N89-22955

MODAL STRAIN ENERGIES IN COSMIC NASTRAN

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SUMMARY

A computer program was developed to take a NASTRAN output file from a normal modes analysis and calculate the modal strain energies of selected elements. The FORTRAN program can determine the modal strain energies for CROD, CBAR, CELAS, CTRMEM, CQDMEM2, and CSHEAR elements. Modal strain energies are useful in estimating damping in structures.

INTRODUCTION

This work was initiated to predict damping in a large passively damped truss structure. The twelve meter truss structure is currently undergoing modal testing in preparation for controls experiments. An estimate of the total damping in the structure is needed for the controls experiments.

The starting point for the computer program was the ANALYZE program (ref. 1). First, the information needed for the strain energies was extracted from a NASTRAN output file (ref. 2). Element information is extracted from the echo of the bulk data and eigenvector information is extracted from the eigenvector tables for each mode. The element stiffness matrices are formed and then multiplied by the appropriate element eigenvector and its transpose and divided by two. The result is the element strain energies for a given mode. With this information, the modal strain energy method can be used to predict the damping in a viscoelastically damped structure. COSMIC NASTRAN can output element strain energies but only for static analyses. To predict the structural damping, the element modal strain energies for a normal modes analysis have to be found.

SYMBOLS

w_x = x displacements in the plane of the plate in the local coordinate system

w_y = y displacements in the plane of the plate in the local coordinate system

$a_1, b_1, c_1, a_2, b_2, c_2$ = six undetermined coefficients

$x_1, y_1, \ldots, x_3, y_3$ = coordinates of the 3 nodes of the triangle in the local coordinate system

$\underset{\sim}{\eta}$ = shape matrix

$\underset{\sim}{\sigma}$ = stress vector

$\underset{\sim}{\epsilon}$ = strain vector

G = shear modulus

E = modulus of elasticity

$\underset{\sim}{k}$ = element stiffness matrix

$\underset{\sim}{\phi}^r$ = element eigenvector for the rth mode

ELEMENT FORMULATION

As mentioned previously, the formulation of the elements comes from the ANALYZE program. The element stiffness matrices are exactly the same as the COSMIC NASTRAN formulation for the CELAS, CBAR, and CROD elements. The formulation for the CTRMEM and CQDMEM2 elements is slightly different than COSMIC NASTRAN. The CSHEAR formulation is very different from the COSMIC NASTRAN formulation. The basis for the derivation of the shear panel is empirical but accurately constructed finite element models produce satisfactory results. The modal strain energy program will produce good results if the shear panel planform is as close to rectangular as possible. The less skewing of the element, the better the results will be.

The triangular membrane element used in this program is a constant strain plate element. The quadrilateral membrane and shear elements are constructed of four (non-overlapping) of the constant strain triangular membrane elements mentioned above. The elements are assumed to be flat plates which means the warping in the elements is ignored. The elements have a fictitious interior node which is later removed by static condensation. Only shear energy is considered in the stiffness of the shear element where the quadrilateral membrane element considers all the energy in the element.

Since the triangular membrane element is the basis for all the other plate elements in this program, the derivation will be given along with how these triangle elements are used to formulate the quadrilateral and shear elements. The linear displacement field in the triangular element can be represented by

$$w_x = a_1 x + b_1 y + c_1 \tag{1}$$

$$w_y = a_2 x + b_2 y + c_2 \tag{2}$$

or in matrix form

$$\underset{\sim}{w} = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{pmatrix} \tag{3}$$

The six unknown coefficients can be uniquely determined by the six boundary conditions at the nodes.

$$\begin{pmatrix} v_1 \\ v_3 \\ v_5 \\ \hline v_2 \\ v_4 \\ v_6 \end{pmatrix} = \left(\begin{array}{ccc|ccc} x_1 & y_1 & 1 & 0 & 0 & 0 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & x_1 & y_1 & 1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{array}\right) \begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ \hline a_2 \\ b_2 \\ c_2 \end{pmatrix} \tag{4}$$

The inversion of the partitioned diagonal matrix involves simply the inversion of the component matrix. The shape matrix $\underset{\sim}{\eta}$ is given by

$$\underset{\sim}{\eta} = \underset{\sim}{x}\,\underset{\sim}{Z}^{-1} \tag{5}$$

where the matrix $\underset{\sim}{x}$ is given by

$$\underset{\sim}{x} = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{pmatrix} \tag{6}$$

and the $\underset{\sim}{Z}$ matrix is given by

$$\underset{\sim}{Z} = \left(\begin{array}{cc} \underset{\sim}{X} & 0 \\ \hline 0 & \underset{\sim}{X} \end{array}\right) \tag{7}$$

The coordinate matrix $\underset{\sim}{X}$ is given by

$$\underset{\sim}{X} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \tag{8}$$

From linear strain-displacement relations, the strains can be written as

$$\epsilon_x = \frac{\partial w_x}{\partial x} = a_1 \tag{9}$$

$$\epsilon_y = \frac{\partial w_y}{\partial y} = b_2 \tag{10}$$

$$\epsilon_{xy} = \frac{\partial w_x}{\partial y} + \frac{\partial w_y}{\partial x} = b_1 + a_2 \tag{11}$$

From the principle of virtual work, the elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \underset{\sim}{\sigma}^{(i)^t} \underset{\sim}{\epsilon}^{(j)}\, dV = \int_V \underset{\sim}{\epsilon}^{(i)^t}\, \underset{\sim}{E}\, \underset{\sim}{\epsilon}^{(j)}\, dV \tag{12}$$

where $\underset{\sim}{\sigma}^{(i)}$ and $\underset{\sim}{\epsilon}^{(j)}$ are the stress and strain matrices corresponding to the unit displacement modes explained in equation 8. Since the linear displacement relation implies constant strain, the integral in equation 12 can be replaced by the volume of the element:

$$k_{ij} = \frac{1}{2}\, |\underset{\sim}{X}|\, t\, \underset{\sim}{\epsilon}^{(i)^t} \underset{\sim}{E}\, \underset{\sim}{\epsilon}^{(j)} \tag{13}$$

where $|\utilde{X}|$ is the determinant of the nodal coordinate matrix which represents twice the area of the element and t is the thickness of the element. Finally, the stiffness matrix of the triangular membrane element is given by

$$\utilde{k} = \frac{1}{2}\,|\utilde{X}|\,t \begin{pmatrix} \utilde{\epsilon}^{(1)^t} & \utilde{E}\utilde{\epsilon}^{(1)} & \utilde{\epsilon}^{(1)^t} & \utilde{E}\utilde{\epsilon}^{(2)} & \cdots & \utilde{\epsilon}^{(1)^t} & \utilde{E}\utilde{\epsilon}^{(6)} \\ \utilde{\epsilon}^{(2)^t} & \utilde{E}\utilde{\epsilon}^{(1)} & \utilde{\epsilon}^{(2)^t} & \utilde{E}\utilde{\epsilon}^{(2)} & \cdots & \utilde{\epsilon}^{(2)^t} & \utilde{E}\utilde{\epsilon}^{(6)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \utilde{\epsilon}^{(6)^t} & \utilde{E}\utilde{\epsilon}^{(1)} & \utilde{\epsilon}^{(6)^t} & \utilde{E}\utilde{\epsilon}^{(2)} & \cdots & \utilde{\epsilon}^{(6)^t} & \utilde{E}\utilde{\epsilon}^{(6)} \end{pmatrix} \tag{14}$$

Equation 14 gives the formulation for the stiffness matrix of the triangular membrane elements. What follows, is how four of these triangular elements are used to construct quadrilateral membrane and shear elements.

The stiffness matrix of the quadrilateral membrane element is determined by breaking it into four component triangles. The fictitious node in the quadrilateral is located by averaging the coordinates of the four nodes as follows

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4} \tag{15}$$

$$y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \tag{16}$$

The stiffness of the four triangles can then be computed by equation 14. Addition of the four stiffness matrices gives a 10 × 10 stiffness matrix with two degrees of freedom included for the fifth node. The force displacement relations of the five node quadrilateral are written as

$$\utilde{R}_Q = \utilde{k}_Q\ \utilde{r}_Q \tag{17}$$

where the subscript refers to the quadrilateral element with five nodes. Equation 17, partitioned to isolate the degrees of freedom of the fifth node can be written as

$$\left(\begin{array}{c} \utilde{R}_{I} \\ \hline \utilde{R}_{II} \end{array}\right) = \left(\begin{array}{c|c} \utilde{k}_{I,I} & \utilde{k}_{I,II} \\ \hline \utilde{k}_{II,I} & \utilde{k}_{II,II} \end{array}\right) \left(\begin{array}{c} \utilde{r}_{I} \\ \hline \utilde{r}_{II} \end{array}\right) \tag{18}$$

Equation 18 can be written as two separate equations

$$\utilde{R}_I = \utilde{k}_{I,I}\ \utilde{r}_I + \utilde{k}_{I,II}\ \utilde{r}_{II} \tag{19}$$

$$\utilde{R}_{II} = \utilde{k}_{II,I}\ \utilde{r}_I + \utilde{k}_{II,II}\ \utilde{r}_{II} \tag{20}$$

Since the fifth node doesn't actually exist in the original model, no external forces can be applied to this node. This condition gives

$$\utilde{r}_{II} = -\utilde{k}_{II,II}^{-1}\ \utilde{k}_{II,I}\ \utilde{r}_I \tag{21}$$

Substitution of equation 21 in equation 19 gives

$$\utilde{R}_I = \left(\utilde{k}_{I,I} - \utilde{k}_{I,II}\ \utilde{k}_{II,II}^{-1}\ \utilde{k}_{II,I}\right) \utilde{r}_I \tag{22}$$

From equation 22 the stiffness matrix of the original quadrilateral membrane element can be written as

$$\underset{\sim}{k} = \underset{\sim}{k}_{\mathrm{II}} - \underset{\sim}{k}_{\mathrm{I,II}}\ \underset{\sim}{k}^{-1}_{\mathrm{II,II}}\ \underset{\sim}{k}_{\mathrm{II,I}} \tag{23}$$

The shear element is also composed of four triangular elements however, the stiffness matrices of the component triangles are determined by considering only the shear strain energy (equation 13).

$$k_{ij} = \frac{1}{2}\ |\underset{\sim}{X}|\ t\ \epsilon^{(i)}_{xy} G \epsilon^{(j)}_{xy} \tag{24}$$

MODAL STRAIN ENERGY PROGRAM

The program starts by reading in all the information it needs from a NASTRAN output file for a normal modes analysis. As the program is set up, it can handle 1,000 of any one type of element for a total of 6,000 elements. A total of 100 materials can be specified but only isotropic materials specified on MAT1 cards are currently accounted for. The model can have 100 properties for any one element type for a total of 600 property cards. These limits can easily be expanded by changing the dimensions of the arrays in the code. The CELAS elements (CELAS1 or CELAS2) must be grounded (fixed) at one end with the other end connected to the structure. There are two ways to do this, one is to leave the second grid point of the CELAS card and its component blank or the second way is to specify a second grid point and component and then fix the second grid point component with an SPC card.

The next step in determining the modal strain energies is to calculate the element stiffness matrices. Using the equations derived above and equations for the CELAS, CBAR, and CROD elements, the stiffness matrices are generated. After this the eigenvector for the current element is extracted from the eigenvector table for a given mode. Then the following equation is used to determine the element modal strain energies for the given mode

$$\mathit{Element\ Modal\ Strain\ Energy} = \frac{1}{2}\underset{\sim}{\phi}^{r^t}\underset{\sim}{k}\underset{\sim}{\phi}^{r} \tag{25}$$

The equation is used for each element for every mode printed in the NASTRAN normal modes analysis.

After the element strain energies are calculated, they are printed in an easy to read format. The modal strain energy program prints out the following quantities for each mode: element ID number (EID), element type (CBAR, CELAS, CROD, CTRMEM, CSHEAR, or CQDMEM2), element strain energy (in consistent units), percent element strain energy of the entire structure, sum of the total element strain energy for each element type, and the total element strain energy for the entire structure. The program also prints one-half the generalized stiffness from the NASTRAN output file as a check. One-half the generalized stiffness should equal the total strain energy for the entire structure.

APPLICATIONS

Viscoelastic materials are seeing widespread use to suppress vibrations in all types of structures. The ability of viscoelastic materials to passively damp vibrations in lightweight structures is well documented. Modal strain energies are useful in estimating the damping in this type of structure. The approach used to predict the modal damping (loss) factors for each mode of the structure is called the modal strain energy method. It states that the ratio of structural loss factor to viscoelastic material loss factor for a given mode of vibration can be estimated as the ratio of elastic strain energy in the viscoelastic to total elastic strain energy in the entire structure when it deforms into the particular undamped mode shape (ref. 3). Mathematically this can be stated as

$$\frac{\eta_s^{(r)}}{\eta_v} = \frac{V_v^{(r)}}{V_s^{(r)}} \tag{26}$$

where

η_s^r = loss factor for the r'th mode of the composite structure

η_v = material loss factor for the viscoelastic material

V_v^r = elastic strain energy stored in the viscoelastic material when the structure deforms in its r'th undamped mode shape

V_s^r = elastic strain energy of the entire composite structure in the r'th mode shape

Computing the undamped mode shapes of the composite structure with the viscoelastic material treated as if it were purely elastic with a real stiffness modulus, the right hand side of equation 26 is calculated as

$$\frac{V_v^r}{V_s^r} = \frac{\sum_{\theta=1}^{n} \underset{\sim}{\phi}_\theta^{r^t} \underset{\sim}{k}_\theta \underset{\sim}{\phi}_\theta^r}{\underset{\sim}{\phi}^{r^t} \underset{\sim}{K} \underset{\sim}{\phi}^r} \tag{27}$$

where

$\underset{\sim}{\phi}^r$ = r'th mode shape vector

$\underset{\sim}{\phi}_\theta^r$ = subvector formed by deleting from $\underset{\sim}{\phi}$ all entries not corresponding to motion of nodes of the θ'th viscoelastic element

$\underset{\sim}{k}_\theta$ = element stiffness matrix of the θ'th viscoelastic element

$\underset{\sim}{K}$ = stiffness matrix of the entire composite structure

n = number of viscoelastic elements in the model

Combining equations 26 and 27 you get (ref. 4)

$$\eta_s^r = \frac{\sum_{\theta=1}^{n} \eta_{v_\theta} \underset{\sim}{\phi}_\theta^{r^t} \underset{\sim}{k}_\theta \underset{\sim}{\phi}_\theta^r}{\underset{\sim}{\phi}^{r^t} \underset{\sim}{K} \underset{\sim}{\phi}^r} \tag{28}$$

This equation states that if you create a NASTRAN model of the damped structure with all elements included except damper elements, and then run a normal modes analysis, you have all the information needed to get the structural loss factor. After you make the NASTRAN run, you run the output through the element modal strain energy program which gives you the percentages of element strain energy to total strain energy for the entire structure. The percentages for the elements that actually possess viscoelastic damping are multiplied by that particular elements material loss factor. These quantities are then summed to give the loss factor for the entire structure.

EXAMPLE PROBLEMS

Three example problems were used to demonstrate the ability of the element strain energy program to accurately output the element modal strain energies. The first problem is a rectangular wing box and it is shown broken up into its numbered elements in figure 1. The rectangular wing box consists of quadrilateral membrane elements for the inboard top and bottom skins, triangular membrane elements for the outboard top and bottom skins, bar elements for the outboard posts, rod elements for all other posts, shear elements for all the ribs and spars, elastic elements provide the inboard top skin attachment points with the inboard bottom skin points rigidly fixed. The strain energy outputs for the second mode of this model are given in table I. This model contains all the element types the strain energy program is capable of handling. Comparing the total structural strain energy with the value for one-half the generalized stiffness shows that the two are in agreement.

The second example is known as the intermediate complexity wing and is just a simplified NASTRAN model of the load carrying portion of a wing. Shown in figure 2 broken into its component elements and their numbering scheme, is a depiction of the wing. The model consists of quadrilateral membrane elements for most of the top and bottom skins, two triangular membrane elements for the outboard corner elements on the top and bottom skins, rod elements for all the posts, and shear elements for all the ribs and spars. The inboard top and bottom skin points rigidly fixed. The strain energy program output for the first mode of the model is given in table II. As you can see, the program accurately produces zero strain energy in all the rod elements for the first bending mode of the wing. The difference in the values for the total structural strain energy and one-half the generalized stiffness can be attributed to the different formulations of the stiffness matrices of the plate membrane and shear elements.

The final example is a part of the Large Space Structures Technology Program at the Flight Dynamics Laboratory. A NASTRAN model of the twelve meter truss structure is shown in figure 3. The elements aren't numbered because of the large number of elements in the model. The model consists of bar elements for the horizontal and vertical elements and rod elements for all the diagonals. The diagonal members contain the viscoelastic dampers on the actual structure. The model is supported at the base with a series of elastic elements. The strain energy output for the second mode is given in table III. This is the first torsion mode of the truss, so most of the strain energy is in the diagonal members. This is verified by the modal strain energy program. The loss factor for the entire structure has been predicted and is awaiting test results for verification.

CONCLUDING REMARKS

A FORTRAN program that calculates element strain energies has been developed and

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verified. This program gives COSMIC NASTRAN a capability that was only previously available for static analysis. Work is currently underway to develop DMAP instructions to calculate modal strain energies directly in NASTRAN. With the ever increasing trend toward lighter structures, damping materials will see increased use in all types of structures. A simple, accurate method, such as the modal strain energy method, to predict structural damping is essential.

REFERENCES

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3. Johnson, C.D., Kienholz, D.A., and Rogers, L.C., "Finite Element Prediction of Damping in Beams with Constrained Viscoelastic Layers," Shock and Vibration Bulletin, No. 50, Part 1, May 1981, pp. 71-82.

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TABLE I - RECTANGULAR WING BOX STRAIN ENERGIES

MODE 2

Element ID	Element Type	Element Strain Energy	% Strain Energy
50001	CBAR	-0.4068E-06	0.0000
50002	CBAR	6.446	0.2810
50003	CBAR	-0.6010E-06	0.0000
500	CELAS	0.2516	0.0110
502	CELAS	0.2749E-01	0.0012
503	CELAS	0.2810E-01	0.0012
504	CELAS	69.68	3.0378
505	CELAS	55.31	2.4114
506	CELAS	50.50	2.2015
507	CELAS	0.5442	0.0237
508	CELAS	0.9635E-01	0.0042
509	CELAS	0.1016	0.0044
50004	CROD	0.0000E+00	0.0000
50005	CROD	0.0000E+00	0.0000
50006	CROD	0.0000E+00	0.0000
50007	CROD	1.088	0.0475
50008	CROD	0.1927	0.0084
50009	CROD	0.2032	0.0089
10001	CTRMEM	117.4	5.1190
10002	CTRMEM	116.7	5.0897

10011	CTRMEM	137.4	5.9880
10012	CTRMEM	117.8	5.1372
20001	CTRMEM	115.1	5.0171
20002	CTRMEM	119.7	5.2185
20011	CTRMEM	132.1	5.7594
20012	CTRMEM	115.9	5.0520
10003	CQDMEM	218.9	9.5449
10004	CQDMEM	166.3	7.2488
20003	CQDMEM	200.6	8.7462
20004	CQDMEM	189.0	8.2413
30001	CSHEAR	29.60	1.2904
30002	CSHEAR	88.67	3.8657
30003	CSHEAR	34.38	1.4987
30004	CSHEAR	80.61	3.5140
30005	CSHEAR	72.90	3.1781
30006	CSHEAR	28.34	1.2355
40001	CSHEAR	12.29	0.5360
40002	CSHEAR	8.467	0.3691
40003	CSHEAR	0.6263E-02	0.0003
40004	CSHEAR	3.315	0.1445
40005	CSHEAR	3.752	0.1636
40006	CSHEAR	0.9805E-05	0.0000

STRAIN ENERGY IN CELAS ELEMENTS = 176.5

STRAIN ENERGY IN CBAR ELEMENTS = 6.446

STRAIN ENERGY IN CROD ELEMENTS = 1.484

STRAIN ENERGY IN CTRMEM ELEMENTS = 972.1

STRAIN ENERGY IN CQDMEM ELEMENTS = 774.9

STRAIN ENERGY IN CSHEAR ELEMENTS = 362.3

TOTAL STRAIN ENERGY = 2294.

GENERALIZED STIFFNESS/2 = 2294.

TABLE II - INTERMEDIATE COMPLEXITY WING STRAIN ENERGIES

MODE 1

Element ID	Element Type	Element Strain Energy	% Strain Energy
120	CROD	0.0000E+00	0.0000
121	CROD	0.0000E+00	0.0000
122	CROD	0.0000E+00	0.0000
123	CROD	0.0000E+00	0.0000
124	CROD	0.0000E+00	0.0000
125	CROD	0.0000E+00	0.0000
126	CROD	0.0000E+00	0.0000
127	CROD	0.0000E+00	0.0000
128	CROD	0.0000E+00	0.0000

129	CROD	0.0000E+00	0.0000
130	CROD	0.0000E+00	0.0000
131	CROD	0.0000E+00	0.0000
132	CROD	0.0000E+00	0.0000
133	CROD	0.0000E+00	0.0000
134	CROD	0.0000E+00	0.0000
135	CROD	0.0000E+00	0.0000
136	CROD	0.0000E+00	0.0000
137	CROD	0.0000E+00	0.0000
138	CROD	0.0000E+00	0.0000
139	CROD	0.0000E+00	0.0000
140	CROD	0.0000E+00	0.0000
141	CROD	0.0000E+00	0.0000
142	CROD	0.0000E+00	0.0000
143	CROD	0.0000E+00	0.0000
144	CROD	0.0000E+00	0.0000
145	CROD	0.0000E+00	0.0000
146	CROD	0.0000E+00	0.0000
147	CROD	0.0000E+00	0.0000
148	CROD	0.0000E+00	0.0000
149	CROD	0.0000E+00	0.0000
150	CROD	0.0000E+00	0.0000
151	CROD	0.0000E+00	0.0000
152	CROD	0.0000E+00	0.0000
153	CROD	0.0000E+00	0.0000
154	CROD	0.0000E+00	0.0000
155	CROD	0.0000E+00	0.0000
156	CROD	0.0000E+00	0.0000
157	CROD	0.0000E+00	0.0000
158	CROD	0.0000E+00	0.0000
1	CTRMEM	0.2817E-01	0.0046
2	CTRMEM	0.2817E-01	0.0046
3	CQDMEM	0.8132E-01	0.0131
4	CQDMEM	0.8132E-01	0.0131
5	CQDMEM	0.6809E-01	0.0110
6	CQDMEM	0.6809E-01	0.0110
7	CQDMEM	0.1561	0.0252
8	CQDMEM	0.1561	0.0252
9	CQDMEM	0.6461	0.1045
10	CQDMEM	0.6461	0.1045
11	CQDMEM	0.7805	0.1262
12	CQDMEM	0.7805	0.1262
13	CQDMEM	0.7629	0.1234
14	CQDMEM	0.7629	0.1234
15	CQDMEM	0.9026	0.1459
16	CQDMEM	0.9026	0.1459
17	CQDMEM	2.573	0.4161
18	CQDMEM	2.573	0.4161
19	CQDMEM	3.169	0.5124
20	CQDMEM	3.169	0.5124
21	CQDMEM	3.299	0.5335
22	CQDMEM	3.299	0.5335
23	CQDMEM	3.306	0.5345

24	CQDMEM	3.306	0.5345
25	CQDMEM	5.762	0.9316
26	CQDMEM	5.762	0.9316
27	CQDMEM	7.520	1.2159
28	CQDMEM	7.520	1.2159
29	CQDMEM	7.967	1.2882
30	CQDMEM	7.967	1.2882
31	CQDMEM	7.312	1.1824
32	CQDMEM	7.312	1.1824
33	CQDMEM	9.504	1.5367
34	CQDMEM	9.504	1.5367
35	CQDMEM	12.89	2.0840
36	CQDMEM	12.89	2.0840
37	CQDMEM	14.20	2.2956
38	CQDMEM	14.20	2.2956
39	CQDMEM	12.59	2.0364
40	CQDMEM	12.59	2.0364
41	CQDMEM	12.67	2.0479
42	CQDMEM	12.67	2.0479
43	CQDMEM	17.60	2.8462
44	CQDMEM	17.60	2.8462
45	CQDMEM	21.24	3.4346
46	CQDMEM	21.24	3.4346
47	CQDMEM	19.38	3.1329
48	CQDMEM	19.38	3.1329
49	CQDMEM	13.58	2.1953
50	CQDMEM	13.58	2.1953
51	CQDMEM	17.25	2.7891
52	CQDMEM	17.25	2.7891
53	CQDMEM	20.66	3.3399
54	CQDMEM	20.66	3.3399
55	CQDMEM	16.06	2.5971
56	CQDMEM	16.06	2.5971
57	CQDMEM	6.645	1.0744
58	CQDMEM	6.645	1.0744
59	CQDMEM	17.18	2.7780
60	CQDMEM	17.18	2.7780
61	CQDMEM	21.31	3.4457
62	CQDMEM	21.31	3.4457
63	CQDMEM	23.16	3.7449
64	CQDMEM	23.16	3.7449
65	CSHEAR	0.4318E-01	0.0070
66	CSHEAR	0.5833E-04	0.0000
67	CSHEAR	0.4419E-01	0.0071
68	CSHEAR	0.6681E-03	0.0001
69	CSHEAR	0.1559E-02	0.0003
70	CSHEAR	0.3317E-01	0.0054
71	CSHEAR	0.3689E-02	0.0006
72	CSHEAR	0.1579E-01	0.0026
73	CSHEAR	0.3236E-02	0.0005
74	CSHEAR	0.6835E-02	0.0011
75	CSHEAR	0.3295E-02	0.0005
76	CSHEAR	0.7111E-02	0.0011

77	CSHEAR	0.3093E-02	0.0005
78	CSHEAR	0.4573E-02	0.0007
79	CSHEAR	0.2292E-02	0.0004
80	CSHEAR	0.4423E-02	0.0007
81	CSHEAR	0.3590E-02	0.0006
82	CSHEAR	0.4746E-02	0.0008
83	CSHEAR	0.1902E-02	0.0003
84	CSHEAR	0.4688E-02	0.0008
85	CSHEAR	0.7538E-02	0.0012
86	CSHEAR	0.8932E-02	0.0014
87	CSHEAR	0.3638E-02	0.0006
88	CSHEAR	0.9771E-02	0.0016
89	CSHEAR	0.9090E-01	0.0147
90	CSHEAR	0.6178E-01	0.0100
91	CSHEAR	0.4218E-01	0.0068
92	CSHEAR	0.6935E-01	0.0112
93	CSHEAR	0.5735	0.0927
94	CSHEAR	0.3672	0.0594
95	CSHEAR	0.1479	0.0239
96	CSHEAR	0.2381	0.0385
97	CSHEAR	0.2449	0.0396
98	CSHEAR	0.4526	0.0732
99	CSHEAR	0.5576	0.0902
100	CSHEAR	0.5448	0.0881
101	CSHEAR	0.3861	0.0624
102	CSHEAR	0.8999E-01	0.0146
103	CSHEAR	0.7155	0.1157
104	CSHEAR	0.1065	0.0172
105	CSHEAR	0.8149	0.1318
106	CSHEAR	1.047	0.1692
107	CSHEAR	1.125	0.1819
108	CSHEAR	1.076	0.1740
109	CSHEAR	1.026	0.1659
110	CSHEAR	0.9453	0.1528
111	CSHEAR	1.735	0.2805
112	CSHEAR	0.5387E-01	0.0087
113	CSHEAR	0.3877	0.0627
114	CSHEAR	0.6420	0.1038
115	CSHEAR	0.7546	0.1220
116	CSHEAR	0.7662	0.1239
117	CSHEAR	0.7926	0.1281
118	CSHEAR	0.5955	0.0963
119	CSHEAR	1.297	0.2097

STRAIN ENERGY IN CROD ELEMENTS = 0.0000E+00

STRAIN ENERGY IN CTRMEM ELEMENTS = 0.5633E-01

STRAIN ENERGY IN CQDMEM ELEMENTS = 600.4

STRAIN ENERGY IN CSHEAR ELEMENTS = 17.97

TOTAL STRAIN ENERGY = 618.5

GENERALIZED STIFFNESS/2 = 619.0

TABLE III - TWELVE METER TRUSS STRAIN ENERGIES

MODE 2

Element ID	Element Type	Element Strain Energy	% Strain Energy
101	CBAR	1.740	6.0085
102	CBAR	1.724	5.9537
103	CBAR	1.127	3.8917
104	CBAR	1.155	3.9862
106	CBAR	0.6792	2.3449
107	CBAR	0.6611	2.2827
108	CBAR	0.3441	1.1880
109	CBAR	0.3522	1.2160
111	CBAR	0.1476	0.5098
112	CBAR	0.1436	0.4956
113	CBAR	0.4434E-01	0.1531
114	CBAR	0.4531E-01	0.1564
116	CBAR	0.7187E-02	0.0248
117	CBAR	0.6948E-02	0.0240
118	CBAR	0.2278E-03	0.0008
119	CBAR	0.2049E-03	0.0007
120	CBAR	0.1767E-01	0.0610
121	CBAR	0.5047E-02	0.0174
122	CBAR	0.4326E-02	0.0149
123	CBAR	0.5652E-02	0.0195
125	CBAR	0.2255E-02	0.0078
126	CBAR	0.3096E-02	0.0107
127	CBAR	0.1182E-02	0.0041
128	CBAR	0.1833E-02	0.0063
130	CBAR	0.3633E-03	0.0013
131	CBAR	0.6576E-03	0.0023
132	CBAR	0.1058E-03	0.0004
133	CBAR	0.2597E-03	0.0009
135	CBAR	0.2277E-04	0.0001
136	CBAR	0.3004E-04	0.0001
137	CBAR	0.4012E-04	0.0001
138	CBAR	0.1046E-04	0.0000
139	CBAR	1.740	6.0085
140	CBAR	1.724	5.9537
141	CBAR	1.127	3.8917
142	CBAR	1.155	3.9862
144	CBAR	0.6792	2.3449
145	CBAR	0.6611	2.2827
146	CBAR	0.3441	1.1880
147	CBAR	0.3522	1.2160
149	CBAR	0.1476	0.5098
150	CBAR	0.1436	0.4956
151	CBAR	0.4434E-01	0.1531
152	CBAR	0.4531E-01	0.1564
154	CBAR	0.7187E-02	0.0248

155	CBAR	0.6948E-02	0.0240
156	CBAR	0.2278E-03	0.0008
157	CBAR	0.2049E-03	0.0007
158	CBAR	0.1767E-01	0.0610
159	CBAR	0.5047E-02	0.0174
160	CBAR	0.4326E-02	0.0149
161	CBAR	0.5652E-02	0.0195
163	CBAR	0.2255E-02	0.0078
164	CBAR	0.3096E-02	0.0107
165	CBAR	0.1182E-02	0.0041
166	CBAR	0.1833E-02	0.0063
168	CBAR	0.3633E-03	0.0013
169	CBAR	0.6576E-03	0.0023
170	CBAR	0.1058E-03	0.0004
171	CBAR	0.2597E-03	0.0009
173	CBAR	0.2277E-04	0.0001
174	CBAR	0.3004E-04	0.0001
175	CBAR	0.4012E-04	0.0001
176	CBAR	0.1046E-04	0.0000
201	CBAR	0.1054E-03	0.0004
202	CBAR	0.9258E-03	0.0032
203	CBAR	0.6164E-03	0.0021
204	CBAR	0.6752E-03	0.0023
205	CBAR	0.5726E-03	0.0020
206	CBAR	0.5726E-03	0.0020
207	CBAR	0.5583E-03	0.0019
208	CBAR	0.5189E-03	0.0018
209	CBAR	0.4704E-03	0.0016
210	CBAR	0.3950E-03	0.0014
211	CBAR	0.3950E-03	0.0014
212	CBAR	0.3261E-03	0.0011
213	CBAR	0.2753E-03	0.0010
214	CBAR	0.2157E-03	0.0007
215	CBAR	0.1482E-03	0.0005
216	CBAR	0.1482E-03	0.0005
217	CBAR	0.8562E-04	0.0003
218	CBAR	0.4871E-04	0.0002
219	CBAR	0.1600E-04	0.0001
220	CBAR	0.8716E-05	0.0000
221	CBAR	0.1055E-03	0.0004
222	CBAR	0.9257E-03	0.0032
223	CBAR	0.6163E-03	0.0021
224	CBAR	0.6755E-03	0.0023
225	CBAR	0.5725E-03	0.0020
226	CBAR	0.5725E-03	0.0020
227	CBAR	0.5583E-03	0.0019
228	CBAR	0.5191E-03	0.0018
229	CBAR	0.4706E-03	0.0016
230	CBAR	0.3950E-03	0.0014
231	CBAR	0.3950E-03	0.0014
232	CBAR	0.3260E-03	0.0011
233	CBAR	0.2754E-03	0.0010
234	CBAR	0.2158E-03	0.0007

235	CBAR	0.1482E-03	0.0005
236	CBAR	0.1482E-03	0.0005
237	CBAR	0.8558E-04	0.0003
238	CBAR	0.4871E-04	0.0002
239	CBAR	0.1596E-04	0.0001
240	CBAR	0.8653E-05	0.0000
241	CBAR	0.1054E-03	0.0004
242	CBAR	0.9258E-03	0.0032
243	CBAR	0.6164E-03	0.0021
244	CBAR	0.6752E-03	0.0023
245	CBAR	0.5726E-03	0.0020
246	CBAR	0.5726E-03	0.0020
247	CBAR	0.5583E-03	0.0019
248	CBAR	0.5189E-03	0.0018
249	CBAR	0.4704E-03	0.0016
250	CBAR	0.3950E-03	0.0014
251	CBAR	0.3950E-03	0.0014
252	CBAR	0.3261E-03	0.0011
253	CBAR	0.2753E-03	0.0010
254	CBAR	0.2157E-03	0.0007
255	CBAR	0.1482E-03	0.0005
256	CBAR	0.1482E-03	0.0005
257	CBAR	0.8562E-04	0.0003
258	CBAR	0.4871E-04	0.0002
259	CBAR	0.1600E-04	0.0001
260	CBAR	0.8716E-05	0.0000
261	CBAR	0.1055E-03	0.0004
262	CBAR	0.9257E-03	0.0032
263	CBAR	0.6163E-03	0.0021
264	CBAR	0.6755E-03	0.0023
265	CBAR	0.5725E-03	0.0020
266	CBAR	0.5725E-03	0.0020
267	CBAR	0.5583E-03	0.0019
268	CBAR	0.5191E-03	0.0018
269	CBAR	0.4706E-03	0.0016
270	CBAR	0.3950E-03	0.0014
271	CBAR	0.3950E-03	0.0014
272	CBAR	0.3260E-03	0.0011
273	CBAR	0.2754E-03	0.0010
274	CBAR	0.2158E-03	0.0007
275	CBAR	0.1482E-03	0.0005
276	CBAR	0.1482E-03	0.0005
277	CBAR	0.8558E-04	0.0003
278	CBAR	0.4871E-04	0.0002
279	CBAR	0.1596E-04	0.0001
280	CBAR	0.8652E-05	0.0000
301	CBAR	0.7572E-01	0.2614
302	CBAR	0.5778E-01	0.1995
303	CBAR	0.9422E-01	0.3253
304	CBAR	0.5765E-01	0.1990
305	CBAR	0.8604E-01	0.2971
306	CBAR	0.5167E-01	0.1784
307	CBAR	0.6932E-01	0.2393

308	CBAR	0.4699E-01	0.1622
309	CBAR	0.5217E-01	0.1801
310	CBAR	0.3316E-01	0.1145
311	CBAR	0.3317E-01	0.1145
312	CBAR	0.2202E-01	0.0760
313	CBAR	0.1538E-01	0.0531
314	CBAR	0.7785E-02	0.0269
315	CBAR	0.3781E-02	0.0131
316	CBAR	0.8850E-03	0.0031
317	CBAR	0.7571E-01	0.2614
318	CBAR	0.5777E-01	0.1995
319	CBAR	0.9421E-01	0.3253
320	CBAR	0.5764E-01	0.1990
321	CBAR	0.8604E-01	0.2971
322	CBAR	0.5167E-01	0.1784
323	CBAR	0.6931E-01	0.2393
324	CBAR	0.4699E-01	0.1622
325	CBAR	0.5218E-01	0.1801
326	CBAR	0.3316E-01	0.1145
327	CBAR	0.3318E-01	0.1145
328	CBAR	0.2202E-01	0.0760
329	CBAR	0.1539E-01	0.0531
330	CBAR	0.7787E-02	0.0269
331	CBAR	0.3783E-02	0.0131
332	CBAR	0.8851E-03	0.0031
333	CBAR	0.7572E-01	0.2614
334	CBAR	0.5778E-01	0.1995
335	CBAR	0.9422E-01	0.3253
336	CBAR	0.5765E-01	0.1990
337	CBAR	0.8604E-01	0.2971
338	CBAR	0.5167E-01	0.1784
339	CBAR	0.6932E-01	0.2393
340	CBAR	0.4699E-01	0.1622
341	CBAR	0.5217E-01	0.1801
342	CBAR	0.3316E-01	0.1145
343	CBAR	0.3317E-01	0.1145
344	CBAR	0.2202E-01	0.0760
345	CBAR	0.1538E-01	0.0531
346	CBAR	0.7786E-02	0.0269
347	CBAR	0.3781E-02	0.0131
348	CBAR	0.8850E-03	0.0031
349	CBAR	0.7571E-01	0.2614
350	CBAR	0.5777E-01	0.1995
351	CBAR	0.9421E-01	0.3253
352	CBAR	0.5764E-01	0.1990
353	CBAR	0.8604E-01	0.2971
354	CBAR	0.5167E-01	0.1784
355	CBAR	0.6931E-01	0.2393
356	CBAR	0.4699E-01	0.1622
357	CBAR	0.5218E-01	0.1801
358	CBAR	0.3316E-01	0.1145
359	CBAR	0.3318E-01	0.1145
360	CBAR	0.2203E-01	0.0760

361	CBAR	0.1539E-01	0.0531
362	CBAR	0.7787E-02	0.0269
363	CBAR	0.3783E-02	0.0131
364	CBAR	0.8852E-03	0.0031
500	CELAS	0.3367E-03	0.0012
501	CELAS	0.2813E-05	0.0000
502	CELAS	0.3367E-03	0.0012
503	CELAS	0.2813E-05	0.0000
504	CELAS	0.3367E-03	0.0012
505	CELAS	0.2812E-05	0.0000
506	CELAS	0.3367E-03	0.0012
507	CELAS	0.2812E-05	0.0000
508	CELAS	0.1465E-16	0.0000
509	CELAS	0.4949E-10	0.0000
510	CELAS	0.1465E-16	0.0000
511	CELAS	0.4949E-10	0.0000
512	CELAS	4.794	16.5529
513	CELAS	0.7808E-10	0.0000
514	CELAS	4.794	16.5529
515	CELAS	0.7808E-10	0.0000
516	CELAS	0.9110E-02	0.0315
517	CELAS	0.8964E-02	0.0309
518	CELAS	0.9110E-02	0.0315
519	CELAS	0.8964E-02	0.0309
520	CELAS	0.9111E-02	0.0315
521	CELAS	0.8964E-02	0.0310
522	CELAS	0.9111E-02	0.0315
523	CELAS	0.8964E-02	0.0310

STRAIN ENERGY IN CELAS ELEMENTS = 9.662

STRAIN ENERGY IN CBAR ELEMENTS = 19.30

TOTAL STRAIN ENERGY = 28.96

GENERALIZED STIFFNESS/2 = 28.96

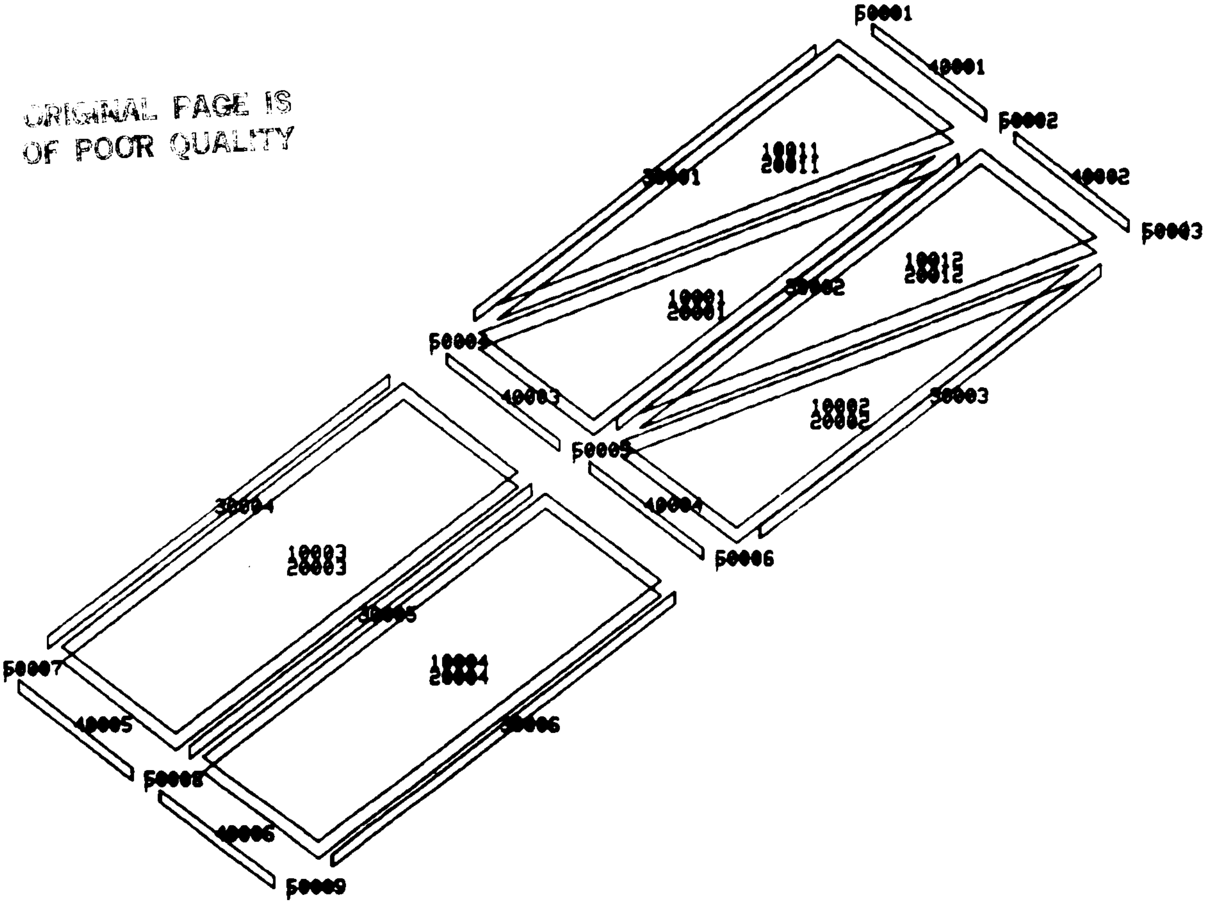

Figure 1 - Rectangular Wing Box Elements

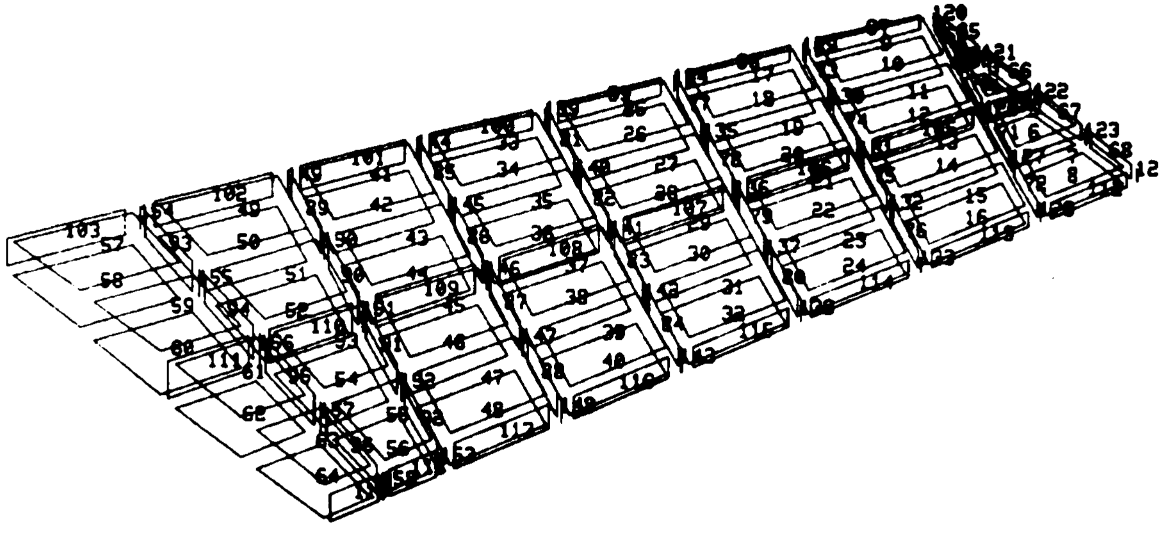

Figure 2 - Intermediate Complexity Wing Elements

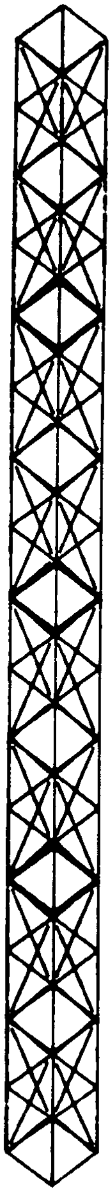

Figure 3 - Twelve Meter Truss Model