \begin{bmatrix} \cos\beta_p & 0 & -\sin\beta_p & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta_p & 0 & \cos\beta_p & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.3.7)$$

In coordinate system S_c, the tool surface and its normal can be represented as:

$$[r_c^{(p)}] = [M_{ca}][M_{ab}][M_{bb'}][M_{bd}][r_d] \tag{3.3.8}$$
$$[n_c^{(p)}] = [L_{ca}][L_{ab}][L_{bb'}][L_{bd}][n_d]$$

where 3 x 3 L matrix is from corresponding 4 x 4 M matrix, excluding the last row and last column. Substituting Eq. (3.3.1), (3.3.3), (3.3.4), (3.3.5), (3.3.6) and (3.3.7) into Eq. (3.3.8), finally we obtain the tool surface Σ_p and its normal in coordinate system S_c as

$$[r_c^{(p)}] = \begin{bmatrix} x_c^{(p)} \\ y_c^{(p)} \\ z_c^{(p)} \\ 1 \end{bmatrix} , \quad [n_c^{(p)}] = \begin{bmatrix} n_{cx}^{(p)} \\ n_{cy}^{(p)} \\ n_{cz}^{(p)} \end{bmatrix} \tag{3.3.9}$$

where

$$\begin{aligned} x_c^{(p)} = {} & u_p\cos\theta_p\sin\alpha(\cos\alpha\cos\psi_n\cos\beta_p + \sin\alpha\sin\psi_n\cos\mu\cos\beta_p \\ & + \sin\alpha\sin\mu\sin\beta_p) + u_p\sin\theta_p\sin\alpha(\sin\psi_n\sin\mu\cos\beta_p - \cos\mu\sin\beta_p) \\ & + u_p\cos\alpha(\cos\alpha\sin\psi_n\cos\mu\cos\beta_p - \sin\alpha\cos\psi_n\cos\beta_p + \cos\alpha\sin\mu\sin\beta_p) \\ & - \left(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n\cos\mu}\right)[\sin\psi_n\cos\mu\cos\beta_p + \sin\mu\sin\beta_p] \end{aligned}$$

$$\begin{aligned} y_c^{(p)} = {} & u_p\cos\theta_p\sin\alpha(\cos\alpha\sin\psi_n - \sin\alpha\cos\psi_n\cos\mu) \\ & - u_p\sin\theta_p\sin\alpha\cos\psi_n\sin\mu - u\cos\alpha(\sin\alpha\sin\psi_n + \cos\alpha\cos\psi_n\cos\mu) \\ & + \left(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n\cos\mu}\right)\cos\mu\cos\psi_n \end{aligned}$$

$$
\begin{aligned}
Z_c^{(p)} = & u_p\cos\theta_p\sin\alpha(\cos\alpha\cos\psi_n\sin\beta_p + \sin\alpha\sin\psi_n\cos\mu\sin\beta_p - \sin\alpha\sin\mu\cos\beta_p) \\
& + u_p\sin\theta_p\sin\alpha(\sin\psi_n\sin\mu\sin\beta_p + \cos\mu\cos\beta_p) \\
& + u_p\cos\alpha(\cos\alpha\sin\psi_n\cos\mu\sin\beta_p - \sin\alpha\cos\psi_n\sin\beta_p - \cos\alpha\sin\mu) \\
& + \left(\frac{p_d}{\cos\alpha} - \frac{b}{\cos\psi_n\cos\mu}\right)(-\cos\mu\sin\beta_p\sin\psi_n + \sin\mu\cos\beta_p)
\end{aligned}
$$

$$
\begin{aligned}
n_{cx}^{(p)} = & \cos\theta_p\cos\alpha(\cos\alpha\cos\psi_n\cos\beta_p + \sin\alpha\sin\psi_n\cos\mu\cos\beta_p \\
& + \sin\alpha\sin\mu\sin\beta_p) + \sin\theta_p\cos\alpha(\sin\psi_n\sin\mu\cos\beta_p - \cos\mu\sin\beta_p) \\
& + \sin\alpha(\sin\alpha\cos\psi_n\cos\beta_p - \cos\alpha\sin\psi_n\cos\mu\cos\beta_p - \cos\alpha\sin\mu\sin\beta_p)
\end{aligned}
$$

$$
\begin{aligned}
N_{cy}^{(p)} = & \cos\theta_p\cos\alpha(\cos\alpha\sin\psi_n - \sin\alpha\cos\psi_n\cos\mu) - \sin\theta_p\cos\alpha\cos\psi_n\sin\beta_p \\
& + \sin\alpha(\sin\alpha\sin\psi_n + \cos\alpha\cos\psi_n\cos\mu)
\end{aligned}
$$

$$
\begin{aligned}
n_{cz}^{(p)} = & \cos\theta_p\cos\alpha(\cos\alpha\cos\psi_n\sin\beta_p + \sin\alpha\sin\psi_n\cos\mu\sin\beta_p - \sin\alpha\sin\mu\cos\beta_p) \\
& + \sin\theta_p\cos\alpha(\sin\psi_n\sin\mu\sin\beta_p + \cos\mu\cos\beta_p) \\
& + \sin\alpha(\sin\alpha\cos\psi_n\sin\beta_p - \cos\alpha\sin\psi_n\cos\mu\sin\beta_p + \cos\alpha\sin\mu\cos\beta_p)
\end{aligned}
$$

3.4 Pinion Tooth Surface

The equation of meshing of the generating surface Σ_p and the helical pinion tooth surface is represented by

$$
\underset{\sim}{N}^{(P)} \cdot \underset{\sim}{V}^{(P1)} = \underset{\sim}{N}^{(P)} \cdot (\underset{\sim}{V}^{(P)} - \underset{\sim}{V}^{(1)}) = 0 \qquad (3.4.1)
$$

We can also use equation based on the fact that the contact normal of generating and generated surfaces must intersect the instantaneous axis of rotation I-I (Litvin, 1968). Thus, we obtain (Fig. 2.2).

$$\frac{X_c - x_c^{(P)}}{n_{cx}^{(P)}} = \frac{Y_c - y_c^{(P)}}{n_{cy}^{(P)}} = \frac{Z_c - z_c^{(P)}}{n_{cz}^{(P)}} \tag{3.4.2}$$

where (X_c, Y_c, Z_c) are the coordinates of a point that lies on axis I-I; $x_c^{(P)}$, $y_c^{(P)}$, $z_c^{(P)}$ are the coordinates of cone surface; n_{cx}, n_{cy} and n_{cz} are the projections of the surface unit normal. From Fig. 3.5, it is known that

$$X_c = S = r_1\phi_p, \quad Y_c = 0$$

Equation (3.4.2), (3.4.3), and (3.3.9) yield

$$\begin{aligned}f_1(u_p,\theta_p,\phi_p) = &-u_p[\cos\theta_p(\cos\mu\cos\beta_p+\sin\mu\sin\psi_n\sin\beta_p)\\ &+\sin\theta_p(\sin\alpha\sin\mu\cos\beta_p-\cos\alpha\cos\psi_n\sin\beta_p-\sin\alpha\sin\psi_n\cos\mu\sin\beta_p)]\\ &+(\frac{d}{\cos\alpha}-\frac{b}{\cos\psi_n\cos\mu})\ [(\cos\theta_p\cos^2\alpha+\sin^2\alpha).[\cos\mu\cos\beta_p\\ &+\sin\psi_n\sin\mu\sin\beta_p]-\sin\theta_p\cos\alpha\cos\psi_n\sin\beta_p]\\ &+r\phi_p[\cos\theta_p\cos\alpha(\cos\alpha\sin\psi_n-\sin\alpha\cos\psi_n\cos\mu)\\ &+\sin\alpha(\sin\alpha\sin\psi_n+\cos\alpha\cos\psi_n\cos\mu)-\sin\theta_p\cos\alpha\cos\psi_n\sin\psi_n]\\ &= 0\end{aligned} \tag{3.4.4}$$

The helical pinion tooth surface can be obtained by transforming the generating tool surface Σ_p to coordinate system S_1, together with equation of meshing. The coordinate transformation in transition from S_c to S_f is represented by the matrix M_{fc} as (Fig. 3.5)

$$[M_{fc}] = \begin{bmatrix} 1 & 0 & 0 & -r_1\phi_p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.4.5}$$

The coordinate transformation in transition from S_f to S_1 is represented by the matrix M_{1f} as (Fig. 3.5).

$$[M_{1f}] = \begin{bmatrix} \cos\phi_p & \sin\phi_p & 0 & 0 \\ -\sin\phi_p & \cos\phi_p & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.4.6)$$

Therefore, the helical pinion surface can be represented in coordinate system S_1 as:

$$[r_1] = [M_{1c}][r_c^{(P)}] = [M_{1f}][M_{fc}][r_c^{(P)}] \qquad (3.4.7)$$

together with Eq. (3.4.4). Substituting Eq. (3.4.5), (3.4.6) and (3.3.9) into Eq. (3.4.7) we have

$$[r_1] = \begin{bmatrix} (x_c^{(P)}-r_1\phi_p)\cos\phi_p+y_c^{(P)}\sin\phi_p \\ (x_c^{(P)}-r_1\phi_p)\sin\phi_p+y_c^{(P)}\cos\phi_p \\ z_c^{(P)} \\ 1 \end{bmatrix} \qquad (3.4.8)$$

Eq. (3.4.8) and (3.4.4) represents the pinion tooth surface where $x_c^{(P)}$, $y_c^{(P)}$ and $z_c^{(P)}$ are expressed in Eq. (3.3.9).

Using the same approach, the unit normal pinion tooth surface can be represented by Eq. (3.4.9) and (3.4.4) where $n_{cx}^{(P)}$, $n_{cy}^{(P)}$ and $n_{cz}^{(P)}$ are expressed in Eq. (3.3.9)

$$[n_1] = \begin{bmatrix} n_{cx}^{(P)}\cos\phi_p + n_{cy}^{(P)}\sin\phi_p \\ -n_{cx}^{(P)}\sin\phi_p + n_{cy}^{(P)}\cos\phi_p \\ n_{cz}^{(P)} \end{bmatrix} \qquad (3.4.9)$$

It is clear that if we set $\beta_p = 0$, the pinion becomes spur pinion. Therefore, crowning spur pinion using conic tool is only a special case of crowning helical pinion.

3.5 Condition of Pinion Non-Undercutting

The problem of undercutting of the helical pinion tooth surface by crowning is related with the appearance on the pinion tooth surface of singular points. From differential geometry, it is known that the surface point is singular if the surface normal is equal to zero at such a point.

Litvin, proposed a method to determine a line on the tool surface whose points will generate singular points on the surface generated by the tool. This line designated by L (Fig. 3.9) must be out of the working part of the tool surface to avoid undercutting of the pinion by crowning.

The limiting line L of the tool surface is determined by the following equations

$$\underset{\sim}{r}_c^{(P)} = \underset{\sim}{r}_c^{(P)}(u_p, \theta_p) \tag{3.5.1}$$

$$f_1(u_p, \theta_p, \phi_p) = 0 \tag{3.5.2}$$

$$F(u_p, \theta_p, \phi_p) = 0 \tag{3.5.3}$$

Vector equation (3.5.1) represents the tool surface [see Eq. (3.3.9)]; equation (3.5.2) is the equation of meshing [see Eq. (3.4.4)] and equation (3.5.3) comes from the requirement of limiting line L as:

$$\begin{vmatrix} \frac{\partial X_c^{(P)}}{\partial u_p} & \frac{\partial X_c^{(P)}}{\partial \theta_p} & V_{cx}^{(cl)} \\ \frac{\partial Z_c^{(P)}}{\partial u_p} & \frac{\partial Z_c^{(P)}}{\partial \theta_p} & V_{cz}^{(cl)} \\ \frac{\partial f_1}{\partial u_p} & \frac{\partial f_1}{\partial \theta_p} & \frac{\partial f_1}{\partial \phi}\frac{\partial \phi}{\partial f} \end{vmatrix} = 0 \tag{3.5.4}$$

where $\underset{\sim}{V}_c^{(cl)}$ is the relative velocity of the tool and generated surface at generating point, represented if coordinate system S_c. Actually, we can write $\underset{\sim}{V}_c^{(cl)}$ as

$$\underset{\sim}{V}_c^{(cl)} = \underset{\sim}{V}_c^{(c)} - \underset{\sim}{V}_c^{(1)} \tag{3.5.5}$$

where $\underset{\sim}{V}_c^{(c)}$ is the velocity of the cutter and $\underset{\sim}{V}_c^{(1)}$ is the velocity of the pinion. From Fig. 2.2, we get

$$\underset{\sim}{V}_c^{(c)} = \begin{bmatrix} -\frac{ds}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_p \\ 0 \\ 0 \end{bmatrix} \tag{3.5.6}$$

$$\underset{\sim}{V}_c^{(1)} = \underset{\approx}{\omega}_p \times \underset{\approx}{r}_c^{(p)} + \overline{O_cO_1} \times \underset{\approx}{\omega}_p \tag{3.5.7}$$

where

$$\underset{\approx}{\omega}_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \qquad \overline{O_cO_1} = \begin{bmatrix} r_1\phi_p \\ -r_1 \\ 0 \end{bmatrix} \tag{3.5.8}$$

Equations (3.5.6), (3.5.7) and (3.5.8) yield

$$\underset{\sim}{V}_c^{(cl)} = \begin{bmatrix} y_c^{(p)} \\ -x_c^{(p)} + r_1\phi_p \\ 0 \end{bmatrix} \omega \qquad (3.5.9)$$

where $X_c^{(p)}$ and $Y_c^{(p)}$ are represented by equations 3.3.9.
Equation (3.5.9), (3.3.9), (3.4.3) and (3.5.4) yield

$$u_p^2\, YW + u_p(r_1 G^3 + u_p^*(WZ + XY)] + u_p^{*2}\, XZ = 0 \qquad (3.5.10)$$

where

$$W = (B^2E + A^2E) + (BD-AF)\, I + \sin^3\theta_p(A^2F-B^2F + 2ABD)$$
$$- \cos^3\theta_p(2AFB + B^2D - A^2D) - \sin\theta_p(2A^2F + ABD - AEI)$$
$$+ \cos\theta_p\ [BEI + 2B^2D + ABF]$$

$$X = [(AF\sin^2\alpha - CE)\cos\theta_p + (AD\sin^2\alpha - AE\cos^2\alpha)\sin\theta_p$$
$$+ (AF\cos^2\alpha - DC)]\ (B\cos\theta_p + A\sin\theta_p + I)$$

$$Y = D\,tg\alpha\cos\theta_p - F\,tg\alpha\sin\theta_p - E\,ctg\alpha$$

$$Z = \cos\mu\cos\psi_n$$

$$U_p^* = \left(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n \cos\mu}\right)$$

$$A = \cos\mu\cos\beta_p + \sin\psi_n \sin\mu\sin\beta_p$$

$$B = \cos\alpha\cos\psi_n \sin\beta_p + \sin\alpha\sin\psi_n \cos\mu\sin\beta_p - \sin\alpha\sin\mu\cos\beta_p$$

$$C = \cos\alpha\cos\psi_n \sin\beta_p$$

$$D = (\cos\alpha\sin\psi_n - \sin\alpha\cos\psi_n \cos\mu)\cos\alpha$$

$$E = (\sin\alpha\sin\psi_n + \cos\alpha\cos\psi_n \cos\mu)\sin\alpha$$

$$F = \cos\alpha\cos\psi_n \sin\mu$$

$$G = D\cos\theta + E - F\sin\theta$$

$$I = \operatorname{ctg}\alpha(\cos\alpha\sin\psi_n \cos\mu\sin\beta_p - \sin\alpha\cos\psi_n \sin\beta_p - \cos\alpha\sin\mu\cos\beta_p)$$

Using Eq. (3.5.10) and taking it into account that $\frac{\partial F}{\partial \theta_p} \neq 0$, we can represented the u_p as a function of θ_p. Undercutting will be avoided if

$$u_p(\theta_p) > \frac{d}{\cos\alpha} \tag{3.5.11}$$

The analysis of Eq. (3.5.10) indicates that the inequality (3.5.11) is satisfied if the condition of helical pinion non-undercutting by a regular rack cutter is satisfied.

3.6 Principal Directions and Curvatures of Tooth Surface

A simplified approach to determine principal directions and curvatures of helical pinion has been proposed by Litvin (Litvin, 1968). The main idea is representing the principal directions and curvatures of generated surface Σ_1 by the principal directions and curvatures of generating surface Σ_p.

Let us determine the principal directions of curvatures of tool surface Σ_p. The tool surface is a cone surface and its principal directions coincide with the direction of the cone generatrix and the direction that is perpendicular to the cone generatrix.

The Rodrigue's formula (Eq. 3.6.1) can be used to obtain the principal curvature and directions

$$\kappa_{I,II}\, \underset{\sim}{V}_r = -\dot{\underset{\sim}{n}}_r \tag{3.6.1}$$

where $\underset{\sim}{V}_r$ is the velocity of a point that moves over a surface and $\dot{\underset{\sim}{n}}_r$ is the derivative of the surface unit normal $\underset{\sim}{n}$, when $\underset{\sim}{n}$ changes its direction due to the motion over the surface.

Using Eq. 3.6.1, the principal directions and curvatures of cone surface Σ_p can be expressed in coordinate system S_d as:

$$e_I^{(P)} = \frac{\partial r_d}{\partial \theta_p} \div \left|\frac{\partial r_d}{\partial \theta_p}\right| = \begin{bmatrix} -\sin\theta \\ 0 \\ \cos\theta \end{bmatrix}, \quad \kappa_I^{(P)} = -\frac{1}{u_p \cos\alpha}$$

$$e_{II}^{(P)} = \frac{\partial r_d}{\partial u_p} \div \left|\frac{\partial r_d}{\partial u_p}\right| = \begin{bmatrix} \cos\theta\sin\alpha \\ -\cos\alpha \\ \sin\theta\sin\alpha \end{bmatrix}, \quad \kappa_{II}^{(P)} = 0 \qquad (3.6.2)$$

The negative sign for κ_I indicates that the curvature center is located on the negative direction of the surface normal. The principal curvatures are invariants with respect to the used coordinate system. Whereas the principal direction will be represented in coordinate system S_f as

$$[e_{I,II}^{(P)}]_f = [L_{fc}][L_{cd}][e_{I,II}^{(P)}]_d \qquad (3.6.3)$$

where $[L_{cd}] = [L_{ca}][L_{ab}][L_{ab'}][L_{b'd}]$, 3 x 3 L matrices can be obtained from corresponding 4 x 4 M matrices [see Eq. (3.3.4), (3.3.5), (3.3.6) and (3.3.7)], L_{fc} is 3 x 3 unitary matrix (see Fig. 3.5).

The determination of principal curvatures and directions for the pinion tooth is based on the following equations (see Litvin, 1968)

$$\tan 2\sigma^{(p1)} = \frac{2b_{13}b_{23}}{b_{23}^2 - b_{13}^2 - (\kappa_I^{(p)} - \kappa_{II}^{(p)})b_{33}} \qquad (3.6.4)$$

$$\kappa_{II}^{(1)} - \kappa_I^{(1)} = \frac{b_{23}^2 - b_{13}^2 - (\kappa_I^{(p)} - \kappa_{II}^{(p)})b_{33}}{b_{33}\cos 2\sigma^{(p1)}} \qquad (3.6.5)$$

$$\kappa_{II}^{(1)} + \kappa_{I}^{(1)} = \kappa_{I}^{(p)} + \kappa_{II}^{(p)} + \frac{b_{13}^{2} + b_{23}^{2}}{b_{33}} \qquad (3.6.6)$$

where $\kappa_I^{(p)}$, $\kappa_{II}^{(p)}$ and $\mathbf{e}_I^{(p)}$, $\mathbf{e}_{II}^{(p)}$ are the principal curvatures and unit vector of principal directions of Σ_p. $\kappa_I^{(1)}$, $\kappa_{II}^{(2)}$ and $\mathbf{e}_I^{(1)}$, $e_{II}^{(1)}$ are the principal curvatures and unit vectors of principal directions of Σ_p. Angle $\sigma^{(p1)}$ is measured counter clockwise from $\mathbf{e}_I^{(p)}$ to $\mathbf{e}_I^{(1)}$ (Fig. 3.10). The coefficient b_{13}, b_{23} and b_{33} have been derived as shown in (Litvin 1968) but modified for the case when a rack cutter generates a gear. The expressions for b_{13}, b_{23}, and b_{33} are as follows:

$$\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = \begin{bmatrix} e_{II}^{(p)} \cdot \boldsymbol{\omega}_p \\ -\mathbf{e}_I^{(p)} \cdot \boldsymbol{\omega}_p \end{bmatrix} - \begin{bmatrix} -\kappa_I^{(p)} & 0 \\ 0 & -\kappa_{II}^{(p)} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{(p1)} & \mathbf{e}_I^{(p)} \\ \mathbf{v}^{(p1)} \cdot & \mathbf{e}_I^{(p)} \end{bmatrix} \qquad (3.6.7)$$

$$b_{33} = [\mathbf{n} \;\; \boldsymbol{\omega}_p \;\; \mathbf{v}_{tr}^{(p)}] + [\mathbf{n} \;\; \boldsymbol{\omega}_p \;\; \mathbf{v}^{(p1)}] - \kappa_I^{(p)}(\mathbf{v}^{(p1)} \cdot e_I^{(p)})^2$$
$$- \kappa_{II}^{(p)}(\mathbf{v}^{(p1)}\mathbf{e}_{II}^{(p)})^2 - \frac{\omega^2}{m_{1p}^2} m'_{1p} \; [\mathbf{n} \;\; \mathbf{r}^{(p1)} \mathbf{k}_f] \qquad (3.6.8)$$

where $m_{1p} = \dfrac{d\phi_p}{ds}$, $m'_{1p} = \dfrac{dm_{1p}}{ds}$.

The vectors used in Eq. (3.6.7) and (3.6.8) are represent in coordinate system S_f (Fig. 3.5) with $\mathbf{i}_f$, $\mathbf{i}_f$, $\mathbf{k}_f$ as its unit vectors of axes. The expressions of the vectors in Eq. (3.6.7) and (3.6.8) are as follows (Fig. 3.5)

$$\underset{\sim}{\omega}_p = \omega \underset{\sim}{k}_f \qquad (3.6.9)$$

is the angular velocity of the pinion being in meshing with the rack cutter.

$$V_{tr}^{(p)} = -\omega r_1 \underset{\sim}{i}_f \qquad (3.6.10)$$

is the transfer velocity of a point on the rack cutter that performs translational motion, r_1 is the radius of pinion centrode.

$$\underset{\sim}{V}_{tr}^{(1)} = \underset{\sim}{\omega}_p \times \underset{\sim}{r}_f^{(p)} = \omega \begin{bmatrix} -y_f \\ x_f \\ 0 \end{bmatrix} = \omega \begin{bmatrix} -(y_c+r_1) \\ x_c-r_1\phi_p \\ 0 \end{bmatrix} \qquad (3.6.11)$$

is the transfer velocity of the pinion

$$\underset{\sim}{V}^{(p1)} = \underset{\sim}{V}^{(p)} - \underset{\sim}{V}^{(1)} = \omega \begin{bmatrix} y_c \\ -x_c+r_1\phi_p \\ 0 \end{bmatrix} \qquad (3.6.12)$$

is the "sliding" velocity - the velocity of a point of rack cutter with respect to the same point of pinion.

Substituting Eq. (3.6.9), (3.6.10), (3.6.11), (3.6.12), (3.6.2) and (3.3.9) into Eq. (3.6.7) and (3.6.8), then substituting Eq. (3.6.7) and (3.6.8) into (3.6.4), (3.6.5) and

(3.6.6), finally we can obtain the principal directions and curvatures from Eq. (3.6.4), (3.6.5), (3.6.6) and (3.6.2). Since the expressions of tool surface and its principal direction are complicated and tedious. It is difficult for us to write down the expression of $\kappa_{II}^{(1)}$, $\kappa_{II}^{(1)}$, $\underset{\sim}{e}_{I}^{(1)}$ and $\underset{\sim}{e}_{II}^{(2)}$. However, a corresponding computer program is developed for determining the principal directions and curvatures of any point of the pinion surface using the algorithm discussed above.

The same approach can be used to determine the principal directions and curvatures of regular helical gear. However, in this case, the generating surface is a plane, and the problem becomes simpler.

3.7 Contact Ellipse and Bearing Contact

The dimensions and orientation of the instantaneous contact ellipse can be determined based on the equations proposed by Litvin. (Litvin 1968). The input data for computation is: $\kappa_{I,II}^{(i)}$ $(i = 1,2)$, $e_{I}^{(1)}$, $\sigma^{(12)}$ and ε, where $\kappa_{I,II}^{(i)}$ are the principal curvature of pinion tooth surface Σ_1 and gear tooth surface Σ_2. $e_{I}^{(1)}$ is the unit vector of the first principal direction of Σ_1. $\sigma^{(12)}$ is the angle formed by the unit vectors of principal directions $\underset{\sim}{e}_{I}^{(1)}$ and $\underset{\sim}{e}_{II}^{(2)}$ (Fig. 3.11) and ε is the elastic approach of the contacting surfaces. The bearing contact is formed by a set of instantaneous contact ellipses that move over the gear tooth surface in the process of meshing.

The axes of the contact ellipse are directed along the η axis and ξ axis, respectively. The orientation of contact ellipse is determined by the angle α, which is angle between η and $e_I^{(1)}$. (Fig. 3.11) The dimensions of ellipse are determined by a* and b* which are the half lengths of major and minor axis of the ellipse respectively. The following equations are used to determine α, a* and b*.

$$
\begin{aligned}
A &= \frac{1}{4}\,[\kappa_\varepsilon^{(1)} - \kappa_\varepsilon^{(2)} - |g_1 - g_2|] \\
B &= \frac{1}{4}\,[\kappa_\varepsilon^{(1)} - \kappa_\varepsilon^{(2)} + |g_1 + g_2|] \\
\text{w}\tan(2\alpha) &= \frac{g_1 \sin 2\sigma^{(12)}}{g_1 - g_2 \cos 2\sigma^{(12)}}
\end{aligned}
\tag{3.7.1}
$$

where

$$
\kappa_\varepsilon^{(1)} = \kappa_I^{(1)} + \kappa_{II}^{(2)}, \quad \kappa_\varepsilon^{(2)} = \kappa_I^{(2)} + \kappa_{II}^{(2)}
$$

$$
g_1 = \kappa_I^{(1)} - \kappa_{II}^{(1)}, \quad g_2 = \kappa_I^{(2)} - \kappa_{II}^{(2)}
$$

and

$$
a^* = \left|\frac{\varepsilon}{A}\right|^{1/2} \quad b^* = \left|\frac{\varepsilon}{B}\right|^{1/2} \tag{3.7.2}
$$

3.8 Simulation of Meshing and Determination of Transmission Errors

The simulation of meshing is a part of the computer aided tooth contact analysis (TCA) program. The simulation of meshing is based on equations that provide the continuous tangency of contacting surfaces.

To simulate the meshing of a crowned helical pinion and regular involute helical gear with misaligned axis, we will use the following coordinate system, as shown in Fig. 3.11 and Fig. 3.12: (i) S_f is rigidly connected to the frame (ii) an auxiliary coordinate system S_h that is also rigidly connected to the frame (iii) S_1 and S_2 are rigidly connected to the pinion and the gear respectively. The relations between S_1 and S_f and between S_2 and S_h are shown in Fig. 3.12 and expressed by M_{f1} and M_{h2} as

$$M_{f1} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.8.1)$$

$$M_{h2} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 & 0 \\ \sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.8.2)$$

It is obvious that the rotation axis of the pinion and gear are $Z_f(Z_1)$ and $Z_h(Z_2)$ respectively. Therefore, the error of assembly of gears can be simulated with orientation and location of coordinate system S_h with respect to S_f. Figure 3.13 shows the orientation of S_h and S_f. When the pinion and gear axes are crossed (Fig. 3.13a) and intersected (Fig 3.13b) with operating center distance $C = O_fO_h$. It must be emphasized that C can be different from the nominal center distance given by $C^o = r_1 + r_2$, where r_1 and r_2 are the radii of the pinion and gear pitch circles. The coordinate transformation from S_h to S_f is represented by $[M_{fh}]$ as follows (Fig. 3.13)

$$[M_{fh}] = \begin{bmatrix} -\cos\Delta\gamma & 0 & \sin\Delta\gamma & 0 \\ 0 & -1 & 0 & C \\ \sin\Delta\gamma & 0 & \cos\Delta\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.8.3)$$

where the gear axes are crossed as shown in Fig. 3.13a.

$$[M_{fh}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -\cos\Delta_f & -\sin\Delta_f & C \\ 0 & -\sin\Delta_f & \cos\Delta_f & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.8.4)$$

where the gear axes are intersected as shown in Fig. 3.13b.

The helical pinion tooth surface and its normal can be expressed in coordinate system S_f as:

$$[r_f^{(1)}] = [M_{f1}][r_1] \qquad (3.8.5)$$
$$[n_f^{(1)}] = [L_{f1}][n_1]$$

where L_{f1} is 3 x 3 matrix from M_{f1}. The helical gear tooth surface and its normal can be expressed in coordinate system S_f as

$$[r_f^{(2)}] = [M_{fh}][M_{h2}][r_2] \qquad (3.8.6)$$
$$[n_f^{(2)}] = [L_{fh}][L_{h2}][r_2]$$

where L_{fh} and L_{h2} is 3 x 3 matrices from M_{fh} and M_{h2}.

For a helical gear (lefthand), $[r_2]$ and $[n_2]$ can be represented as:

$$[r_2] = \begin{bmatrix} -\dfrac{\cos^2\psi_n}{\cos\psi_c}\cos(\phi_G-\psi_n)[-t_G\sin\beta_p+r_2\phi_G] + r_2\sin\phi_G \\ \dfrac{\cos^2\psi_c}{\cos\psi_c}\sin(\phi_G-\psi_c)[-t_G\sin\beta_p+r_2\phi_G] + r_2\cos\phi_G \\ t_G(\cos\beta_p+\sin^2\psi_n\dfrac{\sin^2\beta_p}{\cos\beta_p})-r_2\phi_G\sin^2\psi_n tg\beta_p \end{bmatrix} \tag{3.8.7}$$

$$[n_2] = \begin{bmatrix} -(\cos\psi_n\cos\beta_p\cos\phi_G+\sin\psi_n\sin\phi_G) \\ \cos\psi_n\cos\beta_p\sin\phi_G-\sin\psi_n\cos\phi_G \\ \cos\psi_n\sin\beta_p \end{bmatrix}$$

where t_G and ϕ_G are the surface parameters and ψ_c = arc $tg(tg\psi_n/\cos\beta_p)$ is the pressure angle in the transverse section of the gear tooth.

From Eq. (3.3.9), (3.4.4), (3.4.8), (3.8.5), (3.8.6) and (3.8.7), it is known that. [substituting Eq.(3.4.4) into(3.3.9) to eliminate u_p]:

$$\begin{aligned} \underset{\sim}{r}_f^{(1)} &= \underset{\sim}{r}_f^{(1)}(\phi_p,\theta_p,\phi_1) \\ \underset{\sim}{n}_f^{(1)} &= \underset{\sim}{n}_f^{(1)}(\phi_p,\theta_p,\phi_1) \\ \underset{\sim}{r}_f^{(2)} &= \underset{\sim}{r}_f^{(2)}(\phi_G,t_G,\phi_2) \\ \underset{\sim}{n}_f^{(2)} &= \underset{\sim}{n}_f^{(2)}(\phi_G,t_G,\phi_2) \end{aligned} \tag{3.8.8}$$

The contact of gear tooth surface is simulated in the TCA program by the following equations

$$\underset{\sim}{r}_f^{(1)}(\phi_p,\theta_p,\phi_1) = \underset{\sim}{r}_f^{(2)}(\phi_G,t_G,\phi_2) \tag{3.8.9}$$

$$\underset{\sim}{n}_f^{(1)}(\phi_p,\theta_p,\phi_1) = \underset{\sim}{n}_f^{(2)} = (\phi_G,t_G,\phi_2) \tag{3.8.10}$$

Vector equation (3.8.9) comes from the equality of the position vectors at the contact point and provide three independent equations whereas vector equation (3.8.10) comes from the equality of surface unit normals and provide only two independent equations. Therefore, Eq.(3.8.9) and (3.8.10) yield five independent scalar equations as follows:

$$f_i(\phi_1,\ \theta_p,\phi_p,\phi_2,t_G,\phi_G) = 0 \quad (i = 1,2,\ldots,5) \tag{3.8.11}$$

Equation system 3.8.11 is the expression in implicit form of functions of one variable, i.e. ϕ_1. Using theorem of implicit function, we can obtain

$$\phi_2 = \phi_2(\phi_1) \tag{3.8.12}$$

considering at the neighborhood of $P^o = (\phi_1,\theta_p,\phi_p,\phi_2,t_G,\phi_G)$ where P^o satisfy Eq.(3.8.11) and

$$J = \frac{D(f_1,f_2,f_3,f_4,f_5)}{D(\phi_p,\theta_p,\phi_2,t_G,\phi_G)} \neq 0 \tag{3.8.13}$$

The function of transmission error can be determined by

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (3.8.14)$$

3.9 Modification of Generating Surface Σ_p

Using cone as a tool surface to generate the helical pinion is good way to localize bearing contact. By using the TCA program, it can be shown that the transmission errors caused by gear misalignment are on a very low level. However, the shape of the function of transmission errors is unfavorable as shown in Fig. 3.14(a). This shape of the function of transmission errors will result in interruption and interference with change of meshing gear teeth. A more favorable shape of the function of transmission errors is shown in Fig. 3.14(b). To obtain this shape of transmission error function, we need to modify generating surface Σ_p into a revolute surface. The reason for using revolute surface is similar to the case for crowning a spur pinion (see Litvin and Zhang, 1987).

The surface of revolution is generated by an arc circle with radius ρ. The arc has a common normal with the cone generatrix line at the point M (Fig. 3.15(a)). The circular arc and its normal are expressed in an auxiliary coordinate system S_e as follows

$$X_e = \rho[\cos(\alpha + \beta) - \cos\alpha] + \left(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n \cos\mu}\right)\sin\alpha$$

$$= \left(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n \cos\mu}\right)\sin\alpha - 2\rho\sin\frac{\beta}{2}\sin\left(\alpha + \frac{\beta}{2}\right)$$

$$Y_e = \rho[\sin(\alpha+\beta) - \sin\alpha] + \frac{b}{\cos\psi_n \cos\mu}\cos\alpha \qquad (3.9.1)$$

$$= \frac{b}{\cos\psi_n \cos\mu} \cos\alpha + 2\rho \sin\frac{\beta}{2} \cos(\alpha + \frac{\beta}{2})$$

$$z_e = 0$$

$$[n_e] = \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \\ 0 \end{bmatrix} \qquad (3.9.2)$$

The surface of revolution is generated by rotation of the circular arc about the y_e axis and can be represented in coordinate system S_d as follows

$$[r_d] = [L_{de}][r_e] \;, \quad [n_d] = [L_{de}][r_e] \qquad (3.9.3)$$

where (Fig. 9.2(b))

$$[L_{de}] = \begin{bmatrix} \cos\theta_p & 0 & \sin\theta_p \\ 0 & 1 & 0 \\ -\sin\theta_p & 0 & \cos\theta_p \end{bmatrix}$$

Then we obtain

$$x_d = [(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n \cos\mu})\sin\alpha - 2\rho\sin\frac{\beta}{2}\sin(\alpha + \frac{\beta}{2})]\cos\theta_p$$

$$y_d = \frac{b}{\cos\psi_n \cos\mu} + 2\rho\sin\frac{\beta}{2}\cos(\alpha + \frac{\beta}{2})$$

$$z_d = [(\frac{d}{\cos\alpha} - \frac{b}{\cos\psi_n \cos\mu})\sin\alpha - 2\rho\sin\frac{\beta}{2}\sin(\alpha + \frac{\beta}{2})]\sin\theta_p$$

$$[n_d] = \begin{bmatrix} \cos\theta_p \cos(\alpha + \beta) \\ \sin(\alpha + \beta) \\ \sin\theta_p \cos(\alpha + \beta) \end{bmatrix} \qquad (3.9.4)$$

The installment of the generating surface in coordinate system is the same as described in Section 3.3. The generated crowning pinion tooth surface can be obtained using the same approach as that described in section 3.4.

The meshing of gears using the crowning method described in this section has been simulated by numerical methods. The results of the investigation are illustrated with the following example.

Given: number of pinion teeth $N_1 = 20$, number of gear teeth $N_2 = 40$, diametral pitch in normal section $P_n = 10\ in^{-1}$, pressure angle in normal section $\psi_n = 20^0$, helical angle $\beta = 15°$. The pinion tooth is crowned by revolute surface with generatrix arc $\rho = 30$ in. The revolute surface is deviated from a cone (comparing Σ_p in figure 3.2(b) and (c)). The cone has half apex angle $\alpha = 20^0$ and bottom radius $R = 10.6$ in.

The topology of the pinion tooth surface provides a parabolic type of predesigned transmission errors with $d = 6$ arc seconds (Fig. 2.3 (a)) and a path contact that is directed across the tooth surface (Fig. 3.1).

The influence of gear misalignment has been investigated with the developed computer program and the results of computation are represented in table 3.9.1 and 3.9.2 for crossed and intersected gear axes, respectively. The misalignment of gear axes is 5 arc minutes.

The results of computation show that the resulting function of transmission errors is a parabolic one. Thus the linear function of transmission errors that was caused by gear misalignment has been absorbed by the predesigned parabolic function. Fig. 3.14 show the results of transmissions errors for crossed helical gears with (Table 3.9.1) and without (Table 3.4.1) crowning of the pinion.

TABLE 3.9.1 - TRANSMISSION ERRORS OF CROSSED HELICAL GEARS

ϕ_1 (deg)	-14	-11	-8	-5	-2	1	4
$\Delta\phi_2$ (sec)	-4.99	-1.51	0.65	1.51	1.05	-0.75	-3.87

TABLE 3.9.2 - TRANSMISSION ERRORS OF INTERSECTED HELICAL GEARS

ϕ_1 (deg)	-11	-8	-5	-2	1	4	7
$\Delta\phi_2$ (sec)	-6.15	-2.72	-0.60	0.20	-0.32	-21.9	-5.40

4. CROWNED HELICAL PINION WITH LONGITUDINAL PATH CONTACT

4.1 Basic Concepts and Considerations

A longitudinal path of contact means that the gear tooth surfaces are in contact at a point at every instant and the instantaneous contact ellipse moves along but not across the surface (Fig. 4.1(a)). It can be expected that this type of contact provides improved conditions of lubrication. Until now only the Novikov-Wildhaber's gears could provide a longitudinal path of contact. A disadvantage of this type of gearing is their sensitivity to the change of the center distance and the axes misalignment. The sensitivity to non-ideal orientation of the meshing gears cause a higher level of gear noise in comparison with regular involute helical gears. Litvin et al. (Litvin, 1985) proposed a compromising type of non-conformal helical gears that may be placed between regular helical gears and Novikov-Wildhaber helical gears. The gears of the proposed gear train are the combination of a regular involute helical gear and a specially crowned helical pinion. The investigation of transmission errors for helical gears with a longitudinal path of contact shows that their good bearing contact is accompanied with an undesirable increased level of linear transmission errors. The authors propose to compensate this disadvantage by a predesigned parabolic function of transmission errors, that will absorb the linear function of transmission errors (see section 2.3). The two following methods for derivation of the pinion tooth surface will the modified topology will now be considered.

4.2 Method 1

Consider that two rigidly connected generating surfaces, Σ_G and Σ_p, are used for the generation of the gear and the pinion, respectively (Fig. 4.1(b)). Surface Σ_G is a plane and represents the surface of a regular rack-cutter; surface Σ_p is a cylindrical surface whose cross-section is a circular arc. We may imagine that while surfaces Σ_G and Σ_p

translate, as the pinion and the gear rotate about their axes. To provide the predesigned parabolic function of transmission errors it is necessary to observe the following transmission functions by generation

$$\frac{V}{\omega_G} = r_2 = \text{const}, \quad \frac{V}{\omega_p} = r_2 \left(\frac{N_1}{N_2} - 2a\phi_p\right) = f(\phi_p) \qquad (4.2.1)$$

Here: ω_G and ω_p are the angular velocities of pinion and gear by cutting; V is the velocity of the rack-cutter in translational motion; N_1 and N_2 are the gear and pinion tooth numbers; ϕ_p is the angle of rotation of the pinion by cutting. The generated gears will be in point contact at every instant and transform rotation with the function

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 - a\phi_1^2 \qquad 0 < \phi_1 < \frac{2\pi}{N_1} \qquad (4.2.2)$$

This function relates the angles of rotation of the pinion and the gear, ϕ_1 and ϕ_2, respectively, for one cycle of meshing. The predesigned function of transmission errors is

$$\Delta\phi_2 = -a\phi_1^2 \qquad (4.2.3)$$

It is evident that after differentiation of function (4.2.2) we obtain that the gear ratio ω_2/ω_1 satisfies equation (4.2.1), if ϕ_1 and ϕ_2 are used instead of ϕ_p and ϕ_G.

To apply this method of generation in practice it is necessary to vary the angular velocity of the pinion in the process of its generation that may be accomplished by a computer controlled machine for cutting.

It is obvious that in this method the gear tooth surface is kept as regular skew involute surface as shown in Eq.(2.4.1) and (2.4.2) since its generating surface Σ_G is a plane and generating motion is $V = \omega_G\ r_2$. Now, let us derive the pinion tooth surface equations.

The pinion generating surface Σ_p is a cylindrical surface with circular arc as its cross section and can be represented in an auxiliary coordinate system S_a as follows: (see Fig. 4.2)

$$\underset{\sim}{r}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix} = \begin{bmatrix} R[\cos(\psi_n+\alpha) - \cos\psi_n] \\ R[\sin(\psi_n+\alpha) - \sin\psi_n] \\ t_p \\ 1 \end{bmatrix} \tag{4.2.4}$$

The unit surface normal is represented by

$$\underset{\sim}{n}_a = \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix} = \begin{bmatrix} \cos(\psi_n+\alpha) \\ \sin(\psi_n+\alpha) \\ 0 \end{bmatrix} \tag{4.2.5}$$

From coordinate system S_a to S_c the helical angle β_p is introduced. Based on Fig. 3.8(d) and $[M_{ca}]$ as shown in Eq. (3.3.7), we have

$$\underset{\sim}{r}_c^{(p)} = \begin{bmatrix} x_c^{(p)} \\ y_c^{(p)} \\ z_c^{(p)} \\ 1 \end{bmatrix} = [M_{ca}]\underset{\sim}{r}_a = \begin{bmatrix} R[\cos(\psi_n+\alpha) - \cos\psi_n] - t_p\sin\beta_p \\ R[\sin(\psi_n+\alpha) - \sin\psi_n] \\ R[\cos(\psi_n+\alpha) - \cos\psi_n] + t_p\sin\beta_p \\ 1 \end{bmatrix} \tag{4.2.6}$$

$$\underset{\sim}{n}_c^{(p)} = \begin{bmatrix} n_{cx}^{(p)} \\ n_{cy}^{(p)} \\ n_{cz}^{(p)} \end{bmatrix} = [L_{ca}]\underset{\sim}{r}_a = \begin{bmatrix} \cos(\psi_n+\alpha)\cos\beta_p \\ \sin(\psi_n+\alpha) \\ \cos(\psi_n+\alpha)\sin\beta_p \end{bmatrix} \tag{4.2.7}$$

The generating process is described in the Fig. 3.5 where

$$S = r_2\phi_p\left(\frac{N_1}{N_2} - a\phi_p\right) \tag{4.2.8}$$

The equation of meshing of the generating surface Σ_p and the helical pinion tooth surface is given by:

$$\underset{\sim}{n}^{(p)} \cdot \underset{\sim}{v}^{(p1)} = \underset{\sim}{n}^{(p)} \cdot (\underset{\sim}{v}^{(p)} - \underset{\sim}{v}^{(1)}) = 0 \tag{4.2.9}$$

In the system shown in Fig. 3.5, it can be found that

$$\underset{\sim}{v}^{(p)} = \begin{bmatrix} -\frac{ds}{dt} \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{v}^{(1)} = \begin{bmatrix} -r_1-y_c^{(p)} \\ -S + x_c^{(p)} \\ 0 \end{bmatrix} \omega_p \tag{4.2.10}$$

Taking into account that $\frac{d\phi_p}{dt} = \omega_p$ and from Eq.(4.2.1) we have

$$\underset{\sim}{v}^{(p1)} = \underset{\sim}{v}^{(p)} - \underset{\sim}{v}^{(1)} = \omega_p \begin{bmatrix} -r_2\left(\frac{N_1}{N_2} - 2a\phi_p\right) + r_1+y_c^{(p)} \\ -x_c^{(p)}+r_2\phi_p\left(\frac{N_1}{N_2} - a\phi_p\right) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} y_c^{(p)} + 2r_2 a\phi_p \\ -x_c^{(p)} + r_2\phi_p(\frac{N_1}{N_2} - a\phi_p) \\ 0 \end{bmatrix} \omega_p \qquad (4.2.11)$$

Substituting Eq.(4.2.6), (4.2.7) and (4.2.11) into Eq. (4.2.9). the equation of meshing can be obtained as:

$$\cos(\psi_n+\alpha)\cos\beta_p\{ R[\sin(\psi_n+\alpha)-\sin\psi_n] + 2r_2 a\phi_p\}$$

$$+ \sin(\psi_n+\alpha)\{ -R[\cos(\psi_n+\alpha)-\cos\psi_n] - t\sin\beta_p \} + r_2\phi_p(\frac{N_1}{N_2} - a\phi_p)$$
$$= 0$$

(4.2.12)

Equation of meshing gives the relation among three variables, that is, t_p, α and ϕ_p. It is obvious that from Eq. 4.2.12, the equation of meshing can be rewritten as:

$$t_p = \mathrm{cotan}(\psi_n+\alpha)\mathrm{cotan}(\beta_p) \{ R\ [\sin(\psi_n+\alpha)-\sin\psi_n]+2r_2 a\phi_p\}$$

$$+ \frac{1}{\sin\beta_p} \{-R[\cos(\psi_n+\alpha)-\cos\psi_n] + r_2\phi_p(\frac{N_1}{N_2} - a\phi_p)\}$$

(4.2.13)

The pinion tooth surface can be determined with the same approach as that described in the section 3.4 and represented by Eq.(3.4.8) and (3.4.9) together with Eq.(4.2.6),(4.2.7) and (4.2.13).

4.3 Method 2

The derivation of the crowned pinion tooth surface is based on two stages of synthesis. On the first stage it is assumed the pinion tooth surface Σ_1 is exact conjugate surface to the gear tooth surface which is regular skew involute surface under the condition that the rotation transformed by the pinion and the gear is described in Eq.(4.2.2) with predesigned parabolic function of transmission errors.

On the second stage of synthesis it is necessary to localize the bearing contact and substitute the instantaneous line contact by a point contact. This becomes possible if the pinion tooth surface will be deviated as it is shown in Fig. 4.1(c). Only a narrow strip, L, will be kept while Σ_1 will be changed into Σ_1^1. The deviation of Σ_1^1 with respect to Σ_1 may be accomplished in various ways, for instance, in such a way, that the cross-section of Σ_1^1 is only a circular arc. The generation of Σ_1^1 requires a computer controlled machine to relate the motions of the tool surface and the being generated pinion surface Σ_1^1. The tool surface (it may be only a plane) and Σ_1^1 will be in point contact in the process of generation.

Now, let us derive the pinion tooth surface equations based on the two stages discussed above. First, we can derive the intermediate pinion tooth surface Σ_1 as that generated by gear tooth surface Σ_1. The relation of rotation for the pinion and the gear is shown in Eq.(4.2.2). The gear tooth surface Σ_2 and its unit normal are shown in Eq.(2.4.3) and (2.4.4).

To represent the gear tooth surface Σ_2 in the fixed coordinate system S_f in Fig. 4.3, it is necessary to transform $[r_2]$ and $[n_2]$ into $[r_f^{(2)}]$ and $[n_f^{(2)}]$ as:

$$[r_f^{(2)}] = [M_{f2}][r_2]$$

$$[n_f^{(2)}] = [L_{f2}][n_2] \qquad (4.3.1)$$

where $$[M_{f2}] = \begin{bmatrix} -\cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & -\cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & C \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting Eq.(2.4.3) and(2.4.4) into Eq.(4.3.1) we obtain:

$$[r_f^{(2)}] = \begin{bmatrix} x_f^{(2)} \\ y_f^{(2)} \\ z_f^{(2)} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\cos^2\psi_n}{\cos\psi_c} \cos(\phi_G-\psi_c-\phi_2)[-t_G\sin\beta_p+r_2\phi_G]-r_2\sin(\phi_G-\phi_2) \\ C - \frac{\mathrm{Cos}^2\psi_n}{\mathrm{Cos}\ \psi_c} \mathrm{Cos}(\phi_G-\psi_C\ -\phi_2)[-t_G\sin\beta_p+r_2\phi_G]-r_2\sin(\phi_G-\phi_2) \\ t_G^2\cos\beta_p^2(1+\sin\ \psi_n+g^2\beta_p)-r_2\phi_G\sin\ \psi_n tg\beta_p \\ 1 \end{bmatrix} \qquad (4.3.2)$$

$$[n_f^{(2)}] = \begin{bmatrix} n_{fx}^{(2)} \\ n_{fy}^{(2)} \\ n_{fz}^{(2)} \end{bmatrix} = \begin{bmatrix} \cos\psi_n\cos\beta_p\cos(\phi_G-\phi_2)+\sin\psi_n\sin(\phi_G-\phi_2) \\ -\cos\psi_n\cos\beta_p\sin(\phi_G-\phi_2)+\sin\psi_n\cos(\phi_G-\phi_2) \\ \cos\psi_n\sin\beta_p \end{bmatrix} \qquad (4.3.3)$$

The generating process is shown in Fig. 4.3 where ϕ_1 and ϕ_2 is given by Eq. (4.2.2). The equation of meshing of the generating surface Σ_2 and generated surface Σ_1 is described as:

$$\underset{\sim}{N}^{(2)} \cdot \underset{\sim}{V}^{(21)} = \underset{\sim}{N}^{(2)}(\underset{\sim}{V}^{(2)} - \underset{\sim}{V}^{(1)}) = 0 \tag{4.3.4}$$

The relative velocity $\underset{\sim}{V}^{(21)}$ can be expressed as

$$\underset{\sim}{V}^{(21)} = \underset{\sim}{V}^{(2)} - \underset{\sim}{V}^{(1)} = \underset{\sim}{\omega}_2 \times (\overline{O_2O_1} + \underset{\sim}{r}_f) - \underset{\sim}{\omega}_1 \times \underset{\sim}{r}_f \tag{4.3.5}$$

where

$$\underset{\sim}{\omega}^{(1)} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \omega_1; \quad \underset{\sim}{\omega}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_1 \left[\frac{N_1}{N_2} - 2a\phi_1\right]$$

$$\overline{O_2O_1} = \begin{bmatrix} 0 \\ -C \\ 0 \end{bmatrix} ; \qquad \underset{\sim}{r}_f = \begin{bmatrix} x_f^{(2)} \\ y_f^{(2)} \\ z_f^{(2)} \end{bmatrix} ;$$

and the relation between ω_1 and ω_2 is obtained by taking derivative of Eq. (4.2.2). By rearranging and simplifying Eq. (4.3.5), we obtain

$$\underset{\sim}{V}^{(21)} = \begin{bmatrix} -Y_f^{(2)} \\ X_f^{(2)} \\ 0 \end{bmatrix} \omega_1 \left[\frac{N_2}{N_1} + 1-2a\phi_1\right] + \begin{bmatrix} C \\ 0 \\ 0 \end{bmatrix} \omega_1 \left[\frac{N_1}{N_2} - 2a\phi_1\right] \tag{4.3.6}$$

Substituting Eq.(4.3.6) (4.3.2) and (4.3.3) into Eq.(4.3.4) and simplifying it, we can obtain:

$$\cos(\phi_G-\phi_2-\psi_c) = \cos\psi_c[1-\frac{2a\phi_1}{N_1/N_2}] \qquad (4.3.7)$$

where ψ_c is the pressure angle in transverse section of helical gear. Equation 4.3.7 is the equation of meshing which describes the relation between ϕ_G and ϕ_2. We can rearrange Eq. 4.3.7 as:

$$\phi_G = \phi_2 + \lambda(\phi_1) = \frac{N_1}{N_2}\phi_1-a\phi_1^2 + \lambda(\phi_1) \qquad (4.3.8)$$

where $\lambda(\phi_1) = \psi_c-\arccos \cos\psi_c[1-\frac{2a\phi_1}{1+N_1/N_2}]$

The pinion tooth surface Σ_1 and its normal can be determined by transforming $\underset{\sim}{r}_f^{(2)}$ into coordinate system S_1 as

$$[r_1] = [M_{1f}][\underset{\sim}{r}_f^{(2)}] \qquad (4.3.9)$$

$$[n_1] = [L_{1f}][n_f^{(2)}]$$

where $[M_{1f}] = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Substituting (4.3.2) (4.3.3) into Eq.(4.3.9), we can get

$$[r_1] = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

$$[n_1] = \begin{bmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{bmatrix} \qquad (4.3.10)$$

where

$$x_1 = \frac{\cos^2\psi_n}{\cos\psi_c} \cos(\phi_1+\psi_c-\lambda(\phi_1))[-t_G\sin\beta_p+r_2\phi_G]$$
$$+ r_2\sin(\phi_1-\lambda(\phi_1)) - C\sin\phi_1$$

$$y_1 = \frac{\cos^2\psi_n}{\cos\psi_c} \sin(\phi_1+\psi_c-\lambda(\phi_1)[-t_G\sin\beta_p+r_2\phi_G]$$
$$- r_2\cos(\phi_1-\lambda(\phi_1)) + C\cos\phi_1$$

$$Z_1 = t_G\cos\beta_p(1+\sin^2\psi_n+g^2\beta_p) - r_2\phi_G \sin^2\psi_n t_g\beta_p$$

$$n_{1x} = \frac{\cos\psi_n\cos\beta_p}{\cos\psi_c} \cos(\phi_1-\lambda(\phi_1)+\psi_c)$$

$$n_{1y} = \frac{\cos\psi_n\cos\beta_p}{\cos\psi_c} \sin(\phi_1-\lambda(\phi_1)+\psi_c)$$

$$n_{1z} = \cos\psi_n\sin\beta_p$$

Equations (4.3.10), (4.3.8) and (4.2.2) represent pinion tooth surface Σ_1 in coordinate system S_1. But Σ_1 is only an intermediate surface. Now we come to the second stage, that is, to deviate Σ_1 into Σ_1^1. The surface Σ_1^1 must satisfy such condition that when it is in mesh with gear tooth surface Σ_2 without misalignment, the contact path will be in longitudinal direction. The surface Σ_1^1 will be formed in two steps: (i) the contact path curve in Σ_1 is chosen and kept. (ii) the profile of surface Σ_1 in transverse section is replaced by a smaller circular arc attached on the L in such a way that the original normal of Σ_1 along L is kept.

To choose curve L on Σ_1 we could define that the contact path on the gear tooth surface is in the middle of the tooth, that is

$$r = r_2 \tag{4.3.11}$$

where $r = \sqrt{x_2^2 + y_2^2}$ is the distance of the surface point to gear axis in transverse section. Substituting Eq. (2.4.3) into Eq. (4.3.10) and simplyfing it, we can obtain the relation of surface parameter along the contact path as

$$r_2\phi_G - t_G\sin\beta_p = 0 \tag{4.3.12}$$

Applying Eq.(4.3.12) to Eq.(4.3.10), we can find the curve L on pinion tooth surface Σ_1 as:

$$x_1 = r_2\sin(\phi_1-\lambda(\phi_1)) - C\sin\phi_1$$

$$y_1 = -r_2\cos(\phi_1-\lambda(\phi_1)) + C\cos\phi_1$$

$$z_1 = r_2\mathrm{ctg}\beta_p(\frac{N_1}{N_2}\phi_1 - a\,\phi_1^2 + \lambda(\phi_1)) \tag{4.3.13}$$

where the ϕ_1 is the curve parameter and $\lambda(\phi_1)$ is described in Eq. (4.3.8).

Now we should attach the circular arc to the L described by Eq. (4.3.13) in the transverse section to form new surface Σ_1^1. Also the tangent of the arc at the point on the line L must be perpendicular to the normal of Σ_1 described in Eq. (4.3.10). As shown in Fig. 4.4, the equation of new surface Σ_1^1 are:

$$x_1' = x_1 + r[\cos(\mu+\alpha)-\cos\mu]$$

$$y_1' = y_1 + R[\sin(\mu+\alpha)-\sin\mu] \qquad (4.3.14)$$

$$z_1' = z_1$$

where $\mu = \phi_1 - \lambda(\phi_1)+\psi_c$. Equation (4.3.14) describes the new pinion surface Σ_1'. In Eq. (4.3.14) when $\alpha = 0$, the designed contact path L is obtained.

4.4 Discussion and Example

In section 4.2 and 4.3, two methods have been presented with derivation of the equations of pinion tooth surface. Comparing the two methods for the generation of the pinion tooth surface, it may be concluded that both provide a localized bearing contact, a longitudinal path of contact and predesigned parabolic function of transmission errors. The difference between these methods is that the tool and pinion tooth surfaces are in line contact by applying the first method for generation and in point contact by the second one. The disadvantage of both methods for crowning of the pinion is that the transmission errors caused by gear misalignment are large and it is necessary to envision a high level of the predesigned parabolic function for the absorption of transmission errors. This is illustrated with the following example (the algorithms for simulation have been discussed in Section 3.8).

Given (the data is from ref. 3): pinion tooth number N_1 = 12, gear tooth number N_2 = 94; diametral pitch in normal section P_n = 2 in^{-1}; pressure angle in normal section ψ_n = 30^0; helical angle β = 15^0.

The pinion tooth surface is a crowned surface whose cross-section is an arc of a circle of the radius 0.3584in. The predesigned parabolic function is of the level d = 25 arc seconds (Fig. 2.3(a)).

Consider now that the axes of the gear and the pinion are crossed and the crossing angle is 3 arc minutes. The computer program for the simulation of meshing provides the data of transmission errors that is given in Table 4.1. The data of Table 4.1 shows that the resulting function of transmission errors is a parabolic function. Thus, the linear function of transmission errors caused by misalignment of gear axes has been absorbed by the predesigned parabolic function.

Table 4.2 represents the transmission errors for the same helical gears for the case when the gear axes are intersected and form an angle of 3 arc minutes. The resulting function of transmission errors is again a parabolic function with the level d = 26.2 arc seconds. The relatively high level of transmission errors is the price that must be paid for the longitudinal path of contact. However, the proposed topology of the pinion tooth surface provides a reduction of the level of gear noise since the linear function of transmission errors is substituted by a parabolic function.

TABLE 4.1 TRANSMISSION ERRORS FOR CROSSED HELICAL GEARS

ϕ_1 (deg)	-23	-18	-13	-8	-3	2	7
$\Delta\phi_2$ (sec)	-17.94	-3.06	5.64	8.23	4.84	-4.39	-19.37

TABLE 4.2 TRANSMISSION ERRORS OF INTERSECTED HELICAL GEARS

ϕ_1 (deg)	-20	-15	-10	-5	0	5	10
$\Delta\phi_2$ (sec)	-23.06	-8.50	0.15	2.96	0.00	-8.66	-22.95

5. DEFORMATION OF HELICAL GEAR SHAFT

5.1 Basic Concepts and Considerations

Deformation of gear shafts always exists when the gears are used to transmit power. This is because under the load the force applied on gear tooth surface is transferred to the gear shaft and the shaft is not rigid body. It can be proven that the deformation of gear shaft results in the same effects as misalignment induced by assembly. The misalignment from assembly could be reduced to as little as possible. But the shaft deformation is inevitable. Fortunately, all the transmission errors and shift of bearing contact due to deformation of gear shaft can be compensated by crowning helical pinion tooth surface with the methods discussed in Chapters 3 and 4.

5.2 Force Applied on Gear Shaft

When the pinion and gear mesh a force, between their tooth surfaces, is applied on the contact point. The direction of the force is along the normal of the contact point. For the case of regular helical gears, in the fixed coordinate system S_f as described in section 3.8, the force direction is a constant since the normal of the contact point is a constant (Litvin, 1968) and can be expressed as:

$$\underset{\sim}{n}_c = \begin{bmatrix} \cos\psi_n \cos\beta_p \\ \sin\psi_n \\ \cos\psi_n \sin\beta_p \end{bmatrix} \qquad (5.2.1)$$

As shown in Fig. 5.1, the force applied on the pinion tooth surface is:

$$\underset{\sim}{F} = \underset{\sim}{F}_t + \underset{\sim}{F}_z = -F \begin{bmatrix} \cos\psi_n \cos\beta_p \\ \sin\psi_n \\ \cos\psi_n \sin\beta_p \end{bmatrix} \qquad (5.2.2)$$

where $\underset{\sim}{F}_t = -F \begin{bmatrix} \cos\psi_n \cos\beta_p \\ \sin\psi_n \\ 0 \end{bmatrix}$; $F_z = -F \begin{bmatrix} 0 \\ 0 \\ \cos\psi_n \sin\beta_p \end{bmatrix}$

$\underset{\sim}{F}_t$ is called as transverse force since it is applied on the transverse section of the pinion and the gear and $\underset{\sim}{F}_z$ is called as axial force. Transverse force $\underset{\sim}{F}_t$ is always in tangency with the base circle in tranverse section. Therefore, if the torque transferred by the gears is constant, the magnitude of transverse force is also constant. So is the magnitude of axial force since it has certain ratio with transverse force.

Both transverse force and axial force can be transferred to the axis of gear shaft with resultant torque (designated by $\underset{\sim}{F}_t^*$ and $\underset{\sim}{F}_z^*$) . For the transverse force, the resultant torque is the torque transferred by the gear. For the axial force, the resultant torque is balanced by the support bearings. Assuming the force is applied on the middle section of the gear, after the force is decomposed and transferred to the axis of gear shaft as shown in Fig. 5.1 (b), both the transverse force and axial force will act on the origin of coordinate system S_f. Actually, both forces will cause deformation of the shaft. But since the axial force only cause very small tension or compression of the shaft, it can be neglected.

5.3 Modelling of Shaft Deformation

As shown in Fig. 5.2, the transverse force $\underset{\sim}{F}_t$ is applied on the point A which is the center of the shaft corresponding to the pinion or gear middle cross section. Under the force, the deformation of the shaft is composed of two parts, that is, deflection of shaft at the point A designated by V_p and rotation of the shaft cross section with A as a center designated by

λ_p. The value V_p and λ_p depend on the magnitude of transverse force, geometry and material of shaft and the way how the shaft is supported. (Timoshenko 1973).

Now, let us model the deformation of the helical pinion shaft. For convenience and consistency with the previous chapters, We establish our coordinate system as follows:

(1) As shown in Fig. 5.3a S_f is a fixed coordinate system and rigidly connected to the frame. S_a is an auxiliary coordinate system with the Y_a axis coincident with the direction of transverse force. The angle ψ_c is the pressure angle in the transverse section of helical gear. The matrix $[M_{fa}]$ can transfer vector from S_a to S_f and is expressed as:

$$[M_{fa}] = \begin{bmatrix} \sin\psi_c & \cos\psi_c & 0 & 0 \\ -\cos\psi_c & \sin\psi_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.3.1)$$

(2) Coordinate system S_{f1} and S_a, [Fig. 5.3(b)] are connected to the axis of pinion shaft. Without deformation, S_f' and S_a' are coincident with S_f and S_a respectively. The matrix $[M_{a'f'}]$ can transfer vector from S_f' to S_a, and is expressed as:

$$[M_{a'f'}] = \begin{bmatrix} \sin\psi_c & -\cos\psi_c & 0 & 0 \\ \cos\psi_c & \sin\psi_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.3.2)$$

(3) Coordinate systems S_a and S_a' are not coincident when shaft deformation occurs. As shown in Fig. 5.2, their relation can be

expressed as:

$$M_{aa'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\lambda_p & -\sin\lambda_p & -\nu_p \\ 0 & \sin\lambda_p & \cos\lambda_p & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.3.3)$$

(4) As described in previous chapter, pinion tooth surface can be expressed in coordinate systems S_1 as a position vector $\underset{\sim}{r}_1$ with its normal $\underset{\sim}{n}_1$. The relation of S_1 and S_f' is shown in Fig. 5.3(c) and expressed as:

$$M_{f'1} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.3.4)$$

After the coordinate systems are established, it is easy to find that the deformation of the pinion shaft can be modelled by matrix m_{ff}' as:

$$[M_{ff'}] = [M_{fa}]\ [M_{aa'}]\ [M_{a'f'}]$$

$$= \begin{bmatrix} 1+\cos^2\psi_c(\cos\lambda_p-1) & ,\sin\psi_c\cos\psi_c(\cos\lambda_p-1) & , -\cos\psi_c\sin\lambda_p & , -v_p\cos\psi_c \\ \sin\psi_c\cos\psi_c(\cos\lambda_p-1) & , 1+\sin^2\psi_c(\cos\lambda_p-1) & , -\sin\psi_c\sin\lambda_p & , -v_p\sin\psi_c \\ \sin\lambda_p\cos\psi_c & , \sin\lambda_p\sin\psi_c & , \cos\lambda & , 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5.3.5)

Since λ_p is very small angle, it is reasonable to use λ_p instead of $\sin\lambda_p$ and one instead of $\cos\lambda_p$. Therefore, Eq. (5.3.5) can be written as:

$$[M_{ff'}] = \begin{bmatrix} 1 & 0 & -\lambda_p\cos\psi_c & -V_p\cos\psi_c \\ 0 & 1 & -\lambda_p\sin\psi_c, & -V_p\sin\psi_c \\ \lambda_p\cos\psi_c, & \lambda_p\sin\psi_c, & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.3.6}$$

Also, for the gear shaft, the coordinate systems S_G, $S_{G'}$, S_b, and S_b' are used instead of S_f, S_f', S_a and S_a', and λ_G, and V_G are used instead of λ_p and V_p. Then, using the same ideas, the deformation of gear shaft can be modelled by matrix $[m_{aa'}]$ as:

$$M_{GG'} = \begin{bmatrix} 1+\cos^2\psi_c(\cos\lambda_G-1) & , \sin\psi_c\cos\psi_c(\cos\lambda_G-1), & \cos\psi_c\sin\lambda_G & , V_G\cos\psi_c \\ \sin\psi_c\cos\psi_c(\cos\lambda_G-1), & 1+\sin^2\psi_c(\cos\lambda_G-1) & , \cos\psi_c\sin\lambda_G & , V_G\cos\psi_c \\ -\sin\lambda_G\cos\psi_c & , -\sin\lambda_G\sin\psi_c & , \cos\lambda_G & , 0 \\ 0 & , 0 & , 0 & , 1 \end{bmatrix} \tag{5.3.7}$$

Or considering that λ_G is very small angle:

$$[M_{GG'}] = \begin{bmatrix} 1 & 0 & +\lambda_G\cos\psi_c & V_G\cos\psi_c \\ 0 & 1 & +\lambda_G\sin\psi_c & V_G\sin\psi_c \\ -\lambda_G\cos\psi_c & -\lambda_G\sin\psi_c & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.3.8}$$

It must be emphasized that the gear tooth surfaces are expressed in coordinate system S_2. The relation of S_2 and $S_f{}'$ is shown in Fig. 5.4 and expressed as:

$$[M_{G'2}] = \begin{bmatrix} -\cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & -\cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.3.9}$$

Also as shown in Fig. 5.5, the coordinate system S_f and S_G are not coincident. There is a center distance C between their origins. Therefore M_{fG} is represented as:

$$[M_{fG}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & C \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.3.10}$$

Now it is easy to simulate the performance of the gears with the deformation of their shafts. Assuming, there is a helical pinion in coordinate system S_1 represented by position vector $\underset{\sim}{r}_1$ and normal vector $\underset{\sim}{n}_1$ and a helical gear in coordinate system S_2 represented by $\underset{\sim}{r}_2$ and $\underset{\sim}{n}_2$, we can write the tooth contact equation as:

$$[M_{ff'}]\ [r_{f'1}]\ [\underset{\sim}{r}_1] = [M_{fG}]\ [M_{GG'}]\ [M_{G'2}]\ [\underset{\sim}{r}_2] \tag{5.3.11}$$

$$[L_{ff'}]\ [L_{f'1}]\ [\underset{\sim}{r}_1] = [L_{fG}]\ [L_{GG'}]\ [L_{G'2}]\ [\underset{\sim}{n}_2]$$

where L matrices are 3 x 3 matrices from corresponding 4 x 4 M matrices, crossing 4th row and 4th column and M matrices can be found in this section. Applying the same ideas discussed in Chapter 2 and used in section 3.8, we can find the transmission errors and the shift of the bearing contact by computer aided simulation.

It is interesting to mention that the deformation of the shaft can be expressed as a combination of misalignment with crossing axes, misalignment with intersecting axes, and change of the center distance. This is because, for example

$$[M_{ff'}] = \begin{bmatrix} 1 & 0 & -\lambda_p\cos\psi_c & -V_p\cos\psi_c \\ 0 & 1 & -\lambda_p\sin\psi_c & -V_p\sin\psi_c \\ \lambda_p\cos\psi_c & \lambda_p\sin\psi_c & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\lambda_p\cos\psi_c & 0 \\ 0 & 1 & 0 & 0 \\ \lambda_p\cos\psi_c & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\lambda_p\sin\psi_c & 0 \\ 0 & \lambda_p\sin\psi_c & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -V_p\cos\psi_c \\ 0 & 1 & 0 & -V_p\sin\psi_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.3.12)$$

where the three decomposed matrices are represented by the matrix for crossing axis with small angle $\lambda_p\cos\psi_c$, matrix for intersecting axis with small angle $\lambda_p\sin\psi_c$, and matrix for axis displacement.

5.4 Example and Discussion

The results of investigation in this chapter are illustrated with the following example. Given: number of pinion tooth $N_1 = 20$, number of gear tooth $N_2 = 40$, diametral pitch in normal section $P_n = 10$ in^{-1}, pressure angle in normal section ψ_n = 20^0, helical angle β = 15^0, gear tooth length L = $5/P_n$. Also assume deformation values $V_p = V_G = 0.0125$ in, $\lambda_p = \lambda_G = 2$ min. The computer program for simulation provides the data of transmission errors in Table 5.1

From Table 5.1, it is known that the transmission errors due to gear shaft deformation is an approximately linear function for regular skew involute pinion and gear. Similar to the case of gear axis misalignment, the linear function of transmission errors can be absorbed by predesigned parabolic function of transmission errors that is obtained by crowning helical pinion tooth surface.

TABLE 5.1 TRANSMISSION ERRORS FOR HELICAL GEAR WITH DEFORMED SHAFT

ϕ_1 (deg)	3	6	9	12	15	8	21
$\Delta\phi_2$ (sec)	1.38	3.16	4.74	6.32	7.90	9.48	11.06

6. CONCLUSION

Several methods of crowning helical pinion tooth surface have been developed. The modified pinion tooth surface can provide predesigned parabolic function of transmission errors that are able to absorb linear function of transmission errors induced by misalignment. Also, the modified pinion tooth surface can improve the bearing contact. Principles of computer aided simulation of meshing, contact, and respective computer programs have also been developed. The numerical results of examples of cronwed helical pinion in mesh with regular helical gear show that the ideas of crowning are useful to get favourable bearing contact and allowable transmission errors. But the synthesis of pinion tooth surface should be based on a compromise between the requirements of transmission errors and the patterns of the bearing contact.

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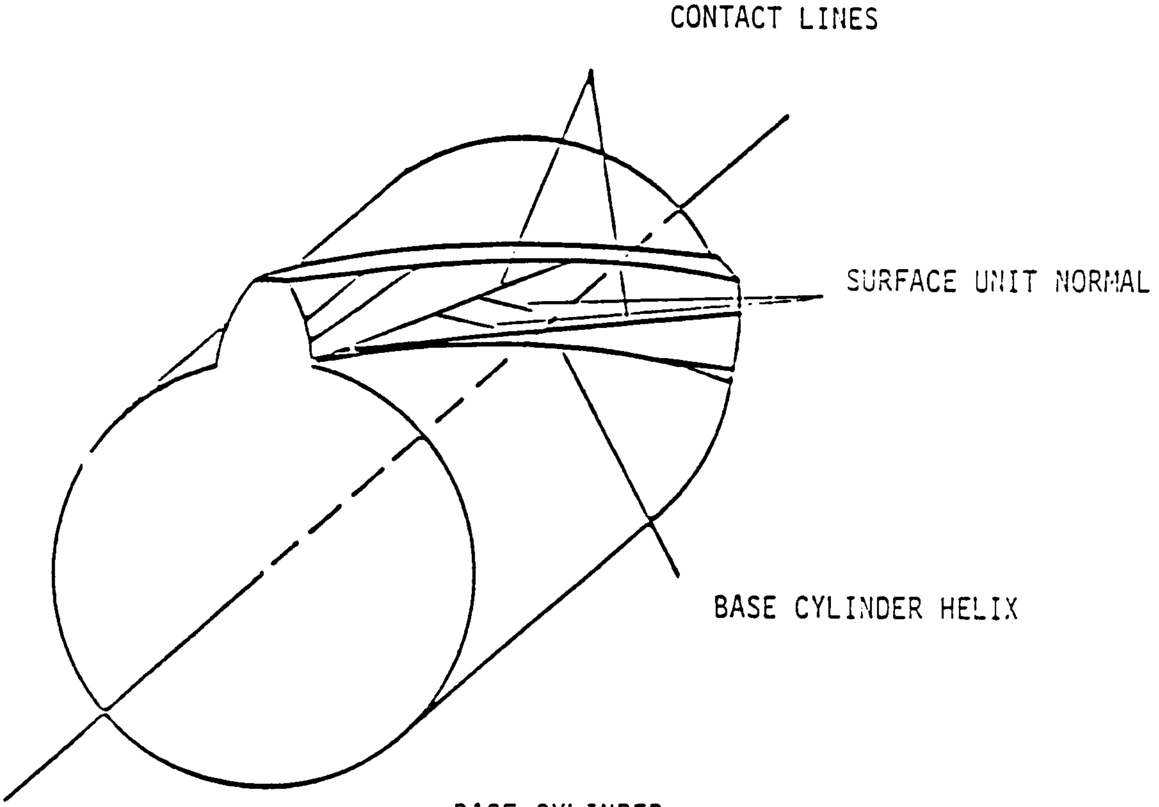


FIG. 1.1 Screw Involute Helical Gear

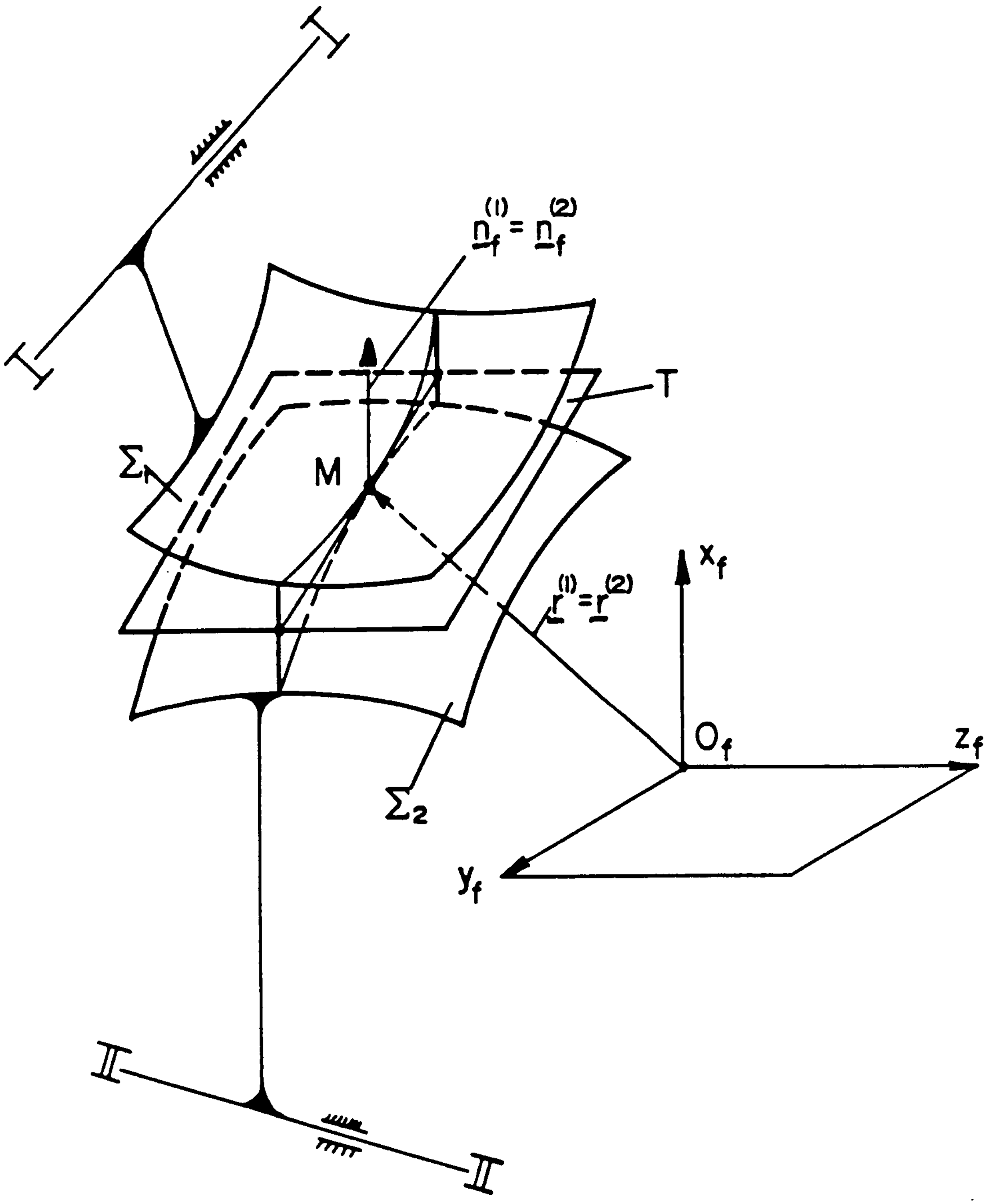


FIG. 2.1 Contacting Tooth Surfaces

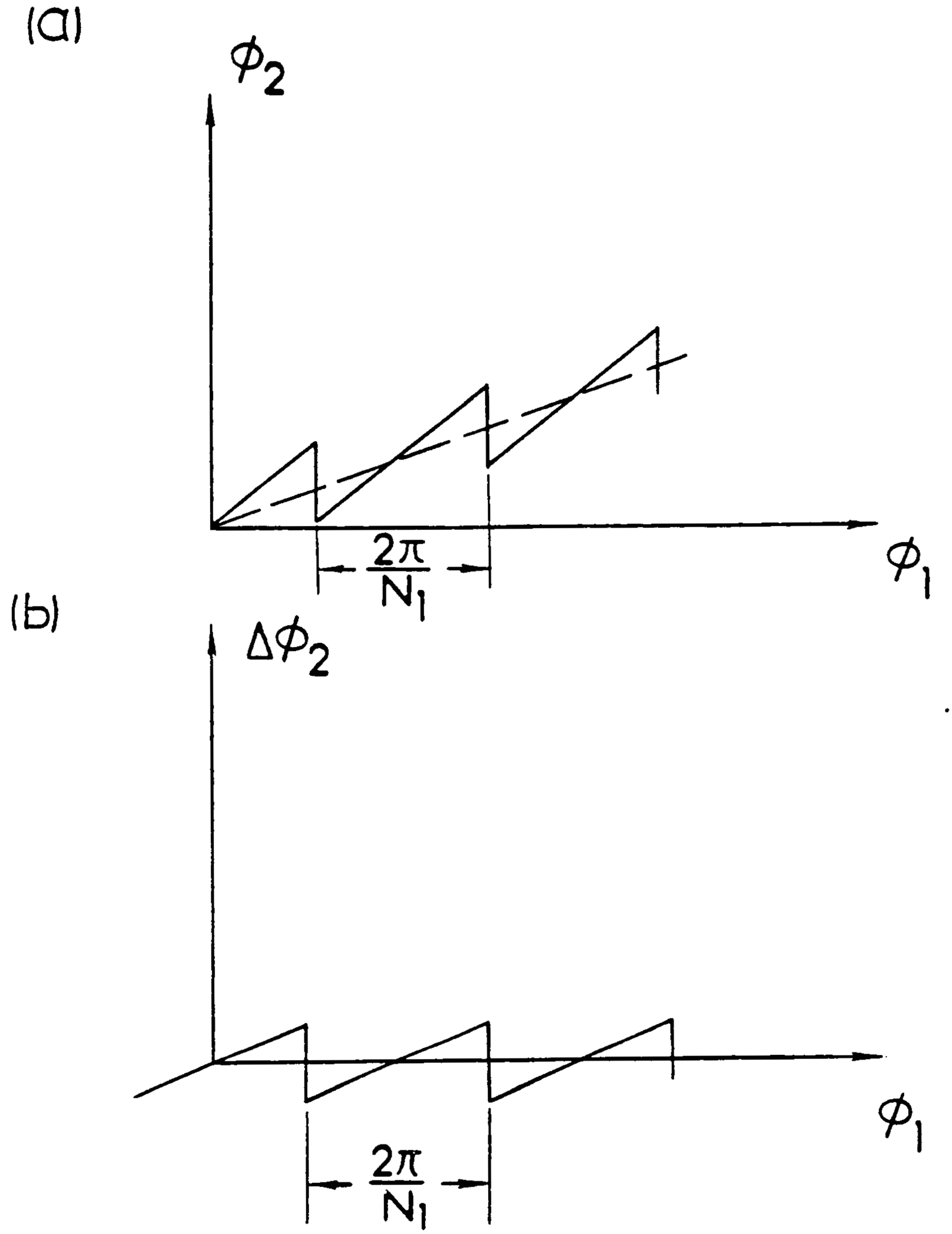


FIG. 2.2 Transmission Error Caused by Gear Misalignment

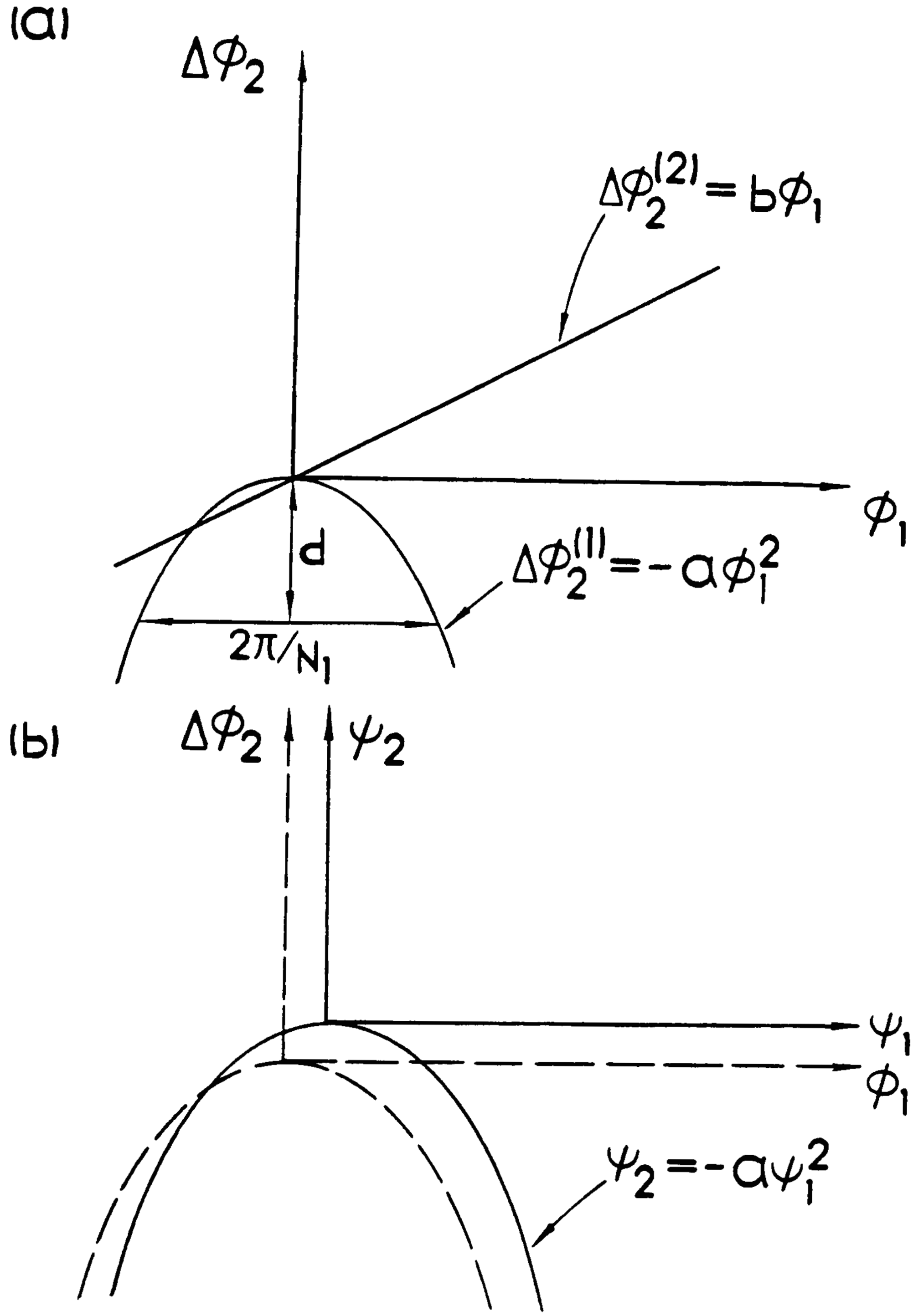


FIG . 2.3 Interaction of Parabolic and Linear Functions

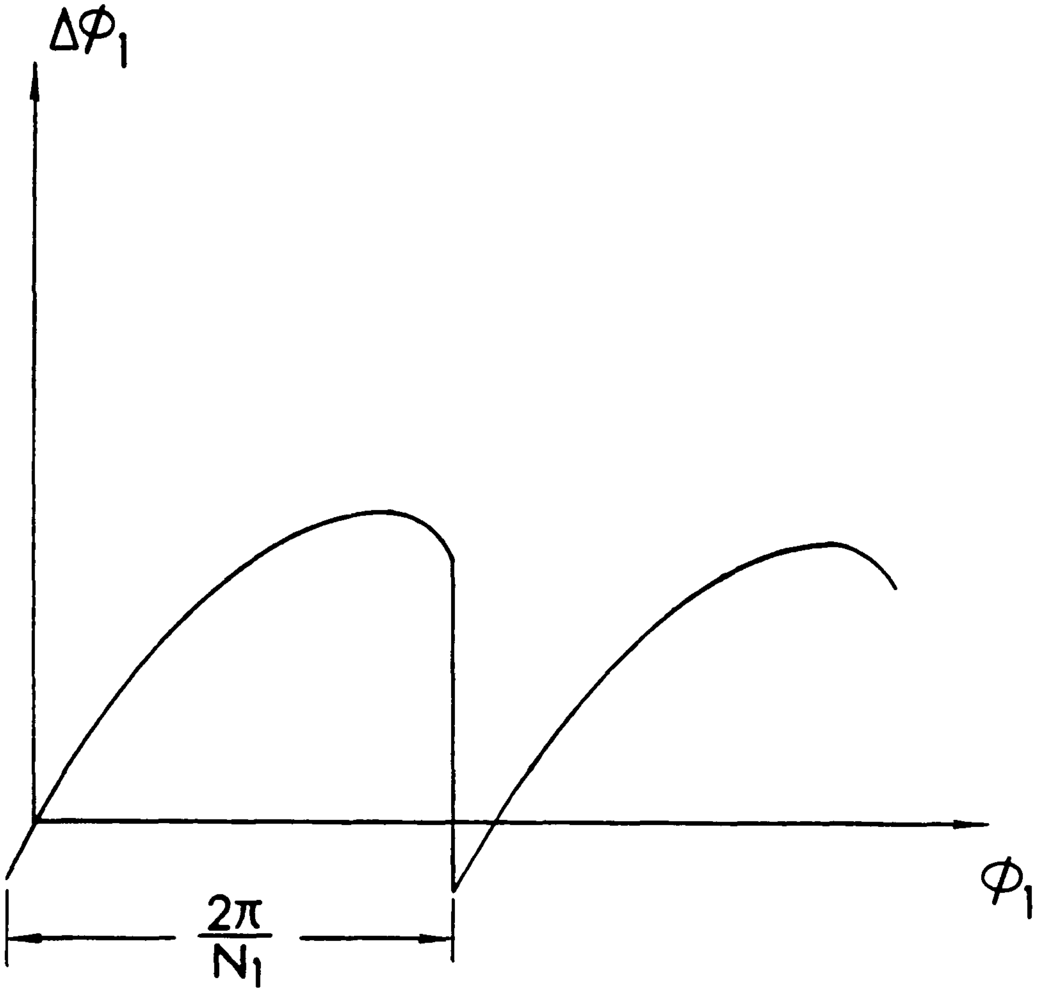


FIG. 2.4 Discontinued Parabolic Function of Transmission Errors

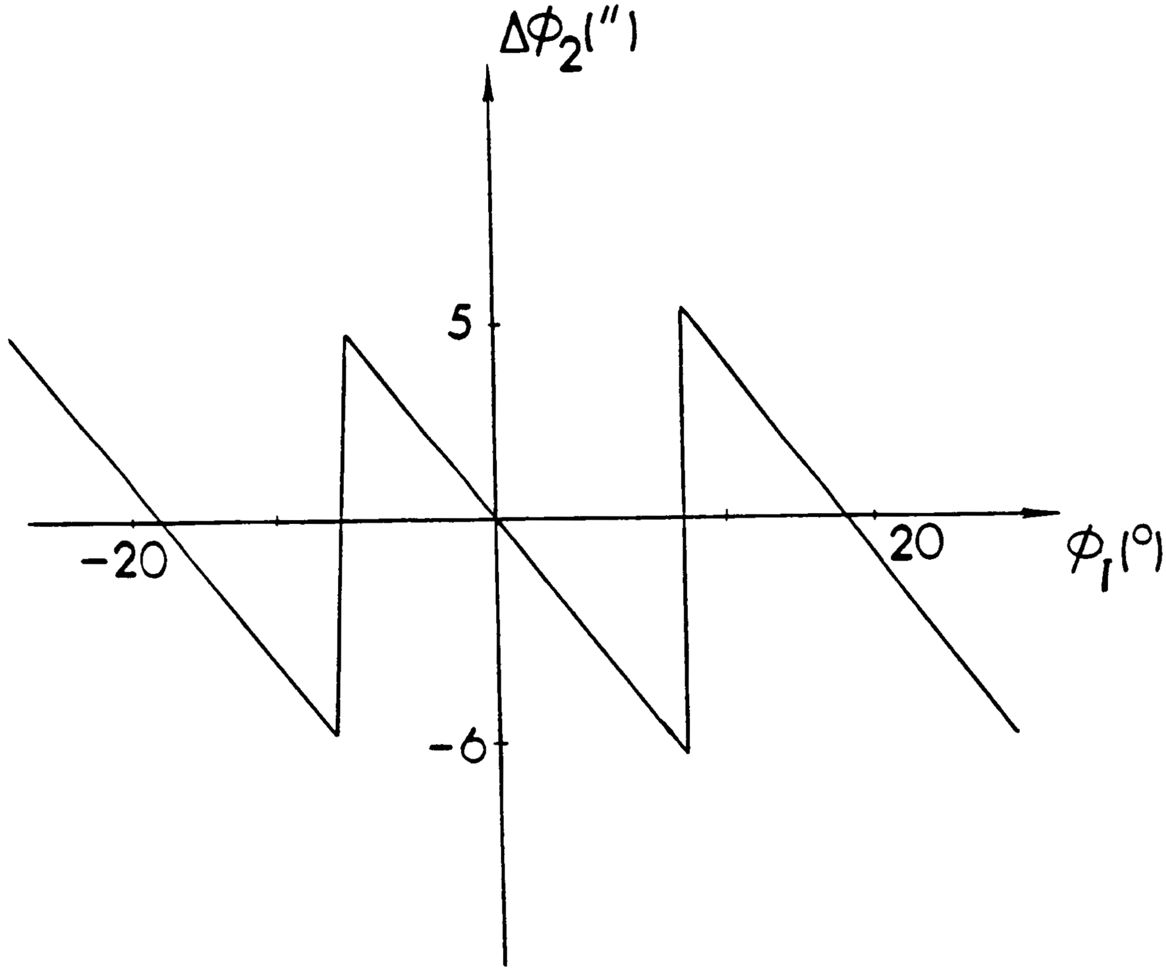


FIG. 2.5 Transmission Error of Helical Gears

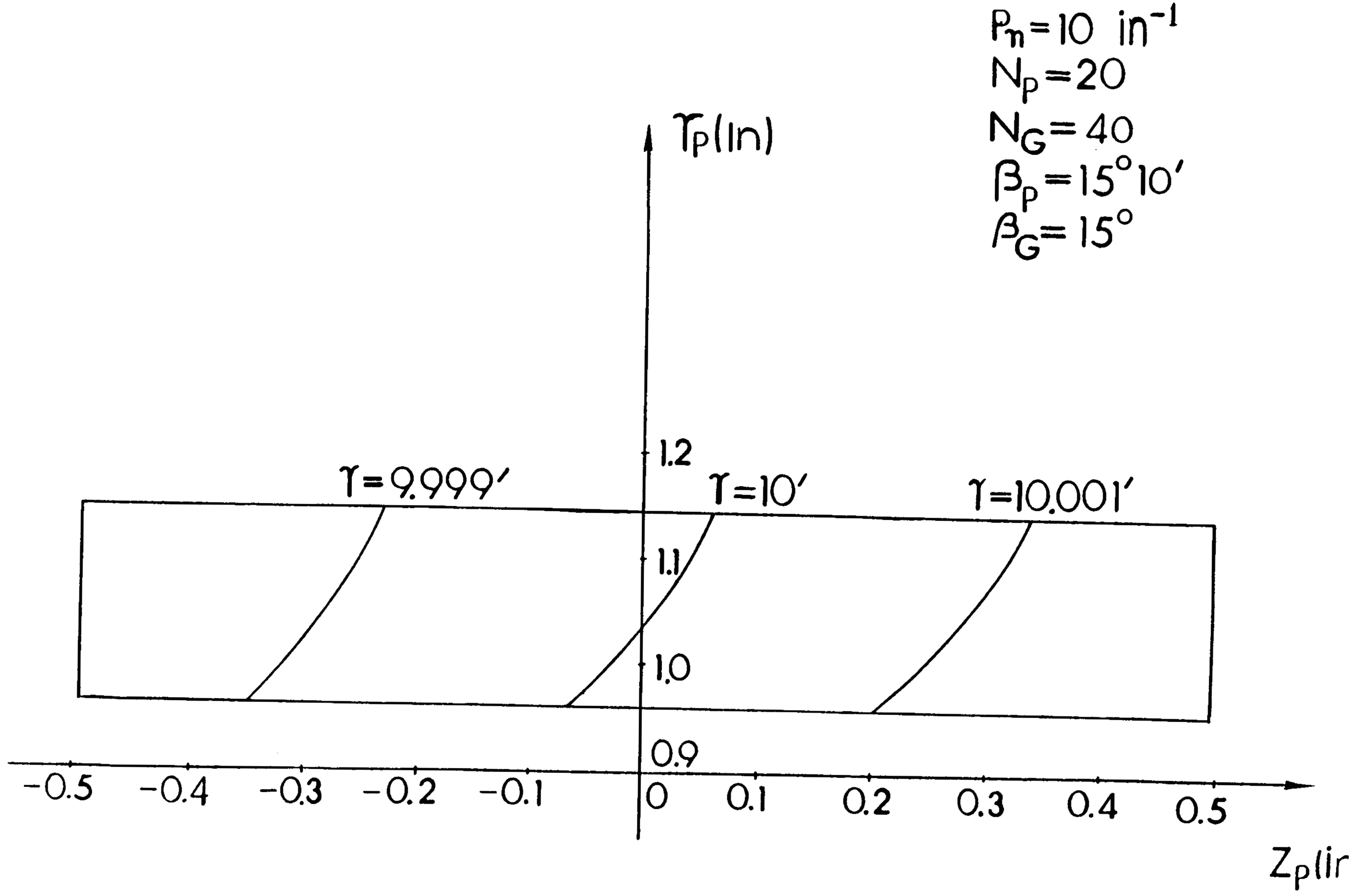


FIG. 2.6 Shift of Contact Path

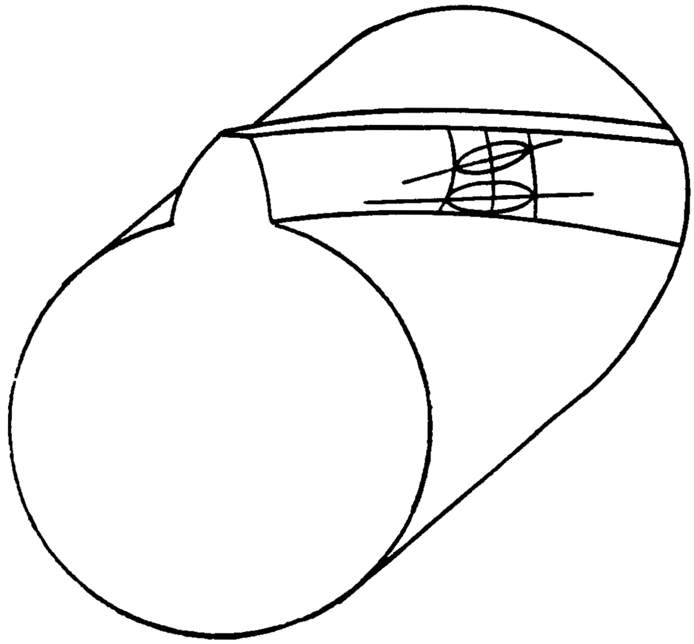


FIG. 3.1 Contact Ellipses on the Pinion Tooth Surface

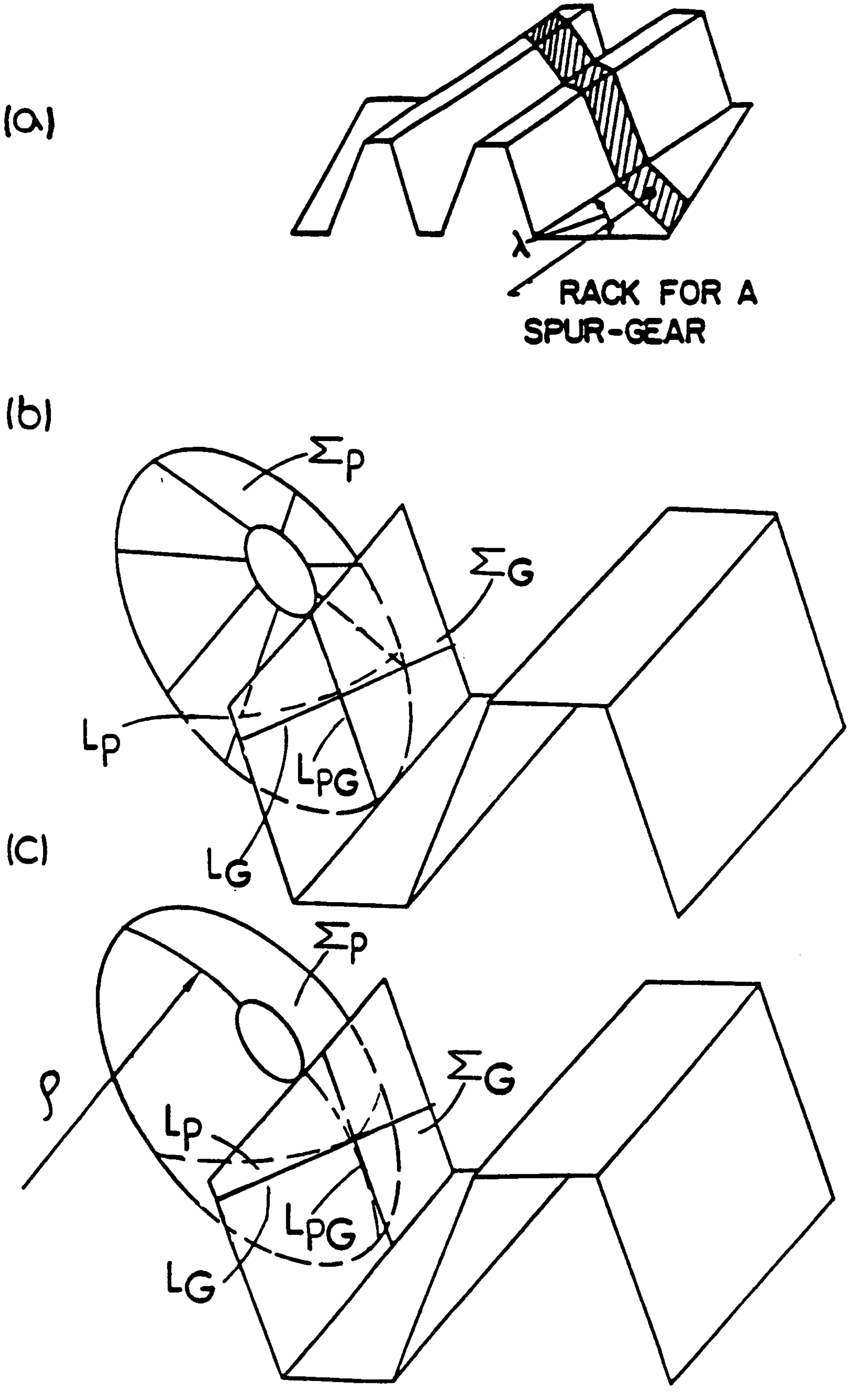


FIG. 3.2 Generating Surfaces

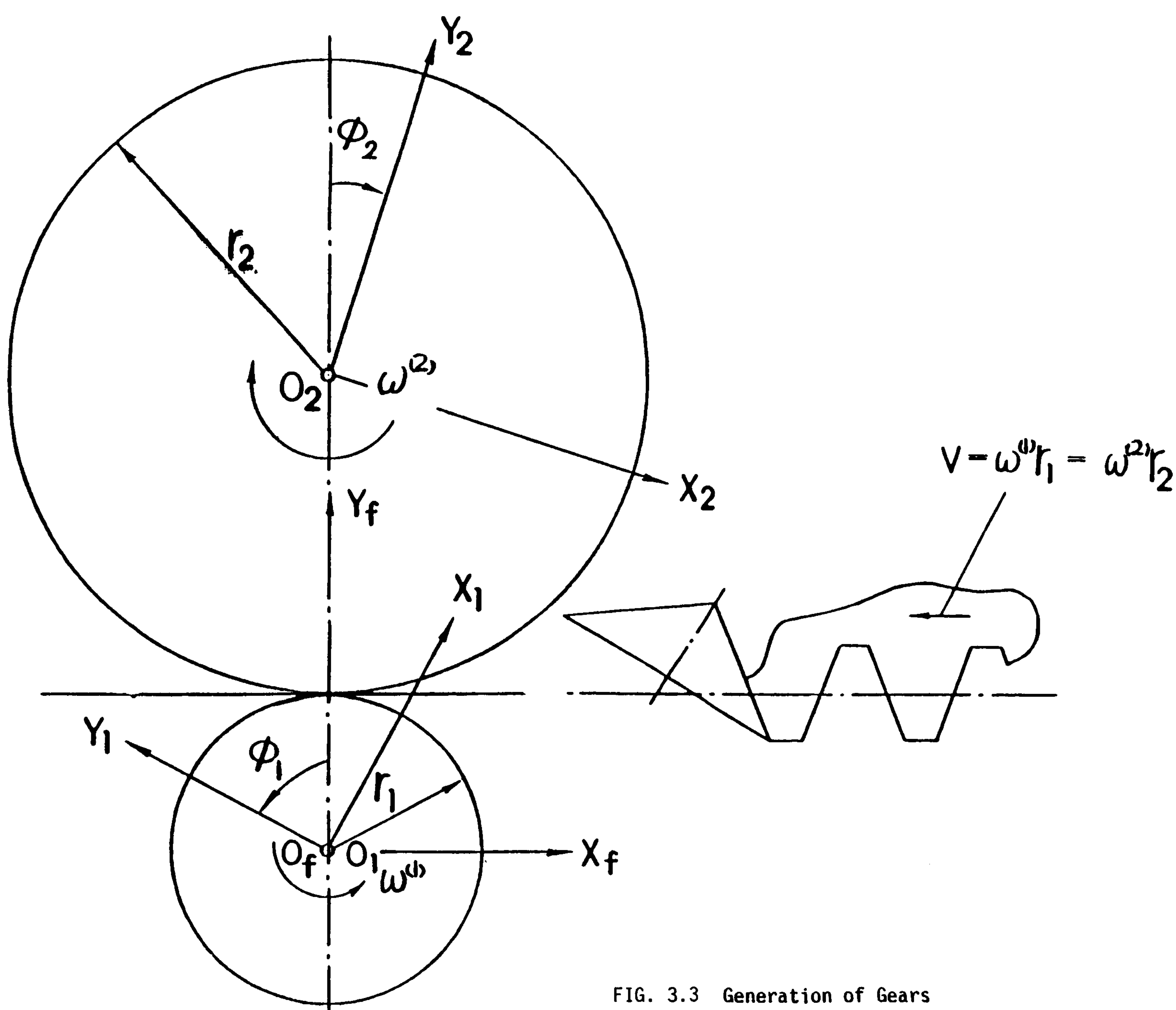


FIG. 3.3 Generation of Gears

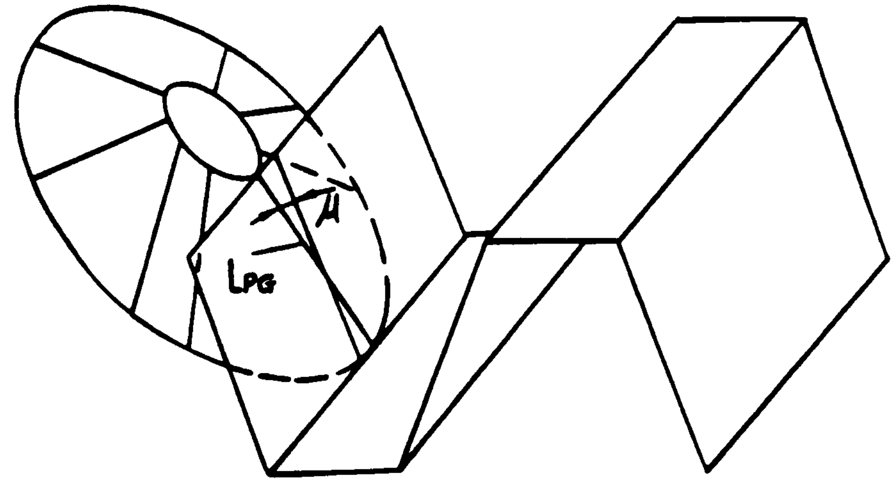


FIG. 3.4 Generating Tool Surfaces

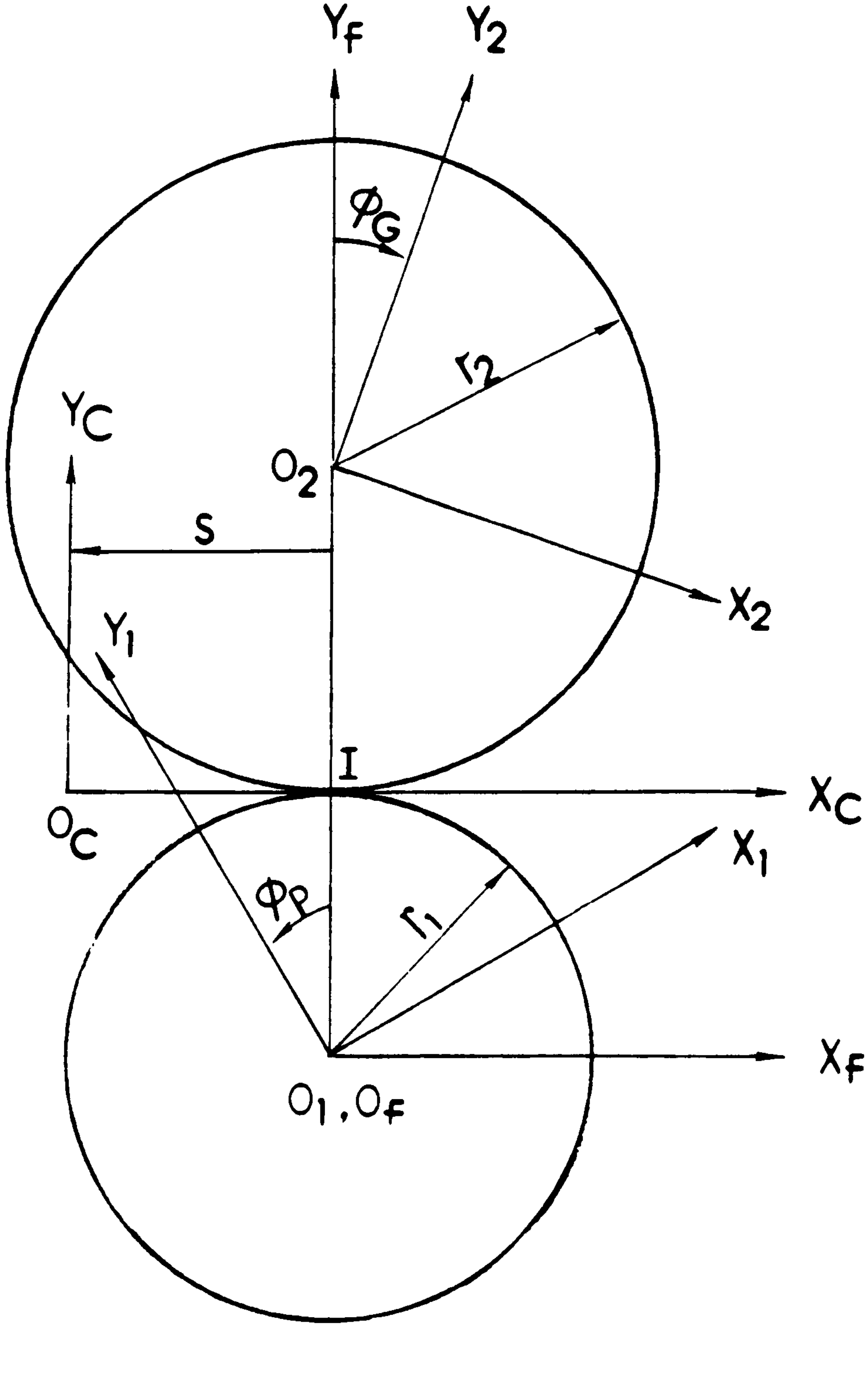
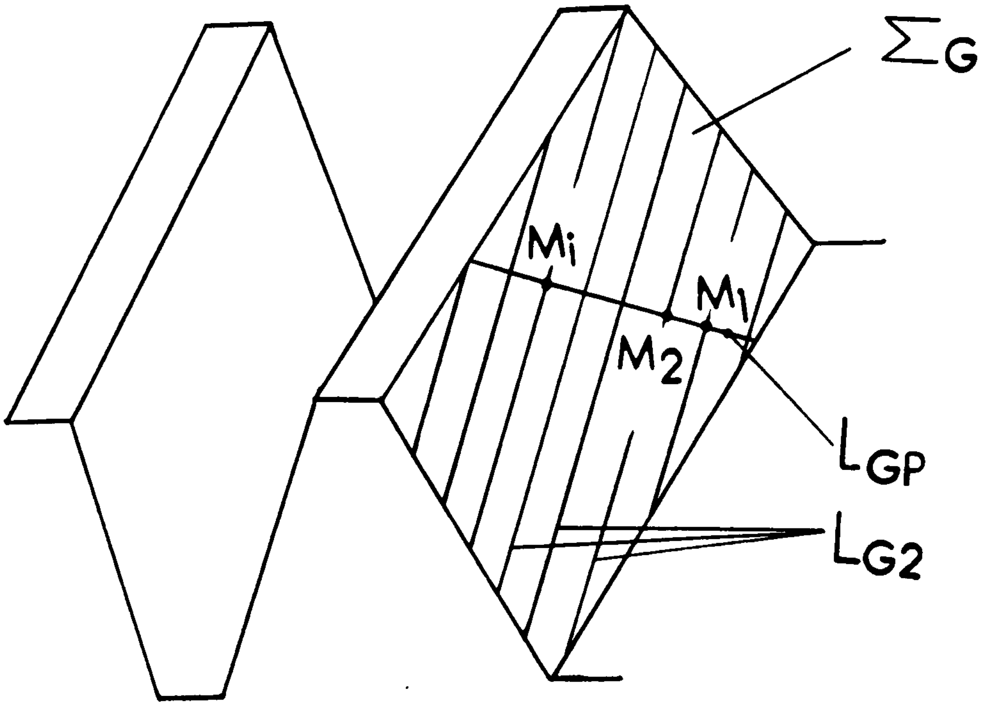


FIG. 3.5 Coordinate Systems for Generating Pinion and Gear Simultaneously

(a)



(b)

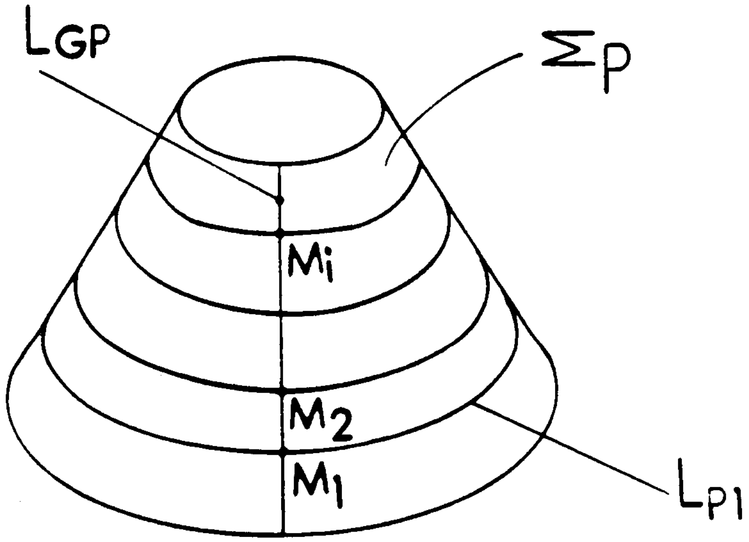


FIG. 3.6 Pinion and Gear Generating Surfaces

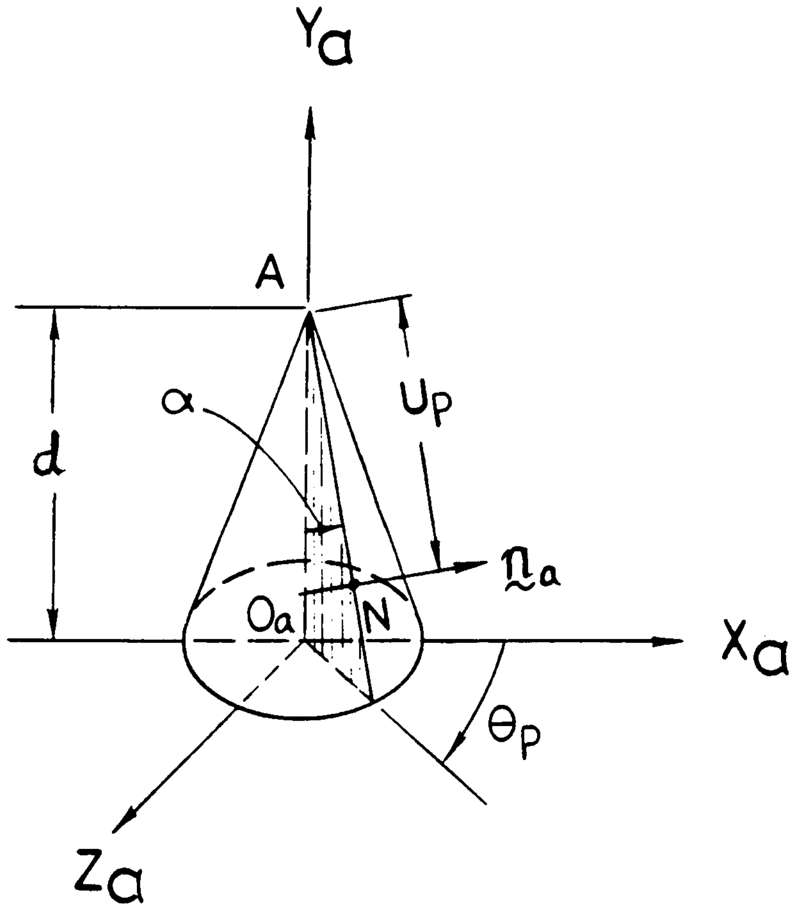


FIG. 3.7 Cone Surface

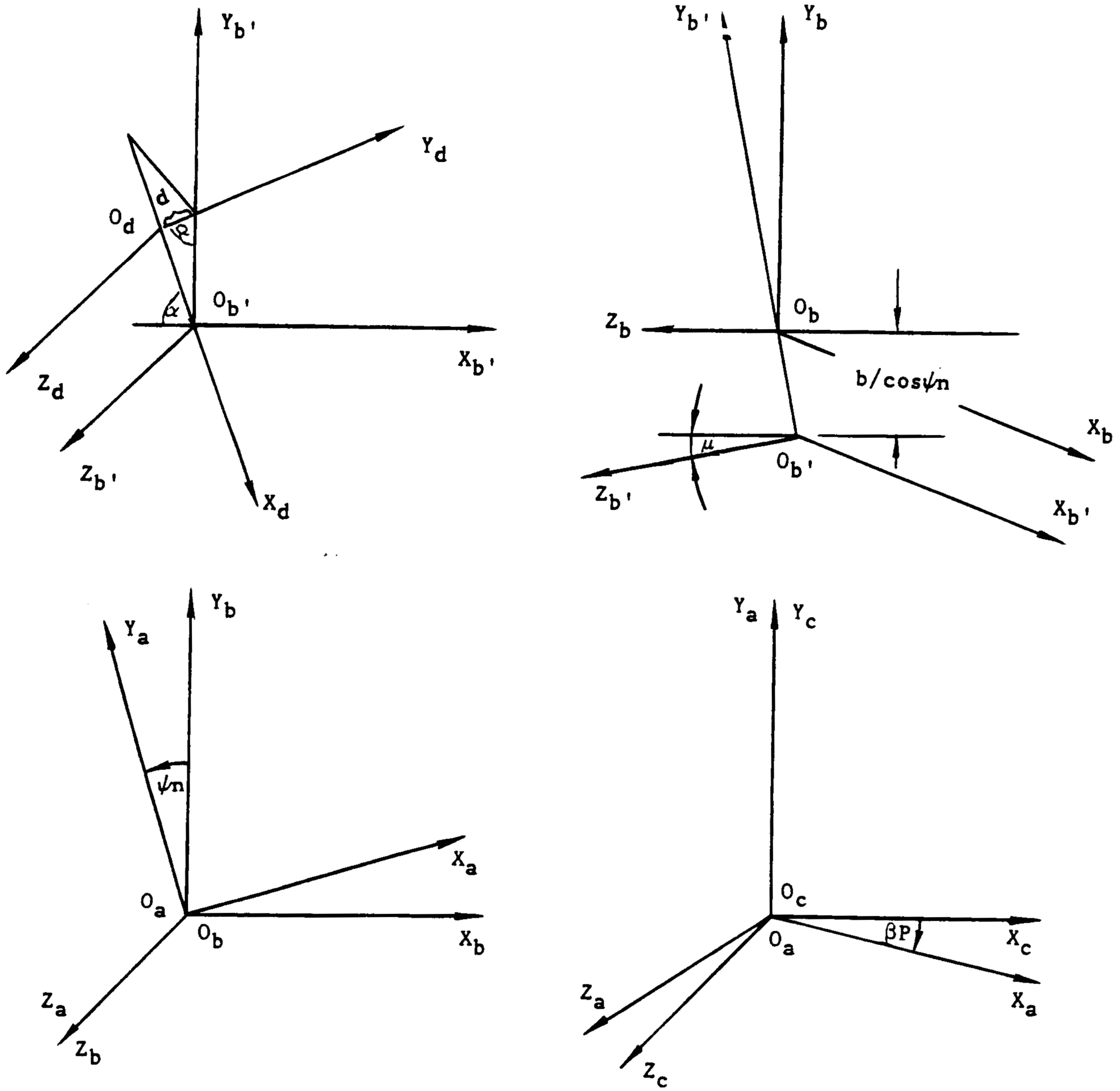


FIG. 3.8 Installment of Conic Tool

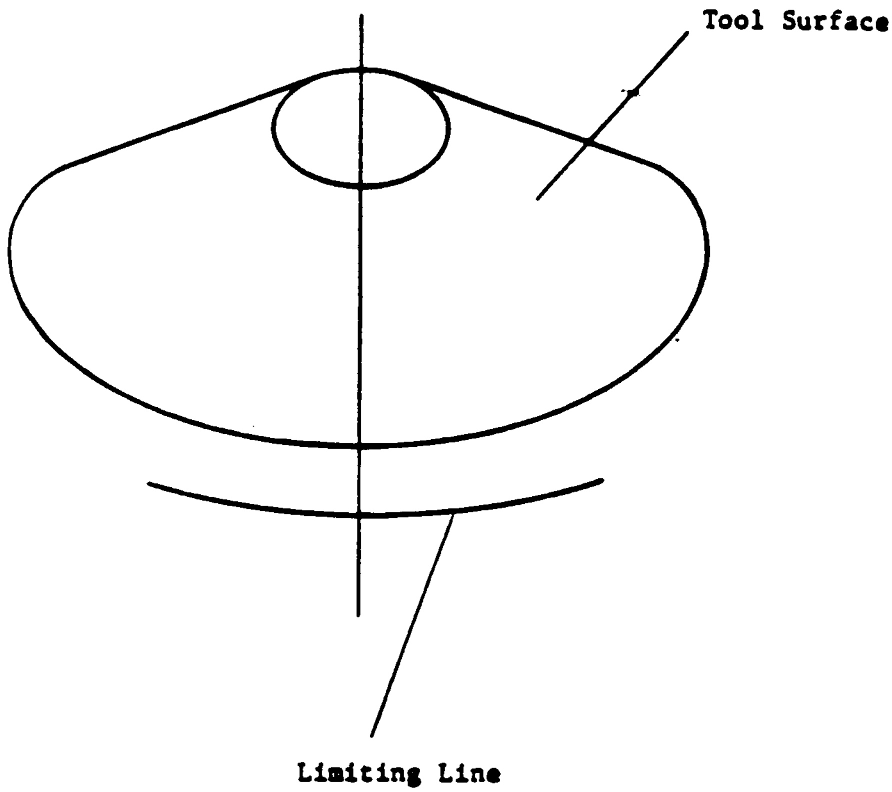


FIG. 3.9 Limiting Line of Tool Surface

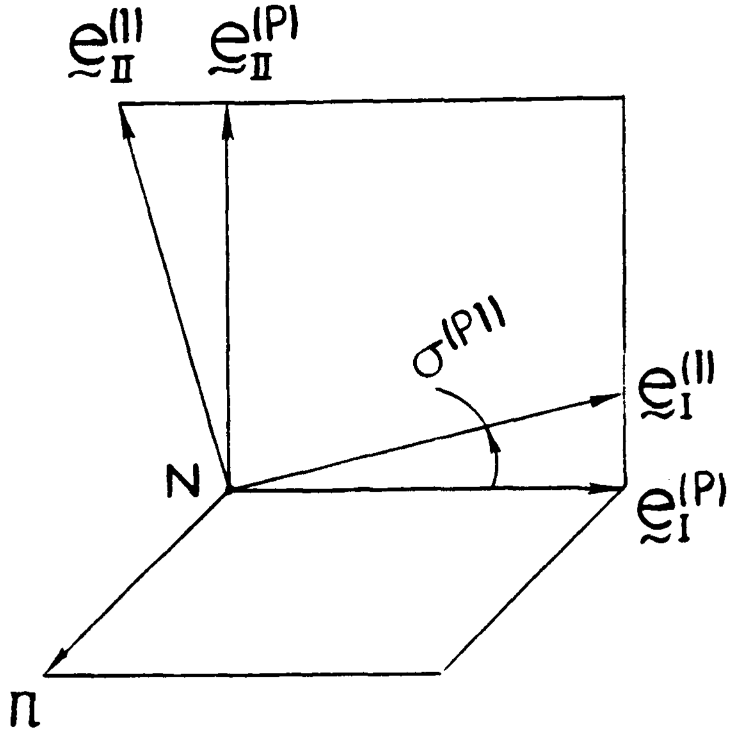


FIG. 3.10 Principal Directions of Generated and Generating Surfaces

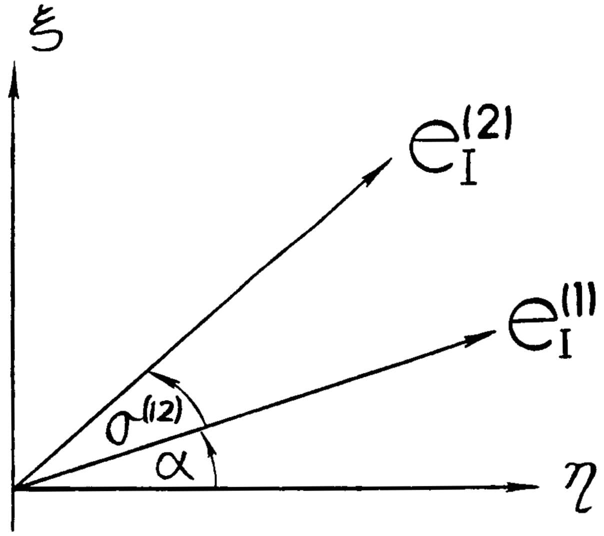


FIG. 3.11 Orientation of Contact Ellipse

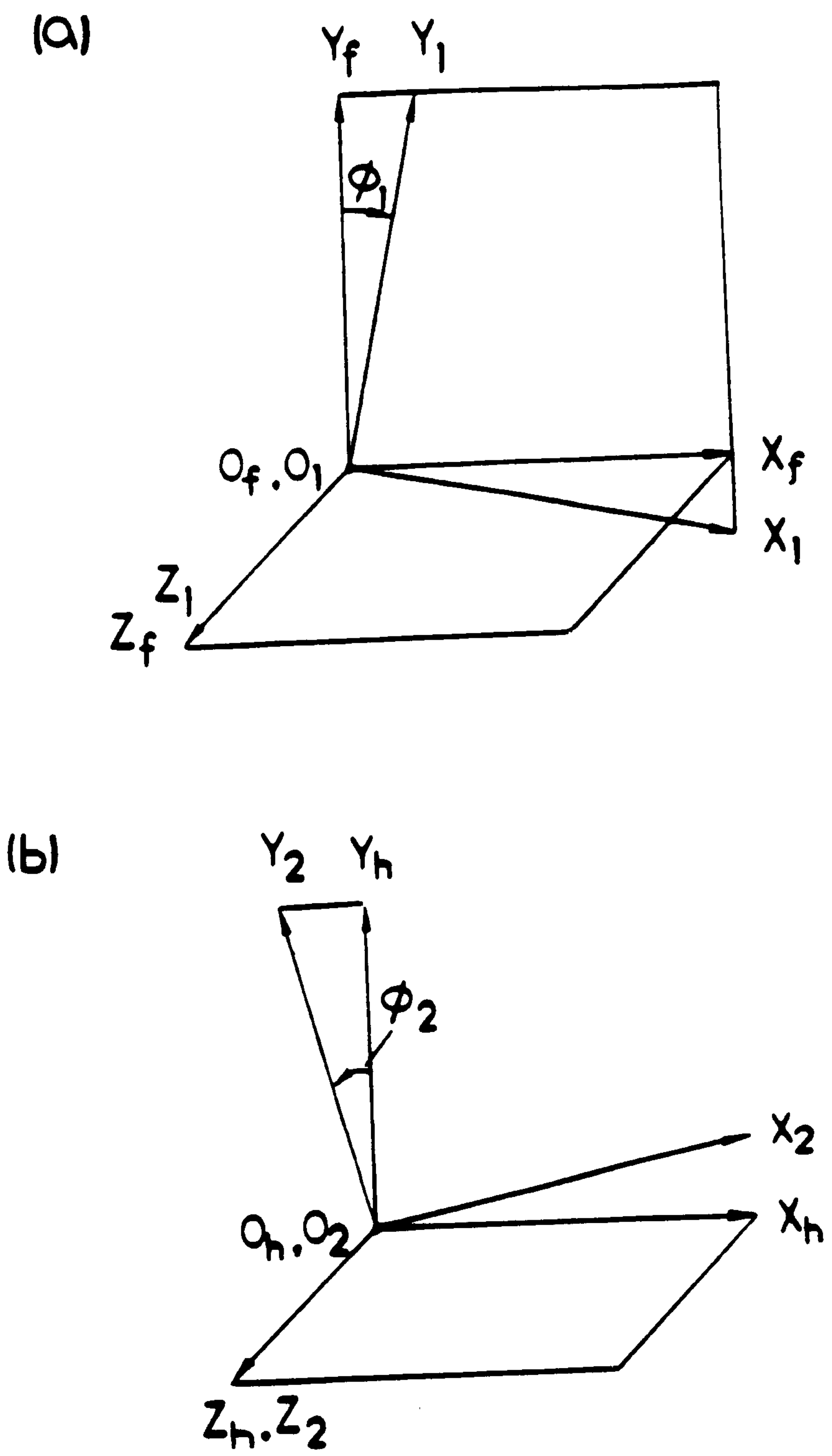


FIG. 3.12 Coordinate Systems for Gear Tooth Rotation

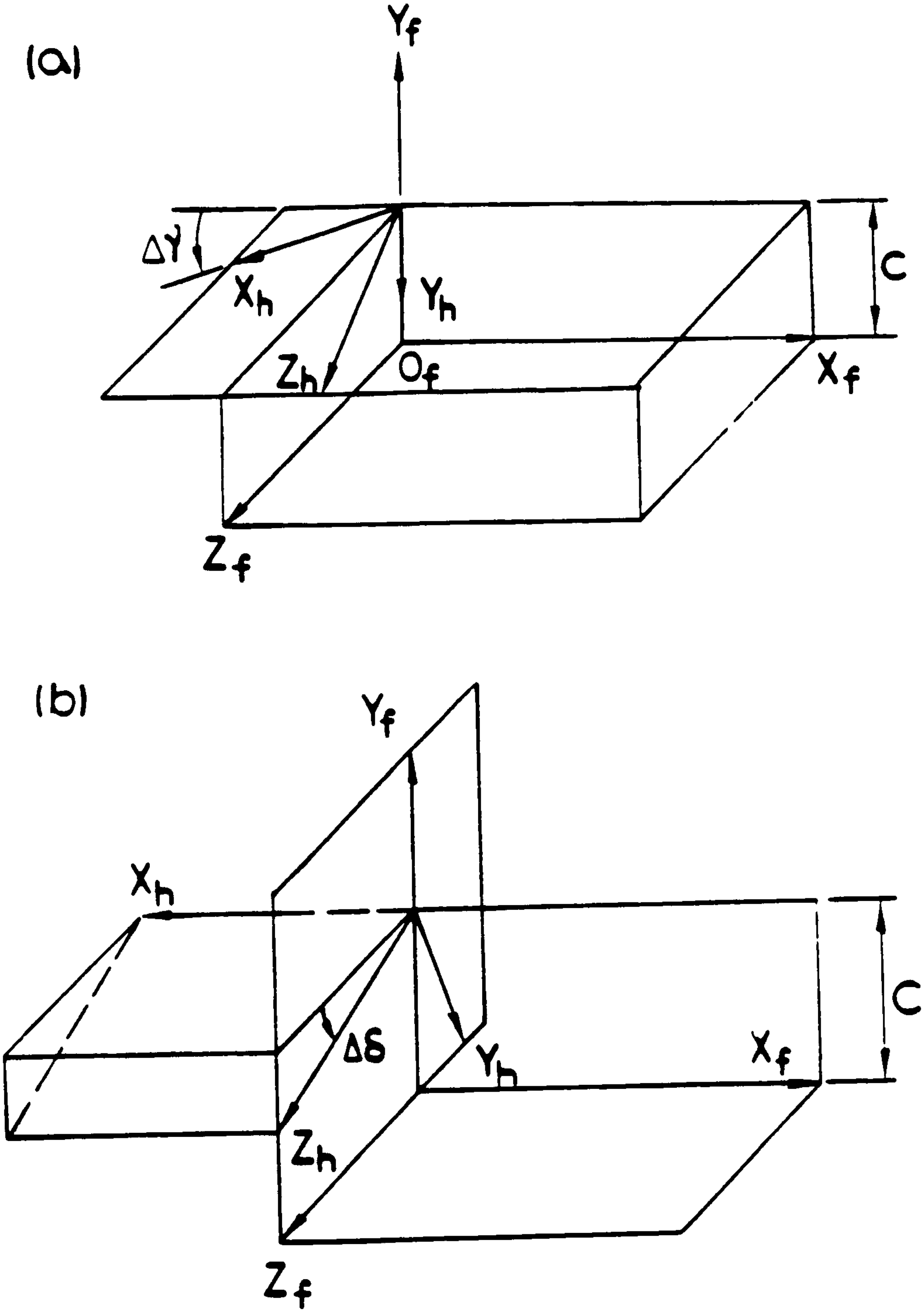


FIG. 3.13 Coordinate Systems for Simulation of Gear Misalignment

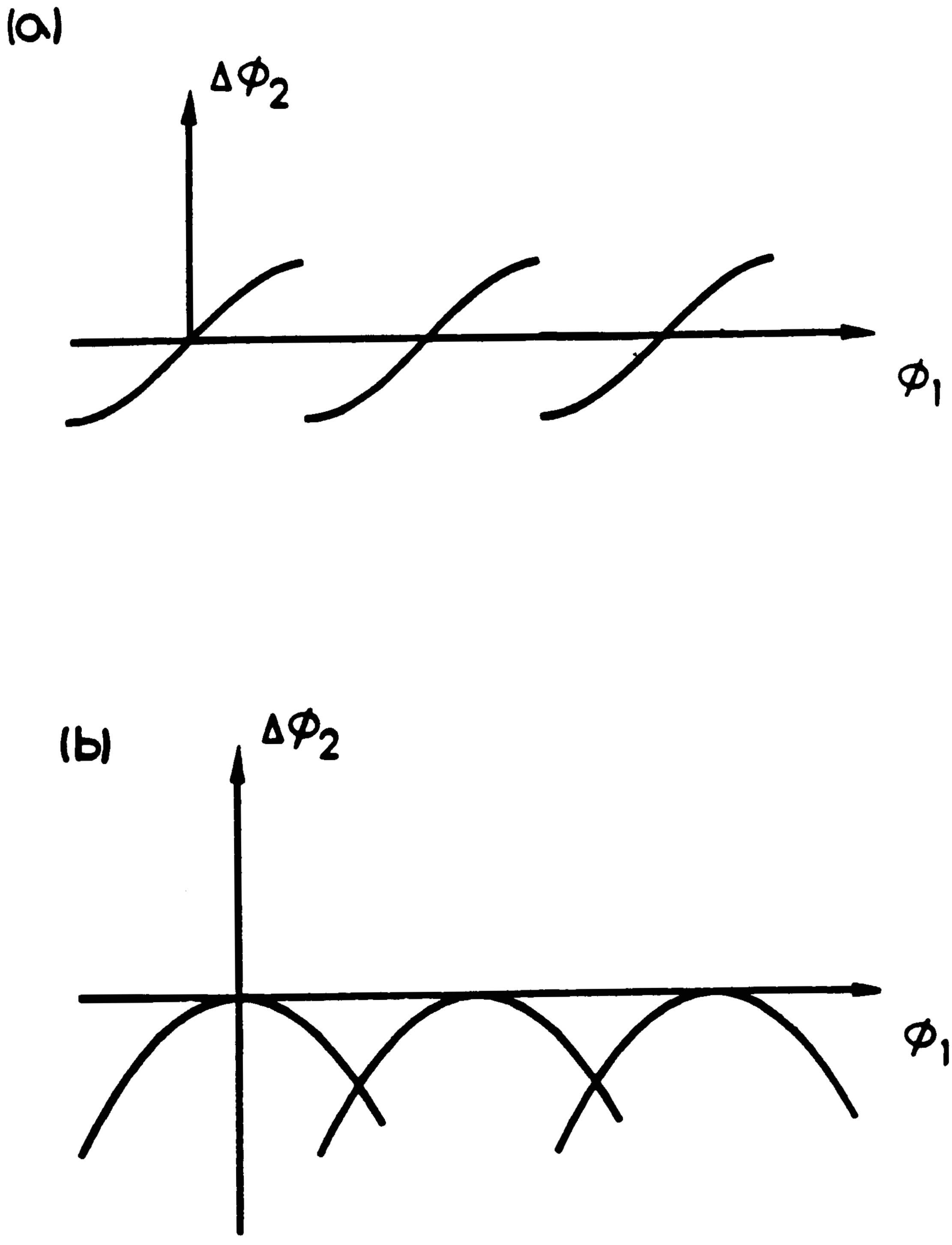


FIG. 3.14 Two Types of Transmission Errors

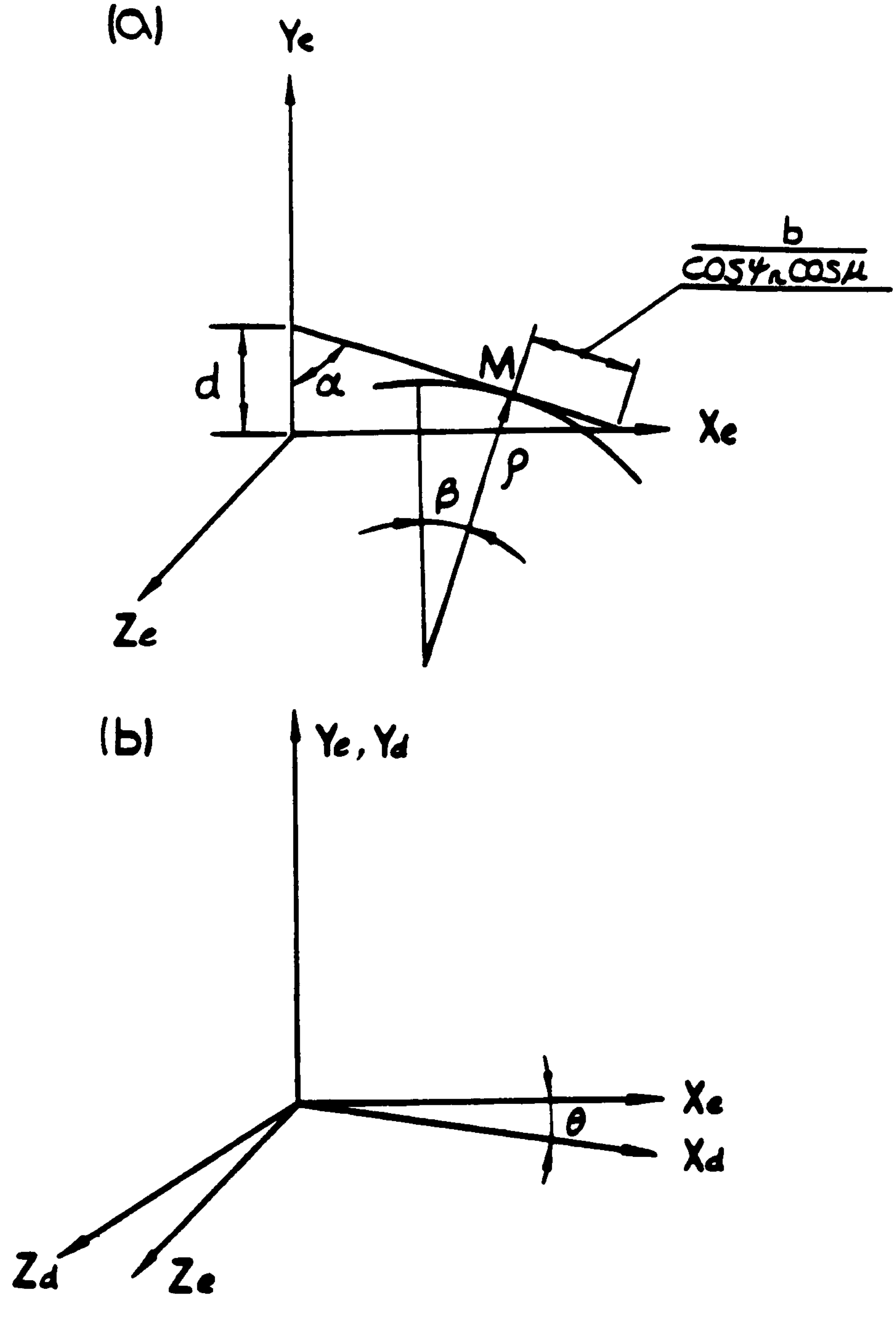


FIG. 3.15 Generation of Surface of Revolution

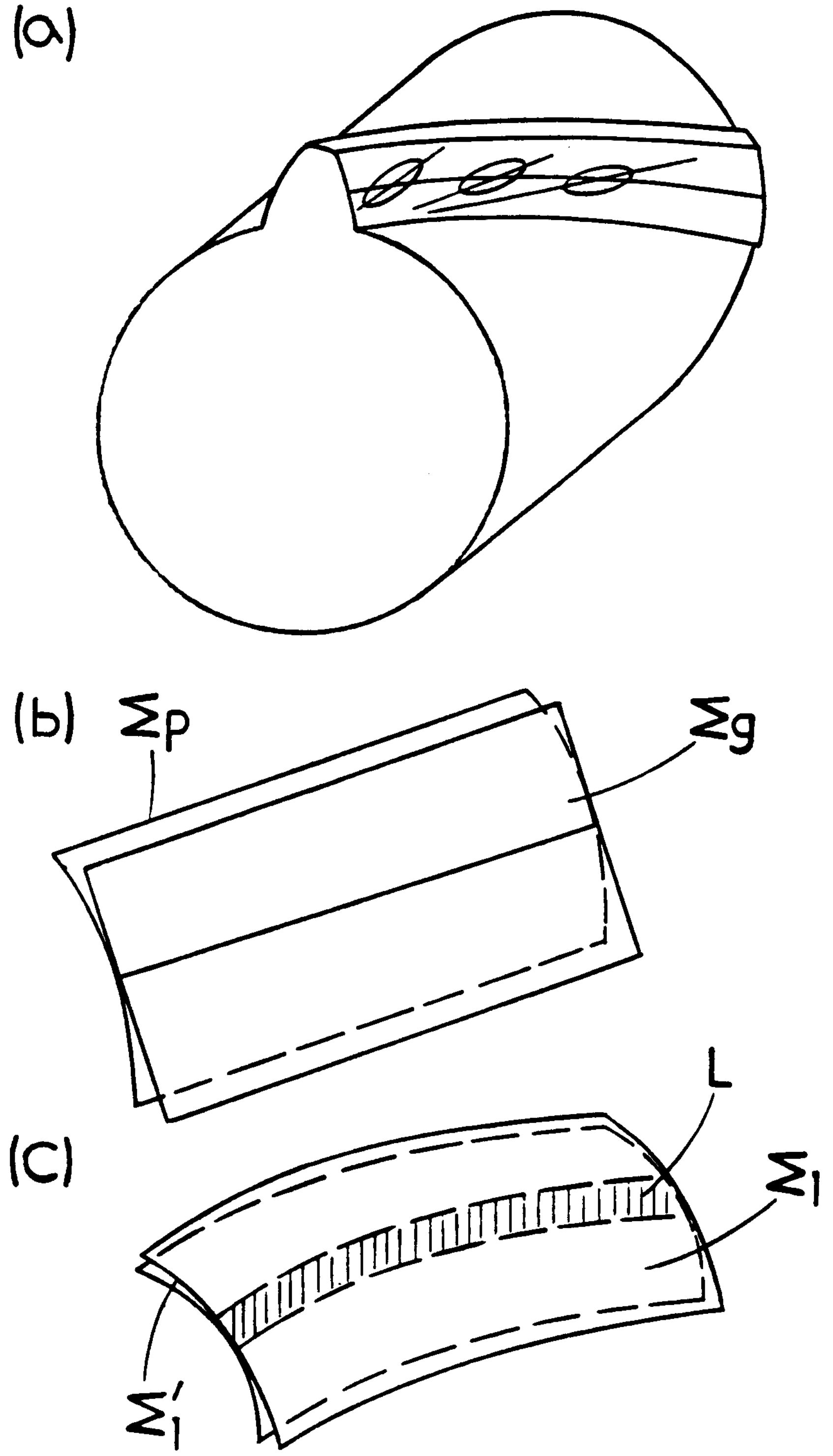


FIG. 4.1 Helical Gear and Generating Tool Surface

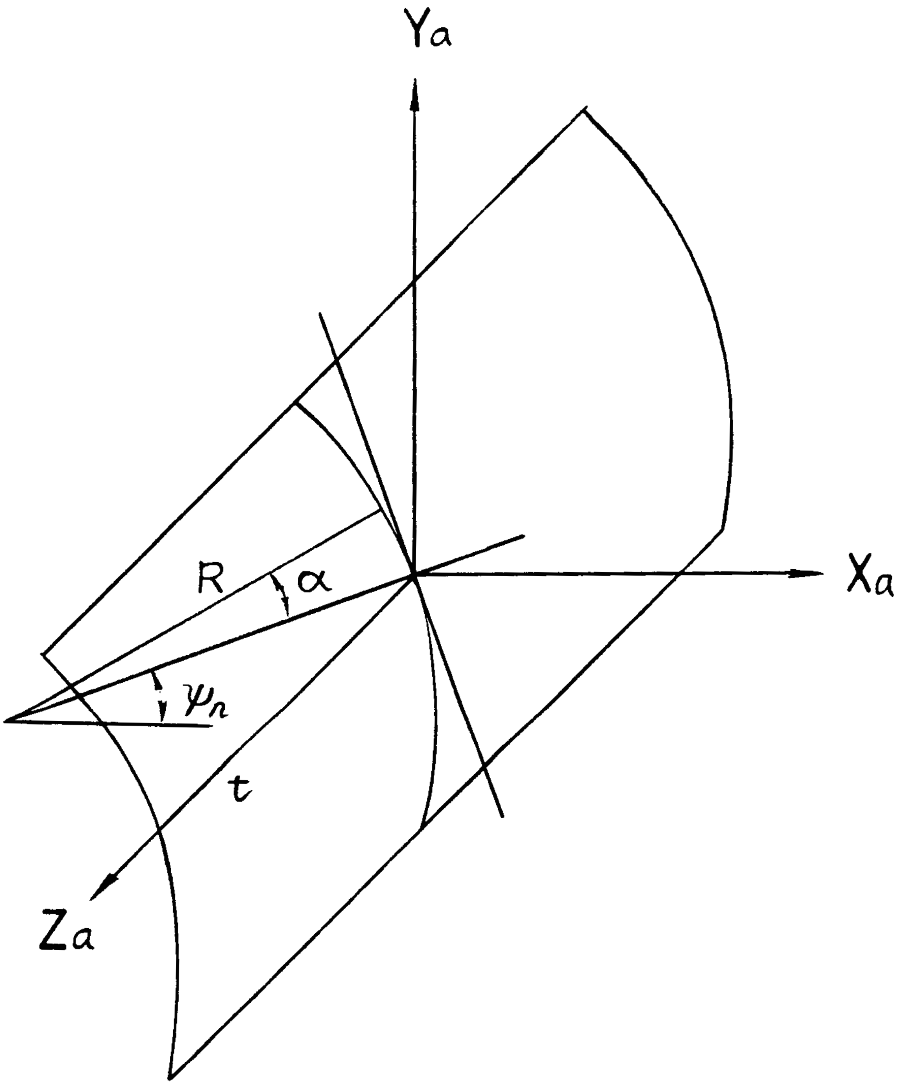


FIG. 4.2 Helical Pinion Generating Surface

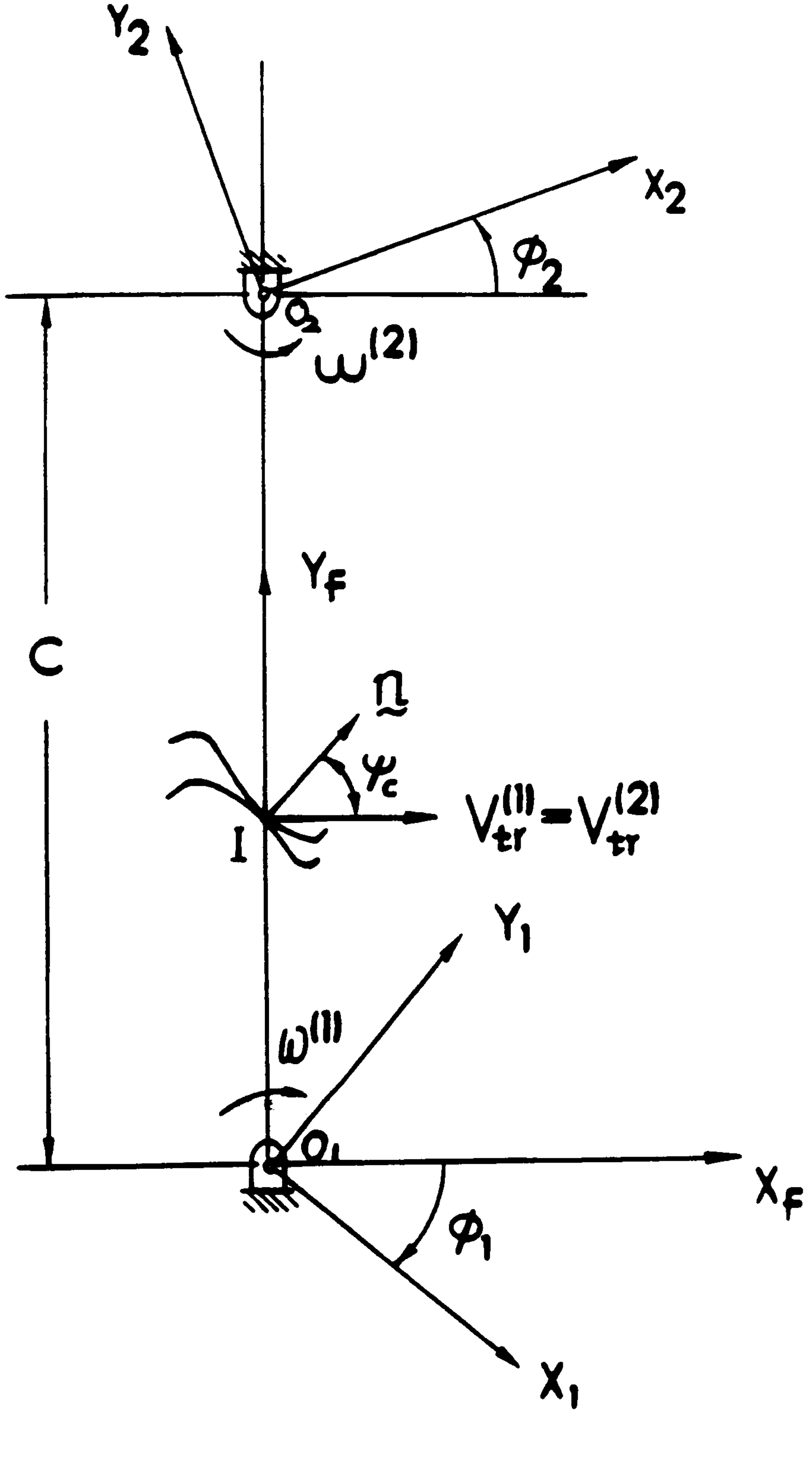


FIG. 4.3 Basic Coordinate Systems for Meshing Gears

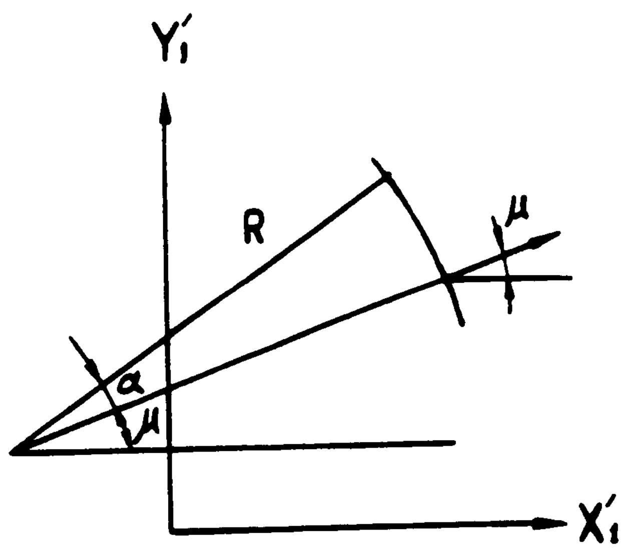


FIG. 4.4 Derivation of Deviated Helical Pinion Tooth Surface

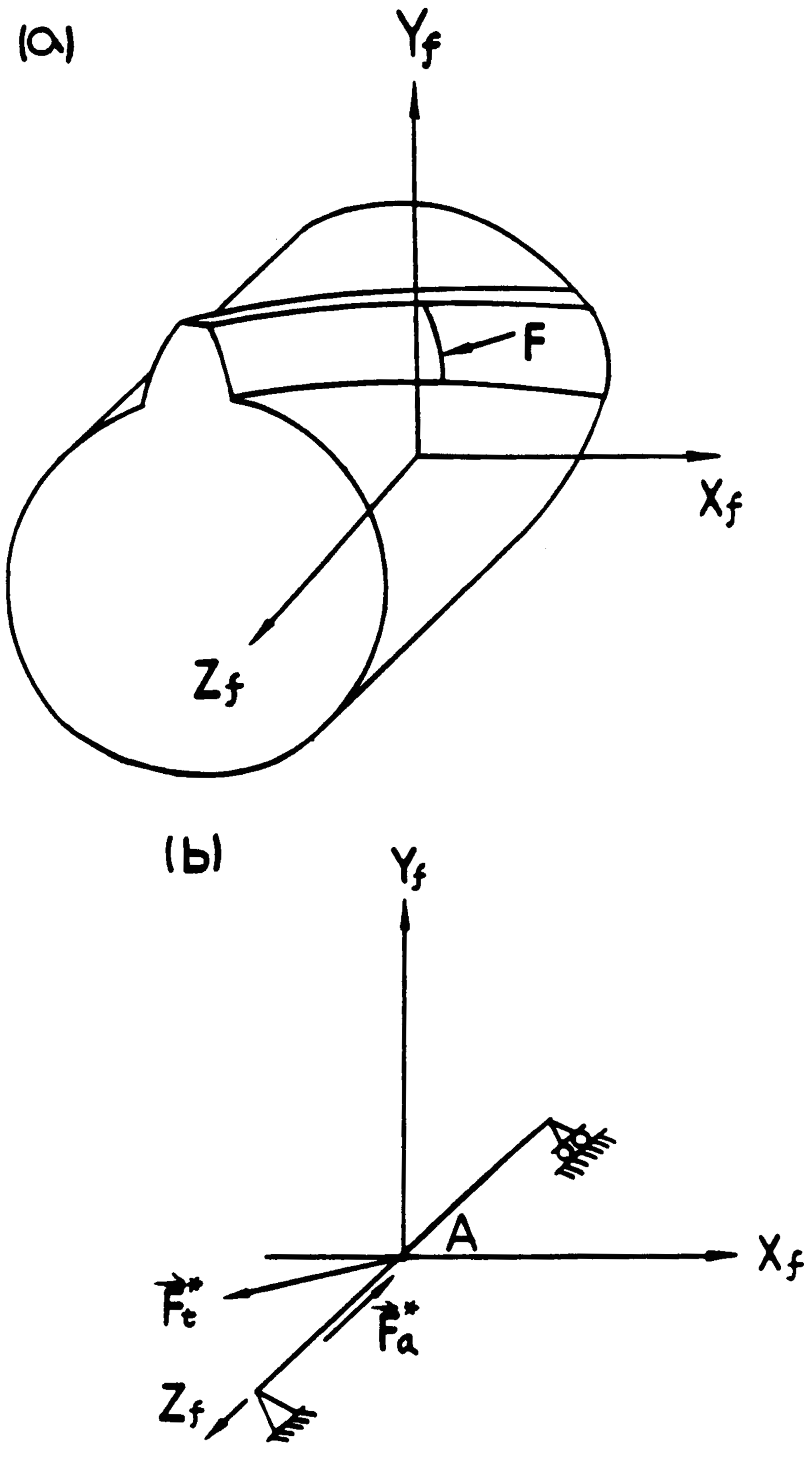


FIG. 5.1 Force Applied on Pinion and Its Shaft

C-2

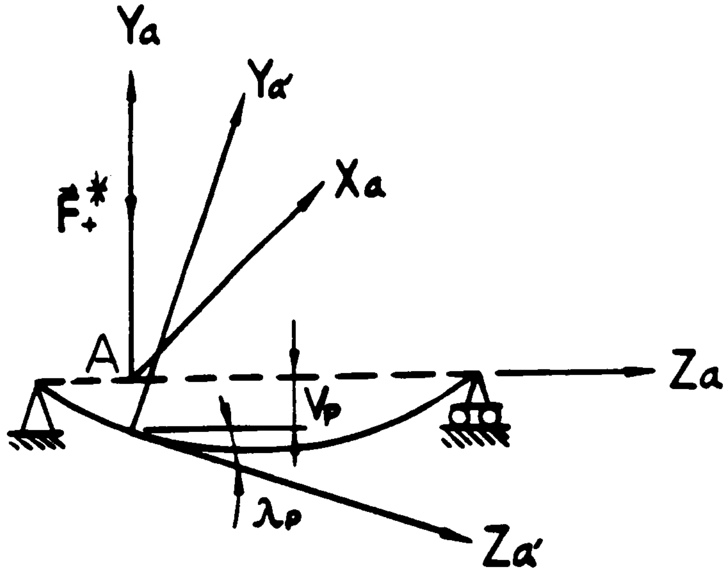


FIG. 5.2 Shaft Deflection

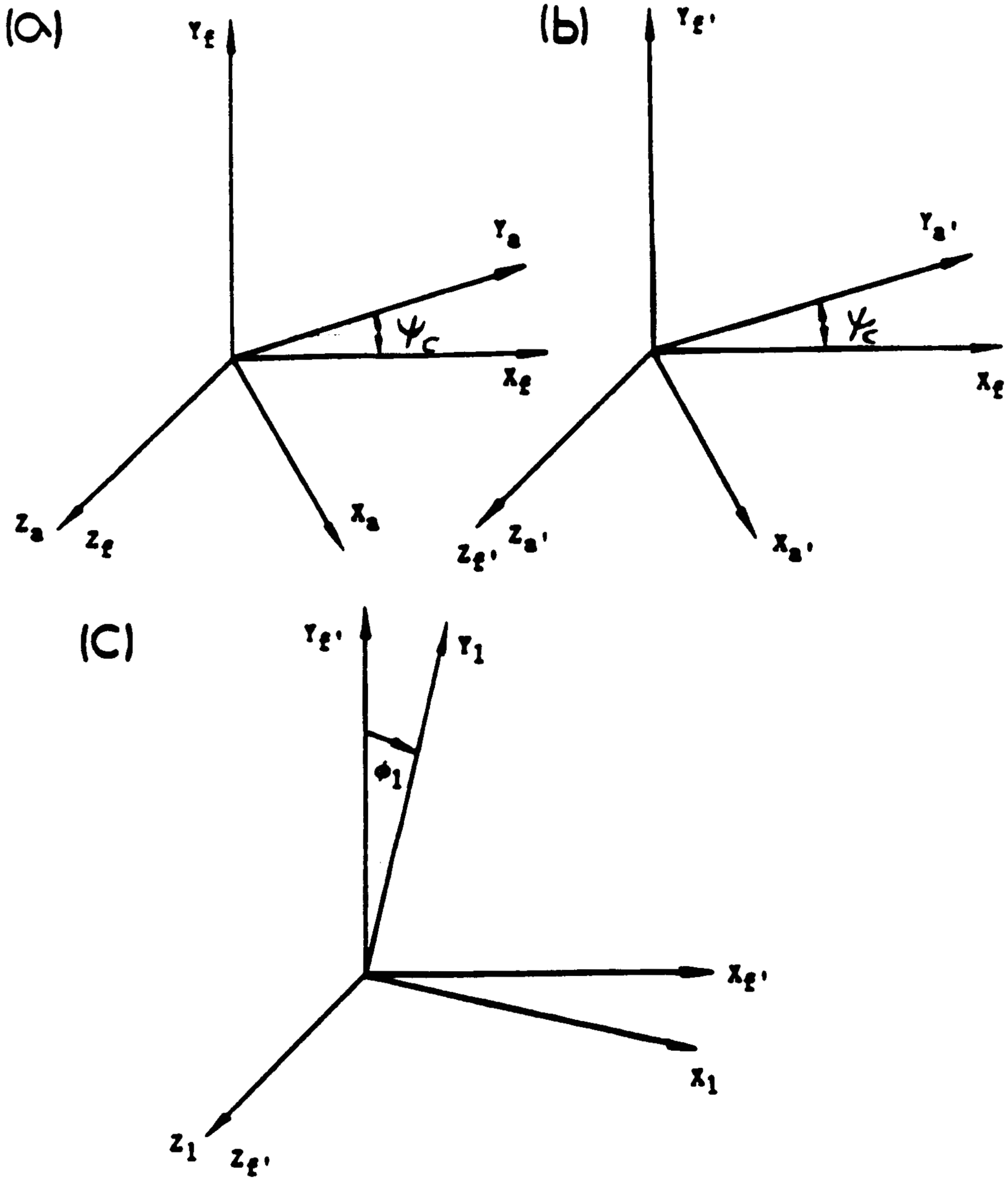


FIG. 5.3 Coordinate Systems for Simulation of Shaft Deformation

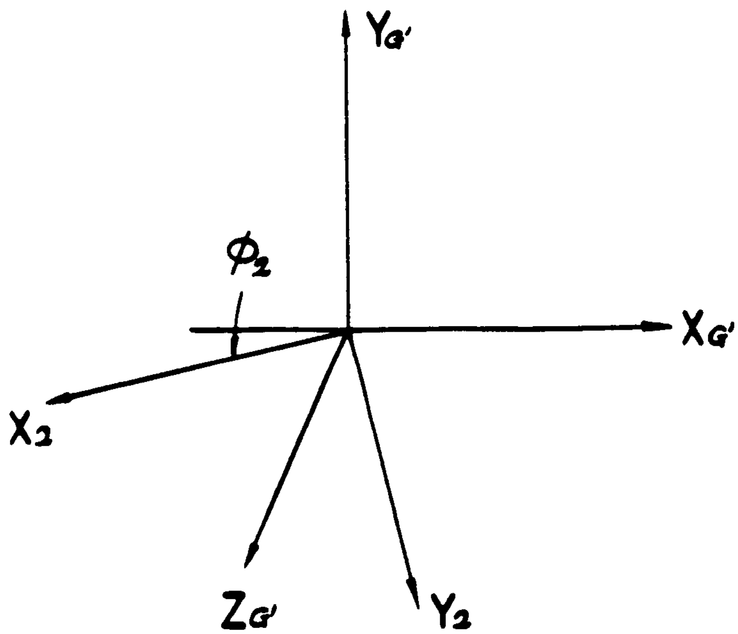


FIG. 5.4 Coordinate Systems for Simulation of Shaft Deformation

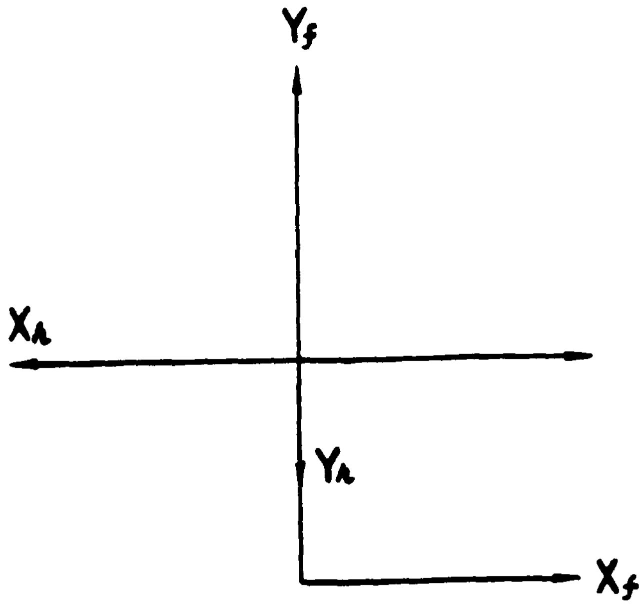
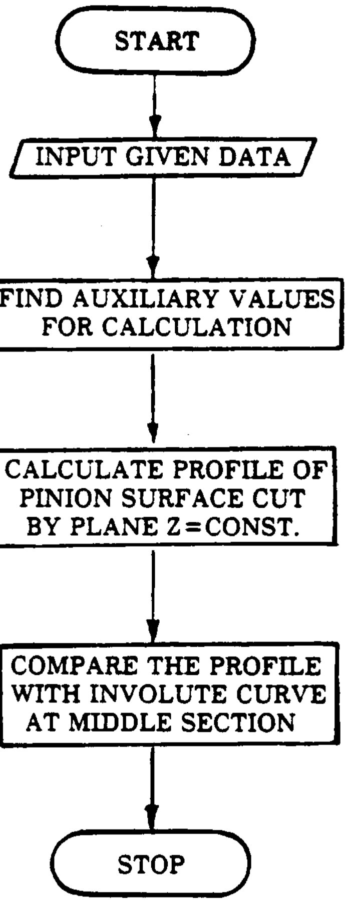


FIG. 5.5 Coordinate Systems for Simulation of Shaft Deformation

FLOWCHART FOR PROGRAM I



```
C... ******************************************************
C... *                                                    *
C... *                    PROGRAM I                       *
C... * SURFACE OF HELICAL PINION GENERATED BY CONE CUTTER *
C... *                                                    *
C... *             AUTHORS: FAYDOR LITVIN                 *
C... *                      JIAO ZHANG                    *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE SURFACE OF A HELICAL PINION
C     WHICH IS GENERATED BY CONE CUTTER
C
C NOTE
C
C   THIS PROGRAME IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILE IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,KSIC,MUO
      DIMENSION X(100),Y(100),ERROR(100),FEEREC(100),UREC(100),
     +          THETAR(100)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=1
C   (3) NSOLVE IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING
C         NONLINEAR EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE CLERANCE OF VARIABLE INCREMENT WHEN FUNCTION IS
C         SOLVED
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NSOLVE=100
      DELTA=1.D-15
      EPSI=1.D-15
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C        KSIN=PRESSURE ANGLE IN NORMAL SECTION (DEGREE);
C        BETAP=HELIX ANGLE (DEGREE);
C        HD=HEIGHT OF DEDENDUM OF PINION;
```

```
C         HA=HEIRHT OF ADDENDUM OF PINION
C         ZCOE=COEFFICIENT OF PINION TOOTH LENGTH (THE LENGTH= ZCOE/PN)
      PN=10.D0
      N1=20
      KSIN=20.D0*DR
      BETAP=20.D0*DR
      HD=1.D0/PN
      HA=1.D0/PN
      ZCOE=10.
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C         RC=RADIUS OF BOTTOM CIRCLE OF CONE;
C         MU=TILT ANGLE TO INSTALL PINION-CUTTING TOOL
      ALP=80.D0*DR
      MU0=DATAN(DSIN(KSIN)*DTAN(BETAP))
      MU=1.*MU0
      RC=1.D0
C   (3) OUTPUT:
C         NZ=NO. OF CROSS SECTIONS WHERE PINION PROFILE IS SIMULATED;
C         NU=NO. OF MAXIMUM POINTS USED FOR SIMULATE PINION PROFILE
C         UIN=INCREMENT OF PINION TOOTH SURFACE COORDINATE U
      NZ=21
      NU=61
      UIN=2.5D0*CH/(NU-1)
C
C  DESCRIPTION OF OUTPUT VARIABLES
C     Z1=DISTANCE BETWEEN CROSS SECTION CONSIDERED AND MIDDLE CROSS
C        SECTION
C     NO=OUTPUT NO.
C     U=TOOL SURFACE COORDINATE
C     THETA=TOOL SURFACE COORDINATE
C     FEE=PINION TOOTH SURFACE GENERATION PARAMETER
C     X1=X COORDINATE OF PINION PROFILE
C     Y1=Y COORDINATE OF PINION PROFILE
C     R1=RADIUS OF PINION PROFILE
C     VSH=AVERAGE DEVIATION SHIFT OF CROSS SECTION PROFILE FROM PROFILE
C           OF GENERAL PINION (INVOLUTE CURVE)
C     VPE=MAXIMUM DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE CURVE
C     VSD=STANDARD DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE
C         CURVE
C
C    THE PROGRAM IS WRITTEN BY JIAO ZHANG
      SK=DSIN(KSIN)
      CK=DCOS(KSIN)
      SB=DSIN(BETAP)
      CB=DCOS(BETAP)
      SM=DSIN(MU)
      CM=DCOS(MU)
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
      KSIC=DATAN(SK/CK/CB)
      CKC=DCOS(KSIC)
      SKC=DSIN(KSIC)
      PT=PN*CB
      RP=N1/2./PT
```

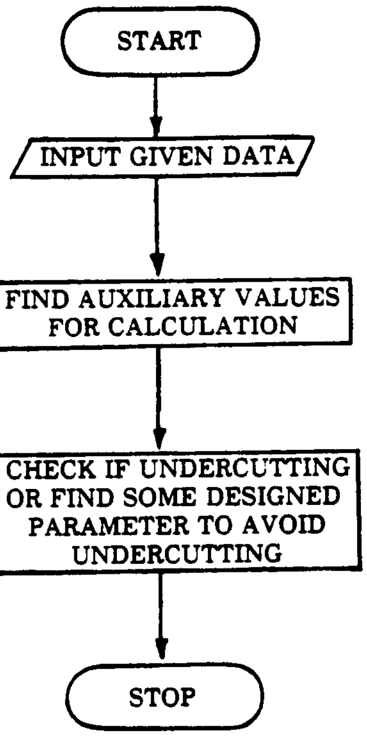
```
      RB=RP*DCOS(KSIC)
      RA=RP+HA
      CL=RC/SA
      CH=HD/CK/CM
      A1=CA*CK*CB+SA*SK*CM*CB+SA*SM*SB
      A2=SK*SM*CB-CM*SB
      A3=CA*SK*CM*CB-SA*CK*CB+CA*SM*SB
      A4=SK*CM*CB+SM*SB
      B1=CA*SK-SA*CK*CM
      B2=CK*SM
      B3=SA*SK+CA*CK*CM
      B4=CK*CM
      C1=CA*CK*SB+SA*SK*CM*SB-SA*SM*CB
      C2=SK*SM*SB+CM*CB
      C3=CA*SK*CM*SB-SA*CK*SB-CA*SM*CB
      C4=-SK*CM*SB+SM*CB
      D1=CM*CB+SK*SM*SB
      D2=SA*SM*CB-SB*(CA*CK+SA*SK*CM)
      D3=CA*CK*SB
      D4=CA*(CA*SK-SA*CK*CM)
      D5=SA*(SA*SK+CA*CK*CM)
      D6=CA*CK*SM
      DO 5 I=1,NZ
      Z1=-ZCOE/PN/2.+ZCOE/PN*FLOAT(I-1)/FLOAT(NZ-1)
      FEEO=Z1*SB/CB/RP
      CFO=DCOS(FEEO)
      SFO=DSIN(FEEO)
      R1=RP
      ERR=0.
      NP=0
      DO 15 J=1,NU
      IF (R1.GT.RA.AND.J.GT.1) GOTO 55
      U=CL-UIN*FLOAT(J-1)
      TEMP1=(Z1-(CL-CH)*C4-U*CA*C3)/U/SA/DSQRT(C1*C1+C2*C2)
      TEMP2=DARSIN(C1/DSQRT(C1*C1+C2*C2))
      TEMP3=DARSIN(TEMP1)
      THETA=TEMP3-TEMP2
      CT=DCOS(THETA)
      ST=DSIN(THETA)
      XC=U*SA*(CT*A1+ST*A2)+U*CA*A3-(CL-CH)*A4
      YC=U*SA*(CT*B1-ST*B2)-U*CA*B3+(CL-CH)*B4
      FEE=(U*(CT*D1+ST*D2)-(CL-CH)*((CT*CA*CA+SA*SA)*D1-ST*D3))/RP/
     +     (CT*D4+D5-ST*D6)
      CF=DCOS(FEE)
      SF=DSIN(FEE)
      X1=XC*CF+YC*SF-RP*FEE*CF+RP*SF
      Y1=-XC*SF+YC*CF+RP*FEE*SF+RP*CF
      R1=DSQRT(X1*X1+Y1*Y1)
      IF (R1.LT.RB) GOTO 15
      NP=NP+1
      X(NP)=X1
      Y(NP)=Y1
      FEEREC(NP)=FEE
      UREC(NP)=U
```

```
      THETAR(NP)=THETA
      XE=X1*CFO+Y1*SFO
      YE=-X1*SFO+Y1*CFO
      YS=YE
      FEET=FEE+FEEO
      DO 25 K=1,NSOLVE
      W=RP*FEET*CKC*DSIN(FEET-KSIC)+RP*DCOS(FEET)-YS
      DWDF=-RP*SKC*DCOS(FEET-KSIC)+RP*FEET*CK*DCOS(FEET-KSIC)
      DF=-W/DWDF
      IF (DABS(W).LT.EPSI.AND.DABS(DF).LT.DELTA) GOTO 65
      FEET=FEET+DF
   25 CONTINUE
   65 CONTINUE
      XS=-RP*FEET*CKC*DCOS(FEET-KSIC)+RP*DSIN(FEET)
      ERROR(NP)=XE-XS
C     WRITE (LP,110) K,W,DF,ERROR(NP),FEET,FEE+FEEO
C 110 FORMAT (1X,'K=',I5,5X,'W=',D15.7,5X,'DF=',D15.7,D15.7,2D25.17)
      ERR=ERR+ERROR(NP)
   15 CONTINUE
   55 VSH=-ERR/FLOAT(NP)
      VPE=0.
      VSD=0.
      DO 35 J=1,NP
      ETEMP=ERROR(J)+VSH
      IF (DABS(ETEMP).GT.VPE) VPE=DABS(ETEMP)
      VSD=VSD+ETEMP**2
   35 CONTINUE
      VSD=DSQRT(VSD/FLOAT(NP))
      WRITE (LP,10) Z1 ,VSH,VPE,VSD
   10 FORMAT(1H1,///'Z1=',F15.7,5X,'VSH=',F15.7,5X,'VPE=',F15.7,5X,
     +       'VSD',D15.7,5X/2X,'NO.',7X,'U',14X,'THETA',10X,'FEE',12X,
     +       'X1',13X,'Y1',13X,'R1',13X,'ERROR')
      DO 45 J=1,NP
      R1=DSQRT(X(J)**2+Y(J)**2)
      ETEMP=ERROR(J)+VSH
      WRITE (LP,20) J,UREC(J),THETAR(J),FEEREC(J),X(J),Y(J),R1,ETEMP
   20 FORMAT (2X,I2,2X,7F15.7)
   45 CONTINUE
    5 CONTINUE
      STOP
      END
$ENTRY
C  FIND AUXILIARY VALUES FOR CALCULATION
      RP=FLOAT(N1)/2.D0/PN
      CL=RC/DSIN(ALP)
      D=CL*DCOS(ALP)
      A1=CL-HDC/DCOS(RKS)
      RPU=RP+HAC
      DO 5 I=1,NL
      Z=FLOAT(I-1)*ZI
      YY=D-Z/DTAN(ALP)
      WRITE (LP,10) Z
   10 FORMAT (1H1/1X,'Z1=',F15.7/1X,'NO',10X,'Y1',13X,'XP',13X,'YP',
     +        13X,'R1',13X,'FEE')
```

```
C  CALCULATE THE PROPILE OF THE SURFACE CUT BY PLANE Z=CONST
      KKK=0
      DO 15 J=1,N
      Y1=YY*FLOAT(J-1)/FLOAT(N-1)
      F=DSQRT(1.-(Z/(D-Y1)/DTAN(ALP))**2)
      FEE=((D-Y1)*F/DCOS(ALP)-A1*(F*DCOS(ALP)*DCOS(ALP)+DSIN(ALP)*DSIN(
     +   ALP)))/RP/(DSIN(ALP)*DCOS(ALP-RKS)-F*DCOS(ALP)*DSIN(ALP-RKS))
      XP(I,J)=(D-Y1)*(DTAN(ALP)*F*DCOS(ALP-RKS+FEE)-DSIN(ALP-RKS+FEE))
     +   +A1*DSIN(FEE-RKS)-RP*FEE*DCOS(FEE)+RP*DSIN(FEE)
      YP(I,J)=-(D-Y1)*(DTAN(ALP)*F*DSIN(ALP-RKS+FEE)+DCOS(ALP-RKS+FEE))
     +   +A1*DCOS(FEE-RKS)+RP*FEE*DSIN(FEE)+RP*DCOS(FEE)
      R1=DSQRT(XP(I,J)**2+YP(I,J)**2)
      IF (R1.GT.RPU) THEN
      JJ=J-1
      XP(I,J)=XP(I,JJ)+(XP(I,J)-XP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
      YP(I,J)=YP(I,JJ)+(YP(I,J)-YP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
      FEE=FEETEM+(FEE-FEETEM)/(R1-R1TEMP)*(RPU-R1TEMP)
      Y1=Y1TEM+(Y1-Y1TEM)/(R1-R1TEMP)*(RPU-R1TEMP)
      R1=DSQRT(XP(I,J)**2+YP(I,J)**2)
      KKK=1
      END IF
      WRITE (LP,20) J,Y1,XP(I,J),YP(I,J),R1,FEE
   20 FORMAT (1X,I4,4F15.7,F15.7)
      IF (KKK.GT.0) GO TO 30
      FEETEM=FEE
      Y1TEM=Y1
      R1TEMP=R1
   15 CONTINUE
   30 NS(I)=J-1
      IF (I.NE.1) GO TO 55
      NS1=NS(1)
      GO TO 5
C PREPARATION OF INTERPLORATION
   55 NS2=NS(I)
      DO 105 L=1,NS2
      J=NS2+1-L
      IF (YP(1,NS1).GT.YP(I,J)) GO TO 110
  105 CONTINUE
  110 NS2=J
      NREC=2
      XERS=0.D0
      DO 115 L=1,NS2
      DO 125 J=NREC,NS1
      IF (YP(1,J).GT.YP(I,L)) GO TO 120
  125 CONTINUE
  120 NREC=J
      J1=J-1
      XERROR(L)=XP(I,L)-XP(1,J1)-(YP(I,L)-YP(1,J1))*(XP(1,J)-XP(1,J1))
     +           /(YP(1,J)-YP(1,J1))
      XERS=XERS+XERROR(L)
  115 CONTINUE
      XERS=-XERS/FLOAT(NS2)
      XPE=0.D0
      SDX=0.D0
```

```
      IF (NDBUG.GT.2) WRITE (LP,80)
   80 FORMAT (1H1,13X,'NO.',4X,'DIVIATION VALUE')
      DO 45 L=1,NS2
      XERROR(L)=XERROR(L)+XERS
      IF (NDBUG.GT.2) WRITE (LP,50) L,XERROR(L)
   50 FORMAT (13X,I3,2F15.7)
      IF (DABS(XERROR(L)).GT.XPE) XPE=DABS(XERROR(L))
      SDX=SDX+XERROR(L)**2
   45 CONTINUE
      SDX=DSQRT(SDX/FLOAT(NS2))
      WRITE (LP,40) XERS,XPE,SDX
   40 FORMAT (///1X,'XSH=',E15.7,5X,'XPE=',E15.7,5X,'SDX=',E15.7)
      IF (NDBUG.GT.2) WRITE (LP,60) NS1,NS2
   60 FORMAT (//5X,'NS1=',I6,6X,'NS2=',I6)
    5 CONTINUE
      STOP
      END
```

FLOWCHART FOR PROGRAM II



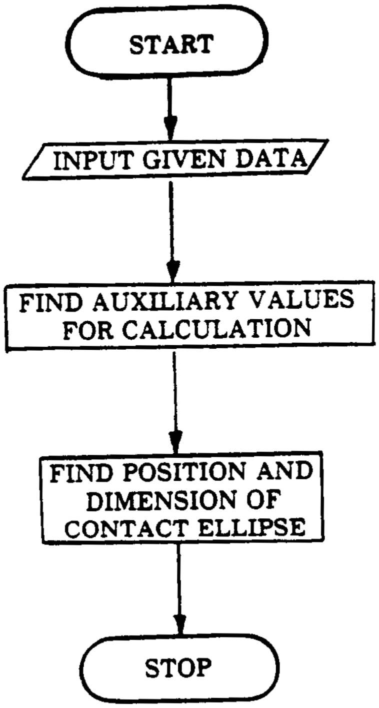
```
C... *******************************************************
C... *                                                     *
C... *                    PROGRAM II                       *
C... *     UDDERCUTTING CONDITION FOR HELICAL PINION       *
C... *             GENERATED BY CONE CUTTER                *
C... *               AUTHOR: FAYDOR LITVIN                 *
C... *                       JIAO ZHANG                    *
C... *                                                     *
C... *                                                     *
C... *******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO FIND THE UDDERCUTTING CONDITIONS FOR A
C     HELICAL PINION GENERATED BY CONE CUTTER
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILE IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C          DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C         KSIN=PRESSURE ANGLE; BETAP=HELIX ANGEL OF PINION;
C         HD=HEIGHT OF DEDENDUM OF PINION
      PN=10.D0
      N1=20
      KSIN=20.D0*DR
      BETAP=30.D0*DR
      HD=1.D0/PN
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C         RC=RADIUS OF BOTTOM CIRCLE OF CONE;
          MU=TILT ANGLE OF MOUNTING TOOL
      ALPHA=20.D0*DR
      RC=0.35D0
      MUO=DATAN(DSIN(KSIN)*DTAN(BETAP))
      MU=0.*MUO
C   (3) PROBLEM: NPROB=ID NO. OF PROBLEM (-1=GIVEN N1 AND HD, FIND IF
C         UDDERCUTTING OCCUR; 0=GIVEN N1, FIND MAXIMUM HD WITHOUT UDDER-
```

```
C         CUTTING; 1=GIVEN HD, FIND MINIMUM N1 WITHOUT UDDERCUTTING);
C         N=NO. OF THETA VALUES USED CALCULATION;
C         DELTHE=INCREMENT OF THETA(DEGREE)
      NPROB= 1
      N=21
      DELTHE=1.DO*DR
C
C DESCRIPTION OF OUTPUT
C
C   OUTPUT IS A STATEMENT BASED ON THE PROBLEM WITHOUT ANY LITERAL
C      PARAMETER
C
C  FIND AUXILIARY VALUES FOR CALCULATION
      SK=DSIN(KSIN)
      CK=DCOS(KSIN)
      SB=DSIN(BETAP)
      CB=DCOS(BETAP)
      SM=DSIN(MU)
      CM=DCOS(MU)
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
      PT=PN*CB
      RP=N1/2./PT
      CL=RC/SA
      CH=HD/CK/CM
      UU=CL-CH
      AA=CM*CB+SK*SM*SB
      BB=CA*CK*SB+SA*SK*CM*SB-SA*SM*CB
      CC=CA*CK*SB
      DD1=CA*SK-SA*CK*CM
      DD=DD1*CA
      EE1=SA*SK+CA*CK*CM
      EE=EE1*SA
      FF1=CK*SM
      FF=FF1*CA
      II=(CA*SK*CM*SB-SA*CK*SB-CA*SM*CB)*CA/SA
      WRITE (LP,90)
   90 FORMAT(1H1)
      IF (NPROB) 5,15,25
C  CHECK IF UNDERCUTTING OCCURS
    5 WRITE (LP,10)
   10 FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
     +        'B**2-4.*A*C',3X,'U1/(RC/SIN(ALPHA))')
      UMIN=5.DO
      DO 45 I=1,N
      NN=(N+1)/2
      THE=DBLE(FLOAT(I-NN))*DELTHE
      ST=DSIN(THE)
      CT=DCOS(THE)
      GG=DD*CT+EE-FF*ST
      WW=(BB*BB+AA*AA)*EE+(BB*DD-AA*FF)*II+ST*ST*ST*(AA*AA*FF-BB*BB*FF
     +    +2.*AA*BB*DD)-CT*CT*CT*(BB*BB*DD-AA*AA*DD+2.*AA*BB*FF)
     +    -ST*(2.*AA*AA*FF+AA*BB*DD-AA*EE*II)+CT*(2.*BB*BB*DD+AA*BB*FF
     +    +BB*EE*II)
```

```
      XX=((AA*FF*SA*SA-CC*EE)*CT+(AA*DD*SA*SA-AA*EE*CA*CA)*ST+(AA*FF*CA
     +    *CA-CC*DD))*(BB*CT+AA*ST+II)
      YY=DD1*SA*CT-FF1*SA*ST-EE1*CA
      ZZ=CM*CK
      A=YY*WW
      B=UU*(WW*ZZ+XX*YY)+RP*GG*GG*GG
      C=UU*UU*XX*ZZ
      D=B*B-4.*A*C
      IF (D.GE.0.) THEN
      U1=(-B-DSQRT(D))/2./A
      U2=(-B+DSQRT(D))/2./A
      U1=U1/CL
      IF (UMIN.GT.U1) UMIN=U1
      U2=U2/CL
      THE=THE/DR
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,D,U1
  100 FORMAT (1X,I4,8F15.7)
      ELSE
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD
      END IF
   45 CONTINUE
      IF (UMIN.LT.1.D0) WRITE (LP,110)
  110 FORMAT (///1X,'UDDERCUTTING WILL OCCUR FOR YOUR DESIGN')
      IF (UMIN.GE.1.D0) WRITE (LP,120)
  120 FORMAT (///1X,'UDDERCUTTING WILL NOT OCCUR FOR YOUR DESIGN')
      GO TO 35
C  DETERMINE THE MAXIMUM ADDENDUM HEIGHT OF RACK CUTTER
  15  WRITE (LP,20)
  20  FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
     +        'B**2-4.*A*C',3X,'ALLOWED RATIO OF HD/(1/PN)')
      UMIN=5.D0
      DO 55 I=1,N
      THE=DBLE(FLOAT(I-NN))*DELTHE
      ST=DSIN(THE)
      CT=DCOS(THE)
      GG=DD*CT+EE-FF*ST
      WW=(BB*BB+AA*AA)*EE+(BB*DD-AA*FF)*II+ST*ST*ST*(AA*AA*FF-BB*BB*FF
     +    +2.*AA*BB*DD)-CT*CT*CT*(BB*BB*DD-AA*AA*DD+2.*AA*BB*FF)
     +    -ST*(2.*AA*AA*FF+AA*BB*DD-AA*EE*II)+CT*(2.*BB*BB*DD+AA*BB*FF
     +    +BB*EE*II)
      XX=((AA*FF*SA*SA-CC*EE)*CT+(AA*DD*SA*SA-AA*EE*CA*CA)*ST+(AA*FF*CA
     +    *CA-CC*DD))*(BB*CT+AA*ST+II)
      YY=DD1*SA*CT-FF1*SA*ST-EE1*CA
      ZZ=CM*CK
      A=XX*ZZ
      B=CL*(WW*ZZ+XX*YY)
      C=CL*CL*YY*WW+CL*RP*GG*GG*GG
      D=B*B-4.*A*C
      TEST=DABS(A/B)
      EPS=1.D-16
      THE=THE/DR
      IF (D.GE.0.) THEN
      IF (TEST.GT.EPS) THEN
      U1=(-B+DSQRT(D))/2./A
```

```
      U2=(-B-DSQRT(D))/2./A
      ELSE
      U1=-C/B
      U2=0.
      END IF
      U1=(CL-U1)*CK*PN
      U2=(CL-U2)*CK*PN
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,D,U1,U2
      ELSE
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,D
      END IF
      IF (UMIN.GT.U1) UMIN=U1
   55 CONTINUE
      WRITE (LP,200) UMIN
  200 FORMAT (///1X,'TO AVOID UDDERCUTTING, IT IS NECESSARY TO KEEP DEDE
     +NDUM OF PINION <=',F10.7,'/PN')
      GO TO 35
C  DETERMINE THE MINIMUM NO. OF TEETH FOR UNUNDERCUTTING
   25 WRITE (LP,30)
   30 FORMAT (1H1/3X,'NO',7X,'THETA',7X,'NO. OF TEETH')
      RNMAX=0.
      DO 65 I=1,N
      THE=DBLE(FLOAT(I-NN))*DELTHE
      ST=DSIN(THE)
      CT=DCOS(THE)
      GG=DD*CT+EE-FF*ST
      WW=(BB*BB+AA*AA)*EE+(BB*DD-AA*FF)*II+ST*ST*ST*(AA*AA*FF-BB*BB*FF
     +     +2.*AA*BB*DD)-CT*CT*CT*(BB*BB*DD-AA*AA*DD+2.*AA*BB*FF)
     +     -ST*(2.*AA*AA*FF+AA*BB*DD-AA*EE*II)+CT*(2.*BB*BB*DD+AA*BB*FF
     +     +BB*EE*II)
      XX=((AA*FF*SA*SA-CC*EE)*CT+(AA*DD*SA*SA-AA*EE*CA*CA)*ST+(AA*FF*CA
     +     *CA-CC*DD))*(BB*CT+AA*ST+II)
      YY=DD1*SA*CT-FF1*SA*ST-EE1*CA
      ZZ=CM*CK
      RR=-(CL*CL*YY*WW+CL*UU*(WW*ZZ+XX*YY)+UU*UU*XX*ZZ)/CL/GG**3
      RN=2.*RR*PT
      THE=THE/DR
      IF (RNMAX.LT.RN) RNMAX=RN
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,RN
   65 CONTINUE
      WRITE (LP,300) RNMAX
  300 FORMAT (/// 1X,'WITHOUT UDDERCUTTING, MINIMUM TOOTH NO. OF PINION
     +IS:',F11.7)
   35 WRITE (LP,400)
  400 FORMAT(1H1)
      STOP
      END
```

FLOWCHART FOR PROGRAM III



```
C... ******************************************************
C... *                                                    *
C... *                     PROGRAM III                    *
C... *  CONTACT ELLIPSIS FOR HELICAL PINION GENERATED BY  *
C... *  CONE CUTTER IN MESHING WITH REGULAR HELICAL GEAR  *
C... *                                                    *
C... *             AUTHORS: FAYDOR LITVIN                 *
C... *                      JIAO ZHANG                    *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO FIND THE SHAPE AND ORIENTAION OF THE CONTACT
C      ELLIPSE WHEN A HELICAL PINION CROWNED BY CONE CUTTER IS IN
C      MESHING WITH A REGULAR HELICAL GEAR
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,MUO,MUT,KSIC,KF,KH,KSP,KQP,KSG,KQG
      DIMENSION CMM(3,4),EFD(3),EHD(3),RND(3),RD(4),EFF(3),EHF(3),
     +          RNF(3),RC(3),RF(3),R1(3)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE IN1UT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C         MPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C         KSIN=PRESSURE ANGLE IN NORMAL SECTION;
C         BETAP=HELIX ANGEL;
C         HD=HEIGHT OF DEDENDUM OF PINION
      PN=10.D0
      N1=20
      MPG=2
      KSIN=20.D0*DR
      BETAP=10.D0*DR
      HD=1./PN
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C         RC=RADIUS OF BOTTOM CIRCLE OF CONE;
```

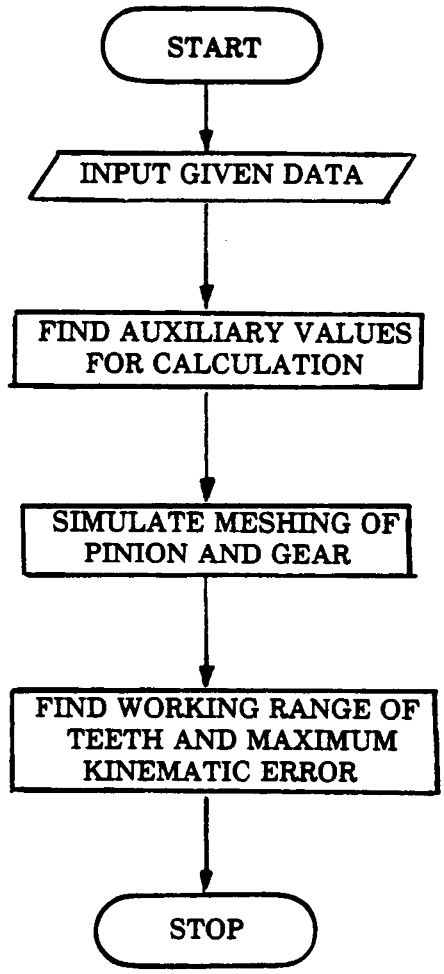
```
C         MU=TILT ANGEL TO INSTALL PINION CUTTING TOOL
      ALPHA=80.D0*DR
      MUO=DATAN(DSIN(KSIN)*DTAN(BETAP))
      MU=0.*MUO
      RC=1.D0
C    (3) DEFORMATION: DEL=CONTACT DEFORMATION AT CONTACT POINT
      DEL=4.D-4
C    (4) OUTPUT: NU=NUMBER OF CONTACT POINTS IN MATING SURFACES FOR US
C         TO CALCULATE CONTACT ELLIPSES)
      NU=101
C
C DESCRIPTION OF OUTPUT PARAMETER
C
C    R1=PINION RADIUS OF CONTACT POINT
C    ALPHA=THE ROTATION ANGLE BETWEEN PRINCIPAL DIRECTION OF PINION
C          TOOTH SURFACE AND AXES OF CONTACT ELLIPSE
C    A=LENGTH OF HALF SHORT AXIS OF CONTACT ELLIPSE
C    B=LENTH OF HALF LONG AXIS OF CONTACT ELLIPSE(ALONG DIRECTION OF
C      GEAR TOOTH LEHGTH)
C    RNF=UNIT NORMAL OF PINION TOOTH SURFACE AT CONTACT POINT
C    EFF=PRINCIPAL DIRECTION OF PINION TOOTH SURFACE AT CONTACT POINT
C    EHF=PRINCIPAL DIRECTION OF PINION TOOTH SURFACE AT CONTACT POINT
C
C  FIND AUXILIARY VALUES FOR CALCULATION
      SK=DSIN(KSIN)
      CK=DCOS(KSIN)
      SB=DSIN(BETAP)
      CB=DCOS(BETAP)
      SM=DSIN(MU)
      CM=DCOS(MU)
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
      KSIC=DATAN(SK/CK/CB)
      CKC=DCOS(KSIC)
      SKC=DSIN(KSIC)
      PT=PN*CB
      RP=N1/2./PT
      RB=RP*DCOS(KSIC)
      RA=RP+1./PN
      N2=N1*MPG
      RG=RP*MPG
      CL=RC/SA
      CH=HD/CK/CM
      A=CL-CH
      CMM(1,1)=CA*CK*CB+SA*SK*CM*CB+SA*SM*SB
      CMM(1,2)=SA*CK*CB-CA*SK*CM*CB-CA*SM*SB
      CMM(1,3)=SK*SM*CB-CM*SB
      CMM(1,4)=-(SK*CM*CB+SM*SB)
      CMM(2,1)=CA*SK-SA*CK*CM
      CMM(2,2)=SA*SK+CA*CK*CM
      CMM(2,3)=-CK*SM
      CMM(2,4)=CK*CM
      CMM(3,1)=CA*CK*SB+SA*SK*CM*SB-SA*SM*CB
      CMM(3,2)=SA*CK*SB-CA*SK*CM*SB+CA*SM*CB
```

```
      CMM(3,3)=SK*SM*SB+CM*CB
      CMM(3,4)=-SK*CM*SB+SM*CB
      D1=CM*CB+SK*SM*SB
      D2=SA*SM*CB-SB*(CA*CK+SA*SK*CM)
      D3=CA*CK*SB
      D4=CA*(CA*SK-SA*CK*CM)
      D5=SA*(SA*SK+CA*CK*CM)
      D6=CA*CK*SM
      UL=CL-2.D0*CH
      UU=CL
      MUT=MUO/DR
      WRITE (LP,110) RP,RA,RB,MUT
110   FORMAT (///1X,'RP=',F15.7,5X,'RA=',F15.7,5X,'RB=',F15.7,5X,
     +              'MUO=',F15.7)
      TH=0.D0
      ST=DSIN(TH*DR)
      CT=DCOS(TH*DR)
      EFD(1)=ST
      EFD(2)=0.D0
      EFD(3)=-CT
      EHD(1)=-CT*SA
      EHD(2)=CA
      EHD(3)=-ST*SA
      RND(1)=CT*CA
      RND(2)=SA
      RND(3)=ST*CA
      CALL MATMUL(CMM,EFD,EFF,3,3)
      CALL MATMUL(CMM,EHD,EHF,3,3)
      CALL MATMUL(CMM,RND,RNF,3,3)
      WRITE (LP,10) TH,(EFF(M),M=1,3),(EHF(M),M=1,3),(RNF(M),M=1,3)
 10   FORMAT(1H1,///3X,'THTEA:',F15.7,' DEGREE'//1X,'EFF:',3F15.7/1X,
     +       'EHF:',3F15.7/1X,'NF :',3F15.7)
      DO 15 J=1,NU
      U=UU-(UU-UL)/FLOAT(NU-1)*FLOAT(J-1)
      KF=-CA/SA/U
      KH=0.D0
      RD(1)=U*CT*SA
      RD(2)=-U*CA
      RD(3)=U*ST*SA
      RD(4)=A
      CALL MATMUL(CMM,RD,RC,3,4)
      FEE=(U*(CT*D1+ST*D2)-A*((CT*CA*CA+SA*SA)*D1-ST*D3))/(CT*D4+D5-ST*
     +     D6)/RP
      RF(1)=RC(1)-RP*FEE
      RF(2)=RC(2)+RP
      RF(3)=RC(3)
      RPP=DSQRT(RF(1)**2+RF(2)**2)
      CALL PROT(RF,R1,FEE)
      TEMP=RC(2)*EFF(1)-RF(1)*EFF(2)
      B13=EHF(3)-KF*TEMP
      B23=-EFF(3)
      B33=-(RNF(1)*RF(1)+RNF(2)*RF(2)+KF*TEMP*TEMP)
      SIGMA=0.5D0*DATAN(2.*B13*B23/(B23*B23-B13*B13-(KF-KH)*B33))
      COE1=(B23*B23-B13*B13-(KF-KH)*B33)/B33/DCOS(2.*SIGMA)
```

```
      COE2=KF+KH+(B13*B13+B23*B23)/B33
      KSP=(COE2-COE1)/2.D0
      KQP=(COE2+COE1)/2.D0
      IF (NDBUG.LT.1) WRITE (LP,20) U,FEE,RPP,(R1(M),M=1,3),
     +                              B13,B23,B33,SIGMA,KSP,KQP
   20 FORMAT(//1X,'U  =',F15.7,5X,'FEE=',F15.7,5X,'R1 =',F15.7/
     +         1X,'XP =',F15.7,5X,'YP =',F15.7,5X,'ZP =',F15.7/
     +         1X,'B13=',F15.7,5X,'B23=',F15.7,5X,'B33=',F15.7/
     +         1X,'SIG=',F15.7,5X,'KSP=',F15.7,5X,'KQP=',F15.7)
      W=-(A-U)
      KSG=0.
      KQG=1./(RG*SK*(CKC/CK/CB)**2+W*CM*CK/SK)
      G1=KSP-KQP
      G2=KSG-KQG
      S1=KSP+KQP
      S2=KSG+KQG
      SIGMGP=-SIGMA
      ALPHA1=0.5D0*DATAN(G2*DSIN(2.*SIGMGP)/(G1-G2*DCOS(2.*SIGMGP)))
      ALPHA2=ALPHA1+SIGMGP
      AA=(S1-S2-DSQRT(G1*G1-2.*G1*G2*DCOS(2.*SIGMGP)+G2*G2))/4.
      BB=(S1-S2+DSQRT(G1*G1-2.*G1*G2*DCOS(2.*SIGMGP)+G2*G2))/4.
      AAA=1./DSQRT(DABS(AA))
      BBB=1./DSQRT(DABS(BB))
      RATIO=BBB/AAA
      ALPHA1=ALPHA1/DR
      ALPHA2=ALPHA2/DR
C     WRITE (LP,130) G1,G2,S1,S2,ALPHA1
C 130 FORMAT (1X,5F15.7)
      WRITE (LP,30) RPP,ALPHA2,AAA,BBB,RATIO
   30 FORMAT (1X,'R1 =',F15.7,5X,'ALP=',F15.7,5X,'A =',F15.7,5X,
     +           'B  =',F15.7,5X,'B/A=',F15.7)
      IF (RPP.GT.RA) GO TO 5
   15 CONTINUE
    5 STOP
      END
C
C
      SUBROUTINE MATMUL(CMM,A,B,N,M)
C  THIS SUBROUTINE IS USED TO MULTIPLY THE MATRIX CMM(N*M) BY THE MATRIX
C     A(M*1). THE RESULT IS STORED IN THE MATRIX B(N*1)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CMM(3,M),A(M),B(N)
      DO 5 I=1,N
    5 B(I)=0.
      DO 15 I=1,N
      DO 15 J=1,M
   15 B(I)=B(I)+CMM(I,J)*A(J)
      RETURN
      END
C
C
      SUBROUTINE PROT(A,B,FEE)
C  THIS SUBROUTINE IS USED TO ROTATE COORDINATE SYSTEM PLANARLY IN XOY
C     THROUGH ANGLE FEE
```

```
DOUBLE PRECISION A(3),B(3),FEE
B(1)=A(1)*DCOS(FEE)+A(2)*DSIN(FEE)
B(2)=A(2)*DCOS(FEE)-A(1)*DSIN(FEE)
B(3)=A(3)
RETURN
END
```

FLOWCHART FOR PROGRAM IV



```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                     PROGRAM IV                     *
C... *  TRANSMISSION ERRORS OF HELICAL PINION GENERATED   *
C... *      BY CUTTER WITH CONE OR REVOLUTE SURFACE       *
C... *        IN MESHING WITH REGULAR HELICAL GEAR        *
C... *                                                    *
C... *              AUTHORS: FAYDOR LITVIN                *
C... *                       JIAO ZHANG                   *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE TRANSMISSION ERRORS OF A
C     PINION GENERATED BY THE CUTTER WITH CONE OR REVOLUTE SURFACE
C     IN MESHING WITH A MISALIGNED REGULAR HELICAL GEAR
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,MUO,KSIC
      DIMENSION Z(99),ANGLE(99),ERROR(99),ERR(99)
      COMMON /BLOCK1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCK2/ S(4,4),CMM(4,4),CMCD(4,4),C,R,RP,RG,A1,HD,RL,KSIC,
     +                ALPHA,CK,SK,CB,SB,CM,SM,CA,SA,CKC,SKC,NTOOL
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=1
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      DELTA=1.D-3
      EPSI=1.D-12
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      NDIM=10
      NE=5
```

```
      DR=DATAN(1.D0)/45.D0
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; NP=PINION TOOTH NUMBER;
C        RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./NP);
C        KSIN=PRESSURE ANGLE IN NORMAL SECTION;
C        BETAP=HELIX ANGLE OF PINION AND GEAR;
C        HD=HEIGHT OF DEDENDUM OF PINION;
C        COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)
      PN=10.D0
      NP=20
      RMPG=2.D0
      KSIN=20.D0*DR
      BETAP=15.D0*DR
      HD=1.D0/PN
      COE=1.000D0
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C        RC=RADIUS OF BOTTOM CIRCLE OF CONE
C        R=RADIUS OF ARC
C        MU=TILT ANGEL OF MOUNTING TOOL
C        NTOOL=TOOL ID NO. (1=CONE SURFACE, 2=REVOLUTE SURFACE)
      NTOOL=2
      ALPHA=20.D0*DR
      RC=10.3527D0
      R=3.0D1
      MUO=DATAN(DSIN(KSIN)*DTAN(BETAP))
      MU= 0.*MUO
C   (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C        NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM -(NG-1)/2 TO
C           (NG-1)/2 TIMES GAMMAI WITH ODD NG);
C        GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
      NMIS=2
      NG=2
      GAMMAI=5.D0
C   (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
      FEEI=1.0D0*DR
C
C  DESCRIPTION OF OUTPUT PARAMERTERS
C
C    FEE1=ROTATION ANGLE OF PINION
C    FEE2=ROTATION ANGLE OF GEAR
C    RP=RADIUS OF PINION CONTACT POINT
C    RG=RADIUS OF GEAR CONTACT POINT
C
C  FIND AUXILIARY VALUES FOR CALCULATION
C
C
      SK=DSIN(KSIN)
      CK=DCOS(KSIN)
      SB=DSIN(BETAP)
      CB=DCOS(BETAP)
      SM=DSIN(MU)
      CM=DCOS(MU)
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
```

```
      KSIC=DATAN(SK/CK/CB)
      CKC=DCOS(KSIC)
      SKC=DSIN(KSIC)
      PT=PN*CB
      RP=NP/2./PT
C     RB=RP*DCOS(KSIC)
C     RA=RP+1./PN
C     NG=NP*RMPG
      RG=RP*RMPG
      RL=RC*CA/SA
C     CH=HD/CK/CM
      A1=RL/CA-HD/CK/CM
      C=(RP+RG)*COE
      NCOEF=360.D0*DR/FEEI/FLOAT(N1)+0.3
      N=NCOEF*2+1
      CALL INTMAT(CMCD,4,4)
      AA=RL*SA*SA/CA-HD/CK/CM
      CMCD(1,1)=CA*CK*CB+SA*SK*CM*CB+SA*SM*SB
      CMCD(1,2)=SA*CK*CB-CA*SK*CM*CB-CA*SM*SB
      CMCD(1,3)=SK*SM*CB-CM*SB
      CMCD(1,4)=-RL*SA*CK*CB-AA*(SK*CM*CB+SM*SB)
      CMCD(2,1)=CA*SK-SA*CK*CM
      CMCD(2,2)=SA*SK+CA*CK*CM
      CMCD(2,3)=-CK*SM
      CMCD(2,4)=-RL*SA*SK+AA*CK*CM
      CMCD(3,1)=CA*CK*SB+SA*SK*CM*SB-SA*SM*CB
      CMCD(3,2)=SA*CK*SB-CA*SK*CM*SB+CA*SM*CB
      CMCD(3,3)=SK*SM*SB+CM*CB
      CMCD(3,4)=-RL*SA*CK*SB+AA*(-SK*CM*SB+SM*CB)
      CALL INTMAT(S,4,4)
      NGG=(NG-1)/2
      DO 505 LL=1,NG
      GAMMA=GAMMAI*FLOAT(LL-NGG)/60.D0*DR
      CG=DCOS(GAMMA/60.*DR)
      SG=DSIN(GAMMA/60.*DR)
      IF (NMIS.EQ.1) THEN
      WRITE (LP,500) COE,GAMMA
  500 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG)      CROSSING ANGLE=',
     +        F5.1,'(M)')
      S(1,1)=CG
      S(1,3)=-SG
      S(3,1)=SG
      S(3,3)=CG
      ELSE
      WRITE (LP,501) COE,GAMMA
  501 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG)      INTERSECTING ANGLE=',
     +        F5.1,'(M)')
      S(2,2)=CG
      S(2,3)=-SG
      S(3,2)=SG
      S(3,3)=CG
      END IF
      DO 205 L=1,2
      DO 5 I=1,NE
```

```
    5 X(I)=0.D0
      IF (NTOOL.EQ.1) X(5)=RL/CA-HD/CK/CM
C     WRITE (6,1100) (X(JK),JK=1,5),RL,CA,HD,CK,CM
C1100 FORMAT (1X,'######',5F15.7/7X,5F15.7)
      DO 15 I=1,N
      X(7)=FEEI*FLOAT(I-(N+1)/2)
      IF (L.EQ.1) X(7)=0.D0
C     X(5)=DARSIN(RP*X(7)/SK/R)
      X(2)=X(7)/RMPG
      CALL NONLIN
      X(8)=X(2)+X(3)
      IF (L.EQ.1) THEN
      XIN=X(8)
      WRITE (LP,10)
   10 FORMAT (////8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
     +        'RP',13X,'RG',F15.7/)
      GO TO 205
      END IF
      X(8)=X(8)-XIN
      X(7)=X(7)/DR
      X(8)=X(8)/DR
      X(9)=(X(8)-X(7)/RMPG)*3600.D0
      Z(I)=X(8)
      ERR(I)=X(9)
      WRITE (LP,20) (X(J),J=7,11)
   20 FORMAT (1X,5F15.7,F15.7)
C     WRITE (LP,30) (X(J),J=1,6)
C  30 FORMAT (1X,'#####',6F15.7)
C     WRITE (LP,30) XP,YP,ZP,XG,YG
   15 CONTINUE
      NT=N-NCOEF
      FII=FEEI/DR/2.D0*FLOAT(NCOEF)
      WRITE (LP,80)
   80 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:',F15.7/)
      DO 55 I=1,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE(I)=Z(KK)-Z(I)
      ERROR(I)=(ANGLE(I)-FII)*3600.D0
      WRITE (LP,60) X(7),X(8),ANGLE(I),ERROR(I)
   60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
   55 CONTINUE
      DO 95 I=1,NT
      ATEMP2=ERROR(I)
      IF (I.NE.1) THEN
      IF (ATEMP1*ATEMP2.LE.0.D0) GOTO 105
      END IF
      ATEMP1=ATEMP2
   95 CONTINUE
      WRITE (LP,160)
  160 FORMAT (//1X,'MESHING IS DISCONTINUOUS')
  105 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.
```

```
      EMIN=0.
      DO 135 J=1,NCOEF
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
      IF (ET.GT.EMAX) EMAX=ET
  135 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,170) Z(I),Z(KK),ET
  170 FORMAT (//1X,'WORKING RANGE FOR ONE TOOTH: ',F7.2,'----',F7.2/
     +          1X,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,'  (S)',I2)
  205 CONTINUE
  505 CONTINUE
      STOP
      END
C
      SUBROUTINE FUNC
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,MUO,KSIC
      COMMON /BLOCK1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCK2/ S(4,4),CMM(4,4),CMCD(4,4),C,R,RP,RG,A1,HD,RL,KSIC,
     +                ALPHA,CK,SK,CB,SB,CM,SM,CA,SA,CKC,SKC,NTOOL
      DIMENSION RD(4),RND(4),RC(4),RNC(4),RF(4)
      CFEE=DCOS(X(3))
      SFEE=DSIN(X(3))
      CT=DCOS(X(4))
      ST=DSIN(X(4))
      CF=DCOS(X(7))
      SF=DSIN(X(7))
      IF (NTOOL.EQ.1) THEN
      RD(1)=X(5)*CT*SA
      RD(2)=RL-X(5)*CA
      RD(3)=X(5)*ST*SA
      RND(1)=CT*CA
      RND(2)=SA
      RND(3)=ST*CA
      ELSE
      CAL=DCOS(ALPHA+X(5))
      SAL=DSIN(ALPHA+X(5))
      SL2=DSIN(X(5)/2.)
      CAL2=DCOS(ALPHA+X(5)/2.)
      SAL2=DSIN(ALPHA+X(5)/2.)
      RD(1)=(A1*SA-2.*R*SL2*SAL2)*CT
      RD(2)=HD/CK/CM*CA+2.*R*SL2*CAL2
      RD(3)=(A1*SA-2.*R*SL2*SAL2)*ST
      RND(1)=CAL*CT
      RND(2)=SAL
      RND(3)=CAL*ST
      END IF
      RD(4)=1.
      CALL MATMUL(CMCD,RD,RC,4,4)
```

```
      CALL MATMUL(CMCD,RND,RNC,3,3)
      X(6)=(RC(1)-RC(2)*RNC(1)/RNC(2))/RP
      RF(1)=RC(1)-RP*X(6)
      RF(2)=RC(2)+RP
      RF(3)=RC(3)
      RF(4)=1.
      CFPF1=DCOS(X(6)+X(7))
      SFPF1=DSIN(X(6)+X(7))
      XPF=RF(1)*CFPF1+RF(2)*SFPF1
      YPF=RF(2)*CFPF1-RF(1)*SFPF1
      ZPF=RF(3)
      XNPF=RNC(1)*CFPF1+RNC(2)*SFPF1
      YNPF=RNC(2)*CFPF1-RNC(1)*SFPF1
      ZNPF=RNC(3)
      CF2FG=DCOS(X(3))
      SF2FG=DSIN(X(3))
      A2=-X(1)*SB+RG*X(2)
      RFG1=CK*CK/CKC*DCOS(X(3)+KSIC)*A2+RG*SF2FG
      RFG2=CK*CK/CKC*DSIN(X(3)+KSIC)*A2-RG*CF2FG
      RFG3=X(1)*(CB+SK*SK*SB*SB/CB)-RG*X(2)*SK*SK*SB/CB
      RNFG1=CK*CB*CF2FG-SK*SF2FG
      RNFG2=CK*CB*SF2FG+SK*CF2FG
      RNFG3=CK*SB
      XGF=RFG1*S(1,1)+RFG2*S(1,2)+RFG3*S(1,3)
      YGF=RFG1*S(2,1)+RFG2*S(2,2)+RFG3*S(2,3)
      ZGF=RFG1*S(3,1)+RFG2*S(3,2)+RFG3*S(3,3)
      XNGF=RNFG1*S(1,1)+RNFG2*S(1,2)+RNFG3*S(1,3)
      YNGF=RNFG1*S(2,1)+RNFG2*S(2,2)+RNFG3*S(2,3)
      ZNGF=RNFG1*S(3,1)+RNFG2*S(3,2)+RNFG3*S(3,3)
C     WRITE (6,100) X(1),X(2),X(3),X(4),X(5)
C     WRITE (6,100) XPF,YPF,ZPF,XGF,YGF,ZGF
C 100 FORMAT (1X,'%%%%%',8F15.7)
C     WRITE (6,100) XNPF,YNPF,ZNPF,XNGF,YNGF,ZNGF
      Y(1)=XPF-XGF
      Y(2)=YPF-YGF-C
      Y(3)=ZPF-ZGF
      Y(4)=YNPF-YNGF
      Y(5)=ZNPF-ZNGF
      X(10)=DSQRT(XPF*XPF+YPF*YPF)
      X(11)=DSQRT(XGF*XGF+YGF*YGF)
C     WRITE (6,20) (Y(II),II=1,5)
C  20 FORMAT (1X,'$$$$',6F15.7)
      RETURN
      END
C
      SUBROUTINE INTMAT (A,N,M)
C  THIS SUBROUTINE IS USED TO INITIATE THE MATRIX, WITH UNIT DIAGONAL
C     ELEMENTS AND NULL OTHER ELEMENTS
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4)
      DO 5 I=1,N
      DO 5 J=1,M
      A(I,J)=0.
      IF (I.EQ.J) A(I,J)=1.
```

```
    5 CONTINUE
      RETURN
      END
C
      SUBROUTINE MATMUL(CMM,A,B,N,M)
C  THIS SUBROUTINE IS USED TO MULTIPLY THE MATRIX CMCD(N*M) BY THE MATRIX
C     A(M*1). THE RESULT IS STORED IN THE MATRIX B(N*1)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CMM(4,4),A(4),B(4)
      DO 5 I=1,N
    5 B(I)=0.
      DO 15 I=1,N
      DO 15 J=1,M
   15 B(I)=B(I)+CMM(I,J)*A(J)
      RETURN
      END
C
C
      SUBROUTINE NONLIN
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCK1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
C
      DO 5 I=1,NC
      CALL FUNC
C     WRITE (6,10) I,(X(J),Y(J),J=1,5)
C  10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
   15 CONTINUE
      GO TO 105
   25 DO 35 J=1,NE
   35 Y1(J)=Y(J)
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (X(J).EQ.0.D0) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
   55 A(K,J)=(Y1(K)-Y(K))/DIFF
   45 CONTINUE
      DO 65 J=1,NE
   65 Y(J)=-Y1(J)
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM,NE,A,Y,IPVT)
      DO 75 J=1,NE
   75 X(J)=X(J)+Y(J)
    5 CONTINUE
  105 RETURN
      END
C
```

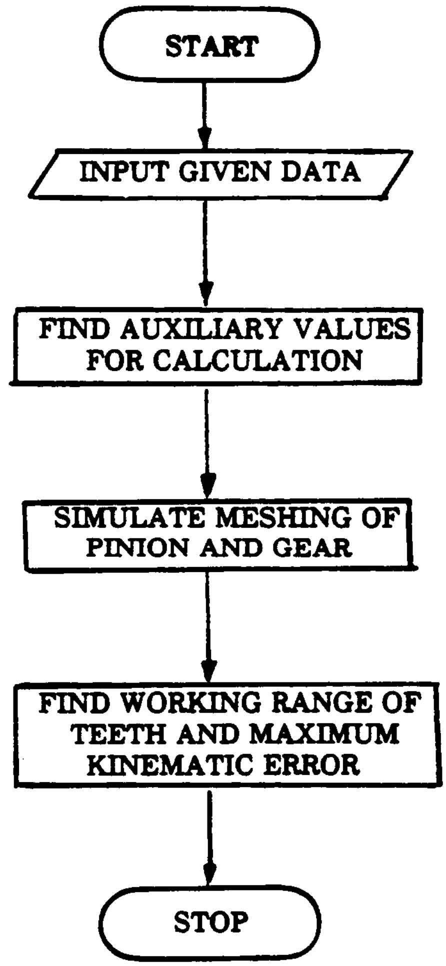
```
C
C
      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C  DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C  AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C  INPUT..
C
C     NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C     N    = ORDER OF THE MATRIX
C     A    = MATRIX TO BE TRIANGULARIZED
C
C  OUTPUT..
C
C     A   CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C         VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C         (PERMUTATION MATRIX)*A=L*U
C
C     COND = AN ESTIMATE OF THE CONDITION OF  A.
C       FOR THE LINEAR SYSTEM  A*X = B , CHANGES IN   A  AND  B
C       MAY CAUSE CHANGES  COND  TIMES AS LARGE IN  X.
C       IF  COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C       PRECISION.  COND  IS SET TO  1.0D+32  IF EXACT
C       SINGULARITY IS DETECTED.
C
C     IPVT      = THE PIVOT VECTOR
C       IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C       IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C  WORK SPACE..   THE VECTOR  WORK  MUST BE DECLARED AND INCLUDED
C       IN THE CALL.  ITS INPUT CONTENTS ARE IGNORED.
C       ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C  THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C     DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
      IPVT(N)=1
      IF (N.EQ.1) GO TO 150
      NM1=N-1
C                               COMPUTE THE 1-NORM OF  A .
      ANORM=0.D0
      DO 20 J=1,N
        T=0.D0
        DO 10 I=1,N
   10   T=T+DABS(A(I,J))
```

```
         IF (T.GT.ANORM) ANORM=T
   20 CONTINUE
C                              DO GAUSSIAN ELIMINATION WITH PARTIAL
C                                   PIVOTING.
      DO 70 K=1,NM1
        KP1=K+1
C                              FIND THE PIVOT.
        M=K
        DO 30 I=KP1,N
          IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
   30   CONTINUE
        IPVT(K)=M
        IF (M.NE.K) IPVT(N)=-IPVT(N)
        T=A(M,K)
        A(M,K)=A(K,K)
        A(K,K)=T
C                               SKIP THE ELIMINATION STEP IF PIVOT IS ZERO.
        IF (T.EQ.0.D0) GO TO 70
C
C                               COMPUTE THE MULTIPLIERS.
        DO 40 I=KP1,N
   40   A(I,K)=-A(I,K)/T
C                               INTERCHANGE AND ELIMINATE BY COLUMNS.
        DO 60 J=KP1,N
          T=A(M,J)
          A(M,J)=A(K,J)
          A(K,J)=T
          IF (T.EQ.0.D0) GO TO 60
          DO 50 I=KP1,N
   50     A(I,J)=A(I,J)+A(I,K)*T
   60   CONTINUE
   70 CONTINUE
C
C  COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C  THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C  SMALL SINGULAR VECTOR.  THIS INVOLVES SOLVING TWO SYSTEMS
C  OF EQUATIONS,  (A-TRANSPOSE)*Y = E  AND  A*Z = Y  WHERE  E
C  IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C  ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C                               SOLVE  (A-TRANSPOSE)*Y = E .
      DO 100 K=1,N
        T=0.D0
        IF (K.EQ.1) GO TO 90
        KM1=K-1
        DO 80 I=1,KM1
   80   T=T+A(I,K)*WORK(I)
   90   EK=1.D0
        IF (T.LT.0.D0) EK=-1.D0
        IF (A(K,K).EQ.0.D0) GO TO 160
  100 WORK(K)=-(EK+T)/A(K,K)
      DO 120 KB=1,NM1
        K=N-KB
        T=0.D0
```

```
      KP1=K+1
      DO 110 I=KP1,N
  110 T=T+A(I,K)*WORK(K)
      WORK(K)=T
      M=IPVT(K)
      IF (M.EQ.K) GO TO 120
      T=WORK(M)
      WORK(M)=WORK(K)
      WORK(K)=T
  120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
  130 YNORM=YNORM+DABS(WORK(I))
C
C                               SOLVE  A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
      ZNORM=0.D0
      DO 140 I=1,N
  140 ZNORM=ZNORM+DABS(WORK(I))
C
C                               ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C                               1-BY-1 CASE..
  150 COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
C
C                               EXACT SINGULARITY
  160 COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C  SOLVES A LINEAR SYSTEM,  A*X = B
C  DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  INPUT..
C
C     NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING  A
C     N    = ORDER OF MATRIX
C     A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C     B    = RIGHT HAND SIDE VECTOR
C     IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C  OUTPUT..
```

```
C
C     B = SOLUTION VECTOR,  X
C
C                                 DO THE FORWARD ELIMINATION.
      IF (N.EQ.1) GO TO 50
      NM1=N-1
      DO 20 K=1,NM1
        KP1=K+1
        M=IPVT(K)
        T=B(M)
        B(M)=B(K)
        B(K)=T
        DO 10 I=KP1,N
   10   B(I)=B(I)+A(I,K)*T
   20 CONTINUE
C                                 NOW DO THE BACK SUBSTITUTION.
      DO 40 KB=1,NM1
        KM1=N-KB
        K=KM1+1
        B(K)=B(K)/A(K,K)
        T=-B(K)
        DO 30 I=1,KM1
   30   B(I)=B(I)+A(I,K)*T
   40 CONTINUE
   50 B(1)=B(1)/A(1,1)
      RETURN
      END
C
C
```

FLOWCHART FOR PROGRAM V



```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                     PROGRAM V                      *
C... *   TRANSMISSION ERRORS OF CROWNED HELICAL PINION IN  *
C... *  MESHING WITH REGULAR HELICAL GEAR WITH PREDESIGNED *
C... *        TRANSMISSION ERRORS AND CONTACT PATH         *
C... *                                                    *
C... *              AUTHORS: FAYDOR LITVIN                 *
C... *                       JIAO ZHANG                    *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C
C   THIS PROGRAM IS USED TO CALCULATE THE TRANSMISSION ERRORS OF A
C     CROWNED HELICAL PINION AND A HELICAL GEAR WHEN THEIR AXES ARE
C     MOUNTED WITH SOME ERRORS. THE MAGNITUDE OF TRANSMISSION ERRORS
C     AND CONTACT PATH ARE PRE-DESIGNED.
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,KK,KSIC
      DIMENSION Z(99),ANGLE(99),ERROR(99),ERR(99),W(99),RPR(99),RGR(99),
     +          ZP(99),ZG(99)
      COMMON /BLOCK1/ X(13),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCK2/ S(4,4),AT(5),C,RP,RG,CK,SK,CB,SB,CKC,KSIC,TB,
     +                COEG1,COEG2,RMGP,AA,DR,KK,R
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=1
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      DELTA=1.D-3
      EPSI=1.D-10
```

```
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      NDIM=10
      NE=5
      DR=DATAN(1.D0)/45.D0
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; NP=PINION TOOTH NUMBER;
C        RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./NP);
C        KSIN=PRESSURE ANGLE IN NORMAL SECTION;
C        BETAP=HELIX ANGLE OF PINION AND GEAR;
C        COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)
      PN=2.D0
      NP=12
      RMPG=94./12.
      KSIN=30.D0*DR
      BETAP=15.D0*DR
      COE=1.000D0
C   (2) PREDESIGN TRANSMISSION ERRORS AND CONTACT PATH:
C        AA=LEVEL OF PREDESIGNED PARABOLIC TRNASMISSION ERRORS IN SECOND
C        KK=COEFFICIENT OF CONTACT PATH DIRECTION (1.D-3=CROSS TOOTH
C           SURFACE; 5.D6=ALONG TOOTH SURFACE)
C        R=RADIUS OF ARC ATTACHED TO CHOSEN CONTACT PATH
      AA=25.D0
      KK=5.D6
      R=0.3584D0
C   (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C        NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM -(NG-1)/2 TO
C           (NG-1)/2 TIMES GAMMAI WITH ODD NG);
C        GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
      NMIS=1
      NG=3
      GAMMAI=3.D0
C   (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
      FEEI=1.0D0*DR
C
C  DESCRIPTION OF OUTPUT PARAMERTERS
C
C    FEE1=ROTATION ANGLE OF PINION
C    FEE2=ROTATION ANGLE OF GEAR
C    RP=RADIUS OF PINION CONTACT POINT
C    RG=RADIUS OF GEAR CONTACT POINT
C    ZP=LENGTH OF PINION CONTACT POINT FROM MIDDLE SECTION
C    ZG=LENGTH OF GEAR CONTACT POINT FROM MIDDLE SECTION
C
C  FIND AUXILIARY VALUES FOR CALCULATION
C
C  DEFINE USEFUL CONSTANTS AND PARAMETERS FOR PINION AND GEAR
      RMGP=1./RMPG
      NG=NP*RMPG+0.5
      AA=AA*(2/3600.*DR*(NP/DR/180.)**2)
      SK=DSIN(KSIN)
      CK=DCOS(KSIN)
      SB=DSIN(BETA)
      CB=DCOS(BETA)
      TB=SB/CB
```

```
      KSIC=DATAN(SK/CK/CB)
      CKC=DCOS(KSIC)
      SKC=DSIN(KSIC)
      PT=PN*CB
      RP=NP/2./PT
      RPB=RP*CKC
      RPA=RP+1.0/PN
      RG=NG/2./PT
      RGB=RG*CKC
      RGA=RG+1.0/PN
      WRITE (LP,56) RP,RPB,RPA,RG,RGB,RGA,AA
   56 FORMAT (1X,'&&&&&&','RP=',F15.7,5X,'RPB=',F15.7,5X,'RPA=',F15.7/
     +        1X,'&&&&&&','RG=',F15.7,5X,'RGB=',F15.7,5X,'RGA=',F15.7)
      CON1=(1.+SK*SK*TB*TB)
      AT(1)=(CK**4/CKC**2*KK**2-SK**4*TB**2)*RG*RG
      AT(2)=(2.*SK*SK*SB*CON1-2.*KK**2*SB*CK**4/CKC**2)*RG
      AT(3)=(-2.*CK*CK/CKC*SKC*KK**2-2.*KK*SK*SK*TB)*RG*RG
      AT(4)=(2.*SB*CK*CK/CKC*SKC*KK**2+2.*KK*CB*CON1)*RG
      AT(5)=CK**4/CKC**2*SB**2*KK**2-CB**2*CON1*CON1
      C=(RP+RG)*COE
      CALL INTMAT(S,4,4)
      NGG=(NG-1)/2
      DO 505 LL=1,NG
      GAMMA=GAMMAI*FLOAT(LL-NGG)
      CG=DCOS(GAMMA/60.*DR)
      SG=DSIN(GAMMA/60.*DR)
      IF (NMIS.EQ.1) THEN
      WRITE (LP,500) COE,GAMMA
  500 FORMAT (1H1,///,1X,'C=',F9.4,'*(RP+RG)     CROSSING ANGLE=',
     +        F8.4,'(M)')
      S(1,1)=CG
      S(1,3)=-SG
      S(3,1)=SG
      S(3,3)=CG
      ELSE
      WRITE (LP,501) COE,GAMMA
  501 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG)     INTERSECTING ANGLE=',
     +        F5.1,'(M)')
      S(2,2)=CG
      S(2,3)=-SG
      S(3,2)=SG
      S(3,3)=CG
      END IF
      NCOEF=IDINT(360.*DR/FEEI/FLOAT(NP)+0.5)
      N=2*NCOEF
      NHALF=(N+1)/2
      XIN=0.
      DO 205 L=1,2
      LSGN=(-1)**L
      DO 15 I=1,NHALF
      LI=NHALF+LSGN*(I-1)
      IF (L.EQ.1) NMIN=LI
      IF (L.EQ.2) NMAX=LI
      X(7)=LSGN*FEEI*FLOAT(I-1)
```

```
      X(4)=X(7)
      X(2)=X(7)/RMPG
      X(3)=0.
      X(5)=(-(AT(2)*X(2)+AT(4))+DSQRT((AT(2)*X(2)+AT(4))**2-4.*AT(5)
     +     *(AT(3)*X(2)+AT(1)*X(2)*X(2))))/2./AT(5)
      X(1)=0.
      CALL NONLIN
      X(8)=X(1)+X(2)
C  FIND INITIAL VALUE OF X(8)
      IF (L.EQ.1.AND.I.EQ.1) THEN
      XIN=X(8)
      WRITE (LP,51) XIN
   51 FORMAT(1X,'XIN=',D15.7)
      GO TO 15
      END IF
C
      X(8)=X(8)-XIN
      W(LI)=X(7)/DR
      Z(LI)=X(8)/DR
      ERR(LI)=(X(8)-X(7)/RMPG)*3600.D0/DR
      RPR(LI)=X(10)
      RGR(LI)=X(11)
      ZP(LI)=X(12)
      ZG(LI)=X(13)
C     WRITE (LP,20) W(LI),Z(LI),ERR(LI),RPR(LI),RGR(LI)
   15 CONTINUE
  205 CONTINUE
      WRITE (LP,10)
   10 FORMAT (////8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
     +        'RP',13X,'ZP',13X,'RG',13X,'ZG',F15.7/)
      DO 25 I=NMIN,NMAX
      WRITE (LP,20) W(I),Z(I),ERR(I),RPR(I),ZP(I),RGR(I),ZG(I)
   20 FORMAT (1X,2F15.7,F12.4,4F15.7)
   25 CONTINUE
      NT=NMAX-NCOEF
      FII=FEEI/DR/RMPG*FLOAT(NCOEF)
      WRITE (LP,80)
   80 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:',F15.7/)
      DO 55 I=NMIN,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/RMPG/DR
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE(I)=Z(KK)-Z(I)
      ERROR(I)=(ANGLE(I)-FII)*3600.D0
C     WRITE (LP,60) X(7),X(8),ANGLE(I),ERROR(I)
C  60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
   55 CONTINUE
      DO 95 I=NMIN,NT
      ATEMP2=ERROR(I)
      IF (I.NE.NMIN) THEN
      IF (ATEMP1*ATEMP2.LE.0.D0) GOTO 105
      END IF
      ATEMP1=ATEMP2
   95 CONTINUE
```

```
      WRITE (LP,160)
  160 FORMAT (//1X,'MESHING IS DISCONTINUOUS')
      GO TO 505
  105 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.
      EMIN=0.
      NTEMP=NCOEF+1
      DO 135 J=1,NTEMP
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
      IF (ET.GT.EMAX) EMAX=ET
  135 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,170) Z(I),Z(KK),ET
  170 FORMAT (//1X,'WORKING RANGE FOR ONE TOOTH: ',F7.2,'----',F7.2/
     +          1X,'THE MAXIMUM KINEMATIC ERROR: ',F12.4,'  (S)',I2)
  505 CONTINUE
      STOP
      END
C
      SUBROUTINE FUNC
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,KK,KSIC
      COMMON /BLOCK1/ X(13),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCK2/ S(4,4),AT(5),C,RP,RG,CK,SK,CB,SB,CKC,KSIC,TB,
     +                COEG1,COEG2,RMGP,AA,DR,KK,R
C     **************************************************
   20 TEMP=CKC*(1.-AA*X(4)/(RMGP+1.))
      IF (DABS(TEMP).GT.1.D0) THEN
      X(4)=X(7)
      GOTO 20
      END IF
C     WRITE (6,220) TEMP,X(4),AA
C 220 FORMAT(1X,'TEMP=',E15.7,'X(4)=',E15.7,'AA=',F15.7)
      RLAM=KSIC-DARCOS(TEMP)
      DLAMDF=-CKC*AA/(RMGP+1.)/DSQRT(1.-TEMP*TEMP)
      FEEGP=X(4)*RMGP-AA/2.*X(4)*X(4)+RLAM
      DFGDFP=RMGP-AA*X(4)+DLAMDF
      CONA=AT(5)
      CONB=AT(2)*FEEGP+AT(4)
      CONC=AT(3)*FEEGP+AT(1)*FEEGP*FEEGP
      COND=CONB*CONB-4.*CONA*CONC
      IF (COND.LT.0.) THEN
      X(4)=X(7)
      GOTO 20
      END IF
      COND=DSQRT(COND)
      TP=(-CONB+COND)/2./CONA
      IF (COND.EQ.0.0) THEN
      DTPDFG=-AT(2)/2./CONA
```

```
      ELSE
      DTPDFG=(-AT(2)-(AT(2)*CONB-2.*CONA*(AT(3)+2.*AT(1)*FEEGP))/COND)
     +        /2./CONA
      END IF
C     ***************************************************
C X(4)=FEEP; X(7)=FEE1
      AP1=X(4)-X(7)
      AP2=AP1-RLAM
      AP3=AP2+KSIC
      AP4=X(4)-RLAM+KSIC
      DAP4DF=1.-DLAMDF
      CAP4=DCOS(AP4)
      SAP4=DSIN(AP4)
      AP=-TP*SB+RG*FEEGP
      DAPDF=(-SB*DTPDFG+RG)*DFGDFP
      MU=X(4)+KSIC-RLAM
      DMUDFP=1.+DLAMDF
      ALP1=R*(DSIN(MU+X(3))-DSIN(MU))
      ALP2=R*(DCOS(MU+X(3))-DCOS(MU))
      XPF=CK*CK/CKC*DCOS(AP3)*AP+RG*DSIN(AP2)-(RP+RG)*DSIN(AP1)+ALP2
      YPF=CK*CK/CKC*DSIN(AP3)*AP-RG*DCOS(AP2)+(RP+RG)*DCOS(AP1)+ALP1
      ZPF=TP*CB*CK*CK/CKC/CKC-SK*SK*TB*RG*FEEGP
C**********************************************************
      DXDA=-DSIN(MU+X(3))
      DYDA=DCOS(MU+X(3))
      DXDF=CK*CK/CKC*(CAP4*DAPDF-SAP4*AP*DAP4DF)+RG*DCOS(X(4)-RLAM)
     +      *(1.-DLAMDF)-(RP+RG)*DCOS(X(4))-ALP1*DMUDFP
      DYDF=CK*CK/CKC*(SAP4*DAPDF+CAP4*AP*DAP4DF)+RG*DSIN(X(4)-RLAM)
     +      *(1.-DLAMDF)-(RP+RG)*DSIN(X(4))+ALP2*DMUDFP
      DZDF=CB*CK*CK/CKC/CKC*DTPDFG*DFGDFP-SK*SK*TB*RG*DFGDFP
      XNP=DYDA*DZDF
      YNP=-DXDA*DZDF
      ZNP=DXDA*DYDF-DYDA*DXDF
      RMN=DSQRT(XNP*XNP+YNP*YNP+ZNP*ZNP)
C     WRITE(6,130) DXDA,DZDA,DXDF,DYDF,DZDF,XNP,YNP,ZNP,RMN,TP,AP
C 130 FORMAT(1X,'$$$',5F15.7/4X,5F15.7)
      XNP=XNP/RMN
      YNP=YNP/RMN
      ZNP=ZNP/RMN
C**********************************************************
      XNPF=XNP*DCOS(X(7))+YNP*DSIN(X(7))
      YNPF=YNP*DCOS(X(7))-XNP*DSIN(X(7))
      ZNPF=ZNP
      CF2FG=DCOS(X(1))
      SF2FG=DSIN(X(1))
      CF2FGK=DCOS(X(1)+KSIC)
      SF2FGK=DSIN(X(1)+KSIC)
      AG=-X(5)*SB+RG*X(2)
      RFG1=CK*CK/CKC*CF2FGK*AG+RG*SF2FG
      RFG2=CK*CK/CKC*SF2FGK*AG-RG*CF2FG
      RFG3=X(5)*CB-AG*SK*SK*TB
      RNFG1=CK*CB*CF2FG-SK*SF2FG
      RNFG2=CK*CB*SF2FG+SK*CF2FG
      RNFG3=CK*SB
```

```
      XGF=RFG1*S(1,1)+RFG2*S(1,2)+RFG3*S(1,3)
      YGF=RFG1*S(2,1)+RFG2*S(2,2)+RFG3*S(2,3)
      ZGF=RFG1*S(3,1)+RFG2*S(3,2)+RFG3*S(3,3)
      XNGF=RNFG1*S(1,1)+RNFG2*S(1,2)+RNFG3*S(1,3)
      YNGF=RNFG1*S(2,1)+RNFG2*S(2,2)+RNFG3*S(2,3)
      ZNGF=RNFG1*S(3,1)+RNFG2*S(3,2)+RNFG3*S(3,3)
C     WRITE (6,100) X(1),X(2),X(3),X(4),X(5)
C     WRITE (6,100) XPF,YPF,ZPF,XGF,YGF,ZGF
C 100 FORMAT (1X,'%%%%%',8E15.7)
C     WRITE (6,100) XNPF,YNPF,ZNPF,XNGF,YNGF,ZNGF
      Y(1)=XPF-XGF
      Y(2)=YPF-YGF-C
      Y(3)=ZPF-ZGF
      Y(5)=XNPF-XNGF
      Y(4)=ZNPF-ZNGF
      X(10)=DSQRT(XPF*XPF+YPF*YPF)
      X(11)=DSQRT(XGF*XGF+YGF*YGF)
      X(12)=ZPF
      X(13)=ZGF
C     WRITE (6,20) (Y(II),II=1,5)
C  20 FORMAT (1X,'$$$$',6F15.7)
      RETURN
      END
C
      SUBROUTINE INTMAT (A,N,M)
C  THIS SUBROUTINE IS USED TO INITIATE THE MATRIX, WITH UNIT DIAGONAL
C     ELEMENTS AND NULL OTHER ELEMENTS
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4)
      DO 5 I=1,N
      DO 5 J=1,M
      A(I,J)=0.
      IF (I.EQ.J) A(I,J)=1.
    5 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE NONLIN
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCK1/ X(13),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
C
      DO 5 I=1,NC
      CALL FUNC
C     WRITE (6,10) I,(X(J),Y(J),J=1,5)
C  10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
   15 CONTINUE
      GO TO 105
   25 DO 35 J=1,NE
   35 Y1(J)=Y(J)
```

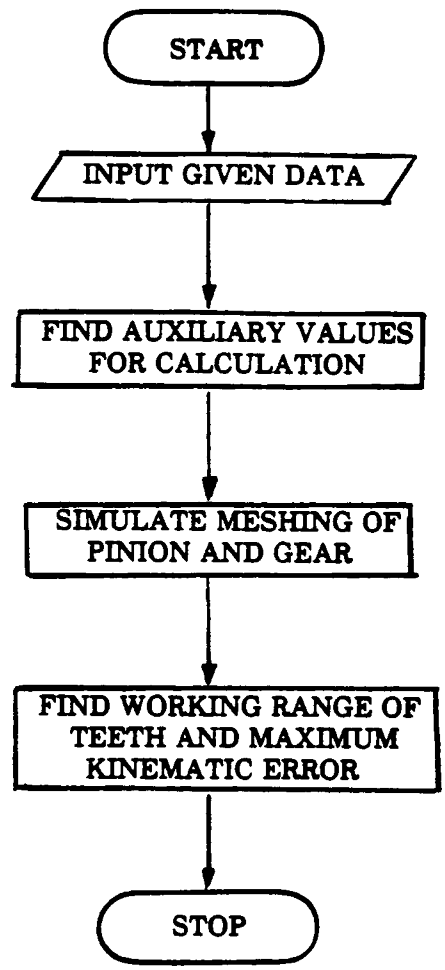
```
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (DABS(X(J)).LT.1.D-12) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
   55 A(K,J)=(Y1(K)-Y(K))/DIFF
   45 CONTINUE
      DO 65 J=1,NE
   65 Y(J)=-Y1(J)
C     DO 205 K=1,NE
C 205 WRITE (6,245) (A(K,J),J=1,NE)
C 245 FORMAT (1X,'¬¬¬',5D15.7)
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM,NE,A,Y,IPVT)
      DO 75 J=1,NE
   75 X(J)=X(J)+Y(J)
    5 CONTINUE
C 105 WRITE (6,20) I
C  20 FORMAT (1X,'I=',I2)
  105 RETURN
      END
C
C
C
      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C  DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C  AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C  INPUT..
C
C     NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C     N    = ORDER OF THE MATRIX
C     A    = MATRIX TO BE TRIANGULARIZED
C
C  OUTPUT..
C
C     A   CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C         VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C         (PERMUTATION MATRIX)*A=L*U
C
C     COND = AN ESTIMATE OF THE CONDITION OF  A.
C       FOR THE LINEAR SYSTEM  A*X = B , CHANGES IN   A  AND  B
```

```
C        MAY CAUSE CHANGES  COND  TIMES AS LARGE IN  X.
C        IF  COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C        PRECISION.  COND  IS SET TO  1.0D+32  IF EXACT
C        SINGULARITY IS DETECTED.
C
C      IPVT      = THE PIVOT VECTOR
C        IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C        IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C   WORK SPACE..   THE VECTOR  WORK  MUST BE DECLARED AND INCLUDED
C        IN THE CALL.  ITS INPUT CONTENTS ARE IGNORED.
C        ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C   THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C      DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
      IPVT(N)=1
      IF (N.EQ.1) GO TO 150
      NM1=N-1
C                             COMPUTE THE 1-NORM OF  A .
      ANORM=0.D0
      DO 20 J=1,N
        T=0.D0
        DO 10 I=1,N
   10   T=T+DABS(A(I,J))
        IF (T.GT.ANORM) ANORM=T
   20 CONTINUE
C                             DO GAUSSIAN ELIMINATION WITH PARTIAL
C                                  PIVOTING.
      DO 70 K=1,NM1
        KP1=K+1
C                             FIND THE PIVOT.
        M=K
        DO 30 I=KP1,N
          IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
   30   CONTINUE
        IPVT(K)=M
        IF (M.NE.K) IPVT(N)=-IPVT(N)
        T=A(M,K)
        A(M,K)=A(K,K)
        A(K,K)=T
C                              SKIP THE ELIMINATION STEP IF PIVOT IS ZERO.
        IF (T.EQ.0.D0) GO TO 70
C
C                              COMPUTE THE MULTIPLIERS.
        DO 40 I=KP1,N
   40   A(I,K)=-A(I,K)/T
C                              INTERCHANGE AND ELIMINATE BY COLUMNS.
        DO 60 J=KP1,N
          T=A(M,J)
          A(M,J)=A(K,J)
          A(K,J)=T
          IF (T.EQ.0.D0) GO TO 60
          DO 50 I=KP1,N
```

```
   50      A(I,J)=A(I,J)+A(I,K)*T
   60    CONTINUE
   70 CONTINUE
C
C  COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C  THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C  SMALL SINGULAR VECTOR.  THIS INVOLVES SOLVING TWO SYSTEMS
C  OF EQUATIONS,  (A-TRANSPOSE)*Y = E  AND  A*Z = Y  WHERE  E
C  IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C  ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C                              SOLVE  (A-TRANSPOSE)*Y = E .
      DO 100 K=1,N
        T=0.D0
        IF (K.EQ.1) GO TO 90
        KM1=K-1
        DO 80 I=1,KM1
   80   T=T+A(I,K)*WORK(I)
   90   EK=1.D0
        IF (T.LT.0.D0) EK=-1.D0
        IF (A(K,K).EQ.0.D0) GO TO 160
  100 WORK(K)=-(EK+T)/A(K,K)
      DO 120 KB=1,NM1
        K=N-KB
        T=0.D0
        KP1=K+1
        DO 110 I=KP1,N
  110   T=T+A(I,K)*WORK(K)
        WORK(K)=T
        M=IPVT(K)
        IF (M.EQ.K) GO TO 120
        T=WORK(M)
        WORK(M)=WORK(K)
        WORK(K)=T
  120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
  130 YNORM=YNORM+DABS(WORK(I))
C
C                              SOLVE  A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
      ZNORM=0.D0
      DO 140 I=1,N
  140 ZNORM=ZNORM+DABS(WORK(I))
C
C                              ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C                              1-BY-1 CASE..
  150 COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
```

```
C
C                                 EXACT SINGULARITY
  160 COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C   SOLVES A LINEAR SYSTEM,  A*X = B
C   DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C   -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C        M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C   INPUT..
C
C      NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING  A
C      N    = ORDER OF MATRIX
C      A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C      B    = RIGHT HAND SIDE VECTOR
C      IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C   OUTPUT..
C
C      B = SOLUTION VECTOR,  X
C
C                                 DO THE FORWARD ELIMINATION.
      IF (N.EQ.1) GO TO 50
      NM1=N-1
      DO 20 K=1,NM1
        KP1=K+1
        M=IPVT(K)
        T=B(M)
        B(M)=B(K)
        B(K)=T
        DO 10 I=KP1,N
   10   B(I)=B(I)+A(I,K)*T
   20 CONTINUE
C                                 NOW DO THE BACK SUBSTITUTION.
      DO 40 KB=1,NM1
        KM1=N-KB
        K=KM1+1
        B(K)=B(K)/A(K,K)
        T=-B(K)
        DO 30 I=1,KM1
   30   B(I)=B(I)+A(I,K)*T
   40 CONTINUE
   50 B(1)=B(1)/A(1,1)
      RETURN
      END
C
C
```

FLOWCHART FOR PROGRAM VI



```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                    PROGRAM VI                      *
C... *  TRANSMISSION ERRORS OF HELICAL GEARS WITH THEIR    *
C... *        AXES DEFORMED BY INTERACTING FORCE          *
C... *                                                    *
C... *              AUTHORS: FAYDOR LITVIN                *
C... *                       JIAO ZHANG                   *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE TRANSMISSION ERRORS OF A
C     HELICAL PINION AND A HELICAL GEAR IN MESHING WHEN THEIR AXES ARE
C     DEFORMED BY INTERACTING FORCE(BOTH PINION AND GEAR ARE NOT CROWNED)
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,MUO,KSIC
      DIMENSION Z(99),ANGLE(99),ERROR(99),ERR(99),W(99),RPR(99),RGR(99),
     +          S1(4,4),S2(4,4),ZP(99)
      COMMON /BLOCK1/ X(12),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM,NCTL,CX2
      COMMON /BLOCK2/ S(4,4),C,RP,RG,CK,SK,CB,SB,CKC,KSIC,ZG,TB,ZNPF,
     +                COEG1,COEG2,CB1,SB1,TB1
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=1
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      DELTA=1.D-4
      EPSI=1.D-13
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      NDIM=10
      DR=DATAN(1.D0)/45.D0
```

```
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; NP=PINION TOOTH NUMBER;
C       RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./NP);
C       KSIN=PRESSURE ANGLE IN NORMAL SECTION;
C       BETAP=HELIX ANGLE OF PINION AND GEAR;
C       PAD=HEIGHT OF ADDENDUM OF PINION;
C       GAD=HEIGHT OF ADDENDUM OF GEAR;
C       ZG=GEAR TOOTH LENGTH (PINION TOOTH LENGTH IS LONGER)
C       COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)
      PN=10.D0
      NP=20
      RMPG=2.D0
      KSIN=20.D0*DR
      BETAP=15.D0*DR
      PAD=1.0/PN
      GAD=1.0/PN
      ZG= 5./PN
      COE=1.000D0
C   (2) SHAFT DEFORMATION:
C       NSIM=MODEL ID NO.(1=SIMPLIFIED DEFORMATION MATRIX;
C                         2=UNSIMPLIFIED DEFORMATION MATRIX)
C       RLAMP=PINION SHAFT ROTATION
C       RLAMG=GEAR SHAFT ROTATION
C       RVP=PINION SHAFT DEFLECTION
C       RVG=GEAR SHAFT DEFLECTION
C
      NSIM=1
      RLAMP= 2./60.*DR
      RLAMG= 2./60.*DR
      RVP=0.0125
      RVG=0.0125
C   (3) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
      FEEI=1.0D0*DR
C
C  DESCRIPTION OF OUTPUT PARAMERTERS
C
C    FEE1=ROTATION ANGLE OF PINION
C    FEE2=ROTATION ANGLE OF GEAR
C    RP=RADIUS OF PINION CONTACT POINT
C    RG=RADIUS OF GEAR CONTACT POINT
C
C  FIND AUXILIARY VALUES FOR CALCULATION
C
C
C  DEFINE USEFUL CONSTANTS AND PARAMETERS FOR PINION AND GEAR
      DELTAB=DR/60.*( 0.D0)
      BETAP1=BETAP+DELTAB
      CLP=DCOS(RLAMP)
      SLP=DSIN(RLAMP)
      CLG=DCOS(RLAMG)
      SLG=DSIN(RLAMG)
      WRITE (LP,4) CLP,SLP,CLG,SLG
    4 FORMAT(1X,4F15.7)
      SK=DSIN(KSIN)
```

```
      CK=DCOS(KSIN)
      SB=DSIN(BETAP)
      CB=DCOS(BETAP)
      TB=SB/CB
      SB1=DSIN(BETAP1)
      CB1=DCOS(BETAP1)
      TB1=SB1/CB1
      KSIC=DATAN(SK/CK/CB)
      CKC=DCOS(KSIC)
      SKC=DSIN(KSIC)
      PT=PN*CB
      RP=NP/2./PT
      RPA=RP+PAD
      RPATOL=RPA-0.0005D0
      RG=RP*RMPG
      RGA=RG+GAD
      RGATOL=RGA-0.0005D0
      WRITE (LP,56) RPATOL,RGATOL
   56 FORMAT (1X,'&&&&&&','RPA=',F15.7,5X,'RGA=',F15.7)
      COEG1=1.+SK*SK*TB*TB
      COEG2=RG/COEG1
      ZNPF=CK*SB1
      C=(RP+RG)*COE
      CALL INTMAT(S,4,4)
      CALL INTMAT(S1,4,4)
      CALL INTMAT(S2,4,4)
      DO 505 LL=1,2
      NSIM=LL
      IF (NSIM.EQ.1) THEN
      WRITE (LP,500)
  500 FORMAT (1H1,///,1X,
     +       'THE CASE OF SIMPLIFIED DEFORMATION MATRIX OF GEAR AXES')
      S(1,3)=(RLAMP+RLAMG)*CKC
      S(2,3)=(RLAMP+RLAMG)*SKC
      S(3,1)=-S(1,3)
      S(3,2)=-S(2,3)
      S(1,4)=(RVP+RVG)*CKC
      S(2,4)=(RVP+RVG)*SKC+C
      S(3,4)=-C*RLAMP*SKC
      ELSE
      WRITE (LP,501)
  501 FORMAT (1H1,///,1X,
     +       'THE CASE OF UNSIMPLIFIED DEFORMATION MATRIX OF GEAR AXES')
      S1(1,1)=1.+CKC*CKC*(CLP-1.)
      S1(1,2)=SKC*CKC*(CLP-1.)
      S1(1,3)=CKC*SLP
      S1(2,1)=S1(1,2)
      S1(2,2)=1.+SKC*SKC*(CLP-1.)
      S1(2,3)=SKC*SLP
      S1(3,1)=-S1(1,3)
      S1(3,2)=-S1(2,3)
      S1(3,3)=CLP
      S1(1,4)=RVP*CKC*CLP
      S1(2,4)=RVP*SKC*CLP
```

```
      S1(3,4)=-RVP*SLP
      S2(1,1)=1.+CKC*CKC*(CLG-1.)
      S2(1,2)=SKC*CKC*(CLG-1.)
      S2(1,3)=CKC*SLG
      S2(2,1)=S1(1,2)
      S2(2,2)=1.+SKC*SKC*(CLG-1.)
      S2(2,3)=SKC*SLG
      S2(3,1)=-S1(1,3)
      S2(3,2)=-S1(2,3)
      S2(3,3)=CLG
      S2(1,4)=RVG*CKC
      S2(2,4)=RVG*SKC+C
      S2(3,4)=0.
      DO 12 MMI=1,3
      DO 12 MMJ=1,4
      S(MMI,MMJ)=0.
      DO 12 MMK=1,4
      S(MMI,MMJ)=S(MMI,MMJ)+S1(MMI,MMK)*S2(MMK,MMJ)
   12 CONTINUE
      END IF
      NCOEF=IDINT(360.*DR/FEEI/FLOAT(NP)+0.5)
      N=3*NCOEF
      NHALF=(N+1)/2
      XIN=0.
      DO 205 L=1,2
      LSGN=(-1)**L
      NCTL=0
      NE=4
      DO 5 I=1,NE
    5 X(I)=0.D0
      X(10)=0.
      X(11)=0.
      DO 15 I=1,NHALF
      IF (NCTL.EQ.2) GOTO 205
      LI=NHALF+LSGN*(I-1)
      IF (L.EQ.1) NMIN=LI
      IF (L.EQ.2) NMAX=LI
      X(7)=LSGN*FEEI*FLOAT(I-1)
      IF (X(10).LE.RPATOL) THEN
       X(4)=-X(7)
      ELSE
       NE=3
       NCTL=NCTL+1
      END IF
      IF (X(11).LT.RGATOL) THEN
       X(2)=X(7)/RMPG
      ELSE
       NCTL=NCTL+1
       CX2=X(2)
      END IF
      X(3)=(ZG-RP*X(4)*SK*SK*TB)/CB/COEG1
      CALL NONLIN
      X(8)=X(1)+X(2)
C  FIND INITIAL VALUE OF X(8)
```

```
      IF (L.EQ.1.AND.I.EQ.1) THEN
      XIN=X(8)
      GO TO 15
      END IF
C
      X(8)=X(8)-XIN
      W(LI)=X(7)/DR
      Z(LI)=X(8)/DR
      ERR(LI)=(X(8)-X(7)/RMPG)*3600.D0/DR
      RPR(LI)=X(10)
      RGR(LI)=X(11)
      ZP(LI)=X(12)
C     WRITE (LP,21) LI,W(LI),Z(LI),ERR(LI),RPR(LI),RGR(LI)
C  21 FORMAT (1X,I2,1X,5F15.7,F15.7)
   15 CONTINUE
  205 CONTINUE
      WRITE (LP,10)
   10 FORMAT (////8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
     +        'RP',13X,'RG',F15.7/)
      DO 25 I=NMIN,NMAX
      WRITE (LP,20) W(I),Z(I),ERR(I),RPR(I),RGR(I)
   20 FORMAT (1X,5F15.7,F15.7)
   25 CONTINUE
      NT=NMAX-NCOEF
      FII=FEEI/DR/2.D0*FLOAT(NCOEF)
      WRITE (LP,80)
   80 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:',F15.7/)
      DO 55 I=NMIN,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE(I)=Z(KK)-Z(I)
      ERROR(I)=(ANGLE(I)-FII)*3600.D0
      WRITE (LP,60) X(7),X(8),ANGLE(I),ERROR(I)
   60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
   55 CONTINUE
      DO 95 I=NMIN,NT
      ATEMP2=ERROR(I)
      IF (I.NE.NMIN) THEN
      IF (ATEMP1*ATEMP2.LE.0.D0) GOTO 105
      END IF
      ATEMP1=ATEMP2
   95 CONTINUE
      WRITE (LP,160)
  160 FORMAT (//1X,'MESHING IS DISCONTINUOUS')
      GO TO 505
  105 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.
      EMIN=0.
      NTEMP=NCOEF+1
      DO 135 J=1,NTEMP
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
```

```
      IF (ET.GT.EMAX) EMAX=ET
  135 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,170) Z(I),Z(KK),ET
  170 FORMAT (//1X,'WORKING RANGE FOR ONE TOOTH: ',F7.2,'----',F7.2/
     +          1X,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,'  (S)',I2)
  505 CONTINUE
      STOP
      END
C
      SUBROUTINE FUNC
C
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLE PRECISION KSIN,MU,MUO,KSIC
      COMMON /BLOCK1/ X(12),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM,NCTL,CX2
      COMMON /BLOCK2/ S(4,4),C,RP,RG,CK,SK,CB,SB,CKC,KSIC,ZG,TB,ZNPF,
     +                COEG1,COEG2,CB1,SB1,TB1
C X(4)=FEEP; X(7)=FEE1
      X(6)=X(4)+X(7)
      CF1FP=DCOS(X(6))
      SF1FP=DSIN(X(6))
      AP=X(3)*SB1+RP*X(4)
      XPF=-CK*CK/CKC*DCOS(X(6)-KSIC)*AP+RP*SF1FP
      YPF=CK*CK/CKC*DSIN(X(6)-KSIC)*AP+RP*CF1FP
      ZPF=X(3)*CB1+AP*SK*SK*TB1
      XNPF=CK*CB1*CF1FP+SK*SF1FP
      YNPF=-CK*CB1*SF1FP+SK*CF1FP
C     ZNPF=CK*SB1
      CF2FG=DCOS(X(1))
      SF2FG=DSIN(X(1))
      CF2FGK=DCOS(X(1)+KSIC)
      SF2FGK=DSIN(X(1)+KSIC)
      AG1=(-ZG*TB+RG*X(2))/COEG1
      RFG1=CK*CK/CKC*CF2FGK*AG1+RG*SF2FG
      RFG2=CK*CK/CKC*SF2FGK*AG1-RG*CF2FG
C     RFG3=ZG
      RTFG1=CK*CK/CKC*( AG1*SF2FGK+COEG2*CF2FGK)-RG*CF2FG
      RTFG2=CK*CK/CKC*(-AG1*CF2FGK+COEG2*SF2FGK)-RG*SF2FG
C     RTFG3=0.
      XGF=RFG1*S(1,1)+RFG2*S(1,2)+ZG*S(1,3)+S(1,4)
      YGF=RFG1*S(2,1)+RFG2*S(2,2)+ZG*S(2,3)+S(2,4)
      ZGF=RFG1*S(3,1)+RFG2*S(3,2)+ZG*S(3,3)+S(3,4)
      XTGF=RTFG1*S(1,1)+RTFG2*S(1,2)
      YTGF=RTFG1*S(2,1)+RTFG2*S(2,2)
      ZTGF=RTFG1*S(3,1)+RTFG2*S(3,2)
C     WRITE (6,100) RTFG1,RTFG2,AG1,COEG2,COEG1,CF2FGK,SF2FGK
C     WRITE (6,100) X(1),X(2),X(3),X(4),X(5)
C     WRITE (6,100) XPF,YPF,ZPF,XGF,YGF,ZGF
C 100 FORMAT (1X,'%%%%%',8E15.7)
C     WRITE (6,100) XNPF,YNPF,ZNPF,XTGF,YTGF,ZTGF
      Y(1)=XPF-XGF
      Y(2)=YPF-YGF
```

```
      Y(3)=ZPF-ZGF
      IF (NCTL.NE.0) THEN
      Y(4)=X(2)-CX2
      ELSE
      Y(4)=XNPF*XTGF+YNPF*YTGF+ZNPF*ZTGF
      END IF
      X(10)=DSQRT(XPF*XPF+YPF*YPF)
      X(11)=DSQRT(RFG1*RFG1+RFG2*RFG2)
      X(12)=ZPF
C     WRITE (6,20) (Y(II),II=1,4)
C  20 FORMAT (1X,'$$$$',6F15.7)
      RETURN
      END
C
      SUBROUTINE INTMAT (A,N,M)
C  THIS SUBROUTINE IS USED TO INITIATE THE MATRIX, WITH UNIT DIAGONAL
C     ELEMENTS AND NULL OTHER ELEMENTS
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4)
      DO 5 I=1,N
      DO 5 J=1,M
      A(I,J)=0.
      IF (I.EQ.J) A(I,J)=1.
    5 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE NONLIN
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCK1/ X(12),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM,NCTL,CX2
C
      DO 5 I=1,NC
      CALL FUNC
C     WRITE (6,10) I,(X(J),Y(J),J=1,4)
C  10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
   15 CONTINUE
      GO TO 105
   25 DO 35 J=1,NE
   35 Y1(J)=Y(J)
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (DABS(X(J)).LT.1.D-12) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
   55 A(K,J)=(Y1(K)-Y(K))/DIFF
   45 CONTINUE
```

```
      DO 65 J=1,NE
   65 Y(J)=-Y1(J)
C     DO 85 K=1,NE
C  85 WRITE (6,104) (A(K,J),J=1,NE),Y(K)
C 104 FORMAT(1X,'A',2X,5E15.7)
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM,NE,A,Y,IPVT)
      DO 75 J=1,NE
   75 X(J)=X(J)+Y(J)
    5 CONTINUE
  105 RETURN
      END
C
C
C
      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C  DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C  AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C  INPUT..
C
C     NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C     N    = ORDER OF THE MATRIX
C     A    = MATRIX TO BE TRIANGULARIZED
C
C  OUTPUT..
C
C     A   CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C         VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C         (PERMUTATION MATRIX)*A=L*U
C
C     COND = AN ESTIMATE OF THE CONDITION OF  A.
C       FOR THE LINEAR SYSTEM  A*X = B , CHANGES IN   A  AND  B
C       MAY CAUSE CHANGES  COND  TIMES AS LARGE IN  X.
C       IF  COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C       PRECISION.  COND  IS SET TO  1.0D+32  IF EXACT
C       SINGULARITY IS DETECTED.
C
C     IPVT      = THE PIVOT VECTOR
C       IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C       IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C  WORK SPACE..   THE VECTOR  WORK  MUST BE DECLARED AND INCLUDED
C       IN THE CALL.  ITS INPUT CONTENTS ARE IGNORED.
C       ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
```

```
C
C  THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C     DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
      IPVT(N)=1
      IF (N.EQ.1) GO TO 150
      NM1=N-1
C                                COMPUTE THE 1-NORM OF  A .
      ANORM=0.D0
      DO 20 J=1,N
        T=0.D0
        DO 10 I=1,N
   10   T=T+DABS(A(I,J))
        IF (T.GT.ANORM) ANORM=T
   20 CONTINUE
C                                DO GAUSSIAN ELIMINATION WITH PARTIAL
C                                     PIVOTING.
      DO 70 K=1,NM1
        KP1=K+1
C                                FIND THE PIVOT.
        M=K
        DO 30 I=KP1,N
          IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
   30   CONTINUE
        IPVT(K)=M
        IF (M.NE.K) IPVT(N)=-IPVT(N)
        T=A(M,K)
        A(M,K)=A(K,K)
        A(K,K)=T
C                                 SKIP THE ELIMINATION STEP IF PIVOT IS ZERO.
        IF (T.EQ.0.D0) GO TO 70
C
C                                 COMPUTE THE MULTIPLIERS.
        DO 40 I=KP1,N
   40   A(I,K)=-A(I,K)/T
C                                 INTERCHANGE AND ELIMINATE BY COLUMNS.
        DO 60 J=KP1,N
          T=A(M,J)
          A(M,J)=A(K,J)
          A(K,J)=T
          IF (T.EQ.0.D0) GO TO 60
          DO 50 I=KP1,N
   50     A(I,J)=A(I,J)+A(I,K)*T
   60   CONTINUE
   70 CONTINUE
C
C  COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C  THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C  SMALL SINGULAR VECTOR.  THIS INVOLVES SOLVING TWO SYSTEMS
C  OF EQUATIONS,  (A-TRANSPOSE)*Y = E  AND  A*Z = Y  WHERE  E
C  IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C  ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C                                 SOLVE  (A-TRANSPOSE)*Y = E .
```

```
      DO 100 K=1,N
        T=0.D0
        IF (K.EQ.1) GO TO 90
        KM1=K-1
        DO 80 I=1,KM1
   80   T=T+A(I,K)*WORK(I)
   90   EK=1.D0
        IF (T.LT.0.D0) EK=-1.D0
        IF (A(K,K).EQ.0.D0) GO TO 160
  100 WORK(K)=-(EK+T)/A(K,K)
      DO 120 KB=1,NM1
        K=N-KB
        T=0.D0
        KP1=K+1
        DO 110 I=KP1,N
  110   T=T+A(I,K)*WORK(K)
        WORK(K)=T
        M=IPVT(K)
        IF (M.EQ.K) GO TO 120
        T=WORK(M)
        WORK(M)=WORK(K)
        WORK(K)=T
  120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
  130 YNORM=YNORM+DABS(WORK(I))
C
C                              SOLVE  A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
      ZNORM=0.D0
      DO 140 I=1,N
  140 ZNORM=ZNORM+DABS(WORK(I))
C
C                              ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C                              1-BY-1 CASE..
  150 COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
C
C                              EXACT SINGULARITY
  160 COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C  SOLVES A LINEAR SYSTEM,  A*X = B
C  DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
```

```
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C      M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  INPUT..
C
C     NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING  A
C     N    = ORDER OF MATRIX
C     A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C     B    = RIGHT HAND SIDE VECTOR
C     IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C  OUTPUT..
C
C     B = SOLUTION VECTOR,  X
C
C                                   DO THE FORWARD ELIMINATION.
      IF (N.EQ.1) GO TO 50
      NM1=N-1
      DO 20 K=1,NM1
        KP1=K+1
        M=IPVT(K)
        T=B(M)
        B(M)=B(K)
        B(K)=T
        DO 10 I=KP1,N
   10   B(I)=B(I)+A(I,K)*T
   20 CONTINUE
C                                   NOW DO THE BACK SUBSTITUTION.
      DO 40 KB=1,NM1
        KM1=N-KB
        K=KM1+1
        B(K)=B(K)/A(K,K)
        T=-B(K)
        DO 30 I=1,KM1
   30   B(I)=B(I)+A(I,K)*T
   40 CONTINUE
   50 B(1)=B(1)/A(1,1)
      RETURN
      END
C
C
```

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16. Abstract

The contents of this report covers: (i) development of optimal geometry for crowned helical gears; (ii) method for their generation; (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned helical gears and (iv) modeling and simulation of gear shaft deflection. The developed method for synthesis is used for determination of optimal geometry for crowned helical pinion surface and is directed to localize the bearing contact and guarantee the favorable shape and low level of the transmission errors. Two new methods for generation of the crowned helical pinion surface have been proposed. One is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. The other is based on crowning pinion tooth surface with predesigned transmission errors. The pinion tooth surface can be generated by a computer controlled automatic grinding machine. The TCA program simulates the meshing and bearing contact of misaligned gears. The transmission errors are also determined. The gear shaft deformation has been modeled and investigated. It has been found that the deflection of gear shafts has the same effects as those of gear misalignment.

17. Key Words (Suggested by Author(s)) Helical gears; Crowned gears; Gear geometry; Tooth contact analysis	18. Distribution Statement Unclassified – Unlimited Subject Category 37

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