NASA Technical Memorandum 102057

Two-Dimensional Numerical Simulation of a Stirling Engine Heat Exchanger

Mounir B. Ibrahim
Cleveland State University
Cleveland, Ohio

and

Roy C. Tew and James E. Dudenhoefer
Lewis Research Center
Cleveland, Ohio

NASA

TWO-DIMENSIONAL NUMERICAL SIMULATION OF A STIRLING ENGINE HEAT EXCHANGER

Mounir Ibrahim
Cleveland State University
Cleveland, Ohio 44115

Roy C. Tew and James E. Dudenhoefer
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

This paper describes the first phase of an effort to develop multidimensional models of Stirling engine components; the ultimate goal is to model an entire engine working space. More specifically, this paper describes parallel plate and tubular heat exchanger models with emphasis on the central part of the channel (i.e., ignoring hydrodynamic and thermal end effects). The model assumes: laminar, incompressible flow with constant thermophysical properties. In addition, a constant axial temperature gradient is imposed. The governing equations, describing the model, have been solved using Crack-Nicloson finite-difference scheme. Model predictions have been compared with analytical solutions for oscillating/reversing flow and heat transfer in order to check numerical accuracy. The simplifying assumptions will later be relaxed to permit modeling of compressible, laminar/turbulent flow that occurs in Stirling heat exchanger.

Excellent agreement has been obtained for the model predictions with analytical solutions available for both flow in circular tubes and between parallel plates. Also the heat transfer computational results are in good agreement with the heat transfer analytical results for parallel plates.

NOMENCLATURE

dp/dx axial pressure gradient, N/m^3

f time-averaged Fanning friction factor =

$$2\sqrt{\overline{\tau_w^2}}\Big/\rho u_{av}^2$$

f_s unidirectional Fanning friction factor

Pr Prandtl number

p pressure, N/m^2

Re Reynolds number = $2 u_{av}(R_o - R_i)/\nu$

R_i half width of the channel, m

R_o half distance between two consecutive plates, m

r radial location, measured from channel centerline, m

T temperature, k

T^* normalized temperature = $T/(\lambda\,\Delta x)$

t time, s

u axial velocity, m/s

u^* normalized axial velocity = u/u_o

u_{av} cross-stream-averaged velocity, m/s

u_o characteristic axial velocity, m/s

x axial distance, m

y cross stream distance, measured from plate centerline, m

α_e effective averaged thermal diffusivity, "the thermal diffusivity that will produce the same axial heat flux as obtained from radial conduction and axial convection heat transfer" =

$$-\frac{\omega}{2\pi\lambda(R_o - R_i)}\int_0^{2\pi/\omega}\int_{R_i}^{R_o} Tu\,dy\,dt,\ m^2/s$$

α_e^* normalized effective averaged thermal diffusivity = $\alpha_e/(\omega\,\Delta x^2)$

Δx tidal displacement "the cross-stream-averaged maximum axial distance which the fluid elements travel during one half period of oscillation" =

$$\frac{1}{R_o - R_i}\int_0^{\pi/\omega}\int_{R_i}^{R_o} u\,dy\,dt,\qquad m$$

Δx^* normalized tidal displacement = $\Delta x/(u_o/\omega)$

$\frac{\partial T}{\partial x} = \lambda$ axial temperature gradient, k/m

μ fluid dynamic viscosity, kg/m/s

ν fluid kinematic viscosity, m^2/s

ρ density, kg/m^3

τ_w wall shear stress, N/m^2

ϕ pressure gradient phase angle, (deg)
    $= \omega t(360/2\pi)$

ω frequency, rad/s

ω^* Valensi number $= [\omega(R_O - R_i)^2]/\nu$

INTRODUCTION

NASA is developing free-piston Stirling engines for space power generation. These engines are designed with computer codes which simulate the oscillating/reversing flow and oscillating pressure level using one-dimensional flow analyses. Confidence is lacking in the resulting characterizations of the various interrelated thermodynamic losses. For example, the use of steady-flow friction factor and heat transfer correlations throughout the engine working space is of questionable validity.

It has not been possible to make accurate in-engine measurements of flows, velocities, temperatures and pressure drops. However, several oscillating-flow rigs are being used to learn more about engine fluid mechanics and heat transfer via dynamic measurements. Multidimensional modeling is another process for achieving better understanding of fluid flow and heat transfer phenomena in Stirling engine working spaces. Such models can be used to study velocity and temperature fields within the engine components; these fields can then be compared with the fields implied by use of the one-dimensional correlations. Heat transfer and fluid friction can also be calculated and compared with correlation assumptions used in one-dimensional codes. It may be possible to validate portions of multidimensional models directly with data from the oscillating flow rigs. Cleveland State University, with support from NASA Lewis Research Center, is working on development of multidimensional models of Stirling engine components. The first stage of this work, reported here, is development of a two-dimensional model of parallel plate and tubular heat exchangers.

One problem encountered by use of any numerical code is the evaluation of the errors in the solution due to the inherently approximate numerical techniques. A practical approach to evaluating such errors is to compare the numerical solution for a restricted version of the problem to an exact analytical solution. This paper reports on comparison of parallel plate heat exchanger model predictions with an exact analytical solution developed by Kurzweg [1]. Kurzweg's solution is for oscillating flow between parallel plates with heat transfer.

ANALYSIS

Assumptions

Figure 1 shows a sketch of the channel flow with the computation domain and boundary conditions. The following assumptions were made:

(1) The flow is laminar, incompressible and has constant thermophysical properties.

(2) Since we are only concerned with the fluid flow in the central part (in the x-direction) of the channel, hydrodynamic and thermal end effects can be ignored. This assumption can be satisfied by using a long channel compared to its width.

(3) As a consequence to assumption (2) all convection terms in the momentum and energy equations can be ignored, except for the $\rho u(\partial T/\partial x)$ term in the energy equation; since it is the only convective driving mechanism for the heat transfer.

(4) The axial temperature gradient is constant. This assumption is consistent with the experimental finding for a similar setup [2] where it was found that the temperature varies linearly with x in almost 80 percent of the central length of the channel.

(5) Under the oscillating flow conditions, heat transfer is controlled by radial conduction and axial convection. Therefore the axial heat conduction can be ignored.

Governing Equations

Applying the above assumptions to the general form of the Navier Stokes equations yields the following set of equations:

Continuity:

$$\frac{\partial u}{\partial x} = 0 \qquad (1)$$

x-Momentum:

$$\rho \frac{\partial u}{\partial t} = -\frac{dp}{dx} + \frac{1}{r^i}\frac{\partial}{\partial r}\left(r^i \mu \frac{\partial u}{\partial r}\right) \qquad (2)$$

Energy:

$$\rho \frac{\partial T}{\partial t} = -\rho u \frac{\partial T}{\partial x} + \frac{1}{r^i}\frac{\partial}{\partial r}\left(r^i \frac{\mu}{Pr} \frac{\partial T}{\partial r}\right) \qquad (3)$$

where i = 1 for circular tubes
    = 0 for flow between parallel plates

The code developed is capable of predicting both circular tube flow and flow between parallel plates; however, the results provided in this paper are only for parallel plates.

The pressure gradient term in Eq. (2) is assumed to be of oscillatory nature according to:

$$\frac{dp}{dx} = \left(\frac{dp}{dx}\right)_{max} \cos \omega t \qquad (4)$$

The above equations are subjected to the following boundary conditions:

from $r = R_o$ to R_i $u = 0$,

at $r = R_o$ $\dfrac{\partial T}{\partial r} = 0$ and   (5)

at $r = 0$ $\dfrac{\partial u}{\partial r} = \dfrac{\partial T}{\partial r} = 0$

Since the computation domain includes both the solid and the fluid, the boundary condition at the solid/fluid interface is automatically satisfied.

It should be noted that the above boundary conditions have been carefully selected for the following reasons:

(1) They match the boundary conditions given in Kurzweg's [1] analysis. Accordingly, the numerical results can be compared with his analytical solution.

(2) The above boundary conditions to great extent could simulate the central element (from both axial and radial directions) of a Stirling engine foil regenerator.

Numerical Method

The above partial differential equations have been transformed into finite difference equations using different schemes, namely, fully explicit, Crank Nicolson and fully implicit. A sensitivity analysis was conducted by comparing the results from each finite difference scheme with the fluid flow analytical solution. It was concluded that the Crank Nicolson scheme is the most accurate; this is consistent with other research findings for unsteady heat equation problems [3].

RESULTS

Comparison with Analytical Solution

One of the main objectives of this work is to develop an accurate and efficient numerical scheme that can be used with confidence in future engine models. Therefore, an extensive grid variation test was conducted for both space and time to obtain grid independent and accurate numerical results.

Table I shows the normalized tidal displacement and normalized effective thermal diffusivity as obtained for water (Pr = 10.26) with $R_o = 2R_i$ and the assumption that the solid has the same thermophysical properties as the fluid. In the table the present work numerical predictions were compared with the analytical solution [1] for ω^* from 0.09 to 9. The numerical results showed the same trend where a maximum α_e^* occurs at $\omega^* = 0.306$ as indicated by the analytical solution for Pr = 10.26. Table I shows differences between the numerical and analytical results of less than 2.0 percent.

Furthermore, the present work velocity profiles for different fluids (water and air) were

checked against the corresponding analytical velocities at several radial locations. Figures 2(a), (b), and (c) show the numerical velocity versus the analytical one at various radial locations in the channel (near wall region, $y/R_o = 0.506$; half distance between the wall and centerline $y/R_o = 0.75$; and centerline, $y/R_o = 1.0$). Results are shown for water, $\omega^* = 0.306$ (Fig. 2(a)); water, $\omega^* = 9.0$ (Fig. 2(b)); and air, $\omega^* = 4.48$ (Fig. 2(c)). The plots show, again the close agreement between the numerical and analytical solutions.

Fluid Flow Results

Velocity profile

Figures 3(a), (b), and (c) show the dimensionless velocity profile u^* versus the dimensionless distance (y/R_o) for water, $\omega^* = 0.306$ (Fig. 3(a)); water, $\omega^* = 9$ (Fig. 3(b)); and air, $\omega^* = 4.48$ (Fig. 3(c)). Upon examining the above figures the following were noted:

(1) For low dimensionless frequency ($\omega^* = 0.306$) the velocity profile has a maximum at the channel centerline.

(2) As ω^* increases a boundary layer starts to develop near the solid surface.

(3) Also, as ω^* increases the flow direction near the wall becomes opposite to that in the core, in certain parts of the cycle.

Friction factor

From the velocity profiles described above one can obtain wall shear stress, friction factor and Reynolds number at any pressure gradient phase angle. In addition, a root-mean-square can be calculated for the wall shear stress as well as the mean cross-sectional axial velocity. From the latter one can calculate friction factor and Reynolds number based upon time averaged quantities, similar to the work done on circular tubes by Chen and Griffin [4]. Chen and Griffin had obtained a correlation for the normalized friction factor (with respect to the corresponding unidirectional flow), f/f_s, based upon an approximate velocity profile given by White [5]. Work is underway to compare present work results with Chen and Griffin's correlation.

Heat Transfer Results

In the channel flow under study, the maximum radial temperature difference, between the channel wall temperature and the bulk fluid temperature, can be obtained by ignoring the radial heat conduction. Accordingly, this maximum radial temperature difference is equal to the axial temperature gradient times the tidal displacement. Now, one can obtain a normalized temperature profile utilizing the maximum radial temperature difference described above.

Figures 4(a) and (b) show the normalized temperature profile versus y/R_o for $\omega^* = 0.306$

3

(water), and $\omega^* = 4.48$ (air) at different pressure gradient phase angles ϕ. It should be noted that the solid is represented in the region $y/R_o = 0$ to 0.5. The temperature fluctuation in the fluid is much larger, as expected, than it is in the solid. Also this ratio between those temperature fluctuations (in fluid/in solid) becomes greater as ω^* increases.

Finally, the maximum effective thermal diffusivity described above in Table I can be better explained by plotting both the velocity and temperature at the same radial location versus ϕ. Figures 5(a) and (b) show the normalized velocity and normalized temperature profiles versus ϕ at $y/R_o = 0.506$, 0.75 and 1.0; for $\omega^* = 0.306$ (water), and $\omega^* = 4.48$ (air) respectively. It should be noted that the value of ω^* chosen for each fluid corresponds to the maximum α_e^* that can be obtained.

It can be seen from the plots that there is always a phase shift between the velocity and temperature profile. It is interesting to notice that the phase shift is almost the same for the above two cases examined. It is believed that this phase shift (approximately 120° at $y/R_o = 1.0$) provides the maximum interaction between axial convection and radial conduction and consequently the maximum net axial heat flux.

CONCLUDING REMARKS

An accurate numerical scheme has been applied to calculate the fluid flow and heat transfer characteristics in an oscillating channel flow. The flow was assumed laminar, incompressible and has constant thermophysical properties. Also a constant axial temperature gradient was imposed on the channel; however the axial heat conduction was assumed negligible.

The numerical scheme has been successful in predicting velocity profiles, tidal displacement, temperature profiles, and effective thermal diffusivity for a wide range of ω^* and different types of fluid (water and air). This shows that the present work numerical studies confirm earlier investigations.

The interaction between axial convection and radial conduction heat transfer, if tuned properly, would result in a considerable enhanced axial heat transfer. These tuned conditions, for the cases examined in the paper, occurred at $\omega^* = 0.306$ (water) and $\omega^* = 4.48$ (air). It is believed that the phase shift between the velocity and temperature profiles are tied in with this tuning process.

In a Stirling engine regenerator, these conditions (at which maximum axial heat transfer occur) should be avoided.

It should be noted that those tuned ω^* are restricted to the channel configuration and the different assumptions explained in the paper. A study is underway to examine other operating conditions (such as type of fluid, wall material, order of magnitude of axial pressure and temperature gradients, etc.) that are relevant to Stirling engines.

This work is a first step in the planned development of a two-dimensional code which can be used to model Stirling engine heat exchangers-tubular heaters, tubular and foil type regenerators, and tubular coolers. As the studies continue, the simplifying assumptions will be changed to include laminar/turbulent flow, compressible working fluid, and different outer thermal boundary conditions.
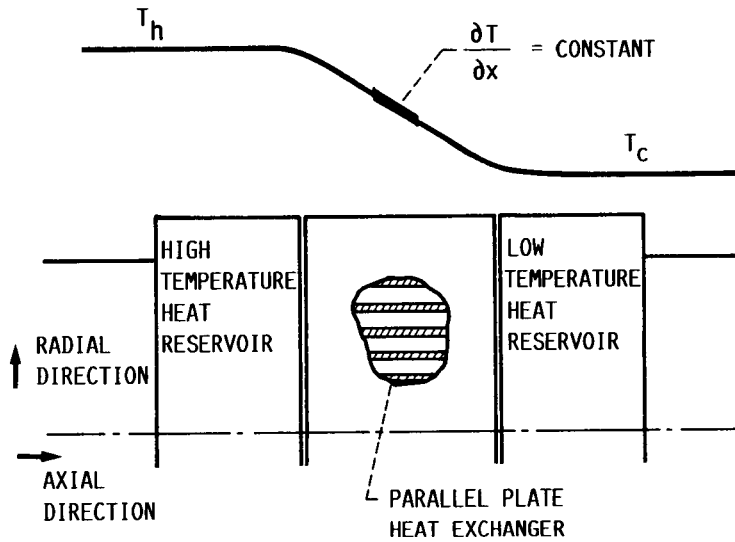
A second study has begun to model the rapid expansion and contraction of oscillating flow entering and exiting the heat exchangers. This work will permit the incorporation of hydrodynamic and thermal end effect conditions to the heat exchanger model.
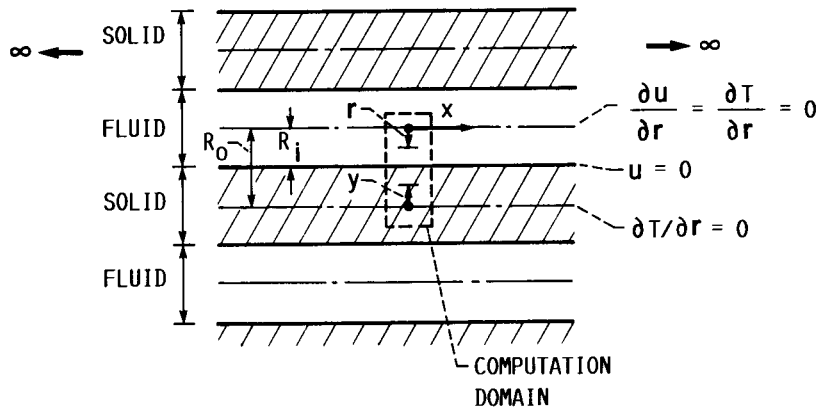
REFERENCES

1. U.H. Kurzweg, "Enhanced Heat Conduction in Oscillating Viscous Flows Within Parallel-Plate Channels," J. Fluid Mech., vol. 156, pp. 291-300, 1985.

2. G. Zhang, "Time Dependent Enhanced Heat Transfer in Oscillating Pipe Flow," Ph.D. Thesis, University of Florida, 1988.

3. D.A. Anderson, J.C. Tannehill and R. H. Pletcher, Computational Fluid Mechanics and Heat Transfer, Washington, D.C.: Hemisphere Publishing Corp., 1984.

4. N.C.J. Chen and F.P. Griffin, "Effects of Pressure-Drop Correlations on Stirling Engine Predicted Performance," in Energy for the Marketplace (18th IECEC), vol. 2, New York: American Institute of Chemical Engineers, 1983, pp. 708-713.

5. F.M. White, Viscous Fluid Flow, New York: McGraw-Hill, 1974.

TABLE I. – COMPARISON BETWEEN PRESENT WORK NUMERICAL RESULTS FOR
Δx^* and α_e^* AND ANALYTICAL SOLUTION [1] FOR DIFFERENT ω^*
[Water (Pr = 10.26) and R_O/R_i = 2.0.]

Valensi number, ω^*	Normalized tidal displacement, Δx^*		Normalized effective averaged thermal diffusivity, α_e^*	
	Present work	Analytical [1]	Present work	Analytical [1]
0.09	0.05941	0.05995	0.02616	0.02568
.16	.1055	.1064	.03707	.03645
.25	.1643	.1658	.04039	.04029
.306	.2019	.2023	.04051	.03979
1.0	.6160	.6181	.02415	.02302
9	1.607	1.61	.00421	.00386

T_h

$\frac{\partial T}{\partial x}$ = CONSTANT

T_c

| HIGH TEMPERATURE HEAT RESERVOIR | | LOW TEMPERATURE HEAT RESERVOIR |

RADIAL DIRECTION

AXIAL DIRECTION

PARALLEL PLATE HEAT EXCHANGER

(a) SKETCH OF OVERALL SYSTEM.

SOLID

FLUID

SOLID

FLUID

$\frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0$

$u = 0$

$\partial T / \partial r = 0$

COMPUTATION DOMAIN

(b) CENTRAL PART SHOWING THE COMPUTATION DOMAIN AND BOUNDARY CONDITIONS.

FIGURE 1. - SKETCH OF A PARALLEL PLATE HEAT EXCHANGER.

6
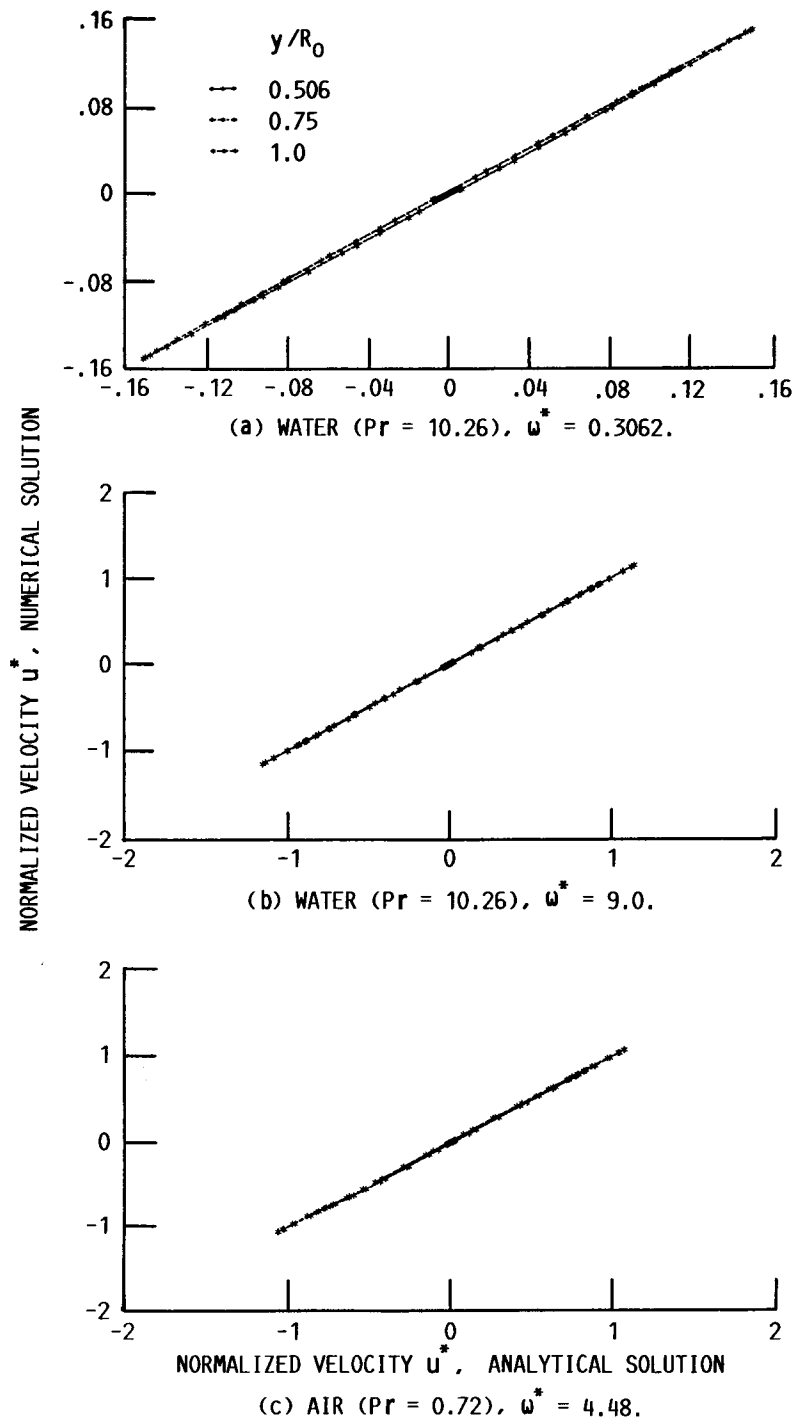
FIGURE 2. - COMPARISON BETWEEN NUMERICAL AND ANALYTI-
CAL VELOCITY PROFILES AT y/R_0 = 0.506 (NEAR WALL),
0.75 AND 1.0 (CENTERLINE) FOR R_0/R_i = 2.0. (NOTE
THAT THE DATA POINTS FOR DIFFERENT y/R_0 ALMOST
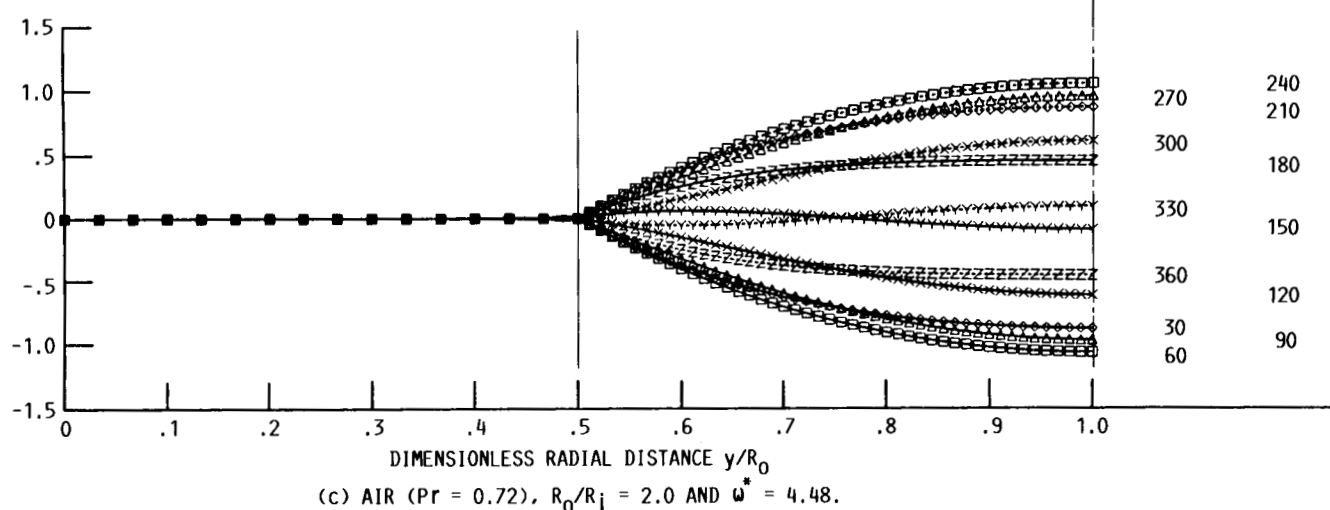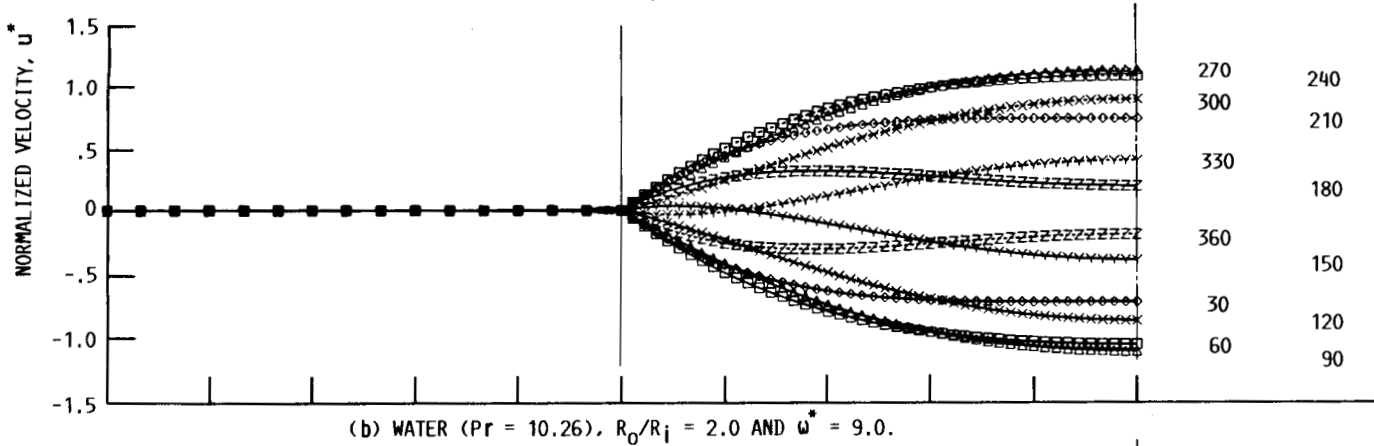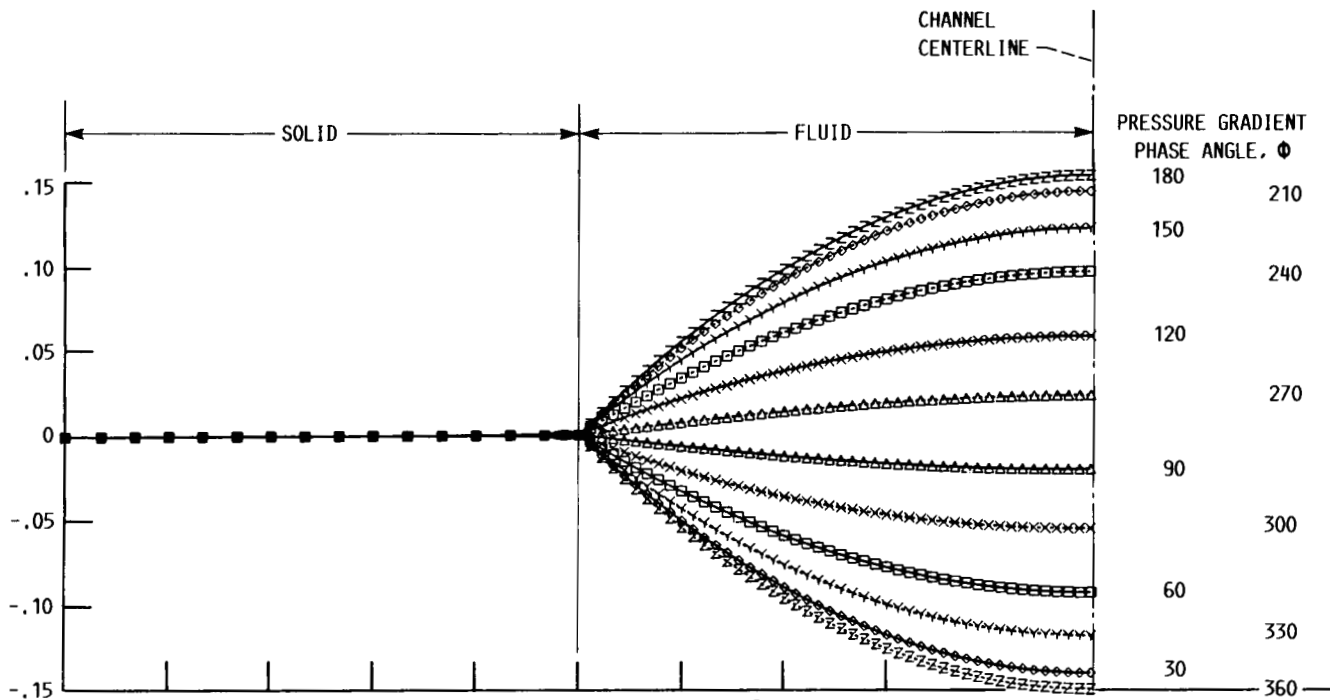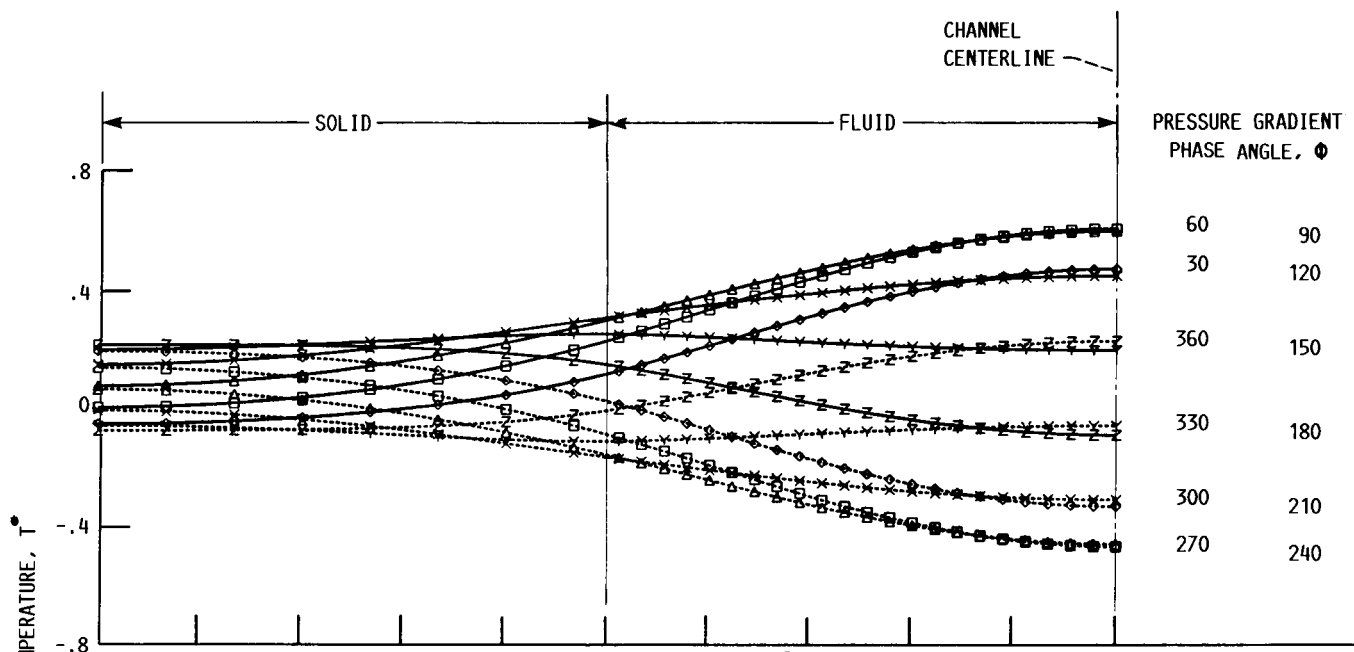COINCIDE AND LIE ON A LINE AT $45°$ WITH THE AXIS).
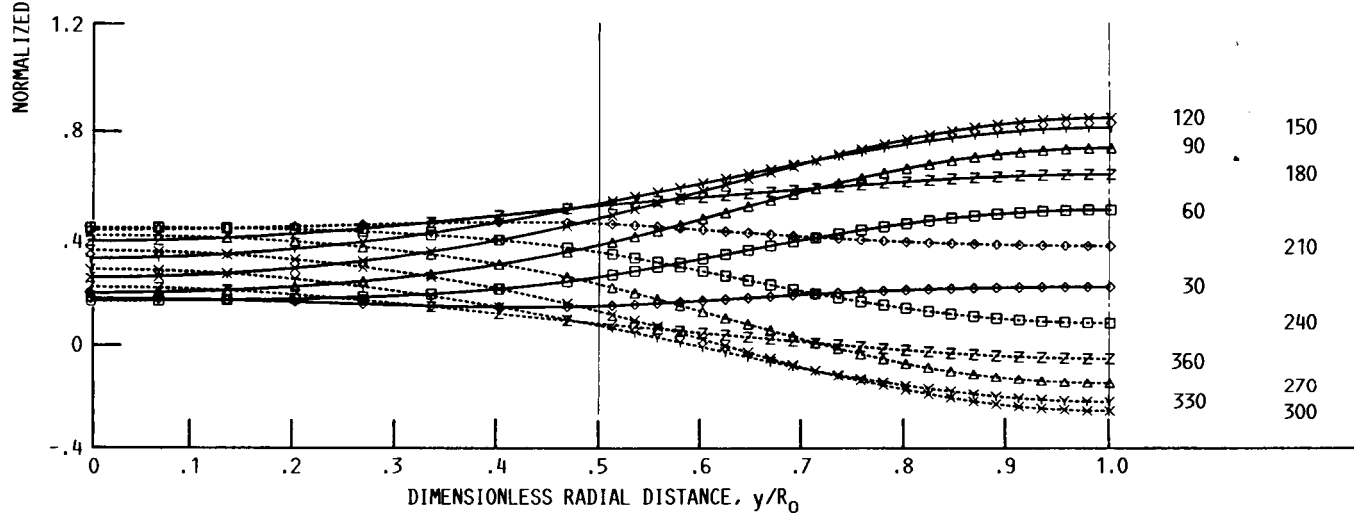
FIGURE 3. - NORMALIZED VELOCITY PROFILES VERSUS DIMENSIONLESS RADIAL DISTANCE AT DIFFERENT PRESSURE GRADIENT PHASE ANGLES.

8

SOLID — FLUID

PRESSURE GRADIENT
PHASE ANGLE, Φ

.8

.4

0

-.4

-.8

NORMALIZED TEMPERATURE, T^*

60	90
30	120
360	150
330	180
300	210
270	240

(a) WATER (Pr = 10.26), R_o/R_i = 2.0 AND ω^* = 0.306.

1.2

.8

.4

0

-.4

120	150
90	180
60	210
30	240
360	270
330	300

0   .1   .2   .3   .4   .5   .6   .7   .8   .9   1.0

DIMENSIONLESS RADIAL DISTANCE, y/R_o

(b) AIR (Pr = 0.72), R_o/R_i = 2.0 AND ω^* = 4.48.

FIGURE 4. - NORMALIZED TEMPERATURE PROFILES VERSUS DIMENSIONLESS RADIAL DISTANCE AT DIFFERENT PRESSURE GRADIENT
ANGLES.

(a) WATER (Pr = 10.26), R_0/R_i = 2.0 AND ω^* = 0.306.

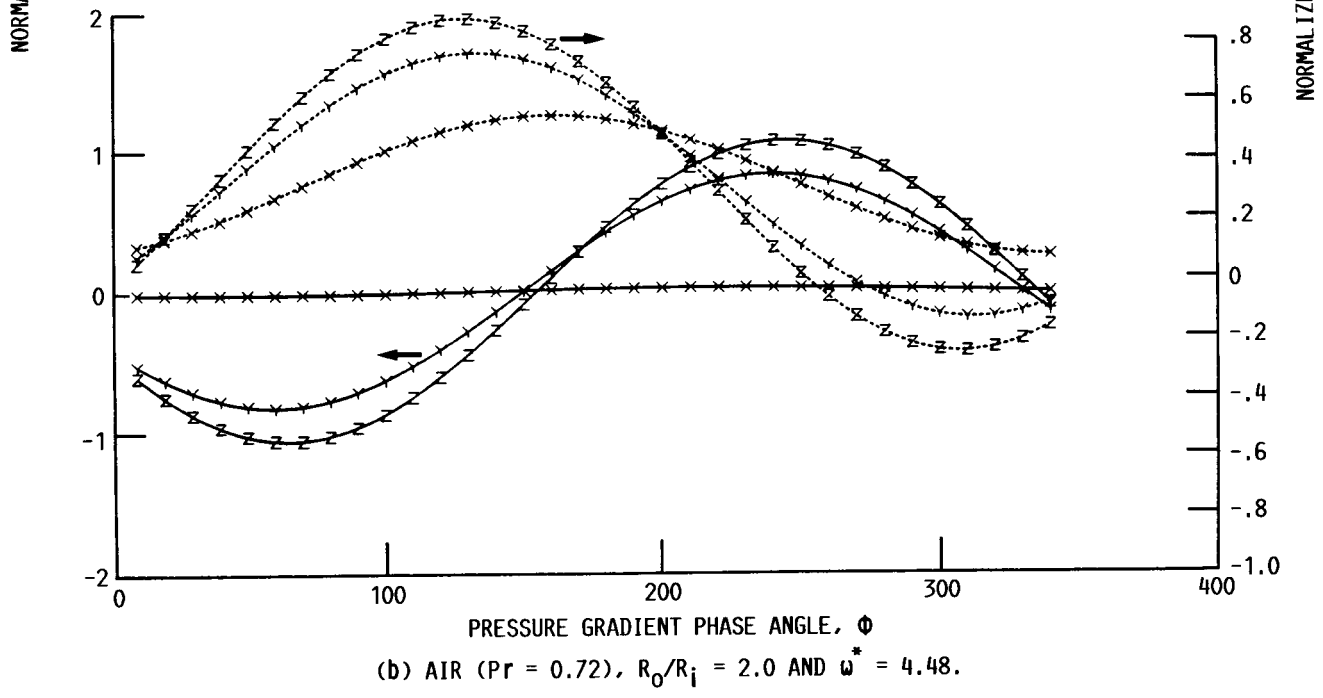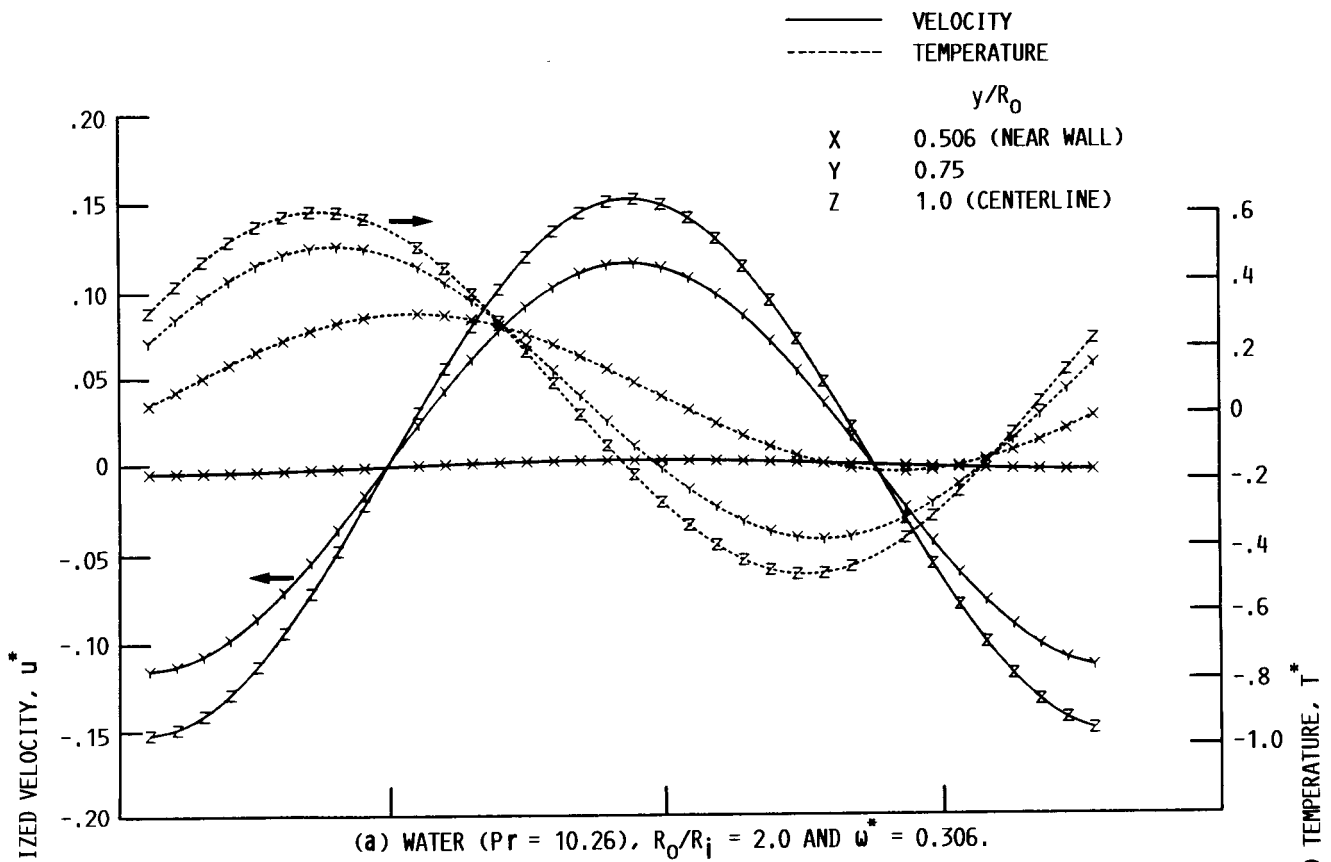(b) AIR (Pr = 0.72), R_0/R_i = 2.0 AND ω^* = 4.48.

FIGURE 5. - NORMALIZED VELOCITY AND TEMPERATURE PROFILES VERSUS PRESSURE GRADIENT PHASE ANGLE AT DIFFERENT RADIAL DISTANCES (y/R_0 = 0.506, 0.75 AND 1.0).

NASA

National Aeronautics and
Space Administration

Report Documentation Page

1. Report No. NASA TM-102057	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Two-Dimensional Numerical Simulation of a Stirling Engine Heat Exchanger		5. Report Date
		6. Performing Organization Code
7. Author(s) Mounir Ibrahim, Roy C. Tew, and James E. Dudenhoefer		8. Performing Organization Report No. E-4815
		10. Work Unit No. 586-01-11
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

16. Abstract

This paper describes the first phase of an effort to develop multidimensional models of Stirling engine components; the ultimate goal is to model an entire engine working space. More specifically, this paper describes parallel plate and tubular heat exchanger models with emphasis on the central part of the channel (i.e. ignoring hydrodynamic and thermal end effects). The model assumes: laminar, incompressible flow with constant thermophysical properties. In addition, a constant axial temperature gradient is imposed. The governing equations, describing the model, have been solved using Crank-Nicloson finite-difference scheme. Model predictions have been compared with analytical solutions for oscillating/reversing flow and heat transfer in order to check numerical accuracy. The simplifying assumptions will later be relaxed to permit modeling of compressible, laminar/turbulent flow that occurs in Stirling heat exchnager. Excellent agreement has been obtained for the model predictions with analytical solutions available for both flow in circular tubes and between parallel plates. Also the heat transfer computational results are in good agreement with the heat transfer analytical results for parallel plates.

17. Key Words (Suggested by Author(s)) Oscillating flow Heat transfer Heat exchanger Stirling engine	18. Distribution Statement Unclassified – Unlimited Subject Category 34	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No of pages 12	22. Price* A02