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RITZ METHOD FOR TRANSIENT RESPONSE IN SYSTEMS HAVING UNSYMMETRIC STIFFNESS

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BUTLER ANALYSES

If the choice of generalized coordinates for determining the transient response of a non-symmetric structure were not eigenvectors but were modes of deformations due to operating loads, there would be certain advantages. Among these would be: 1. the economy of requiring only a small number of modes, 2. the avoidance of having either to cull out certain non-participating modes or to retain the non-participating modes at the expense of having to operate with larger order matrices, and 3. the confidence of getting well converged solutions. Using load response modes as generalized coordinates is properly classified as the Ritz Method.

The interest in the case of non-symmetric stiffness derives from structures with active control systems. The assymmetry comes from the sensor being situated at a different location than the actuator. Loads that would be typical of those used in the design of control systems are: externally applied forces and pressures, vernier jets whose firing is commanded by the control system, and constraints from appendages assigned to attach points. Such a solution method is developed here as a DMAP ALTER packet to the Statics Rigid Format in NASTRAN. The Inertia Relief Rigid Format would appear to be a more natural host for such an ALTER packet, because it carries out full processing of mass. Its prohibition against boundary constraints disqualifies it, hence Statics with modifications to process mass is used as host.

THEORETICAL APPROACH

The theory will be organized in 5 parts. It will describe the mathematical decisions on using the original stiffness matrices in the development of fundamentals first. It will describe the generation of harmonics from each fundamental, second. Then it will develop the adjoint vectors third. Having a full complement of primary and adjoint generalized vectors, the next step is to orthogonalize them and finally to integrate them into an actual solution by constructing the generalized mass and generalized stiffness and reconciling the form of the generalized damping.

The methods presented herein are an outgrowth of a new non-collocated sensor actuator analysis method under development by H. P. Frisch at the NASA/GSFC, Code 712.[1] The motivation for the NASTRAN/DMAP implementation presented herein is to provide a working capability which can be used for both current practical applications and for the evaluation of the new analysis method prior to its inclusion into the general purpose multi-flexible body data preparation program FEMDA.

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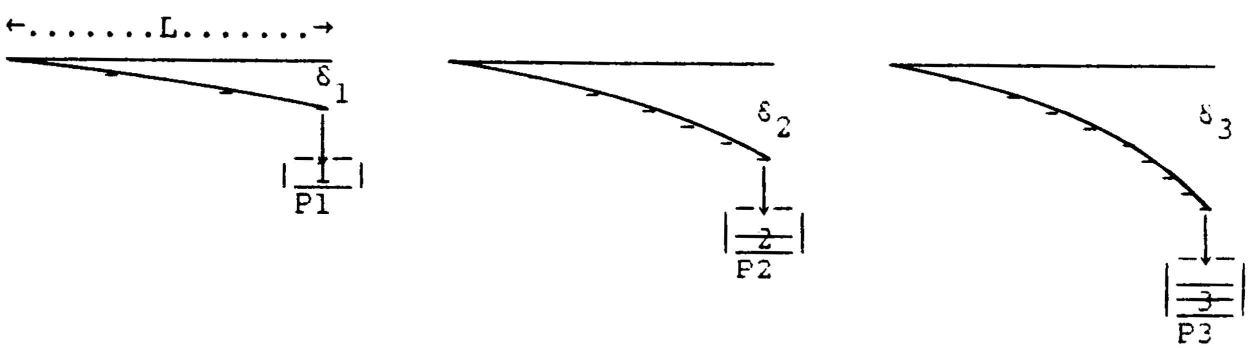
1. Frish, H.P. "IAC Program FEMDA, Theory and User's Guide, Interface from Structural Analysis Output Data to Input Data for Multi Flexible Body Dynamics Analysis," NASA Tech Brief Draft, June 13, 1988. (Call author for status info 301-286-8730)

RITZ METHOD FOR TRANSIENT RESPONSE
IN SYSTEMS HAVING UNSYMMETRIC STIFFNESS

FUNDAMENTAL MODES

A structure must be defined according to its elastic distribution, its damping distribution, its mass distribution, its boundary conditions and its complement of those loading conditions which are active during its operation under a control system. Each such loading that can be applied independently of some of the other loadings should be treated as a distinct deformation-producing condition; i.e. as one producing a unique static response shape. Within the theory of linear elasticity the magnitudes of static response to a static loading varies directly as the magnitude of the loading, so a simple unit magnitude of load is all that is necessary to establish the <u>shape</u> of an individual mode into which the structure deforms. A sketch of a cantilevered beam with an end load illustrates the point. The loads are graduated from 1 unit, to 2 units, to 3 units.



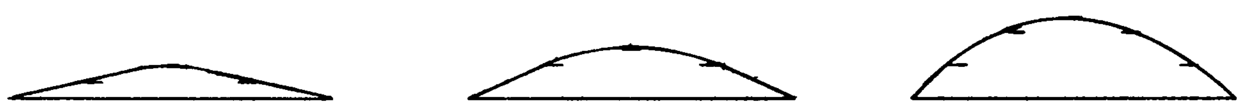
$$\frac{\delta_1}{P_1} = \frac{\delta_2}{P_2} = \frac{\delta_3}{P_3} = \frac{\delta}{P} = \frac{L^3}{3EI}$$

The ratio of deformation to load stays constant at $L^3/(3EI)$ as the load varies. The same holds true at positions other than the tip deflections. For instance, the deflection at an interior point, such as L/s where $1 < s < \infty$, is

$$\delta\left(\frac{L}{s}\right) = \frac{PL^3}{6EIs^2}\left(3 - \frac{1}{s}\right) \text{, therefore}$$

$$\frac{\delta\left(\frac{L}{s}\right)}{P} = \frac{L^3}{6EIs^2}\left(3 - \frac{1}{s}\right) \text{,}$$

which shows that the ratio of deformation to load at an interior point stays constant as load varies. The notion of shape that is independent of amplitude can be easily depicted by a sketch of a violin string being played in its fundamental mode.



All three show the string sounding the same pitch (frequency) but at different loudnesses (amplitude). All 3 deformations are considered to have the same shape.

Therefore, in order to get started, a controlled structure must be exercised with a unit load for each loading condition, which can vary in magnitude independently of other loads. This will produce, what will be called, the set of fundamental modes for the complement of loading conditions. It was assumed at the start of this derivation that the set of loads produced a set of unique static modes. This assumption needs to be tested against a criterion for uniqueness. The criterion that is germain here is linear independence. The modes need to be put on an equal footing for such a test by normalizing them uniformly. There is a choice of methods for normalizing at this point. Because this is a dynamics problem, one would be inclined toward mass normalization, but for now a simple Euclidean length will suffice to put the modes on an equal footing to establish linear independence. Later, when the modes are orthogonalized, they will be normalized to mass. Normalizing to maximum or to the amplitude of a common position will be excluded, because of the bias that would be introduced in the linear independence check.

Initially static modes are to be obtained by solving n loadings.

(1) $$\left[K_{LL}\right]\left[\left\{u_1\right\},\left\{u_2\right\},\ldots\ldots,\left\{u_n\right\}\right]_L = \left[\left\{P_1\right\},\left\{P_2\right\},\ldots\ldots,\left\{P_n\right\}\right]_L$$

$$===> \quad \left[K_{LL}\right]\left[U_{Lj}\right] = \left[P_{Lj}\right].$$

Each of the $\left\{u_j\right\}$ of [U] will be individually normalized by its Euclidean length. Compute the individual normalizing constants according to

(2) $$\left[\left\{u_j\right\}^T\left\{u_j\right\}\right] = n_j.$$

Each term of $\left\{u_j\right\}$ will now be divided by the square root of the normalizing constant n_j. Name this normalized fundamental

(3) PHI sub j, $\left\{\phi_j\right\}$; i.e. $\frac{1}{\sqrt{n_j}}\left\{u_j\right\} \equiv \left\{\phi_j\right\}$.

In order to use these static deformations as generalized coordinates, they should be linearly independent. Now we are faced with the decision as to what criterion of linear independence to use and what tolerance to allow. The classical definition of linear independence is that the Gramian > 0. The

Gramian[2] is a determinant of the matrix of the dot products of each coordinate into every other coordinate. Translated to the context in which we want to consider it, assume that the group of vectors $\phi_1 - - - - \phi_n$ is a set to be tested, then the Gramian for them is

$$(4) \qquad G(\phi_1, \phi_2, \ldots\ldots, \phi_n) = \begin{vmatrix} \phi_1^T\phi_1 & \phi_1^T\phi_2 & \phi_1^T\phi_3 \cdots\cdots & \phi_1^T\phi_n \\ \phi_2^T\phi_1 & \phi_2^T\phi_2 & \phi_2^T\phi_3 \cdots\cdots & \phi_2^T\phi_n \\ \cdots & \cdots & \cdots & \cdots \\ \phi_n^T\phi_1 & \phi_n^T\phi_2 & \phi_n^T\phi_3 \cdots\cdots & \phi_n^T\phi_n \end{vmatrix} .$$

If $G(\phi_1 \ldots\ldots \phi_n) = 0$, the set of ϕ_n are a dependent set, but if it is > 0 the set of ϕ_n are linearly independent. In a finite number system, such as that under which a digital computer operates, the establishment of a true zero is difficult, if not impossible. Consequently the decision as to what criterion to use rests on the practical consideration of how much impurity to allow in the set that we want to use as the expansion functions. If the Gramian is just slightly greater than zero, it implies that yes a functional relationship can be set up for one in the set with respect to the others, because imperfections creep in to contaminate the zero computation. Now if all ϕ's were normalized to unity and constituted a truly linearly independent set, then the value of the Gramian would be unity.

$$G(\text{unity normalized } \phi) = 1.$$

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2. TENSOR ANALYSIS, by I. S. Sokolnikoff; pp 6; John Wiley, 1951

Any departure from 1 in the Gramian of this unity normalized set means that some imperfection is creeping in and the further the descent from 1 towards zero implies more and more imperfections away from true linear independence. So What! Our goal is to have a good set of vectors so that when we expand our solution in them, we will get good accuracy and get good convergence. If we have a set of linearly independent vectors but too few of them to span the range of actions in which our structure will operate, we will fail to converge close enough to a correct solution. If we admit too many vectors that are almost independent, but do have some imperfections, the answers will contain biased emphases and will definitely contribute amplitudes that are too great in some modes. In further consideration of the practical factors that will govern our decision, we ask, "What will it cost to find out if there is linear independence?" In the case of the Gramian, the computations involve the creation of a full matrix of vector dot products that must then be decomposed and finally the product of all diagonal terms of the decomposed matrix forms the Gramian determinant. Decompositions are expensive, so a method other than using the Gramian would be worth while to investigate.

Another approach is to look at the ingredients of the Gramian i.e. the individual matrix dot products. By definition the dot product of 2 vectors **A** and **B** is

$\mathbf{A} \cdot \mathbf{B} = |A| x |B| \cos \theta$, where θ is the generalized angle between the 2 vectors. This can be extended to vectors in N-dimensional space. Set up a criterion based upon the size of the angle that a trial vector ϕ makes with each of those ϕ_n that have already been judged linearly independent.

(5) $\theta_{k,n} = \cos^{-1} \left[\frac{\phi_n}{|\phi_n|}\right]^T \left[\frac{\phi_k}{|\phi_k|}\right] \geq \tau$, where τ is some

threshhold value. Ideally τ would be $\pi/2$ for an orthogonal set with perfect linear independence. The poorest possible value would be that for which ϕ_n and ϕ_k are coincident, i.e. zero angle. This criterion can be rephrased by saying that it tests how well cos θ compares with cos $\pi/2$

(6) $\left(\cos \theta_{k,n}\right) - \left(\cos \frac{\pi}{2}\right) < \kappa.$

The desire is to hold the angle between test vectors to be somewhere between a threshhold and $\pi/2$. If the threshhold angle were $\pi/3$, $\kappa = \cos \pi/3 = .5$, then the test would require the cosine of the angle between test vectors to be less than 0.5 :

$$\left| \left[\frac{\phi_n}{|\phi_n|}\right]^T \left[\frac{\phi_k}{|\phi_k|}\right] \right| < 0.5 = \kappa \quad \text{for all } 1< n < k.$$

Once κ has been decided upon, the test is carried out against every ϕ_n for $n < \kappa$.

In the case of the cosine test, the matrix of dot products would have to be formed as in the case of the Gramian, but no decomposition need be done. The absolute value of each term is compared to κ.

The Gramian test has the advantage of making its decision by comparing only one number against a threshold while the cosine test involves comparison of every ratio in a column against a threshhold. The term by term processing involves only simple operations; the net result is that the cosine test is much less expensive than the Gramian test. The cosine test has been chosen for this method.

HARMONICS

Modes, determined solely on the basis of static loads, are questionable to apply without supplement to the solution of dynamics, because they are devoid of inertia effects. Supplemental modal vectors can be generated by finding the deformation due to forces derived from accelerating masses distributed through the structure by an amplitude equal to the vector of elastic deformation. Call the deformation from inertia effects, accelerated by amplitudes derived from the fundamental mode, the first harmonic. A second harmonic can be generated by the scheme used to generate the first harmonic, except that the the inertias are now accelerated through amplitudes derived from the first harmonic. Similarly, a third harmonic can be generated from the acceleration of mass through the amplitudes of the second harmonic, etc. Eventually the upper harmonics will tend towards congruence, so there will be an nth harmonic beyond which no distinct modes will be added. The measure to be used for finding the useful limit will be linear independence.

The set of linearly independent modes, consisting of a group of fundamentals plus groups of harmonics associated with each fundamental, when normalized to the Euclidean length, then orthogonalized, will be used as modes of generalized coordinates in expanding the behavior of a structure under the management of a control system. The mathematics of these inertia modes follows. The mass of the structure $\left[M_{LL}\right]$ will be accelerated by an amplitude distributed spatially according to the shape of the normalized - but not orthogonalized - fundamental mode $\{\phi_f\}$ to create an inertia forcing for the first harmonic $\{F_{h1}\}$; i.e.

(7) $$\{F_{h1}\} = [M_{LL}]\{\ddot{\phi}_f\} \text{ , where}$$

(8) $$\{\ddot{\phi}_f\} = \left\{\phi_f \frac{d^2}{dt^2} \cos \omega t\right\} = \{\phi_f(-\omega^2 \cos \omega t)\} \text{ .}$$

The shape of the deformation through which the mass will be accelerated is established by ϕ_f. The effect of the term $(\omega^2 \cos \omega t)$ is to merely amplify the shape as a function of time. Our interest, at this stage of the derivation, is only in the shape and not the total dynamic response, therefore the forcing can be treated as a statics problem with the spatial distribution of the set of accelerations, limited to any instant of time, therefore

$|\ddot{\phi}_f| \simeq |\phi_f|$ and the resulting static force is

(9) $$\{F_{h1}\} = [M_{LL}]\{\phi_f\}.$$

Apply this force to the structure and solve for the response.

(10) $$[K_{LL}]\{u_{h1}\} = \{F_{h1}\} = [M_{LL}]\{\phi_f\} \text{ ,}$$

from which the response can be explicitly isolated:

(11) $$\{u_{h1}\} = [K_{LL}]^{-1}[M_{LL}]\{\phi_f\}.$$

The $\{u_{h1}\}$ so obtained will be normalized by the Euclidean length and will be tested for linear independence , which if accepted, will be named $\{\phi_{h1}\}$ and will augment the complement of Ritz modes. This first harmonic will have a shape of deformation sufficiently unique to make it worth while to consider it as the basis for accelerating the mass through <u>its</u> spatial behavior to obtain a second harmonic, similar to the way that the fundamental was used in the generation of the first harmonic. Mathematically the method of forming the second harmonic follows the pattern already established for the first harmonic. Form the forcing

(12) $$\{F_{h2}\} = [M_{LL}]\{\ddot{\phi}_{h1}\}.$$

Let $\{\ddot{\phi}_{h1}\} = \{\phi_{h1}\} \frac{d^2}{dt^2}(\cos \omega t)$ and limit the value to its static amplitude $|\ddot{\phi}_{h1}| \simeq |\phi_{h1}|$, then solve for the static response $\{u_{h2}\}$ from

(13) $$[K_{LL}]\{u_{h2}\} = \{F_{h2}\} = [M_{LL}]\{\phi_{h1}\} .$$

Extract $\{u_{h2}\}$ explicitly.

(14) $$\{u_{h2}\} = [K_{LL}]^{-1}[M_{LL}]\{\phi_{h1}\} .$$

The $\{u_{h2}\}$ so obtained will be normalized by its Euclidean length and will be tested for linear independence, which if accepted, will be named $\{\phi_{h2}\}$ and will augment the complement of Ritz modes. A question arises as to the extent to which a candidate harmonic should be tested for linear independence. Should it be tested against <u>every</u> other vector established up to this point, or should the candidate <u>harmonic</u> be tested for linear independence only against <u>its</u> parent fundamental and the harmonics that are spawned from that fundamental alone? From an algorithmic standpoint the latter route is favored, because all quantities stay within an inner loop. The merit of this decision can be tested and be replaced if need be. It is known that as the recursion steps are carried out for higher harmonics, the deformations will tend toward congruence so there is a definite need for testing each new candidate harmonic against its parent and siblings. If there is no physical risk for so limiting the linear independence check to the family associated with just one fundamental, it will be opted for here.

A certain pattern starts to appear from the development of these two harmonics. The matrix product $\left[K_{LL}\right]^{-1}\left[M_{LL}\right]$ is used repeatedly. Consequently, it can be generated once and saved for recall in the generation of any level of harmonics of any fundamental. Name this matrix product

(15) $[SOLi] = \left[K^{\star}_{LL}\right]^{-1}\left[M_{LL}\right]$, where i can take on the BCD

character for the primary or the adjoint mode, and * can take on either blank for primary or T (for transpose) for the adjoint. Discussion of adjoint mode generation will be taken up subsequently.

Capitalizing on the pattern that has been revealed, all first harmonics for all fundamentals can be generated in a series of matrix operation as follows, as adapted for the primaries:

(15A) $$\left[\{u_1\},\{u_2\},\{u_3\},\ldots,\{u_i\},\ldots,\{u_n\}\right]_{Sh1} = \left[K_{LL}\right]_S^{-1}\left[M_{LL}\right]\left[\{\phi_1\},\{\phi_2\},\{\phi_3\},\ldots,\{\phi_i\},\ldots,\{\phi_n\}\right]_{Sf}$$

This can be compressed into

(15B) $$\left[U_{H1}\right]_S = \left[K_{LL}\right]_S^{-1}\left[M_{LL}\right]\left[\phi_F\right]_S$$

Equations (15,15A,15B) represent all possible inertia response raw data for forming first harmonic modes of primary fundamentals.

Normalization of these responses can also be performed on all vectors treated as a matrix. The Euclidean length can be extracted from the multiplication of $\left[U_{H1}\right]$ by itself.

(15C) $$\left[U_{H1}\right]^T\left[U_{H1}\right] = \left[U_{H1}SQ\right].$$

Strip off the diagonal, take the inverse of each, followed by its square root, to form a diagonal matrix of scale factors mode by mode.

(15D) $$\left[{}^{\backslash}U_{H1}SQ_{\backslash}\right]^{-.5} \text{======>} \left[SCAL_{H1}\right].$$

Apply this matrix of scale factors to the first harmonic responses, of fundamental inertia loadings, as a post multiplication operation to get a set of candidate normalized harmonic modes.

(15E) $$\left[U_{H1}\right]\left[SCAL_{H1}\right] = \left[\phi_{H1}^{CA}\right].$$

This matrix of candidate first harmonics needs to be tested for linear independence. Testing will proceed in two parts. All first harmonics will be tested against all other first harmonics, then all first harmonics will be tested against all fundamental modes. Start with the harmonics by themselves. The cosine test involves taking the dot product of every mode against every other mode. This is done in a single matrix operation.

$$\text{(15F)} \qquad \left[\phi_{H1}^{CA}\right]^T \left[\phi_{H1}^{CA}\right] = \left[DOT_{H1}\right].$$

Examining the $\left[DOT_{H1}\right]$ matrix closely, one can recognize that the first row represents the dot product of the harmonic (first in sequence) against each of those in the set of harmonics. The second row represents the dot product of the harmonic (second in sequence) against each of the set of harmonics. Et cetera. Consequently, the next step is to strip off one row to examine how well this candidate harmonic holds up in the cosine test versus other first harmonics. Next another row is stripped off and this candidate is tested and so on until all candidates have been examined. If the vectors had not been initially normalized to length, it would have been necessary to do so at this point to form the cosines. As a consequence, the matrix DOT_{H1} consists of all cosine terms. Going back now to the first row, some detail will reveal a pattern for systematizing all of the candidates.

Select one term at a time starting with the 2nd and take its absolute value then compare that value with κ. Shift the index to the 3rd and do the same. Continue until either a value greater than κ is encountered or until the the end of the row is reached. If any term tests greater than κ, catalog the row number and proceed to the next row. All successful candidates will be eligible to be tested against the matrix of fundamental modes. The harmonic vs. fundamental test will patterned after the harmonic vs. harmonic test. All successful candidate harmonics will be held in reserve to form the basis of 2nd harmonics before they will be merged with, but in sequence after, the fundamental modes.

The number of successful first harmonic modes may be fewer than those in the set of fundamentals. This does not matter, because the method of computing harmonics is independent of the size of the order of the vectors from which they are derived. The generation of second harmonic modes and higher will proceed along the pattern just outlined for first harmonic modes, except that after the responses to inertia loads have been computed, the successful modes from which they were derived, will be merged into the matrix of previous Ritz modes. Now the $i^{\underline{th}}$ set of harmonics will be tested for linear independence vs. not only themselves but against all other Ritz vectors including fundamentals and all previous order successful harmonics up through the $(i-1)^{\underline{th}}$.

The generation of higher harmonics will be subject to a choice of two limitations. The analyst may want to limit the maximum number of harmonics to be admitted for any particular investigation because of, say, a study in a low frequency domain. He can invoke such control by giving the value of the maximum number of harmonics to the parameter MODSPEC. Declining to assign a value to MODSPEC will cause the number of harmonics to be limited by those that pass the linear independence check.

ADJOINT MODES

For the non-symmetric problem, adjoint vectors are required to obtain the reduced order coefficient matrices. There are no set rules for introducing the adjoint basis. In the spirit of Lanczos method, a trial method is introduced and refined. For lack of any better trial scheme, the starting matrices of the adjoint system that will be used here will consist of the transpose of the original $\left[K_{LL}\right]$ matrix; i.e. $\left[K_{LL}^{T}\right]$, (which will also be non-symmetric), and the original load vectors $\left\{P_{L}\right\}$.

The static solution of this adjoint system under the n original loads yields a set of responses $\left\{v_{j}\right\}$:

$$(16) \qquad \left[K_{LL}^{T}\right]\left[\left\{v_{1}\right\},\left\{v_{2}\right\},\ldots\ldots,\left\{v_{n}\right\}\right] = \left[\left\{P_{1}\right\},\left\{P_{2}\right\},\ldots\ldots,\left\{P_{n}\right\}\right]$$

$$====\rangle \quad \left[K_{LL}^{T}\right]\left[V_{Lj}\right] = \left[P_{Lj}\right] \ .$$

Each of the $\left\{v_{j}\right\}$ of [V] will be individually normalized by its Euclidean length. Compute the individual normalizing constants according to

$$(17) \qquad \left[\left\{v_{j}\right\}^{T}\left\{v_{j}\right\}\right] = \alpha_{j}.$$

Each term of $\left\{v_{j}\right\}$ will now be divided by the square root of the normalizing constant α_{j}. Name this normalized fundamental

$$(18) \qquad \text{PSI sub j, } \left\{\psi_{j}\right\}; \quad \text{i.e. } \frac{1}{\sqrt{\alpha_{j}}}\left\{v_{j}\right\} \equiv \left\{\psi_{j}\right\}.$$

Each adjoint fundamental will be checked for linear independence against all other adjoint fundamentals. If any adjoint fundamental mode fails the linear independence check, and if its primary companion passed, both primary and adjoint will be discarded. This is necessary in order to retain a uniform sequence when setting up generalized mass and generalized stiffness. The cosine test will be used to certify linear independence. Inertial harmonics of adjoint fundamentals will be generated in the same manner as those of the primary fundamentals. Once again the linear independence of the adjoint harmonics will be checked only against its parent and siblings, instead of the currently established set of **ALL** Ritz modes.

ORTHOGONALIZATION

The solution of the differential equations is enhanced if the generalized coordinates used to span the response space are orthogonal. Our set of linearly independent vectors can be orthogonalized. At this point we have a pair of bases vectors $\{\phi\}$ and $\{\psi\}$ that are each separately linearly independent. After orthogonalizing they will be given the symbols $\{\zeta\}$ and $\{\Omega\}$ respectively. Options for orthogonalization and for associated constraint weightings were given due consideration. Before deciding on what options to choose, it will help to review the ultimate application.

The dynamic equation in metric coordinates is:

$$[M]\{\ddot{x}\} + [B]\{\dot{x}\} + [K]\{x\} = F(t). \tag{19}$$

Transform the metric coordinates to the primary bases vectors as generalized coordinates. Assume that the orthogonalization of ϕ into ζ has already taken place. Let

(20) $\{x\} = [\zeta]\{\xi\}$, giving

$$[M][\zeta]\{\ddot{\xi}\} + [B][\zeta]\{\dot{\xi}\} + [K][\zeta]\{\xi\} = F(t).$$

At this point the columns only of the coefficient matrices **M**, **B**, and **K** have been transformed to generalized form. Next the rows of the matrices must be transformed. In the symmetric case[3] this is done by using the transform of the symmetric modes. However, in this the unsymmetric case, we pre-multiply by the transpose of the <u>adjoint</u> bases vectors. Assume that the orthogonalization of ψ into Ω has taken place.

$$(22) \quad [\Omega]^T[M][\zeta]\{\ddot{\xi}\} + [\Omega]^T[B][\zeta]\{\dot{\xi}\} + [\Omega]^T[K][\zeta]\{\xi\} = [\Omega]^T F(t).$$

The desire is to decouple the equations as much as possible. To start with, we want the generalized mass to be a unity matrix; i.e.

$$(23) \quad [\Omega]^T[M][\zeta] \equiv \begin{bmatrix} \diagdown I \diagdown \end{bmatrix}.$$

Constraining the generalized mass to unity will affect the normalization to such an extent that the generalized coordinates will have been transformed to final form. Thus, much of the

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3. Dynamics of Structures, by W. C. Hurty & M. F. Rubinstein, pp 135, Prentice Hall, 1964

control in obtaining diagonalized damping and diagonalized stiffness will have been removed, so it appears that the generalized stiffness matrix may be coupled. Since $\left[\diagdown I \diagdown\right]$ is a square matrix, the requirement of equation (23) implies that the order of the adjoint vectors $\boldsymbol{\Omega}$ be the same as the order of the primary vectors $\boldsymbol{\zeta}$.

A. Use Gram-Schmidt method for orthogonalizing the Primary Ritz modes and apply the simple constraint of unit diagonalizing with no weighting.

B. But in the case of Adjoint Ritz modes the orthogonalized set will be expanded in terms of the complete set of adjoint bases with the dual constraint of equation (23) having mass weighting.

Mathematically these statements translate into building the normalized vectors as follows.

A. Self-Orthogonalization

To start with, the simple Gram-Schmidt method sets up a matrix of undetermined coefficients to involve an increasing number of bases vectors $\boldsymbol{\phi}$ in the content of the orthogonalized vectors $\boldsymbol{\zeta}$.

$$\begin{aligned}
\{\zeta_1\} &= \{\Phi_1\} \\
\{\zeta_2\} &= a_{11}\{\Phi_1\} + \{\Phi_2\} \\
\{\zeta_3\} &= a_{21}\{\Phi_1\} + a_{22}\{\Phi_2\} + \{\Phi_3\} \\
&\vdots \\
\{\zeta_{i+1}\} &= a_{i1}\{\Phi_1\} + a_{i2}\{\Phi_2\} + \cdots + a_{ii}\{\Phi_i\} + \{\Phi_{i+1}\} \\
&\vdots \\
\{\zeta_n\} &= a_{n-1,1}\{\Phi_1\} + a_{n-1,2}\{\Phi_2\} + \cdots + a_{n-1,n-1}\{\Phi_{n-1}\} + \{\Phi_n\}.
\end{aligned} \tag{24}$$

The self normalizing constraint is written in matrix notation as:

$$(25) \qquad \left[\zeta_{Li}\right]^{T}\left[\zeta_{Lj}\right] = \left[{}^{\backslash}\delta_{ij}{}_{\backslash}\right]$$

One can better appreciate the earlier topic of having to decide on sequencing during generation of ingredients of the bases in light of the character of orthogonalization. If the fundamentals are sequenced together at first, then the initial orthogonalized vectors will contain a minimum of higher harmonics in their expansion. We ask, Is this good or bad? If inertia effects dominate the dynamic behavior of a structure under certain loads, probably sequencing harmonics in earlier might help. But in this study the option was taken to group the fundamentals ahead of the harmonics instead of layering one fundamental and all of its higher harmonics on top of a second fundamental and all of its higher harmonics et cetera.

Trace the effect of the orthogonality constraint on achieving a solution for the undetermined coefficients. Operate on the first two equations of the set in equation (24).

$$(26) \qquad \{\zeta_1\} = \{\phi_1\}. \qquad \{\zeta_2\} = a_{11}\{\phi_1\} + \{\phi_2\}.$$ Impose the single orthogonality constraint of equation (25).

$$(27) \qquad \{\zeta_1\}^{T}\{\zeta_2\} = 0 \xRightarrow{\text{Subs for } \zeta_2} \{\zeta_1\}^{T}\left\{a_{11}\{\phi_1\} + \{\phi_2\}\right\} = 0 \xRightarrow{\text{Multiply}}$$

$$(28) \qquad \{\zeta_1\}^{T}\{\phi_1\}a_{11} + \{\zeta_1\}^{T}\{\phi_2\} = 0. \qquad a_{11} = (-)\,\frac{\{\zeta_1\}^{T}\{\phi_2\}}{\{\zeta_1\}^{T}\{\phi_1\}}\,.$$

Substitute from (26) $a_{11} = (-)\dfrac{\{\phi_1\}^T\{\phi_2\}}{\{\phi_1\}^T\{\phi_1\}}$. The coefficient

a_{11} is now expressed entirely in terms of the set of known normalized bases vectors $\{\phi\}$. Substitute a_{11} into the equation above and now $\{\zeta_2\}$ is known. Turn to the third vector.

(29) $\{\zeta_3\} = a_{21}\{\phi_1\} + a_{22}\{\phi_2\} + \{\phi_3\}$. Two orthogonality

conditions are imposed between ζ_3 and the previously found ζ_1 and ζ_2.

(30)

$$\{\zeta_1\}^T\{\zeta_3\} = 0 \quad \xrightarrow{\text{Subs for } \zeta_3} \quad \{\zeta_1\}^T\left\{a_{21}\{\phi_1\} + a_{22}\{\phi_2\} + \{\phi_3\}\right\} = 0 \quad \Longrightarrow$$

$$\{\zeta_2\}^T\{\zeta_3\} = 0 \qquad \{\zeta_2\}^T\left\{a_{21}\{\phi_1\} + a_{22}\{\phi_2\} + \{\phi_3\}\right\} = 0$$

(31)

$$\{\zeta_1\}^T\{\phi_1\}a_{21} + \{\zeta_1\}^T\{\phi_2\}a_{22} = (-)\{\zeta_1\}^T\{\phi_3\}$$

$$\{\zeta_2\}^T\{\phi_1\}a_{21} + \{\zeta_2\}^T\{\phi_2\}a_{22} = (-)\{\zeta_2\}^T\{\phi_3\}$$

This gives two algebraic equations in the two unknowns a_{21} and a_{22}.
Substituting these coefficients makes $\{\zeta_3\}$ known. Turn to the $i+1^{th}$ vector.

$$\text{(32)} \qquad \{\zeta_{i+1}\} = a_{1i}\{\phi_1\} + a_{2i}\{\phi_2\} + \dots + a_{ii}\{\phi_i\} + \{\phi_{i+1}\} =$$

$$= \left[\{\phi_1\},\{\phi_2\},\dots\{\phi_i\},\{\phi_{i+1}\}\right]\begin{Bmatrix} a_{1i} \\ a_{2i} \\ \cdot \\ a_{qi} \\ \cdot \\ a_{ii} \\ 1 \end{Bmatrix} = [\phi]_{i+1} \lfloor a \rfloor^T_{i+1}$$

Assemble the i constraint conditions in preparation for substituting ζ_{i+1} from equation (32) into them.

$$\text{(33)} \qquad \{\zeta_1\}^T\{\zeta_{i+1}\} = \{\zeta_2\}^T\{\zeta_{i+1}\} = \dots = \{\zeta_i\}^T\{\zeta_{i+1}\} = 0,$$

which can be combined into

$$\text{(33A)} \qquad \left[\{\zeta_1\}^T,\{\zeta_2\}^T,\{\zeta_3\}^T,\dots\{\zeta_i\}\right] \{\zeta_{i+1}\} = \begin{Bmatrix} 0 \\ \cdot \\ 0 \\ \cdot \\ 0 \end{Bmatrix}.$$

Now substitute the expansion in terms of $\{\phi\}$.

$$\text{(34)} \qquad \left[\{\zeta_1\}^T,\{\zeta_2\}^T,\dots,\{\zeta_i\}^T\right] \left[\{\phi_1\},\{\phi_2\},\dots\{\phi_i\},\{\phi_{i+1}\}\right]\begin{Bmatrix} a_{1i} \\ a_{2i} \\ \cdot \\ a_{qi} \\ \cdot \\ a_{ii} \\ 1 \end{Bmatrix} = 0.$$

Which can be compressed into

$$\text{(35)} \qquad [\zeta]^T_i[\phi]_{i+1}\lfloor a \rfloor^T_{i+1} = 0.$$

But by introducing partitions into $[\phi]$ and $\{a\}$ it can be written more intuitively as

$$\text{(36)} \quad [\zeta]^T_i\left[[\phi]_i \vdots \{\phi\}_{i+1}\right]\begin{Bmatrix} a_i \\ \cdot \\ 1 \end{Bmatrix} = 0. \quad \Longrightarrow \quad \left[[\zeta]^T_i[\phi]_i \vdots [\zeta]^T_i\{\phi\}_{i+1}\right]\begin{Bmatrix} a_i \\ \cdot \\ 1 \end{Bmatrix} = 0.$$

$$\Longrightarrow [\zeta]^T_i[\phi]_i\{a\}_i + [\zeta]^T_i\{\phi\}_{i+1} = 0. \Longrightarrow [\zeta]^T_i[\phi]_i\{a\}_i = (-)[\zeta]^T_i\{\phi\}_{i+1}$$

from which all of the undetermined coefficients $a_i, a_2, \ldots, a_i$ can be computed and substituted into equation (32) to evaluate ζ_{i+1}.

For solutions of succeeding vectors i+2, i+3, ...n; the square coefficient matrix $[\zeta]_i^T[\phi]_i$ increases incrementally in order up to n x n, so that results from the previous calculation might be considered for saving. Then the increments can be merged into the salvaged core to continue on. Details of the strategy will be given in the CODING document.

B. Dual Orthogonalization

The dual orthogonalization of the adjoint bases are organized into a full matrix of undetermined coefficients for the expansion of normalized vectors Ω as components of the raw adjoint bases ψ.

$$(37) \qquad \begin{Bmatrix} \{\Omega_{L1}\} \\ \{\Omega_{L2}\} \\ \{\Omega_{L3}\} \\ \vdots \\ \{\Omega_{Ln}\} \end{Bmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & & b_{2n} \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3n} \\ \vdots & & & & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \cdots & b_{nn} \end{pmatrix} \begin{Bmatrix} \{\psi_{L1}\} \\ \{\psi_{L2}\} \\ \{\psi_{L3}\} \\ \vdots \\ \{\psi_{Ln}\} \end{Bmatrix} .$$

This can be written in more conventional form as

$$(38) \qquad \left[\{\Omega_{L1}\}, \{\Omega_{L2}\}, \cdots \{\Omega_{Ln}\}\right] = \left[\{\psi_{L1}\}, \{\psi_{L2}\}, \cdots \{\psi_{Ln}\}\right] \begin{pmatrix} b_{11} & b_{21} \cdots & b_{n1} \\ b_{12} & b_{22} \cdots & b_{n2} \\ b_{13} & b_{23} \cdots & b_{n3} \\ \vdots & & \vdots \\ b_{1n} & b_{2n} \cdots & b_{nn} \end{pmatrix} .$$

Note the the b_{ij}'s in equation (38) are the transpose of the b_{ji}'s in equation (37). Equation (38) can be condensed to

(39) $\left[\Omega_{Ln}\right] = \left[\psi_{Lp}\right]\left[\beta_{pn}\right]$, in which $[\beta]$ is the transpose of the b_{ij}'s.

The mass orthogonality constraint, according to the transformation requirements of the dynamic differential equation (23), is a relationship between the normalized adjoint and the normalized primary bases; i.e. $\Omega^T_{Li} M_{LL} \zeta_{Lj} = \delta_{ij}$. Since the M and ζ are known, their transposes can be taken immediately, so it becomes better strategy to transpose the constraint equation in order to obtain Ω in non-transposed form; i.e.

(40) $$\left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left[\Omega_{Li}\right] = \left[{}^{\backslash}\delta_{ji\backslash}\right].$$

Now substitute from the expansion equation (39) into the mass orthogonality constraint equation (40). Confine to one index at a time. Set i = 1.

(41) $$\left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left\{\Omega_{L1}\right\} = \left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left[\psi_{Lp}\right]\left\{\beta_{p1}\right\} = \delta_{j1} = \begin{cases} 1 & \text{for } j = 1 \\ 0 & \text{for } j \neq 1 \end{cases}$$

This will produce a solution to the first column of β_{p1}. Next set i = 2 and substitute for Ω_{L2} from equation (39).

(42) $$\left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left\{\Omega_{L2}\right\} = \left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left[\psi_{Lp}\right]\left\{\beta_{p2}\right\} = \delta_{j2} = \begin{cases} 1 & \text{for } j = 2 \\ 0 & \text{for } j \neq 2 \end{cases}$$

This will produce a solution to the second column of β. A pattern is now apparent for solving for the complete content of β in a single operation, by recognizing the coefficient $\left[\zeta_{Lj}\right]^T\left[M_{LL}\right]\left[\psi_{Lp}\right]$ is the same in every equation; only the columns of unknown β's and the columns of the δ_{ji} change. Combine the columns into matrices; i.e.

(43) $$\left[\zeta_{Lj}\right]^T \left[M_{LL}\right]\left[\psi_{Lp}\right]\left[\beta_{pi}\right] = \left[{}^{\backslash}I_{\backslash}\right].$$

Isolate β_{pi}.

(44) $$\left[\beta_{pi}\right] = \left[{}^{\backslash}I_{\backslash}\right]\left[\zeta_{Lj}^{T} M_{LL} \psi_{Lp}\right]^{-1}.$$

Substitute into the defining equation for normalized adjoints (39).

(45) $$\left[\Omega_{Ln}\right] = \left[\psi_{pn}\right]\left[\beta_{pn}\right] = \left[\psi_{Lp}\right]\left[{}^{\backslash}I_{\backslash}\right]\left[\zeta_{Lj}^{T} M_{LL} \psi_{Lp}\right]^{-1}.$$

All quantities are now derived for getting a solution of the dynamic differential equation by using Ritz modal vectors. One thing not taken up however, was the definition of damping so as to give as sparse a generalized damping matrix as possible. The other topic that still needs addressing is data recovery. These topics will be reserved for an extension to the basics as developed here. Details of converting this theory to DMAP coding has been published in a report entitled "RITZ MODES FOR UNSYMMETRIC MATRICES--DMAP CODING OF THE THEORY" by Thomas B. Butler.

The coding was done in 3 steps. (1) Fundamental Primary and Adjoint modes were obtained from a DMAP ALTER to the Statics Rigid Format. The listing of this code is attached as Appendix A. (2) Harmonics for the Primary and Adjoint sets were coded as a pure DMAP approach. The listing of this code is attached as Appendix B. (3) Orthogonalization was coded as a pure DMAP approach. The listing of this code is attached as Appendix C.

A simple demonstration problem was used to certify the method and the coding. It was run twice with a different threshhold value of Kappa to exercise a number of different paths. Details of the demonstration problem are given in Appendix D. Generalized Mass and Generalized Stiffness that were produced in the two different runs are given in Appendix E. One run set the linear independence threshhold Kappa to 0.007. Only the four fundamentals passed the linear independence test, so the generalized mass and stiffness are only of order 4 x 4. Kappa was deliberately set high to a value of 0.95 in the second run so as to admit 5 harmonics in addition to the four fundamentals. Resulting generalized mass and stiffnesses are of order 9 x 9.

The generalized mass in both cases was practically unity. Off diagonal terms were at least 14 orders of magnitude less than those on the diagonal. Marked differences show up in the generalized stiffneses for the two cases. When a mathematically logical value of Kappa is used as in the .007 case, the terms on the diagonal dominate the off-diagonal terms, implying very weak coupling between Ritz modes. This weak coupling could very well justify the use diagonal matrices and so benefit from a decoupled solution. When an improbable value of Kappa is used as in the 0.95 case, off-diagonal terms are large. This observation can lead to a manageable criterion for completeness of the modes.

CONCLUSIONS

The method is workable. DMAP coding has been automated to such an extent by using the device of bubble vectors,[4] that it is useable for analyses in its present form. This feasibility study demonstrates that the Ritz Method is so compelling as to warrant coding its modules in FORTRAN and organizing the resulting coding into a new Rigid Format.

Even though this Ritz technique was developed for unsymmetric stiffness matrices, it offers advantages to problems with symmetric stiffnesses. If used for the symmetric case the solution would be simplified to one set of modes, because the adjoint would be the same as the primary. Its advantage in either type of symmetry over a classical eigenvalue modal expansion is that information density per Ritz mode is far richer than per eigenvalue mode; thus far fewer modes would be needed for the same accuracy and every mode would actively participate in the response. Considerable economy can be realized in adapting Ritz vectors for modal solutions. This new Ritz capability now makes NASTRAN even more powerful than before.

4. Butler, T. G. and Famidi, F. R. "Bubble Vector in Automatic Merging", Proceedings of the Fifteenth NASTRAN Colloquium, 1987

APPENDIX A

DMAP CODING FOR

PRIMARY AND ADJOINT FUNDAMENTALS

```
$$ ADVISABLE TO INCLUDE THIS NASTRAN CARD IN JOBS.
$
$    NASTRAN  MAXFILES = 60, FILES = (INPT,INP1,INP2,INP3,INP4)
$ RITZFUND.DMP $ DMAP ALTER FOR GENERATING FUNDAMENTAL RITZ VECTORS $
ALTER   2 $  ALTERS FOR 1988 VERSION OF NASTRAN
PARAML       CONTK/ /*PRESENCE*/ / / /V,N,NOCONTK $ CONTK IS A DMAP INPUT $
$$                 $ NOCONTK = -1 IF CONTK IS MISSING  $
PRTPARM  / /0/C,N,NOCONTK $DB
COND         ERROR4,NOCONTK $ ABORTS IF USER OMITS CONTK.
$$  AFTER GP3
ALTER 26,26  $  REPLACES PARAM STATEMENT WITH ONE THAT ENABLES MASS GENERATION
PARAM        //*ADD*/NOMGG/1/0 $ ALERTS EMG TO GENERATE MGG $
ALTER   39 $ AFTER EMA OUTPUTS KGGX FOR STIFFNESS
ADD          CONTK,KGGX/NSKGG/ $ THIS IS THE NON-SYMMETRIC STIFFNESS.
ALTER 45   $ AFTER EMA OUTPUTS MGG
PURGE     MNN,MFF,MAA/NOMGG $
ALTER     61,61 $ DON'T PURGE QG.
PURGE     KRR,KLR,QR,DM/REACT/GM/MPCF1/GO,KOO,LOO,PO,OUUV/OMIT/PS,
          KFS,KSS/SINGLE $
ALTER     62,62 $ ADD MGG TO EQUIV
EQUIV     KGG,KNN/MPCF1/MGG,MNN/MPCF1 $
ALTER     65,69 $ REPLACE MCE2 & SCE1 WITH NON-SYM OPN'S
VEC       USET/GNVEC/*G*/*M*/*N* $ 1'S ON N
PARTN     KGG,GNVEC, /KMM,KNM,KMN,KNNBAR/-1 $ -1 MEANS THAT GNVEC IS USED
$$  FOR BOTH ROW AND COL PARTNG, BUT DOES NOT MEAN THAT KGG IS SYMMETRIC
MPYAD     KNM,GM,KNNBAR/KNN1/0/+1/+1 $
MPYAD     GM,KMN,KNN1/KNN2/+1/+1/+1 $
```

```
MPY3      GM,KMM,KNN2/KNN/0 $
MCE2      USET,GM, ,MGG, , /KWASTE,MNN, ,  $
LABEL     LBL2 $
EQUIV     KNN,KFF/SINGLE/MNN,MFF/SINGLE $
COND      LBL3,SINGLE $
VEC       USET/NFVEC/*N*/*F*/*S* $ 1'S ON S
PARTN     KNN,NFVEC, /KFF,KSF,KFS,KSS/-1 $
SCE1      USET, ,MNN, , / , , ,MFF, , $
ALTER     71,71 $ ADD MFF TO EQUIV
EQUIV     KFF,KAA/OMIT/MFF,MAA/OMIT $
ALTER  73,82 $ REPLACE SMP1,RBMG1,RBMG2,RBMG3 WITH UNSYM OPN'S
VEC       USET/FAVEC/*F*/*A*/*O* $
PARTN     KFF,FAVEC, /KAABAR,KOA,KAO,KOO/-1 $
DECOMP    KOO/KOL,KOU/0/0/S,N,KOMIN/S,N,KODET/S,N,KOPWR/S,N,DOSING $ UNSYM
$$  DECOMP OF KOO
FBS       KOL,KOU,KOA/GO/0/-1 $
MPYAD     KAO,GO,KAABAR/KAA/0/+1/+1 $
SMP2      USET,GO,MFF/MAA $
LABEL     LBL5 $
EQUIV     KAA,KLL/REACT/MAA,MLL/REACT $
COND      LBL6,REACT $
VEC       USET/ALVEC/*A*/*L*/*R* $
PARTN     KAA,ALVEC, /KLL,KRL,KLR,KRR/-1 $
RBMG1     USET, ,MAA/ , , ,MLL,MLR,MRR $
LABEL     LBL6 $
OUTPUT1, ,,,,/ /-1/4 $ SET THE DEFAULT LABEL
OUTPUT1   MLL,KLL,, ,/ /0 /4 $ FOR ORTHOGONALIZATION AND FINAL CHECK
DECOMP    KLL/LLL,ULL/0/0/S,N,KLMIN/S,N,KLDET/S,N,KLPWR/S,N,KLSING $
COND      LBL7,REACT $
FBS       LLL,ULL,KLR/DM/0/-1 $
MPYAD     KRL,DM,KRR/X/0/+1/+1 $
DECOMP    X/XL,XU/0/0/S,N,XMIN/S,N,XDET/S,N,XPWR/S,N,XSING $
```

RITZ METHOD FOR TRANSIENT RESPONSE
IN SYSTEMS HAVING UNSYMMETRIC STIFFNESS

```
DECOMP   KRR/KRLL,KRUU/0/0/S,N,KRMIN/S,N,KRDET/ S,N,KRPWR/S,N,KRSING $
PARAMR   //C,N,DIV/ / / /V,N,RBEPS/V,N,XDET/V,N,KRDET $
PRTPARM  //0/C,N,RBEPS $$ RIGID BODY TRANSFORMATION CHECK
LABEL    LBL7 $
$$USER INPUT = CONTK, A matrix of control properties of the same order as KLL.
$$    = SCVEC, A vector of a leading 1 followed by zeroes of order equal to
$$      the number of loading subcases.
$$    = SCADJ, A vector of leading zeroes followed by a trailing 1 of order
$$      equal to the number of loading subcases.
$$    = KAPPA, A real double precision parameter for setting the tolerance of
$$      the cosine linear independence check.
$$    = MODSPEC, A parameter for setting the integer maximum of harmonics
$$      to be generated.
$$    = LONGONE, A Vector of ones whose length is greater than twice the
$$      number of subcases times MODSPEC plus one. L= 2(SC)(1 + MODSPEC)
$+++++++++++++++++++++++++++++++++++++++
FBS      LLL,ULL,MLL/SOLP  $ INERTIA COEFFICIENT FOR HARMONIC RITZ P VECTORS
$$                           PER EQUATION (15)
$   INTRODUCE THE SOL'N OF THE TRANSPOSE OF KLL TO DEVELOP INVERSE VECTORS
TRNSP    KLL/KLLT/ $
DECOMP   KLLT/TLLL,TULL/0/0 $
FBS      TLLL,TULL,MLL/SOLA $ INERTIA COEFFICIENT FOR HARMONIC RITZ A VECTORS
$$                            PER EQUATION (15)
$++++++++++++++++++++++++++++++++++++++++++
ALTER 88,92 $ REPLACE SSG3 WITH NON SYM OPN'S
FBS      LLL,ULL,PL/ULV/ $
FBS      TLLL,TULL, PL /TULV/ $PAY THE PRICE OF DOING FBS ON INVERSE TOO
COND     NOUOOV,OMIT $
FBS      KOL,KOU,PO/UOOV/ $
LABEL    NOUOOV $
```

```
$
$$  CALCULATION OF RULV IN CONNECTION WITH IRES WILL BE IGNORED BECUZ IT WOULD
$$  BE ADVISABLE TO DO THIS CHECKING WITH SYMMETRIC MATRICES.
$ ======> USER SHOULD ESCHEW USE OF IRES <=======
$ THE EPSILON SUB E CHECK WILL BE DONE IN DBL PREC BECUZ PARAMD IS NOW AVAILABLE
$
MPYAD    KLL,ULV,PL/DELPL/0/-1/+1 $
MPYAD    ULV,DELPL, /DELWORK/+1/+1 $
MPYAD    PL,ULV, /ALLWORK/+1/+1 $
SCALAR   DELWORK/ /1/1/ /V,N,EPSNUM $
SCALAR   ALLWORK/ /1/1/ /V,N,EPSDEN $
PARAMD   / /*DIV*/V,N,EPSUBE/V,N,EPSNUM/V,N,EPSDEN $
PRTPARM  //0/C,N,EPSUBE $$ RIGID BODY TRANSFORMATION CHECK
PARAM    / /*ADD*/V,N,ADJCYC/+1/0 $ VALUE OF PARAM REMAINS POSITIVE DURING
$$                                      PROCESSING OF PRIMARIES.
COPY     LONGONE/CLONONE/ 0  $
ADD      LONGONE,CLONONE/LONGNULL/(-1.0,0.0) $
PARAML   SCVEC/ /*TRAILER*/2/V,N,VECRO $ ROW SIZE IS READ FROM SCVEC
PARAML   ULV/ /*TRAILER*/1/V,N,ZCOL $ COL SIZE IS READ FROM ULV
PARAM    / /*EQ*/V,N,LODNO/V,N,VECRO/V,N,ZCOL $ LODNO IS NEGATIVE IF VECRO=ZCOL
COND     ADJLUP,LODNO $ CONTINUE IF SCVEC AND ULV AGREE
JUMP     ERROR4 $ ABORT IF SCVEC AND ULV DON'T AGREE
FILE     FALCCLI= SAVE/FALRRLI= SAVE $
LABEL    ADJLUP $ TOP OF LOOP PRIOR TO FORMATION OF EITHER PRIM OR ADJ FUND
COND     ADJTRN,ADJCYC $ ADJCYC INITIALLY IS POS TO GIVE PRIORITY TO PRIMARY
COPY     ULV/FLV/ 0 $ FLV WILL REMAIN AN INTERNAL DATA BLOCK TO THIS FUND LOOP
EQUIV    ULV,CLONFLV $ CLONFLV IS INTERNAL. EQUIV WILL BE BROKEN AT TOP'O LOOP
JUMP     PRIMSEG $ GO AROUND THE AJOINT PREP
LABEL    ADJTRN $
COPY     TULV/FLV/ 0 $ FLV IS INTERNAL
EQUIV    TULV,CLONFLV $ CLONFLV IS INTERNAL
LABEL    PRIMSEG $
```

```
MPYAD    CLONFLV,FLV, /FLSQ/+1 $
DIAGONAL FLSQ/SCALF/*DIAGONAL*/C,N,-0.5 $
MPYAD    FLV,SCALF, /FMOD/ 0 $ CANDIDATE MODE NORMALIZED TO EUCLID LGTH.
COPY     FMOD/FCLON/ 0 $
MPYAD    FCLON,FMOD, /FDOT/+1 $ MATRIX ORDERS = ZCOL.
MATPRN   FDOT,,,,// $DB
COPY     SCVEC/SCVECI/ 0 $ ROW BUBBLE STARTING FROM HEAD
COPY     SCVEC/LIVECI/ 0 $ COL BUBBLE STARTING FROM HEAD
COPY     SCVEC/MODPARTN/ 0 $ DUMMY TO BE USED FOR SWITCHING WITHIN LOOPS
ADD      LIVECI,SCVEC/FALCCLI/ /(-1.0,0.0) $ NULL BUT SAME LENGTH AS SCVEC
COPY     FALCCLI/FALRRLI/ -1 $ NULL SAME LGTH AS SCVEC
PARAM    / /*MPY*/V,N,ROCNT/ 1 / 0 $ RESET ROW COUNT TO 0 BEFORE LI CHECK
LABEL    LIRLUP $ TOP OF ROW PORTION OF LINEAR INDEPENDENCE LOOP
PARAM    / /*MPY*/V,N,NORFAL/+1/-1 $ SET DEFAULT TO NEG TO JUMP OVER FAIL BOOK
PARTN    FDOT, ,SCVECI/ ,ROCAI, , /+7/+2 $
PARAM    / /*ADD*/V,N,ROCNT/V,N,ROCNT/ 1 $ ROW COUNT MONITOR INCREMENTED BY ONE
PARAM    / /*MPY*/V,N,CLCNT/ 1 /V,N,ROCNT $SET COL COUNT=ROW COUNT PRIOR LI CHK
LABEL    LICLUP $ TOP OF COLUMN PORTION OF LINEAR INDEPENDENCE LOOP
PARAM    / /*ADD*/V,N,CLCNT/V,N,CLCNT/ 1 $ COL COUNT MONITOR INCREMENTED BY ONE
SCALAR   ROCAI/ /1/V,N,CLCNT/ /V,N,RCF $ COSINE TERM TO BE TESTED
PARAMD   / /*ABS*/V,N,COSRCF/V,N,RCF $GETS ABSOLUTE VALU OF COS (ROW,COL) TERM
PARAMD   //*LE*/ /V,Y,KAPPA/V,N,COSRCF////V,N,LICHK $LICHK =-1 IF KAPPA < COSRC
PARTN    LIVECI, ,SCADJ/CDUM, , ,/+7/+1 $ HAVE BUBL VEC TO TRACK.TRIM TRAIL ZER
MERGE    CDUM, , , , ,SCVEC/LIVECJ/+7/+1 $BUBBLE INCREMENTED AWAY FROM HEAD
COND     FALBOOK,LICHK $ CATALOG FAILURE POSITION
JUMP     MORCLI $ SKIP AROUND CATALOGING IF TEST WAS SUCCESSFUL
LABEL    FALBOOK $
PARAM    / /*MPY*/V,N,NORFAL/V,N,LICHK/ -1 $ SETS SIGNAL ONLY WHEN A COL FAILS.
$$                    HAS OPPOSITE SIGN TO LICHK.  POSSIBLE REPEATS ARE O.K.
PRTPARM  / /0/C,N,ROCNT $ ROW # OF CANDIDATE WHICH FAILED LI TEST
PRTPARM  / /0/C,N,CLCNT $ COL # OF CANDIDATE WHICH FAILED LI TEST
ADD      FALCCLI,LIVECJ/FALCCLJ/ $ ACCUM OF COL POS'NS OF FAILURES
```

```
SWITCH    FALCCLI,FALCCLJ/ / V,N,LICHK $
LABEL     MORCLI $ CONTINUE LI TESTING IN THIS ROW EVEN AFTER A COL FAILS
SWITCH    LIVECI,LIVECJ/ / -1 $
PARAM     / /*EQ*/V,N,LICDUN/V,N,CLCNT/V,N,ZCOL $ LICDUN = -1 IF CLCNT = ZCOL
COND      NUROW,LICDUN $ JUMP OUTSIDE OF COL LOOP IF @ LAST COL
REPT      LICLUP,999 $$$$$$$$$$ END OF COLUMN LOOP
$
LABEL     NUROW $
$
COND      GUDRO,NORFAL $ JUMP IF NO COLS IN CURRENT ROW HAD A LI FAILURE
ADD       FALRRLI, SCVECI/FALRRLJ/ $ ACCUMULATED ROW POSN'S OF FAILED ROWS
SWITCH    FALRRLI,FALRRLJ/ / -1 $ SWITCH ONLY IF THIS ROW FAILED,ELSE STAYS
LABEL     GUDRO $
PARAM     / /*SUB*/V,N,ROTEST/V,N,ZCOL/1 $ DECREMENT ZCOL BY ONE FOR ROTEST
PARAM     / /*EQ*/V,N,LIRDUN/V,N,ROCNT/V,N,ROTEST $ LIRDUN = -1,IF ROCNT=ROTEST
COND      KLENUP,LIRDUN $
JUMP      MOROW $ GO AROUND CLEAN UP IF MORE ROWS REMAIN TO BE TESTED
LABEL     KLENUP $
PARAML    FALCCLI/ /*TRAILER*/C,N,6/V,N,FALCDENS $ DENSITY OF THE COL FAIL VEC
PARAML    FALRRLI/ /*TRAILER*/C,N,6/V,N,FALRDENS $ DENSITY OF THE ROW FAIL VEC
PARAM     / /*LE*/V,N,DENSLK/V,N,FALCDENS/V,N,FALRDENS $ IF COL DENS </= ROW DEN:
$$                 DENSITY SELECTION PARAMETER IS NEGATIVE
PRTPARM   / /C,N,0/C,N,DENSLK $DB
COND      OTHER,DENSLK $ SWITCH MODPARTN TO THE VECTOR W LWR DENSITY
SWITCH    FALRRLI,MODPARTN/ /-1 $
JUMP      MODSET $
LABEL     OTHER $
SWITCH    FALCCLI,MODPARTN/ /-1 $
LABEL     MODSET $
PARTN     FMOD,MODPARTN, /PHI, , , /+7/+2 $SURVIVORS OF LI TEST
PARTN     SCALF, ,MODPARTN/PFVEC,,,/+7/+1 $ CLUSTER VECTOR FOR MERGING FUND-
$$                                         AMENTAL MODES WITH HARMONICS
```

```
MERGE     PFVEC,,,,,LONGNULL/HEADPF/+7/+1 $PUT PFVEC AT THE HEAD OF A LONG VEC
PARTN      LONGONE, ,HEADPF/SHORTONE,,,/+7/+1 $ PARTITION LONGONE DOWN BY
PERMANENT
MERGE     SHORTONE,,,,,LONGNULL/HEDSHORT/+7/+1 $APPEND PERMANENT-SIZE NULL TO
TAIL
ADD       HEDSHORT,LONGONE/NEGTAL/(1.0,0.0)/(-1.0,0.0) $PERMANENT-SIZE NEG @ TAIL
$
$  PROVIDE FOR THE POSSIBILITY OF THE CULLING VECTOR CONTAINING 1'S IN THE END
$  POSITIONS, WHICH WOULD DESTROY THE FUNCTION OF THE SHIFTING VECTORS. CONVERT
$  SCALF TO ALL ONES.
$
MERGE     SCALF,,,,,SCADJ/COLSCAL/+7/+2/+2 $ SETS TRAILER TO RECTANGULAR
TRNSP     COLSCAL/SCALPRO $ CONVERT COL TO ROW
MPYAD     COLSCAL,SCALPRO, /SQUID/0 $SQUARE MTX OF READ D.F. IN PREP FOR DIAG
DIAGONAL SQUID/FULU/*COLUMN*/0.0 $FULU IS A CLUSTER OF ALL 1'S. LGTH=FMOD
PARTN     FULU,,MODPARTN/FUNPART,TOSS,,/+7/+1/+2/+2 $LGTH 1ST=PHI,TOSS=COMP WRT
FMOD
MERGE     FUNPART,,,,,FULU/HEDCLUS/+7/+1 $FORM CLUSTER OF FUNPART @ VEC HEAD
MERGE     TOSS,,,,,FULU/HEDTOSS/+7/+1/+2 $FORM CLUSTER OF TOSS @ VEC HEAD
ADD        FULU,HEDTOSS/TALCLUS/(1.0,0.0)/(-1.0,0.0) $ CAP ZEROES @ HEAD OF
FUNPART
$
COND      ADJWRAP,ADJCYC $
$
COPY      PHI/PHIPI/ 0 $
ADD       NEGTAL, /LONGPRMI/(-1.0,0.0) $ TAIL CLUSTER = PERMANENT PRIMARY MODES
PARTN     SCVEC, ,HEDCLUS/,HMHED,,/+7/+1 $ HEAD SHIFTER TRIMMED TO LGTH = PHIPI
PARTN     SCADJ, ,TALCLUS/,HMTAL,,/+7/+1 $ TAIL SHIFTER TRIMMED TO LGTH = PHIPI
JUMP      ADJKUNT $
LABEL     ADJWRAP $
COPY      PHI/PHIAI/ 0 $
ADD       NEGTAL, /LONGARMI/(-1.0,0.0) $ TAIL CLUSTER = PERMANENT ADJOINT MODES
```

```
PARTN    SCVEC, ,HEDCLUS/,ADJHED,,/+7/+1 $ SHIFTER TRIMMED TO LGTH = PHIAI
PARTN    SCADJ, ,HEDCLUS/,ADJTAL,,/+7/+1 $ SHIFTER TRIMMED TO LGTH = PHIAI
$
JUMP     HARMONY $
$
LABEL    MOROW $
PARAM    / /*MPY*/V,N,CLCNT/1/0 $ RESET COLUMN COUNT TO ZERO
PARTN    SCVECI, ,SCADJ/RDUM, , , /+7/+1 $ TRIM TRAILING ZERO
MERGE    RDUM, , , , ,SCVEC/SCVECJ/+7/+1 $ BUBBLE INCREMENTED AWAY FROM HEAD
SWITCH   SCVECI,SCVECJ/ / -1 $
COPY     SCVECI/LIVECI/ 0 $ COL BUBBLE INDEX ALIGNED WITH ROW TRACKER
$
REPT      LIRLUP,999 $
$
LABEL    ADJKUNT $
PARAM    / /*MPY*/V,N,ADJCYC/1/-1 $
$
REPT      ADJLUP,999 $
$
LABEL    HARMONY   $
$
OUTPUT1, ,,,,/ /-1/0 $ CALLS THE DEFAULT LABEL. NEEDED FOR REWINDS LATER.
OUTPUT1  PHIPI,PHIAI,SOLP,SOLA, / /0/0 $MANY CALLS TO BE MADE IN HARMONIC PHASE
OUTPUT1, ,,,,/ /-1/1 $ SETS THE DEFAULT LABEL
OUTPUT1  HMHED,HMTAL,ADJHED,ADJTAL, / /0/1 $
OUTPUT1, ,,,,/ /-1/2 $ SETS THE DEFAULT LABEL
OUTPUT1  LONGNULL,LONGPRMI,LONGARMI,LONGONE, / /0/2 $
$
ALTER 154,154 $ REMOVE OPTIMIZATION LOOP TO PREVENT O'FLOW OF CEITBL
$
ENDALTER $ END OF ALTER FOR RITZ FUNDAMENTAL MODES
```

APPENDIX B

DMAP CODING FOR
PRIMARY AND ADJOINT HARMONICS

```
APP  DMAP $ FOR EXECUTION AFTER RITZFUND. INPUTS FROM INPT,INP1,INP2
BEGIN $  PROGRAMMED FOR 1988 VERSION OF NASTRAN. OUTPUT TO INP3
FILE LONGPRMI=SAVE/LONGARMI=SAVE/PGENI=SAVE/AGENI=SAVE/FULU=SAVE $
FILE PHHED=SAVE/PHTAL=SAVE/AHHED=SAVE/AHHED=SAVE/AHTAL=SAVE $
PARAM    / /*MPY*/V,N,PRIMCYC/+1/-1 $ CONTROL PARAM FOR PRIMARY 1ST HARMONIC
PARAM    / /*ADD*/V,N,ADJCYC/+1/ 0 $ CONTROL PARAM FOR ADJUNCT 1ST HARMONIC
PARAM    / /*ADD*/V,N,NUPGEN/+1/ 0 $ CONTROL PARAM FOR PRIMARY HIGHER HARMONICS
PARAM    / /*ADD*/V,N,NUAGEN/+1/ 0 $ CONTROL PARAM FOR ADJUNCT HIGHER HARMONICS
PARAM    / /*MPY*/V,N,HARMNO/ 1/ 0 $ SET THE HARMONIC COUNTER TO ZERO.
$
LABEL    HMNICGEN $ TOP OF LOOP FOR HARMONIC GENERATION %%%%%%%%%%%
$
PARAM    / /*ADD*/V,N,HARMNO/V,N,HARMNO/ 1 $INCREMENT THE HARMONIC COUNT BY ONE
COND     PRIMPREP,PRIMCYC $
COND     ADJPREP,ADJCYC $
COND     PHMNPREP,NUPGEN $
COND     AHMNPREP,NUAGEN $
LABEL    PRIMPREP $
INPUTT1  /, , , , /-1/0 $
INPUTT1  /HLV,,,, /0/0 $ READ PHIPI INTO HLV
COPY     HLV/PHIPI/ 0 $ THIS IS THE ROOT FOR THE 1ST MERGE OF HARMONICS TO PRIM
EQUIV    HLV,TESTER/PRIMCYC  $ HLV AND TESTER ARE INTERNAL NAMES OF GENERATOR.
$$       LATER ON, EQUIV WILL BE BROKEN AT TOP'O LOOP.
INPUTT1  /KMMTX,,,, /1/0 $ SKIP PASSED 2ND DB AND READ SOLP INTO KMMTX
INPUTT1  /, , , , /-1/1 $
INPUTT1  /HEDVECI,TALVECI,,, /0/1 $READ HMHED INTO HEDVECI & HMTAL INTO TALVECI
```

RITZ METHOD FOR TRANSIENT RESPONSE
IN SYSTEMS HAVING UNSYMMETRIC STIFFNESS

```
INPUTT1   /, , , , /-1/1 $
INPUTT1   /PGENI,PHTAL,,,/0/1 $READ HMHED INTO PGENI;READ HMTAL INTO PHTAL.
COPY      PGENI/PHHED/ 0 $ DUMMY STATEMENT TO FOOL THE COMPILER
ADD       HEDVECI,TALVECI/FULU $ FLUFF TO HELP FIAT LOCATE THE REAL FULU
JUMP      HMYBUS $ GO AROUND THE AJOINT PREP
LABEL     ADJPREP $
INPUTT1   /,,,,/-1/0 $ REWIND FROM PREVIOUS PASS THRU LOOP AND POSITION @ 1ST DB
INPUTT1   /HLV,,,, /1/0 $SKIP PASSED 1ST DB AND READ PHIAI INTO HLV
COPY      HLV/PHIAI/0 $THIS IS THE ROOT FOR THE 1ST MERGE OF HARMONICS TO ADJOINT
EQUIV     HLV,TESTER/ADJCYC  $ HLV & TESTER ARE INTERNAL NAMES OF GENERATOR
INPUTT1   /KMMTX,,,, /1/0 $ SKIP PASSED 3RD DB AND READ SOLA INTO KMMTX
INPUTT1   /, , , , /-1/1 $
INPUTT1   /HEDVECI,TALVECI,,,/2/1 $READ ADJHED INTO HEDVECI & ADJTAL INTO TALVECI
INPUTT1   /, , , , /-1/1 $
INPUTT1   /AGENI,AHTAL,,,/2/1 $SKIP 2DB & READ ADJHED > AGENI;READ ADJTAL >PHTAL.
COPY      AGENI/AHHED/ V,N,ADJCYC $
JUMP      HMYBUS $
LABEL     PHMNPREP $
COPY      PGENI/HLV/ 0 $
EQUIV     PGENI,TESTER/NUPGEN  $ TESTER IS INTERNAL NAME OF GENERATOR
INPUTT1   /,,,, /-1/0 $ REWIND FROM PREVIOUS PASS THRU LOOP AND POSITION @ 1ST DB
INPUTT1   /KMMTX,,,, /2/0 $ SKIP PASSED 1ST 2 DB'S AND READ SOLP INTO KMMTX
EQUIV     PHHED,HEDVECI/NUPGEN/PHTAL,TALVECI/NUPGEN $
JUMP      HMYBUS $
LABEL     AHMNPREP $
COPY      AGENI/HLV/ 0 $
EQUIV     AGENI,TESTER/NUAGEN  $ TESTER IS INTERNAL NAME OF GENERATOR
INPUTT1   /,,,, /-1/0 $ REWIND FROM PREVIOUS PASS THRU LOOP AND POSITION @ 1ST DB
INPUTT1   /KMMTX,,,, /3/0 $ SKIP PASSED 1ST 3 DB'S AND READ SOLA INTO KMMTX
EQUIV     AHHED,HEDVECI/NUAGEN/AHTAL,TALVECI/NUAGEN $
LABEL     HMYBUS $
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
PARAM     / /*MPY*/V,N,INITIAL/+1/-1 $ NEWLY GENERATED CANDIDATE GOING TO
$$                                      INITIAL TEST.  SET TO -1.
MPYAD     KMMTX,HLV, /UHC/0/1/0/2 $ CANDIDATES OF THE INERTIAL RESPONSES
PURGE     KMMTX $
COPY      UHC/CLONUHC/ 0 $
MPYAD     CLONUHC,UHC, /UHCSQ/ +1/+1/0/2 $
DIAGONAL UHCSQ/SCALH/*DIAGONAL*/-0.5 $ VECTOR OF EUCLIDEAN LENGTHS.
MPYAD     UHC,SCALH, /PHIHC/0/1/0/2 $ MATRIX OF CANDIDATE FIRST HARMONIC MODES.
COPY      PHIHC/CANDIDAT/ 0 $ USE GENERALIZED LOOP NAMES
$$
LABEL     LIPREP $ TOP OF LOOP FOR LINEAR INDEPENDENCE CHECKING %%%%%%%%%%%%
PARAM     / /*MPY*/V,N,RFALNO/1/-1 $ SET DEFAULT TO NEGATIVE.
$$
MPYAD     CANDIDAT,TESTER, /CVST/ +1/+1/0/2 $ HARMONICS AGAINST THE GENERATORS.
MATPRN    CVST,,,,/ / $
COND      RECT0,INITIAL $ SUBSTITUTE A REDUCED FULU IF REMNANT < GENERATOR
DIAGONAL CVST/FULU/*COLUMN*/0.0 $ALL ONE VEC FOR HARM VS HARM (CVST IS SQUARE)
LABEL     RECT0 $
EQUIV     HLV,TESTER/MODSPEC $BREAK EQUIV W TESTER
EQUIV     PGENI,TESTER/MODSPEC/AGENI,TESTER/MODSPEC $BREAK EQUIV W TESTER
COPY      HEDVECI/HMROWI/ 0 $ ROW BUBBLE STARTING FROM HEAD
ADD       HMROWI,HEDVECI/FALHRI/ /(-1.0,0.0) $ NULL. SAME LENGTH AS HEDVECI
COPY      FALHRI/FALHCI/ 0 $ NULL. SAME LENGTH AS HEDVECI
PARAM     / /*MPY*/V,N,CLKNT/ 1/ 0 $ RESET COL COUNT TO 0 BEFORE LI CHECK
PARAM     / /*MPY*/V,N,ROKNT/ 1/ 0 $ RESET ROW COUNT TO 0 BEFORE LI CHECK
PARAML    CVST//*TRAILER*/2/V,N,HROW $ ROWS IN CVST
COND      RECTT,INITIAL $ JUMP IF ON RECTANGULAR CYCLE
PARAM     / /*SUB*/V,N,HROW/V,N,HROW/1 $ REDUCE ROW TEST VALUE 1 FOR TRIANGLE
LABEL     RECTT $
PARAML    CVST//*TRAILER*/1/V,N,HCOL $ COLS IN CVST
$
LABEL     HLIRLUP $ TOP OF ROW PORTION OF LINEAR INDEPENDENCE CHECK %%%%%%%%%%
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
$
COPY      HEDVECI/HMCOLI/ 0 $ COL BUBBLE STARTING FROM HEAD
PARTN     CVST, ,HMROWI/ ,ROCH, , /+7/+2 $
PARAM     / /*ADD*/V,N,ROKNT/V,N,ROKNT/ 1 $ ROW COUNT MONITOR INCREMENTED BY ONE
COND      RECT1,INITIAL $
PARAM     / /*MPY*/V,N,CLKNT/1/V,N,ROKNT $ SET COL COUNT=ROW COUNT IF TRIANGLE
LABEL     RECT1 $
$
LABEL     HLICLUP $ TOP OF COLUMN PORTION OF LI LOOP %%%%%%%%%%%%%%%%%%%%%%%%%%
$
PARAM     / /*ADD*/V,N,CLKNT/V,N,CLKNT/ 1 $ INCREMENT THE COLUMN COUNT
SCALAR    ROCH/ /1/V,N,CLKNT/ /V,N,RCH $ RCH IS DBL PREC FETCH OF COSINE
$$ TERM IN ROW POSITION INDEXED BY THE CONSTANT PARAMETER '1' AND IN THE COL
$$ POSITION INDEXED BY THE VARIABLE PARAMETER CLKNT.
PARAMD    / /*ABS*/V,N,COSRCH/V,N,RCH $ GETS ABS. VAL. OF COS(ROW,COL) TERM
PARAMD    / /*LE*/ /V,Y,KAPPA/V,N,COSRCH/ / / /V,N,LIHZK $
COND      CATALOG,LIHZK $ GO TO CATALOGING IF LIHZK IS NEGATIVE
JUMP      MORHCOL $ JUMP TO MORE COL PROCESSING IF TEST PASSED
LABEL     CATALOG $
COND      RECT2,INITIAL $
PARAM     / /*MPY*/V,N,RFALNO/V,N,LIHZK/ -1 $ SETS SIGNAL ONLY WHEN A COL FAILS.
$$                   RFALNO TAKES ON OPPOSITE SIGN TO LIHZK. REPEATS ARE OK.
LABEL     RECT2 $
PRTPARM   / /0/C,N,ROKNT $ ROW # OF CANDIDATE TERM WHICH FAILED LI TEST
PRTPARM   / /0/C,N,CLKNT $ COL # OF CANDIDATE TERM WHICH FAILED LI TEST
COND      RECT3,INITIAL $
ADD       FALHCI,HMCOLI/FALHCJ/ $ ACCUMULATION OF COL POS'NS OF FAILURES
SWITCH    FALHCI,FALHCJ/ /V,N,LIHZK  $
$
```

```
LABEL    MORHCOL $ CONTINUE LI TESTING IN THIS ROW EVEN AFTER A COL FAILS
$
PARAM    / /*EQ*/V,N,LICDON/V,N,CLKNT/V,N,HCOL $ LICDON = -1 IF CLKNT = HCOL
COND     GNROW,LICDON $ JUMP OUTSIDE OF COL LOOP IF @ LAST COL.
PARTN    HMCOLI, ,TALVECI/DUMMY,,,/+7/+1 $ TRIM TRAILING ZERO
MERGE    DUMMY, , , , ,HEDVECI/HMCOLJ/+7/+1 $ BUBBLE INCREMENTED AWAY FROM HEAD
SWITCH   HMCOLI,HMCOLJ/ / -1 $
REPT     HLICLUP,999 $ $$$$$$$$$$$$$$$  END OF COLUMN LOOP
$
LABEL    GNROW $ CONSIDER TESTING ANOTHER ROW
$
COND     GODRO,RFALNO $ JUMP IF NO COLS IN CURRENT ROW HAD A LI FAILURE
LABEL    RECT3 $ CATALOG ROW FAILURE
ADD      FALHRI, HMROWI/FALHRJ/ $ ACCUMULATED ROW POS'NS OF FAILED ROWS
SWITCH   FALHRI,FALHRJ/ / -1 $ SWITCH ONLY IF THIS ROW FAILED, ELSE STAYS
COND     RECT4,INITIAL $ BYPASS IF ON RECTANGULAR ROUTE
PARAM    / /*MPY*/V,N,RFALNO/V,N,RFALNO/-1 $ RESET TO NEGATIVE.
LABEL    RECT4 $
$
LABEL    GODRO $
$
PARAM    / /*EQ*/V,N,LIRDON/V,N,ROKNT/V,N,HROW $ LIRDON = -1 IF ROKNT = HROW
COND     CLENUP,LIRDON $ JUMP OUT OF LI CHECKING IF MTX IS COMPLETELY EXAMINED
PARAM    / /*MPY*/V,N,CLKNT/1/0 $ RESET COLUMN COUNT TO ZERO
PARTN    HMROWI, ,TALVECI/DUMMY,,,/+7/+1 $ TRIM TRAILING ZERO
MERGE    DUMMY,,,,,HEDVECI/HMROWJ/+7/+1 $BUBBLE INCREMENTED AWAY FROM HEAD
SWITCH   HMROWI,HMROWJ/ /-1 $
$
REPT      HLIRLUP,999 $ END OF ROW PORTION OF LINEAR INDEPENDENCE LOOP%%%%%%%%%
$
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
LABEL    CLENUP $
$
COND     RECT5,INITIAL $
PARAML   FALHCI/ /*TRAILER*/C,N,6/V,N,DENSFALC $ DENSITY OF THE COL FAIL VEC
PARAML   FALHRI/ /*TRAILER*/C,N,6/V,N,DENSFALR $ DENSITY OF THE ROW FAIL VEC
PARAM    / /*LE*/V,N,SLKDENS /V,N,DENSFALC/V,N,DENSFALR $ IF COL DENS
$$       </= ROW DENS, THE DENSITY SELECTION PARAMETER IS NEGATIVE.
COND     UTHER,SLKDENS $SET MODEPART TO THE VECTOR WITH LWR DENSITY
LABEL    RECT5 $
COPY     FALHRI/MODEPART/ 0 $
JUMP     MODESET $
LABEL    UTHER $
COPY     FALHCI/MODEPART/ 0 $
LABEL    MODESET $
PARAML   MODEPART/ /*TRAILER*/C,N,6/V,N,MODENSY $ DENSITY OF MODEPART
PRTPARM  / /0/C,N,MODENSY $
PARAM    / /*GE*/V,N,FILLED/V,N,MODENSY/10000 $ FILLED=-1 IF MODEPART IS FULL
COND     FOLD,FILLED $
JUMP     FLEDGE  $
LABEL    FOLD $ SAVE AND GO
PARAM      /   /*GT*/V,N,SUMHUM/V,N,HARMNO/2   $   SUMHUM=-1   IF 1ST HARMS OF P & 
PASSED
PRTPARM  / /0/C,N,SUMHUM $
COND     ORTHOG,SUMHUM $ IF 1ST HARMS FAIL RESTORE ORIGINAL NAMES TO OUTPUT
INPUTT1  /, , , , /-1/0 $
INPUTT1  /PHIPI,PHIAI,,,/0/0 $ FUNDS W/O HARMONICS
INPUTT1  /, , , , /-1/1 $
INPUTT1  /HEADPHI,TAILPHI,,,/0/1 $TRACKERS W/O HMNIC.HMHED=HEADPHI.HMTAL=TAILPH
JUMP     NOBIZNEZ $ COPY OUT AS THEY CAME IN
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
LABEL     FLEDGE $
PARTN     CANDIDAT,MODEPART,/REMNANT,,,/+7/+2 $SURVIVORS OF LI TEST
$$ PROVIDE FOR THE POSSIBILITY OF THE CULLING VECTOR CONTAINING 1'S IN THE
$$ ENDS, WHICH WOULD DESTROY THE SHIFTING VECTORS. CONVERT SCALH TO ALL ONES.
COND      RECT6,INITIAL $
JUMP      TRIA6 $
LABEL     RECT6 $
MERGE     SCALH,,,,,TALVECI/COLSCAL/+7/+2/+2 $ SETS TRAILER TO RECTANGULAR
TRNSP     COLSCAL/SCALPRO $ CONVERT COL TO ROW
MPYAD     COLSCAL,SCALPRO, /SQUID/0 $ SQAURE MTX OF REAL S.P. IN PREP FOR DIAG
DIAGONAL  SQUID/FULU/*COLUMN*/0.0 $FULU IS A CLUSTER OF ALL ONES LGTH=CANDIDAT
LABEL     TRIA6 $
PARTN     FULU,,MODEPART/HMYPART,TOSS,,/7/1/2/2 $1'S.LGTH 1ST =REMNANT.TOSS=COMPI
MERGE     HMYPART,,,,,FULU/HEDCLUS/+7/+1/2 $FORM CLUSTER OF HMYPART @ VEC HEAD
PARTN     HEDVECI,,HEDCLUS/,HEDVECJ,,/7/1/2/2 $NU SHIFTER HAS LGTH=REMNANT
MERGE     TOSS,,,,,FULU/HEDTOSS/7/1/2 $FORM CLUSTER OF TOSS @ VEC HEAD
ADD       FULU,HEDTOSS/TALCLUS/(1.0,0.0)/(-1.0,0.0) $CAP ZEROES @ HEAD OF HMYPAR
PARTN     TALVECI,,TALCLUS/,TALVECJ,,/7/1/2/2 $NU SHIFTER HAS LGTH=REMNANT
SWITCH    HEDVECI,HEDVECJ/ / -1 $
SWITCH    TALVECI,TALVECJ/ / -1 $
PRTPARM   / /0/C,N,INITIAL $
COND      TEST2,INITIAL $ If INITIAL IS NEGATIVE GO TO 2nd LI TEST
JUMP      DEPOT $READY FOR MERGING AND GENERATING (1.ADJ 2.NU PHMNY 3.NU AHMNY)
LABEL     TEST2 $
$Test whether one or less columns of REMNANT are left. SET PARAMETER IF SO.
PARAML    REMNANT/ /*TRAILER*/1/V,N,REMCOL $
PARAM     / /*LE*/V,N,SALVAGE/V,N,REMCOL/1 $
COND      DEPOT,SALVAGE $
SWITCH    REMNANT,CANDIDAT/ /V,N,INITIAL $
COPY      CANDIDAT/TESTER/V,N,INITIAL $
PARAM     / /*MPY*/V,N,INITIAL/V,N,INITIAL/ -1 $
REPT       LIPREP,1 $ MAKE A 2ND PASS FOR THE LI TESTS ON HARMONICS ALONE.
```

```
$
LABEL     DEPOT $ STAGING POINT.WRAPUP.MERGE.HARMONIC GENERATION.
COND      QUIV1,PRIMCYC $
JUMP      CHKADJ $
LABEL     QUIV1 $
INPUTT1   /, , , , /-1/2 $
INPUTT1   /LONGPRMI,,,, /1/2 $SKIP 1 DB & READ IN LONGPRMI
EQUIV     LONGPRMI,LONGRMI/PRIMCYC/PHIPI,PHII/PRIMCYC $
JUMP      MERGBUS $
LABEL     CHKADJ $
COND      QUIV2,ADJCYC $
JUMP      CHKPGEN $
LABEL     QUIV2 $
INPUTT1   /, , , , /-1/2 $
INPUTT1   /LONGARMI,,,, /2/2 $SKIP 2 DB & READ IN LONGARMI
EQUIV     LONGARMI,LONGRMI/ADJCYC/PHIAI,PHII/ADJCYC $
JUMP      MERGBUS $
LABEL     CHKPGEN $
COND      QUIV3,NUPGEN $
JUMP      CHKAGEN $
LABEL     QUIV3 $
EQUIV     LONGPRMI,LONGRMI/NUPGEN/PHIPI,PHII/NUPGEN $
JUMP      MERGBUS $
LABEL     CHKAGEN $
COND      QUIV4,NUAGEN $
JUMP      MERGBUS $
LABEL     QUIV4 $
EQUIV     LONGARMI,LONGRMI/NUAGEN/PHIAI,PHII/NUAGEN $
LABEL     MERGBUS $
INPUTT1   /, , , , /-1/2 $
INPUTT1   /LONGNULL,,,,/0/2 $READ IN LONGNULL
INPUTT1   /LONGONE,,,,/2/2 $ SKIP 2 DB & READ IN LONGONE
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
MERGE    HMYPART,,,,,LONGNULL/LHRMHED/+7/+1 $ INCREMENT = LENGTH OF NEW HARMONIC
PARTN    LONGONE, ,LHRMHED/DUMMY,,,/+7/+1 $ PARTITION LONGONE DOWN BY HARMONIC
MERGE    DUMMY,,,,,LONGNULL/HEDSHRTH/+7/+1 $APPEND HARMONIC-SIZE NULL TO TAIL
ADD      LONGONE,HEDSHRTH/LHRMTAL/ /(-1.0,0.0) $HARMONIC-SIZE CLUSTER @ TAIL
PARTN    LONGRMI,,LHRMHED/DUMMY,,,/+7/+1 $TRIM A HARMONIC INCREMENT OF ZEROES
$$         FROM THE HEAD OF LONGRMI.
MERGE    DUMMY,,,,,LHRMTAL/BBLRM/+7/+1 $PLUG A HARMONIC INCREMENT OF ZEROES
$$         ONTO THE TAIL OF LONGRMI.
ADD      BBLRM,LHRMTAL/LONGRMJ/ $ CLUSTER=NUHARM INC + ACCEPTED
PARTN    LHRMTAL,,LONGRMJ/ ,TRIMRM,,/+7/+1 $ THIS IS THE PARTITIONING VECTOR
$$         NEEDED FOR COMBINING THE HARMONIC TO THE MATRIX OF ACCEPTED MODES
$Test whether one or less columns of REMNANT are left. SET PARAMETER IF SO.
$
PARAML   REMNANT/ /*TRAILER*/1/V,N,REMCOL $
MERGE    PHII, ,REMNANT,,TRIMRM,/PHIJ/+7/+2 $ MERGED!!
SWITCH   LONGRMJ,LONGPRMI/ /V,N,PRIMCYC $
SWITCH   LONGRMJ,LONGPRMI/ /V,N,NUPGEN $
SWITCH   LONGRMJ,LONGARMI/ /V,N,ADJCYC $
SWITCH   LONGRMJ,LONGARMI/ /V,N,NUAGEN $
SWITCH   PHIJ,PHIPI/ /V,N,PRIMCYC $
SWITCH   PHIJ,PHIPI/ /V,N,NUPGEN $
SWITCH   PHIJ,PHIAI/ /V,N,ADJCYC $
SWITCH   PHIJ,PHIAI/ /V,N,NUAGEN $
SWITCH   HEDVECI,PHHED/ /V,N,PRIMCYC $
SWITCH   HEDVECI,AHHED/ /V,N,ADJCYC $
SWITCH   HEDVECI,PHHED/ /V,N,NUPGEN $
SWITCH   HEDVECI,AHHED/ /V,N,NUAGEN $
SWITCH   TALVECI,PHTAL/ /V,N,PRIMCYC $
SWITCH   TALVECI,AHTAL/ /V,N,ADJCYC $
SWITCH   TALVECI,PHTAL/ /V,N,NUPGEN $
SWITCH   TALVECI,AHTAL/ /V,N,NUAGEN $
$
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
PARAM    //*EQ*/V,N,HARMDONE/C,Y,MODSPEC/V,N,HARMNO $ IF # HARM=MODSPEC =>DONE
COND     ORTHOG,HARMDONE $JUMP OUTSIDE HARMONIC LOOP IF HARMONICS ARE DONE
$
COND     PRIMOUT,PRIMCYC $
JUMP     ADJHRMNY $
LABEL    PRIMOUT $
SWITCH   REMNANT,PGENI/ /V,N,PRIMCYC $
PARAM    //*MPY*/V,N,PRIMCYC/V,N,PRIMCYC/ -1 $ RESET PRIMCYC TO POSITIVE
PARAM    //*MPY*/V,N,ADJCYC/+1/-1 $ ENABLE THE LOOP FOR THE ADJOINT 1ST HARM
JUMP     HLOOPEND $
LABEL    ADJHRMNY $
COND     ADJOUT,ADJCYC $
JUMP     PHMNZ $
LABEL    ADJOUT $
SWITCH   REMNANT,AGENI/ /V,N,ADJCYC $-------
PARAM    //*MPY*/V,N,ADJCYC/V,N,ADJCYC/ -1 $ RESET ADJCYC TO POSITIVE
PARAM    //*MPY*/V,N,NUPGEN/+1/-1 $ ENABLE THE LOOP FOR THE PRIM HIGHER HARM
JUMP     HLOOPEND $
LABEL    PHMNZ $
COND     PHMNOUT,NUPGEN $
JUMP     AHMNZ $
LABEL    PHMNOUT $
SWITCH   REMNANT,PGENI/ /V,N,NUPGEN $--------
PARAM    //*MPY*/V,N,NUPGEN/V,N,NUPGEN/ -1 $ RESET NUPGEN TO POSITIVE
PARAM    //*MPY*/V,N,NUAGEN/+1/-1 $ ENABLE THE LOOP FOR THE ADJ HIGHER HARM
JUMP     HLOOPEND $
LABEL    AHMNZ $
SWITCH   REMNANT,AGENI/ /V,N,NUAGEN $-------
PARAM    //*MPY*/V,N,NUAGEN/V,N,NUAGEN/ -1 $ RESET NUAGEN TO POSITIVE
PARAM    //*MPY*/V,N,NUPGEN/+1/-1 $ ENABLE THE LOOP FOR THE PRIM HIGHER HARM
LABEL    HLOOPEND $
$
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
PARAM    / /*LE*/V,N,TESTOVER/V,N,REMCOL/1 $ TESTOVER =-1 IF  REMCOL (</=) 1
$$                            THIS IS THE EXIT  IN CASE FEWER HARMONICS PASS
$$                            THE LI TEST THAN ARE SPECIFIED BY MODSPEC
COND     ALTCHK,TESTOVER $ EXIT WHEN HARMONICS ARE EXHAUSTED PRIOR TO MODSPEC
JUMP     USUAL  $
LABEL    ALTCHK $
COND     USUAL,ADJCYC $ ADJOINT GETS A CHANCE TO GENERATE A SINGLE
COND     ORTHOG,NUPGEN $ PREVENT ANOTHER HARMONIC TO BE GENERATED FROM A SINGLE
COND     USUAL,NUAGEN $ ADJHRM GETS A CHANCE TO GENERATE A SINGLE
JUMP     ORTHOG $
$
LABEL     USUAL $
PURGE    KMMTX/MODSPEC $
REPT      HMNICGEN,999 $ END OF HARMONIC GENERATOR LOOP %%%%%%%%%%%%%%%%%%%%%%%
$
LABEL    ORTHOG $
$
MERGE    PHHED, , , , ,LONGNULL/LONGHED1/+7/+1 $**START OF HEADPHI CONSTRUCTION
PARTN    LONGONE, ,LONGHED1/DUMMY, , , /+7/+1 $ LUMP TO MERGE ON HEAD
MERGE    DUMMY, , , , ,LONGNULL/MISSTAIL/+7/+1 $ ONE MISSING FROM TAIL
ADD      LONGONE,MISSTAIL/LONGTAL1/ /(-1.0,0.0) $**START OF TAILPHI CONSTRUCTIO
PARTN    LONGONE, ,LONGPRMI/ ,DUMMY, ,/+7/+1 $ALL ONES OF LGTH=ACCEPTED VECTORS
MERGE    DUMMY, , , , ,LONGNULL/HEADER/+7/+1 $HEAD CLUSTER OBVERSE OF LONGPRMI
PARTN    LONGHED1, ,HEADER/ ,HEADPHI, , /+7/+1 $ SAVE FOR DELIVERY TO ORTHOG
PARTN    LONGTAL1, ,LONGPRMI/ ,TAILPHI, , /+7/+1 $ SAVE FOR DELIVERY TO ORTHOG
LABEL    NOBIZNEZ $ GET OUT WITH OUTPUT SAME AS INPUT
OUTPUT1, ,,,,/ /-1/3 $ SET DEFAULT LABEL
OUTPUT1  PHIPI,HEADPHI,TAILPHI,PHIAI, / /0/3 $
LABEL    PRINTOUT $
PRTPARM  //0/C,N,HARMNO $
LABEL    FINIS $
END   $ FINISH OF DMAP PROGRAM FOR RITZ HARMONICS
```

APPENDIX C

RITZRTHG.DMP
SELF AND DUO ORTHOGONALIZATION

```
NASTRAN MAXFILES = 60,FILES = (INP3,INP4)
APP       DMAP $ PROGRAMMED FOR 1988 NASTRAN. OUTPUT TO PUNCH FILE.
$$  EXECUTES AFTER BOTH RITZFUND AND RITZHARM TO ORTHOGONALIZE RITZ MODES
BEGIN $ $$ORTHOG.DMP
INPUTT1   /, , , , /-1/3 $
INPUTT1   /PHIPI,HEADPHI,TAILPHI,PHIAI, /0/3 $
PARTN     PHIPI,HEADPHI, / , ,PHI1, /+7/+2 $
COPY      PHI1/ZETA1/ 0 $
PARTN     HEADPHI, ,TAILPHI/DUMMY, , , /+7/+1 $
MERGE     DUMMY, , , , ,HEADPHI/BBLHI/+7/+1 $
PARTN     PHIPI,BBLHI, / , ,PHI2, /+7/+2 $
MPYAD     PHI1,PHI2, /NUM/+1/-1 $.
COPY      PHI1/CLONPHI1/ 0 $
MPYAD     CLONPHI1,PHI1, /DEN/+1/+1 $
SCALAR    NUM/ /1/1/ / /V,N,SPXNUM $
SCALAR    DEN/ /1/1/ / /V,N,SPXDEN $
PARAMR    / /*DIVC*/ / / /V,N,A11/V,N,SPXNUM/V,N,SPXDEN $
ADD       PHI1,PHI2/ZETA2/V,N,A11 $ SINGLE PREC.WON'T TAKE DBL PREC!!!
PARTN     TAILPHI, ,HEADPHI/DUMMD, , ,/+7/+1 $
MERGE     DUMMD,,,,,TAILPHI/BBLTI/+7/+1 $
ADD       BBLTI,TAILPHI/PTALCLUI/ $
PARTN     TAILPHI, ,PTALCLUI/,BUILDI, , /+7/+1 $
MERGE     ZETA1, ,ZETA2, ,BUILDI, /ZETAI/+7/+2 $
MPYAD     ZETAI,PHIPI, /COEFI/+1/+1 $
ADD       BBLHI,HEADPHI/PHEDCLUI/ $
PARAM     / /*ADD*/V,N,ROWCOW/2/0 $
PARAML    PHIPI/ /*TRAILER*/1/V,N,PCOL $
```

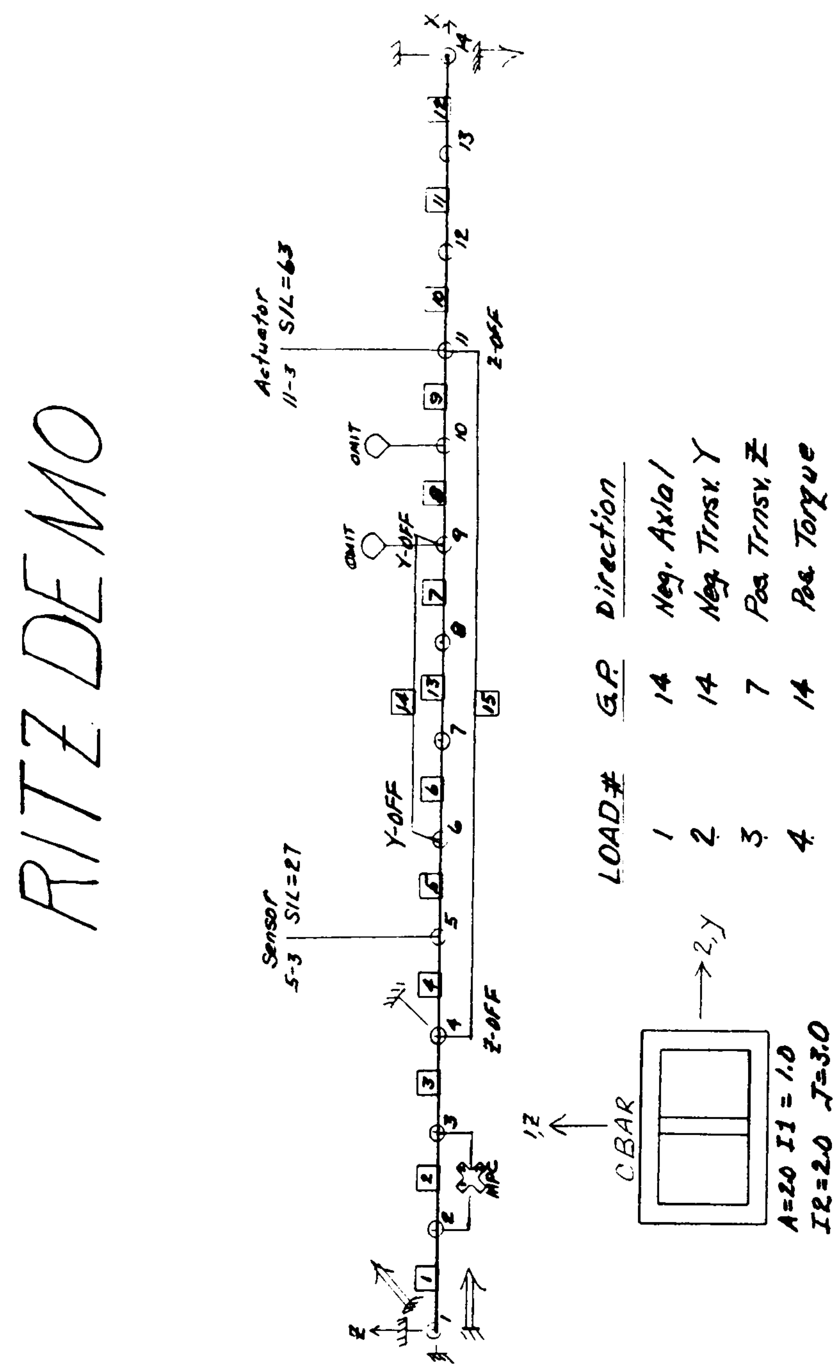
```
LABEL     ORTHLUP $ TOP OF SELF ORTHOGONALIZATION LOOP
PARAM     / /*ADD*/V,N,ROWCOW/V,N,ROWCOW/1 $
PARTN     COEFI,PHEDCLUI, / , ,CAI, /+7/+2 $.
PARTN     BBLHI, ,TAILPHI/DUMVEC, , , /+7/+1 $
MERGE     DUMVEC, , , , ,HEADPHI/BBLHJ/ +7/+1 $
PARTN     COEFI,BBLHJ, / , ,CFI, /+7/+2 $
SOLVE     CAI,CFI/AIN/-1/-1/2 $
PARTN     BBLTI, ,HEADPHI/DMY, , , /+7/+1 $
MERGE     DMY, , , , ,TAILPHI/BBLTJ/ +7/+1 $
ADD       PTALCLUI,BBLTJ/PTALCLUJ/ $
PARTN     TAILPHI, ,PTALCLUJ/ ,BUILDJ,,/+7/+1 $
PARTN     PTALCLUJ, ,TAILPHI/ ,UNIT, , /+7/+1/ /2 $ UNIT IS RECTANGULAR S.P.
MERGE     AIN,UNIT, , , ,BUILDJ/AJN/+7/+2 $
ADD       PHEDCLUI,BBLHJ/PHEDCLUJ/ $
PARTN     PHIPI,PHEDCLUJ, / , ,PHIZ, /+7/+2 $
MPYAD     PHIZ,AJN, /ZETAX/0$
MERGE     ZETAI, ,ZETAX, ,BUILDJ, /ZETAJ/+7/+2 $
MPYAD     ZETAJ,PHIPI, /COEFJ/+1/+1 $
SWITCH    ZETAI,ZETAJ/ / -1 $
SWITCH    BBLHI,BBLHJ/ / -1 $
SWITCH    PHEDCLUI,PHEDCLUJ/ / -1 $
SWITCH    BBLTI,BBLTJ/ / -1 $
SWITCH    PTALCLUI,PTALCLUJ/ / -1 $
SWITCH    BUILDI,BUILDJ/ / -1 $
SWITCH    COEFI,COEFJ/ / -1 $
PARAM     / /*EQ*/V,N,SELFDUN/V,N,ROWCOW/V,N,PCOL $
COND      DUALORTH,SELFDUN $
REPT      ORTHLUP,999 $
LABEL     DUALORTH $
COPY      ZETAI/CLONZETA/ 0 $
MPYAD     CLONZETA,ZETAI, /ZSQ/+1/+1 $
MATPRN    ZSQ,,,,// $
```

RITZ MODES FOR UNSYMMETRIC MATRICES
DMAP CODING OF THE THEORY

```
$
$ START OF DUAL ORHTOGONALIZATION OF ADJOINT MODES.
$
INPUTT1    /, , , ,/-1/4
INPUTT1    /MLL,KLL,,,/0/4 $
MPYAD      ZETAI,MLL, /ZEM/+1 $
MPYAD      ZEM,PHIAI, /KOEF/0 $
DIAGONAL   MLL/UNITY/*SQUARE*/0.0 $
SOLVE      KOEF,UNITY/BETA/-1/+1/+2/+2 $
MPYAD      PHIAI,BETA, /OMEGA/0 $
MPYAD      OMEGA,MLL, /MEGM/+1/+1 $
MPYAD      MEGM,ZETAI, /GENMASS/0 $
MPYAD      OMEGA,KLL, /MEGK/+1/+1 $
MPYAD      MEGK,ZETAI, /GENSTIF/0 $
MATPRN     GENMASS,ZETAI,OMEGA,GENSTIF,// $
OUTPUT3    ZETAI,OMEGA,GENMASS,GENSTIF,//0/C,Y,N1=ZZZ/
           C,Y,N2=MEG/C,Y,N3=MMM/C,Y,N4=KKK $
END $ FINISH OF ORTHOGONALIZATION OF RITZ VECTORS
```

APPENDIX D

DEMONSTRATION PROBLEM FOR UNSYMMETRIC RITZ



APPENDIX E

GENMASS KAPPA = .007

```
1.00000E+00 -1.57700E-21 -1.75392E-21 -4.23773E-26
7.33943E-25  1.00000E+00  5.52286E-26  1.84410E-23
4.86876E-22 -1.76725E-24  1.00000E+00 -1.41962E-23
6.08346E-27 -4.10067E-23  6.50197E-24  1.00000E+00
```

GENMASS KAPPA = 0.95

```
 1.00000E+00 -3.41619E-17 -2.08085E-17 -7.77767E-17 -3.55682E-15
        6.80886E-17  7.19268E-16 -3.59782E-16  1.85463E-15
-2.09800E-17  1.00000E+00 -2.66490E-16  5.34076E-15  4.56150E-17
       -1.0807            2732E-14  1.36521E-14 -1.98969E-13
-7.27244E-16  3.14776E-15  1.00000E+00  1.58474E-15  6.56536E-16
       -7.71337E-15 -3.17466E-14  1.96339E-14 -4.29536E-14
-6.40719E-18 -1.66050E-15  5.80912E-17  1.00000E+00  4.28422E-19
        1.45089E-15  7.28284E-15 -5.87202E-15  1.86094E-14
-1.15957E-15  1.05575E-16  4.21965E-17  1.36209E-16  1.00000E+00
       -2.06182E-16 -1.70354E-15  4.34321E-16 -2.34621E-15
-4.61736E-17  4.20176E-15 -3.42951E-16  1.05352E-14  5.74604E-17
        1.00000E+00  2.34472E-14  1.11612E-14 -1.95622E-13
-1.94247E-18 -4.76531E-16  3.61424E-17 -7.44025E-16  4.46676E-18
        1.22754E-15  1.00000E+00 -1.59921E-15  2.45933E-14
 2.42787E-18  8.22934E-16 -2.48630E-17  1.54808E-15 -1.46353E-17
       -6.75527E-16 -7.19174E-16  1.00000E+00 -1.18324E-14
 1.14403E-18 -1.60473E-17 -1.99198E-18  6.85738E-18 -1.30097E-18
        3.33071E-17 -6.84577E-18 -9.50914E-17  1.00000E+00
```

GENSTIF KAPPA = .007

```
 1.04682E+01  6.15654E-04 -9.26258E-04 -2.98838E-10
-3.08635E-04  4.61758E-02  2.74302E-08  5.87860E-08
-8.90200E-04 -5.23407E-08  1.36253E-02 -3.94594E-08
-8.76229E-10  2.54442E-05 -1.91582E-05  2.57602E+01
```

GENSTIF KAPPA = 0.95

```
 3.39196E+02 -6.69263E-01 -2.44002E-01 -9.04235E-01 -3.12387E+02
              1.17373E+00  2.61338E+01 -1.37057E+00  2.05697E+01
-1.08090E-02  3.15758E-01 -5.48427E-03  1.31833E-01  1.07321E-02
             -1.70708E-01  8.53201E-01  6.03132E-01 -2.68652E+00
-3.09520E-02  1.61339E-02  2.06822E-02  2.84852E-02  2.85713E-02
             -3.69205E-02 -2.37364E-01  1.00253E-01 -6.03664E-01
-8.47928E-01  1.11503E+02 -8.23610E+00  6.49374E+02  1.19001E+00
             -8.02691E+02  3.80773E+02  3.22937E+02 -4.57428E+03
-3.07444E+02  6.23311E-01  2.25336E-01  8.38877E-01  2.89818E+02
             -1.08889E+00 -2.42260E+01  1.27283E+00 -1.90819E+01
 7.98571E-01 -1.04804E+02  7.74130E+00 -5.92276E+02 -1.12001E+00
              7.54470E+02 -3.57896E+02 -3.03535E+02  4.29948E+03
 1.98390E-01  2.15040E-02 -8.83532E-03  2.23315E-02 -1.85797E-01
             -2.80937E-02  3.86155E-01 -1.58374E-02  6.00169E-02
 6.36402E-02  1.95278E+00 -1.21436E-01  4.85570E+00 -5.51377E-02
             -6.20209E+00  6.10810E+00  5.60222E+00 -7.46195E+01
 1.76325E-02 -2.24893E+00  1.66211E-01 -5.93498E+00 -2.44872E-02
              7.57854E+00 -7.62053E+00 -6.48352E+00  9.22406E+01
```