FATIGUE STRENGTH REDUCTION MODEL: RANDOM3 and RANDOM4 USER MANUAL

Prepared by :

Lola Boyce, Ph.D., P.E.
Thomas B. Lovelace

APPENDIX 2
of Annual Report
of Project Entitled
Development of Advanced Methodologies
for Probabilistic Constitutive Relationships
of Material Strength Models

NASA Grant No. NAG 3-867

Prepared for :

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center
Cleveland, OH 44135

The Division of Engineering
The University of Texas at San Antonio
San Antonio, TX 78285
January, 1989

FATIGUE STRENGTH REDUCTION MODEL: RANDOM3 and RANDOM4 USER MANUAL

Prepared by :

Lola Boyce, Ph.D., P.E.
Thomas B. Lovelace

APPENDIX 2
of Annual Report
of Project Entitled
Development of Advanced Methodologies
for Probabilistic Constitutive Relationships
of Material Strength Models

NASA Grant No. NAG 3-867

Prepared for :

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center
Cleveland, OH 44135

The Division of Engineering
The University of Texas at San Antonio
San Antonio, TX 78285
January, 1989

TABLE OF CONTENTS

1.0 INTRODUCTION

This User Manual documents the FORTRAN programs RANDOM3 and RANDOM4. They are based on fatigue strength reduction, using a probabilistic constitutive model. They predict the random lifetime of an engine component to reach a given fatigue strength (see Section 2.0, Theoretical Background).

Included in this Manual are details regarding the theoretical backgrounds of RANDOM3 and RANDOM4, input data instructions and sample problems illustrating the use of RANDOM3 and RANDOM4 . Appendix A gives information on the physical quantities, their symbols, FORTRAN names and both SI and U.S. Customary units. Appendix B and C include photocopies of the actual computer printout corresponding to the sample problems. Appendices D and E detail the IMSL, Version 10 [1], subroutines and functions called by RANDOM3 and RANDOM4 and SAS/GRAPH [2] programs that can be used to plot both the probability density functions (p.d.f.) and the cumulative distribution functions (c.d.f.).

2.0 THEORETICAL BACKGROUND

Fatigue strength data are usually presented as cycles to failure for each of several stress amplitudes, the familiar S-N diagram. Results indicate that for lower stress amplitudes the cycles (or time) to failure increases. Thus, a power curve fit through the data yields a monotonically decreasing curve. In general, this curve is represented as

$$S = [N/C']^{-1/m'} \tag{6}$$

where the primitive variables in this equation are as follows: S is the applied constant amplitude alternating stress at failure or fatigue strength, N is number of cycles, C' is a material parameter that varies from specimen to specimen and m' is a material constant.[3] Equation (6) can be written in terms of "cycles to reach a given fatigue strength" as

$$N = C'S^{-m'} \tag{7}$$

Recently another fatigue strength reduction model has been proposed that takes into account the effect of temperature as well as other parameters that affect strength.[4] The general form of the constitutive relationships for this model is applied to the constituents of high temperature composite materials. Specifically, it is applied herein for the case of a single material constituent. The mechanical property of interest is fatigue strength which is expressed in terms of primitive variables, including the general categories of temperature, mechanical cycles and mean stress. For these categories, the relationship becomes

$$\frac{S}{S_o} = \left[\frac{T_F - T}{T_F - T_o}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m \left[\frac{\log N_{MF} - \log N_M}{\log N_{MF} - \log N_{MO}}\right]^q \tag{8}$$

where S is the applied constant amplitude alternating stress at failure (fatigue strength) at current (or operating) temperature, T, mean stress, σ, and mechanical cycle, N_M. S_O is fatigue strength at reference temperature, T_O (usually room temperature), reference mean stress (or residual stress), σ_O, and reference mechanical cycle, N_{MO}. Also, T_F is the final or melting temperature of the material, S_F is the final or tensile strength of the material, and N_{MF} is the final mechanical cycle or lifetime. Empirical parameters, n, m, and q, are determined from available experimental data or estimated from anticipated behavior of the particular product term.[5] Note that the term containing mechanical cycles is expressed in terms of the log of cycles rather than cycles. This formulation is attractive when N_M and N_{MO} are small compared to N_{MF}. The equation may be solved for N_M, or the "cycles to reach a given fatigue strength." The expression is

$$N = 10 \exp\left[\log N_{MF} - \left[(\log N_{MF} - \log N_{MO})\left[\frac{S}{S_o\left[\frac{T_F - T}{T_F - T_o}\right]^n\left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m}\right]^{1/q}\right]\right] \tag{9}$$

For values typical of a cast nickel base-superalloy subjected to typical loads and temperatures, equation (9) indicates increasing life for decreasing temperature, decreasing tensile mean stress, and decreasing applied alternating stress. It indicates decreasing life for increasing temperature, decreasing compressive mean stress, and increasing applied alternating stress. Therefore, equation (9) predicts observed trends in general.

Probabilistic analysis, via simulation, yields the distribution of the dependent random variable, cycles, N. A probability density function (p.d.f.) of cycles is generated using the maximum penalized likelihood method for RANDOM3. For RANDOM4, a p.d.f. of cycles is generated using the maximum entropy method. Maximum entropy uses Jaynes' principle which says that "the minimally prejudiced distribution is that which maximizes the entropy subjected to the constraints supplied by the given information."[6]

3.0 INPUT DATA

Data input for RANDOM3 and RANDOM4 is user friendly and easy to manipulate (see, for example, the file entitled NORMAL.INP, in Section 4.0). The first twelve lines of input have the same format, 2E12.4 and the last two lines differ. The last two lines of input have the formats I3,2X,I3,2X,2E12.4,2X,I3 and I3, respectively. A brief, line by line description is given along with an example for each line (NOTE: the ruler is to aid the user in formatting and is not a part of the input). A table listing the physical quantities, their units and symbols is given in Appendix A.

1. Random Number Generator Seed, ISEED, and Sample Size, NTOT

EXAMPLE:

```
12345678901234567890123456789O
          1          40
```

2. Ultimate Tensile Strength, SF

EXAMPLE:

```
12345678901234567890123456789O
   900.0000    45.0000
```

3. Log of Final Cycle, NMF

EXAMPLE:

```
12345678901234567890123456789O
     8.0000      0.8000
```

4. Reference Fatigue Strength, SO

EXAMPLE:

```
12345678901234567890123456789O
   500.0000    25.0000
```

5. Log of Reference Cycle, NMO

EXAMPLE:

```
12345678901234567890123456789O
     7.0000      0.7000
```

6. Current Fatigue Strength, S

EXAMPLE:

```
1234567890123456789012345678 90
     250.0000       12.0000
```

7. Residual Compressive Stress, SIGO

EXAMPLE:

```
1234567890123456789012345678 90
      20.0000        1.0000
```

8. Current Mean Stress, SIG

EXAMPLE:

```
1234567890123456789012345678 90
     150.0000        7.5000
```

9. Temperature Exponent, XXN, Stress Exponent, XXM, and Cycle Exponent, XXQ

EXAMPLE:

```
1234567890123456789012345678 90
       0.5000        0.0150
```

10. Melting Temperature, TF

EXAMPLE:

```
1234567890123456789012345678 90
    1500.0000       75.0000
```

11. Reference Temperature, TO

EXAMPLE:

```
1234567890123456789012345678 90
      20.0000        0.6000
```

12. Current Temperature, T

EXAMPLE:

```
12345678901234567890123456789O
    850.0000      25.0000
```

13. The DESPL[1] parameters are NODE, INIT, ALPHA, EPS, and MAXIT and are entered in that order as follows:

EXAMPLE:

```
12345678901234567890123456789012345678901
 21    0        20.0000      1.0E-05   30
```

14. The DESPL parameter, IOPT, is entered as follows:

EXAMPLE:

```
1234567890
  2
```

4.0 SAMPLE PROBLEMS FOR RANDOM3 AND RANDOM4

The objective of these programs is to predict the random lifetime to reach a given fatigue strength for an engine component. The theory is based on fatigue strength reduction, using a probabilistic constitutive model. The only difference between RANDOM3 and RANDOM4 is the method used to generate p.d.f. estimates. RANDOM3 uses maximum penalized likelihood, while RANDOM4 uses maximum entropy (see Section 2.0, Theoretical Background). RANDOM3 and RANDOM4 input parameters are given in Table A2.1.

TABLE A2.1 RANDOM3 and RANDOM4 input (SI units)

FORTRAN Name	Distribution Type	Mean	Standard Deviation (Value)	(% of Mean)
SF	normal	900.0	45.0	(3%)
NMF	lognormal	8.0	0.8	(10%)
SO	lognormal	500.0	25.0	(5%)
NMO	lognormal	7.0	0.7	(10%)
S	lognormal	250.0	12.5	(5%)
SIGO	lognormal	-20.0	-1.0	(1%)
SIG	lognormal	150.0	7.5	(5%)
XXN	normal	0.5	0.015	(0.3%)
XXM	normal	0.5	0.015	(0.3%)
XXQ	normal	0.5	0.015	(0.3%)
TF	normal	1500.0	45.0	(3%)
TO	normal	20.0	0.6	(3%)
T	normal	850.0	25.5	(3%)

The input is entered in the following format in a file entitled NORMAL.INP.

```
12345678901234567890123456789012345678901234567890
                1                  40
         900.0000           45.0000
           8.0000            0.8000
         500.0000           25.0000
           7.0000            0.7000
         250.0000           12.5000
          20.0000            1.0000
         150.0000            7.5000
           0.5000            0.0150
        1500.0000           75.0000
          20.0000            0.6000
         850.0000           25.5000
21          0                20.00          1.0E-05     30
 2
```

Execution of RANDOM3 and RANDOM4 (source code entitled NR3.FOR and NR4.FOR, respectively) produces files entitled RANDM33 and RANDM44. These give intermediate results (see Appendices B and C). Execution also produces plotfiles entitled PLOT1 and PLOT2 (see Appendices B and C). These files are used to plot the X and Y axes of the probability density function (p.d.f.) and the cumulative distribution function (c.d.f.), respectively, generated by RANDOM3 and RANDOM4. The plots are drawn from the plotfiles by the SAS/GRAPH graphing program (see Appendix D). These plots for the sample problem are shown Figures 1, 2, 3, and 4. This same sample problem has been reported in Boyce and Chamis.[7] There, however, it utilized U.S. Customary units and older versions of RANDOM3 and RANDOM4 (using IMSL Version 9.2 subroutines).

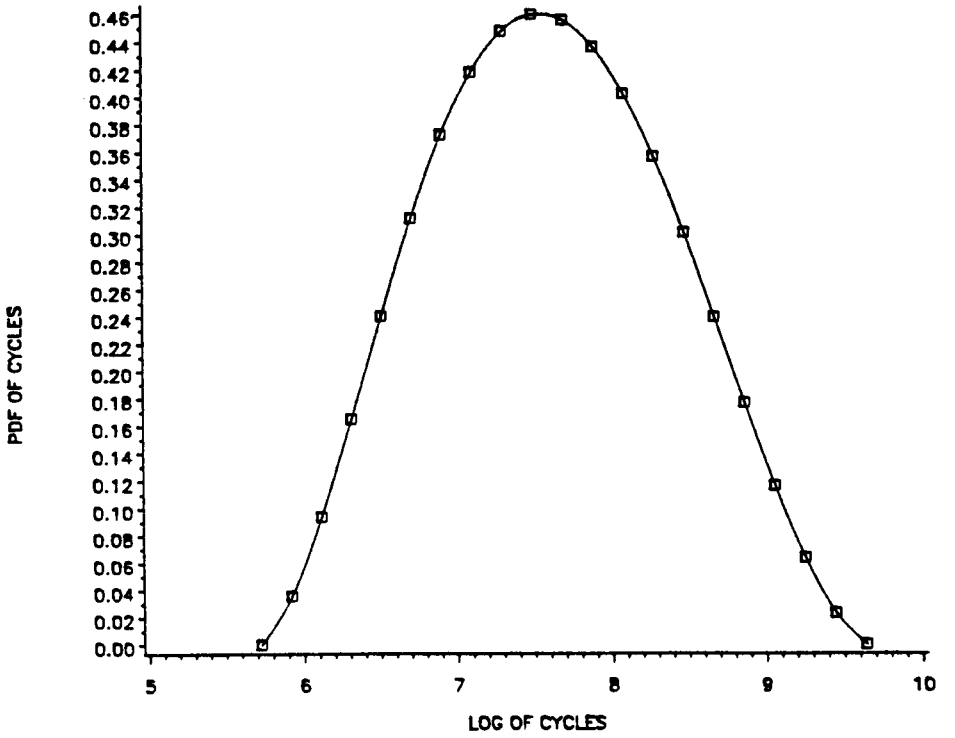


Fig. A2.1 p.d.f. of log of mechanical cycles for fatigue strength reduction model, using maximum penalized likelihood method of p.d.f. generation.

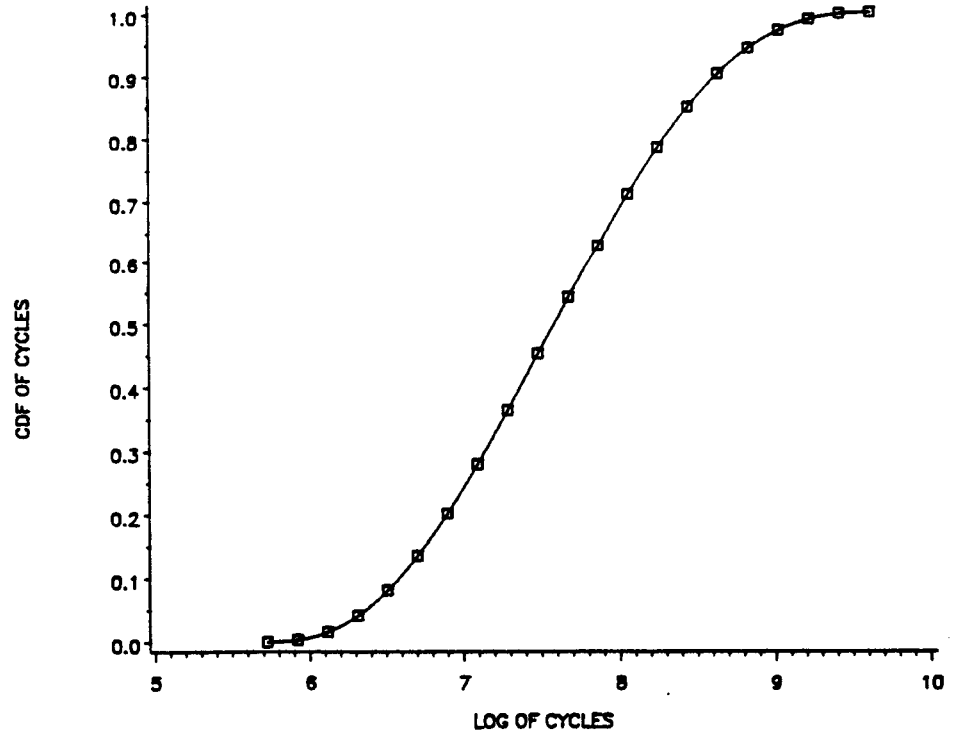


Fig. A2.2 c.d.f. of log of mechanical cycles for fatigue strength reduction model, using maximum penalized likelihood method of p.d.f. generation.

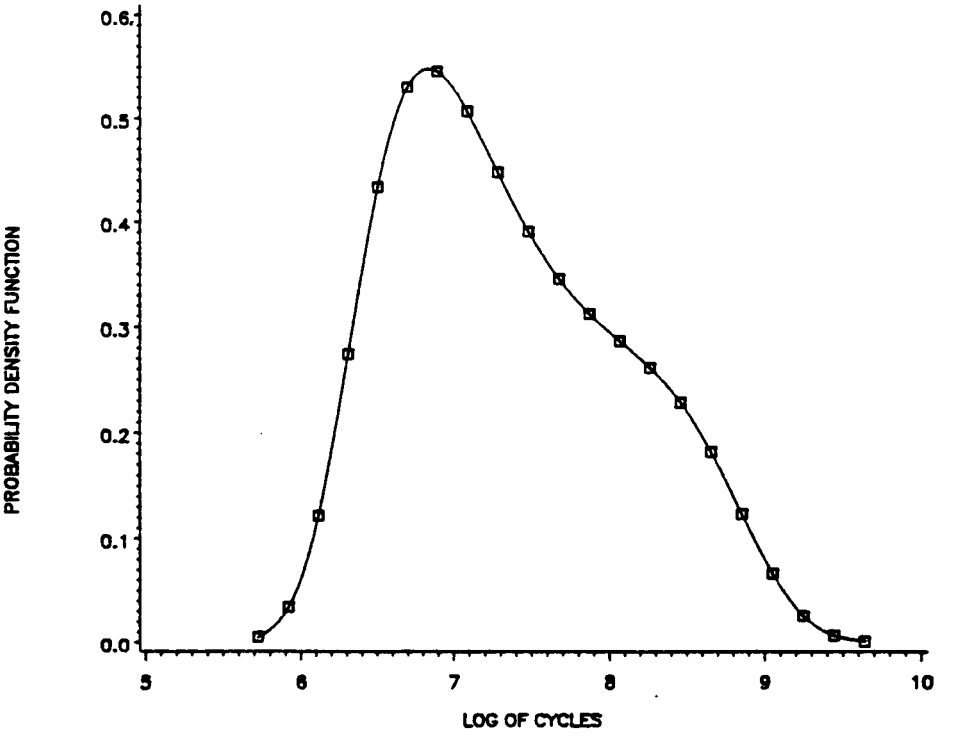


Fig. A2.3 p.d.f. of log of mechanical cycles for fatigue strength reduction model, using maximum entropy method of p.d.f. generation.

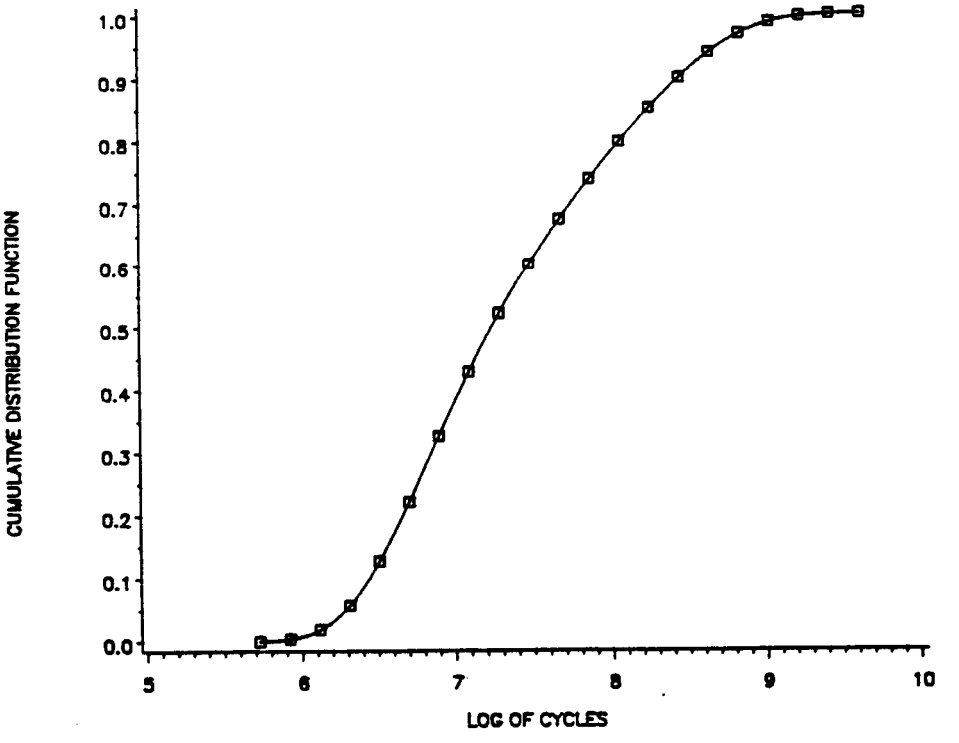


Fig. A2.4 c.d.f. of log of mechanical cycles for fatigue strength reduction model, using maximum entropy method of p.d.f. generation.

5.0 REFERENCES

[1] IMSL, "STAT/LIBRARY, FORTRAN Subroutines for Statistical Analysis", Houston, Texas

[2] SAS Institute, Inc., SAS/GRAPH User's Guide, Version 5 Edition, Cary NC: SAS Institute, Inc., 1985, 596 pp.

[3] Madsen, H.O., "Bayesian Fatigue Life Prediction," Probabilistic Methods in the Mechanics of Solids and Structures, S. Eddwertz and N.C. Lind, Eds., Proceedings of the IUTAM Symposium, Stockholm, Sweden, 1984,pp. 395-406.

[4] Hopkins, D.A. and Chamis, C.C., "A Unique Set of Micromechanics Equations for High Temperature Metal Matrix Composites," NASA TM87154, Nov., 1985.

[5] Chamis, C.C. and Hopkins, D.A., "Thermoviscoplastic Nonlinear Constitutive Relationships for Structural Analysis of High Temperature Metal Matrix Composites," NASA TM 87291, Nov., 1985.

[6] Siddall, J.N., "A Comparison of Several Methods of Probabilistic Modeling," Proceedings of the Computers in Engineering Conference, ASME, San Diego, CA, Vol. 4, 1982, pp. 231-238.

[7] Boyce, L. and Chamis, C.C., "Probabilistic Constitutive Relations for Cyclic Material Strength Models," Proceedings, 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, 1988.

6.0 APPENDIX A

PHYSICAL QUANTITIES, SYMBOLS, AND UNITS

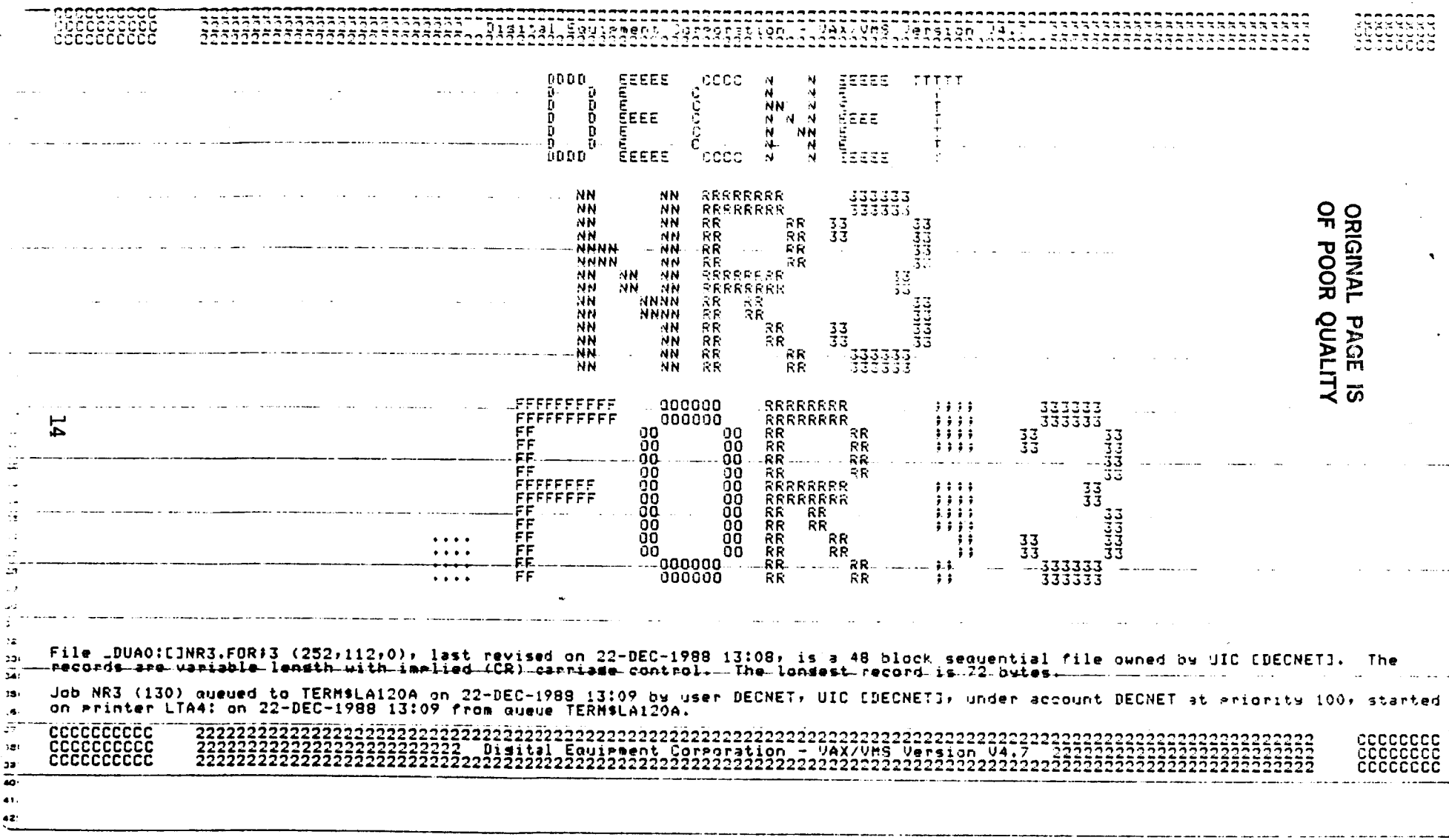
The physical quantities, their symbols and units for the fatigue crack growth model are given in the following table.

Table A2.2 Physical quantities, symbols, and units for fatigue crack growth model for RANDOM3 and RANDOM4.

Physical Quantity	Theory Symbol	FORTRAN Name	Units SI	Units U.S.
Ultimate Tensile Strength	SF	SF	MPa	ksi
Final Cycle (lifetime)	N_{MF}	NMF	dimensionless	
Reference Fatigue Strength	SO	SO	MPa	ksi
Reference Cycles	N_{MO}	NMO	dimensionless	
Current Fatigue Strengths	S	S	MPa	ksi
Residual Compressive Stress	σ_o	SIGO	MPa	ksi
Current Mean Stress	σ	SIG	MPa	ksi
Empirical Material Parameters	n	XXN	dimensionless	
	m	XXM	dimensionless	
	q	XXQ	dimensionless	
Melting Temperature	TF	TF	°C	°F
Reference Temperature	TO	TO	°C	°F
Current Temperature	T	T	°C	°F

7.0 APPENDIX B

RANDOM3 SAMPLE PROBLEM: SOURCE, INPUT AND OUTPUT FILES



File _DUA0:[]NR3.FOR;3 (252,112,0), last revised on 22-DEC-1988 13:08, is a 48 block sequential file owned by UIC [DECNET]. The records are variable length with implied (CR) carriage control. The longest record is 72 bytes.

Job NR3 (130) queued to TERM$LA120A on 22-DEC-1988 13:09 by user DECNET, UIC [DECNET], under account DECNET at priority 100, started on printer LTA4: on 22-DEC-1988 13:09 from queue TERM$LA120A.

CCCCCCCCCC 2222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222 CCCCCCCC
CCCCCCCCCC 22222222222222222222222 Digital Equipment Corporation - VAX/VMS Version V4.7 2222222222222222222222222 CCCCCCCC
CCCCCCCCCC 2222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222222 CCCCCCCC

```
JOB,JN=COMP3,US=USAQ530,RT=60,T=30,MFL=300000.
ACCOUNT,USW=LOLAB.
DELETE,PDN=NR3BLD,ID=SMBOYCE.
CFT77,LIST.
REWIND,DN=$BLD.
BUILD,I=0,OBL=0.
SAVE,DN=$NBL,PDN=NR3BLD,ID=SMBOYCE.
DELETE,PDN=NR3BLD,ID=SMBOYCE,ED=-1.
/EOF
C CHAMIS MICROMECHANICS CONSTITUTIVE EQUATIONS;
C RANDOMIZED AND APPLIED TO FATIGUE STRENGTH
      INTEGER NTOT, ISEED, M, INIT, NMISS, MAXIT, NODE
      REAL XM, XS, YM, YS, EPS, P, RWKSP(7612), ALPHA
      COMMON /WORKSP/ RWKSP
      DIMENSION SF(10000),XLNMF(10000),SO(10000)
      DIMENSION XLNMO(10000),S(10000)
      DIMENSION SIGO(10000),SIG(10000)
      DIMENSION XXN(10000),XXM(10000),XXQ(10000)
      DIMENSION XNM(10000),BNDSX(10000)
      DIMENSION TF(10000),TO(10000),T(10000)
      DIMENSION STAT(10000), DENS(10000),DISTX(10000)
      DIMENSION BNDS(10000),BB(999),FF(999)
      DIMENSION XX(999),PP(999)
      DIMENSION XXXX(999),PPPP(999)
      DIMENSION BBBB(999),FFFF(999)
      DIMENSION C(10000)
      EXTERNAL RNLNL, RNSET, RNNOR, DESPL, IWKIN
 1001 FORMAT(5E12.4)
 1002 FORMAT(I12,I12)
 1003 FORMAT(I4,I4)
 1004 FORMAT(I4)
 1009 FORMAT(I3,2X,I3,2X,2E12.4,2X,I3)
 1010 FORMAT(I3)
 1011 FORMAT(2E12.4)
C LOGNORMAL ULTIMATE TENSILE STRENGTH, SF
      READ (3,1002) ISEED,NTOT
      WRITE(6,1002) ISEED,NTOT
      READ (3,1011) XM,XS
      WRITE(6,1011) XM,XS
      YS = SQRT(LOG(1.0+(XS/XM)**2.))
      YM = LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SF)
      WRITE(6,2020)
 2020 FORMAT(' LOGNORMAL SF')
      WRITE(6,1001)(SF(I),I=1,NTOT)
C LOGNORMAL LOG OF FINAL CYCLE, XLNMF
      WRITE(6,1002) ISEED,NTOT
      READ (3,1011) XM,XS
      WRITE(6,1011) XM,XS
      YS = SQRT( LOG(1.0+(XS/XM)**2))
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,XLNMF)
      WRITE(6,2021)
 2021 FORMAT(' LOGNORMAL XLNMF')
      WRITE( 6,1001) (XLNMF(I),I=1,NTOT)
C LOGNORMAL FATIGUE STRENGTH AT REFERENCE CONDITIONS, SO
      WRITE(6,1002) ISEED,NTOT
      READ (3,1011) XM, XS
      WRITE (6,1011) XM, XS
      YS = SQRT( LOG(1.0+(XS/XM)**2))
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
```

```
      CALL RNLNL(NTOT,YM,YS,S0)
      WRITE(6,2022)
 2022 FORMAT(' LOGNORMAL S0')
      WRITE( 6,1001) (S0(I),I=1,NTOT)
C LOGNORMAL LOG OF REFERENCE CYCLES, XLNMO
      WRITE(6,1002) ISEED,NTOT
      READ (3,1011) XM, XS
      WRITE(6,1011) XM, XS
      YS = SQRT(LOG(1.0+(XS/XM)**2))
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,XLNMO)
      WRITE (6,2023)
 2023 FORMAT (' LOGNORMAL XLNMO')
      WRITE (6,1001)(XLNMO(I),I=1,NTOT)
C LOGNORMAL FATIGUE STRENGTH AT CURRENT CONDITIONS, S
      WRITE(6,1002) ISEED,NTOT
      READ (3,1011) XM, XS
      WRITE(6,1011) XM, XS
      YS = SQRT(LOG(1.0+(XS/XM)**2.))
      YM = LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,S)
      WRITE (6,2024)
 2024 FORMAT(' LOGNORMAL S')
      WRITE(6,1001)(S(I),I=1,NTOT)
C DEFINE RANDOM STRESSES
C LOGNORMAL REFERENCE STRESS, SIGO
      WRITE(6,1002)ISEED,NTOT
      READ (3,1011) XM, XS
      WRITE(6,1011) XM, XS
      YS=SQRT(LOG(1.0+(XS/XM)**2.))
      YM=LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SIGO)
C CHANGE SIGO TO NEGATIVE VALUES FOR COMPRESSIVE
C RESIDUAL STRESSES
      DO 201 I = 1,NTOT
      SIGO(I)=-SIGO(I)
  201 CONTINUE
      WRITE(6,2036)
 2036 FORMAT(' LOGNORMAL SIGO')
      WRITE(6,1001)(SIGO(I),I=1,NTOT)
C LOGNORMAL CURRENT STRESS, SIG
      WRITE(6,1002)ISEED,NTOT
      READ (3,1011) XM, XS
      WRITE(6,1011) XM, XS
      YS=SQRT(LOG(1.0+(XS/XM)**2.))
      YM=LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SIG)
      WRITE(6,2037)
 2037 FORMAT(' LOGNORMAL SIG')
      WRITE(6,1001)(SIG(I),I=1,NTOT)
C NORMAL EXPONENTS, XXN,XXM,XXQ
      WRITE(6,1002)ISEED,NTOT
      READ (3,1011) YM, YS
      WRITE(6,1011) YM, YS
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,XXN)
      DO 202 I=1,NTOT
      XXN(I)=YS*XXN(I)+YM
  202 CONTINUE
      WRITE(6,2025)
```

```
 2025 FORMAT(' NORMAL XXN')
      WRITE(6,1001)(XXN(I),I=1,NTOT)
      WRITE(6,1002)ISEED,NTOT
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,XXM)
      DO 203 I=1,NTOT
      XXM(I)=YS*XXM(I)+YM
  203 CONTINUE
      WRITE(6,2026)
 2026 FORMAT(' NORMAL XXM')
      WRITE(6,1001)(XXM(I),I=1,NTOT)
      WRITE(6,1002)ISEED,NTOT
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,XXQ)
      DO 204 I=1,NTOT
      XXQ(I)=YS*XXQ(I)+YM
  204 CONTINUE
      WRITE(6,2027)
 2027 FORMAT(' NORMAL XXQ')
      WRITE(6,1001)(XXQ(I),I=1,NTOT)
C NORMAL TEMPERATURES, TF,TO,T
C NORMAL FINAL (MELTING) TEMPERATURE, TF
      WRITE(6,1002)ISEED,NTOT
      READ(3,1011) YM, YS
      WRITE(6,1011) YM, YS
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,TF)
      DO 205 I=1,NTOT
      TF(I)=YS*TF(I)+YM
  205 CONTINUE
      WRITE(6,2046)
 2046 FORMAT(' NORMAL TF')
      WRITE(6,1001)(TF(I),I=1,NTOT)
C NORMAL REFERENCE TEMPERATURE, TO
      WRITE(6,1002)ISEED,NTOT
      READ (3,1011) YM, YS
      WRITE(6,1011) YM, YS
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,TO)
      DO 206 I=1,NTOT
      TO(I)=YS*TO(I)+YM
  206 CONTINUE
      WRITE(6,2047)
 2047 FORMAT(' NORMAL TO')
      WRITE(6,1001)(TO(I),I=1,NTOT)
C NORMAL CURRENT TEMPERATURE,T
      WRITE(6,1002)ISEED,NTOT
      READ(3,1011) YM, YS
      WRITE(6,1011) YM, YS
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,T)
      DO 207 I=1,NTOT
      T(I)=YS*T(I)+YM
  207 CONTINUE
      WRITE(6,2048)
 2048 FORMAT(' NORMAL T')
      WRITE(6,1001)(T(I),I=1,NTOT)
C CALCULATE LOG OF CURRENT CYCLES, LOG XNM
      DO 102 I=1,NTOT
      RS=((SF(I)-SIG(I))/(SF(I)-SIGO(I)))**XXM(I)
      TEMP=((TF(I)-T(I))/(TF(I)-TO(I)))**XXN(I)
      XNM1=(S(I)/(SO(I)*TEMP*RS))**(1./XXQ(I))
      XNM2=(XLNMF(I)-((XLNMF(I)-XLNMO(I))*XNM1))
      IF(XNM2.LT.0.0)XNM2=0.0
```

```
            XNM(I)=XNM2
  102 CONTINUE
      WRITE(6,2028)
 2028 FORMAT(' LOG OF CYCLES TO REACH MEAN FATIGUE STR =    '//.
     1'250 MPA')
      WRITE(6,1001)(XNM(I),I=1,NTOT)
C SORT LOG OF CYCLES
      CALL SORT(XNM,NTOT)
      WRITE(6,2029)
 2029 FORMAT(' SORTED LOG OF CYCLES')
      WRITE(6,1001)(XNM(I),I=1,NTOT)
C CALCULATE PDF OF LOG OF CURRENT CYCLES,LOG XNM
      READ(3,1009)NODE,INIT,ALPHA,EPS,MAXIT
      WRITE(6,985)
  985 FORMAT(' DESPL PARAMETERS')
      WRITE(6,1009)NODE,INIT,ALPHA,EPS,MAXIT
      BNDS(1)=XNM(1) - 0.05*XNM(1)
      BNDS(2)=XNM(NTOT) + 0.05*XNM(NTOT)
      WRITE(6,979)BNDS(1),BNDS(2)
  979 FORMAT(' BNDS(1),BNDS(2)=',E12.4,1X,E12.4)
      CALL DESPL(NTOT,XNM,NODE,BNDS,INIT,ALPHA,MAXIT,EPS,DENS,STAT,
     1NMISS)
      WRITE(6,980)
  980 FORMAT(' PDF OF LOG OF CURRENT CYCLES,LOG XNM,Y AXIS OF PDF PLOT')
      WRITE(6,1001)(DENS(I),I=1,NODE)
      WRITE(6,981)
  981 FORMAT(' OUTPUT STATISTICS')
      WRITE(6,1001)(STAT(I),I=1,4)
      WRITE(6,982)
  982 FORMAT(' NUMBER OF MISSING VALUES')
      WRITE(6,1010)NMISS
C
C CALCULATE WINDOW WIDTH, HH
C
      HH=(BNDS(2)-BNDS(1))/(NODE-1)
C
C CALCULATE VALUES OF LOG OF CURRENT CYCLES AT WHICH PDF IS ESTIMATED;
C ALSO CALLED 'NODE' VALUES
C
      DO 6001,I=1,NODE-2
      BNDS(I+2)=BNDS(1) + (I*HH)
 6001 CONTINUE
      WRITE(6,983)
  983 FORMAT(' LOG OF CURRENT CYCLES, LOG XNM')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
C
C REORDER BNDS FOR PLOTTING
C
      SAVE1 = BNDS(2)
      SAVE2 = BNDS(NODE)
      BNDS(NODE)=BNDS(2)
      DO 6002, I=1, NODE-2
      BNDS(I+1)=BNDS(I+2)
 6002 CONTINUE
      BNDS(NODE-1)=SAVE2
      BNDS(NODE)=SAVE1
      WRITE(6,984)
  984 FORMAT(' ORDERED LOG OF CURRENT CYCLES, LOG,XNM,
     1X AXIS PDF, CDF PLOT')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
C WRITE LOG OF CURRENT CYCLES AND PDF OF LOG OF CURRENT CYCLES,
C   LOG XNM TO PLOT FILES
      WRITE(34,990)
  990 FORMAT(' (E12.4,1X,E12.4)')
```

```
      WRITE(35,991)(BNDS(J),DENS(J),J=1,NODE)
  991 FORMAT(E12.4,1X,E12.4)
C
C CALCULATE CDF OF LOG OF CURRENT CYCLES
C
      READ(3,1010)IOPT
      WRITE(6,992)
  992 FORMAT(' GCDF PARAMETERS')
      WRITE(6,1010)IOPT
      XO=BNDS(1)
      DO 6003 I=1,NODE
      P=GCDF(XO,IOPT,NODE,BNDS,DENS)
      BNDSX(I)=XO
      XO=XO+HH
      DISTX(I)=P
 6003 CONTINUE
      WRITE(6,994)
  994 FORMAT(' CDF OF LOG OF CURRENT CYCLES, LOG XNM,
     1Y AXIS OF PDF, CDF PLOT')
      WRITE(6,1001)(DISTX(I),I=1,NODE)
C
      WRITE(6,993)
  993 FORMAT(' ORDERED LOG OF CURRENT CYCLES, LOG XNM,
     1X AXIS OF PDF, CDF PLOT')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
      WRITE(6,1001)(BNDSX(I),I=1,NODE)
C
C WRITE LOG OF CURRENT CYCLES AND CDF OF LOG OF CURRENT
C TO THE PLOT FILES
C
      WRITE(35,990)
      WRITE(35,991)(BNDS(J),DISTX(J),J=1,NODE)
      STOP
      END
      SUBROUTINE SORT(Y,N)
      DIMENSION Y(10000)
      N1=N-1
      DO 1 I=1, N1
      J=I+1
      DO 2 K=J, N
      IF (Y(I).LT.Y(K))GO TO 2
      TEMP=Y(I)
      Y(I)=Y(K)
      Y(K)=TEMP
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
C-----------------------------------------------------------------------
C
C  IMSL Name:  D3SPL/DD3SPL (Single/Double precision version)
C
C  Computer:   IBM/SINGLE
C
C  Revised:    November 1, 1985
C
C  Purpose:    Nonparametric probability density function estimation
C              estimation by the penalized likelihood method.
C
C  Usage:      CALL D3SPL (NOBS, X, NODE, BNDS, INIT, ALPHA, MAXIT, EPS,
C                          DENS, STAT, HESS, LDHESS, ILOHI, DENEST, B,
C                          IPVT, WK2)
C
C  Arguments:
C     NOBS   - Number of observations.  (Input)
```

```
C     X      - Vector of length NOBS containing the random sample of
C              responses.  (Input)
C     NODE   - Number of mesh nodes for the discrete pdf estimate.
C              (Input)
C     BNDS   - Vector of length 2 containing the minimum and maximum
C              values for X(i) in BNDS(1) and BNDS(2), respectively.
C              (Input)
C     INIT   - Initialization option.  (Input)
C     ALPHA  - Positive penalty weighting factor which controls the
C              smoothness of the estimate.  (Input)
C     MAXIT  - Maximum number of iterations allowed in the iterative
C              procedure.  (Input)
C     EPS    - Convergence criterion.  (Input)
C     DENS   - Vector of length NODE containing the estimated values of
C              the discrete pdf at the NODE equally spaced mesh nodes.
C              (Input/output if INIT=1, Output otherwise)
C     STAT   - Vector of length 4 containing out statistics.  (Output)
C              STAT(1) and STAT(2) contain the log-likelihood and the
C              log-penalty terms, respectively.  STAT(3) and STAT(4)
C              contain the estimated mean and variance for the
C              estimated density.
C     HESS   - Seven by NODE-2 hessian matrix (and its factorization).
C              (Output)
C     LDHESS - Leading dimension of HESS exactly as specified in the
C              dimension statement in the calling program.  (Input)
C     ILOHI  - NODE by 2 matrix containing the indices for the risk set
C              at each node value.  (Output)
C     DENEST - NODE by 3matrix containing the gradient vector, among
C              other quantities.  (Output)
C     B      - Vector of length NODE containing the NODE values.
C              (Output)
C     IPVT   - Pivot vector of length NODE-2.  (Output)
C     WK2    - Work vector of length NODE-2.  (Output)
C
C  Chapter:  STAT/LIBRARY Density and Hazard Estimation
C
C  Copyright:  1985 by IMSL, Inc.  All Rights Reserved.
C
C  Warranty:  IMSL warrants only that IMSL testing has been applied
C             to this code.  No other warranty, expressed or implied,
C             is applicable.
C-----------------------------------------------------------------------
C
      SUBROUTINE D3SPL (NOBS, X, NODE, BNDS, INIT, ALPHA, MAXIT, EPS,
     &                  DENS, STAT, HESS, LDHESS, ILOHI, DENEST, B,
     &                  IPVT, WK2)
C                                SPECIFICATIONS FOR ARGUMENTS
      INTEGER    NOBS, NODE, INIT, MAXIT, LDHESS, ILOHI(NODE,*),
     &           IPVT(*)
      REAL       ALPHA, EPS, X(*), BNDS(2), DENS(*), STAT(4),
     &           HESS(LDHESS,*), DENEST(NODE,*), B(*), WK2(*)
C                                SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER    I, IMPTR, IPTR, ITER, K, KM1, KM2, KP1, KP2, M, MOLD,
     &           NER, NOB1
      REAL       BK, BKM1, BSMALL, CK, CKM1, CKM2, CKMCM1, CKP1, CKP2,
     &           CONS, EPS1, FACTOR, FK, FKM1, FKM2, FKP1, H, H2, H3,
     &           SUM, TEMP, WK(4)
      DOUBLE PRECISION SUM1, SUM2, SUM3
C                                SPECIFICATIONS FOR SAVE VARIABLES
      INTEGER    MINCR(8)
      SAVE       MINCR
C                                SPECIFICATIONS FOR INTRINSICS
      intrinsic  alog,amaxi,max0,min0,mod,sqrt
```

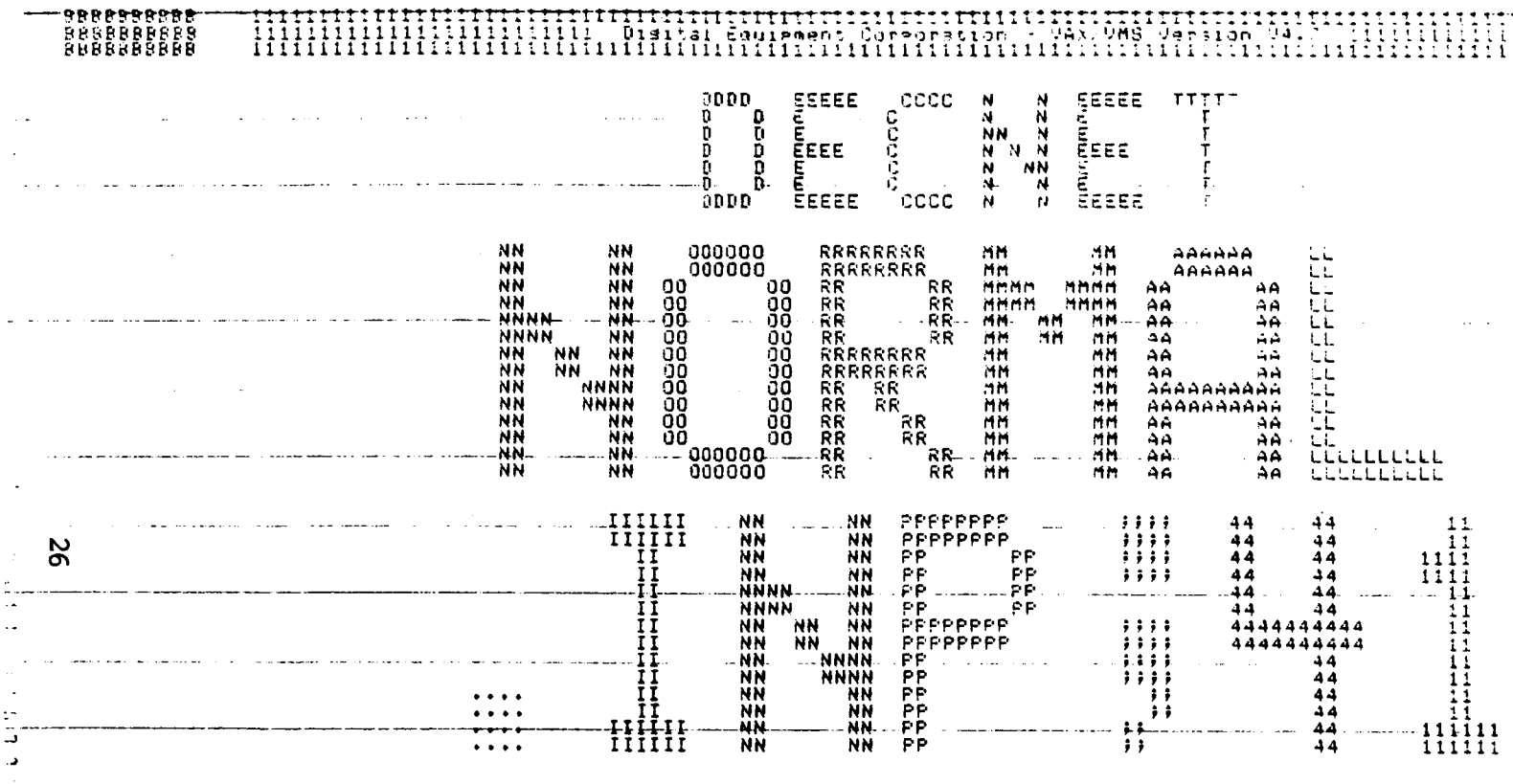
```
      INTRINSIC ALOG, AMAX1, MAXO, MINO, MOD, SQRT
      INTEGER   MAXO, MINO, MOD
      REAL      ALOG, AMAX1, SQRT
C                                SPECIFICATIONS FOR SUBROUTINES
      EXTERNAL  E1MES, E1POP, E1PSH, E1STI, E1STR, SADD, SAXPY,
     &          SCOPY, SHPROD, SSCAL, D2SPT, L2TRB, LFSRB
C                                SPECIFICATIONS FOR FUNCTIONS
      EXTERNAL  ISMIN, N1RCD, SDOT, SNRM2, SSUM
      INTEGER   ISMIN, N1RCD
      REAL      SDOT, SNRM2, SSUM
C
      DATA MINCR/5, 9, 17, 33, 65, 129, 253, 100001/
C
      CALL E1PSH ('D3SPL ')
C                                Error checks
      NER = 1
      IF (NOBS .LT. 1) THEN
         CALL E1MES (5, 1, 'After removing all missing (NaN, not a '//
     &              'number) values from X there are no valid '//
     &              'observations.  At least one valid observation '//
     &              'is necessary.')
      END IF
      IF (NODE .LE. 4) THEN
         CALL E1STI (1, NODE)
         CALL E1MES (5, 2, 'NODE = %(I1).  The number of mesh '//
     &              'nodes, NODE, must be an odd integer greater '//
     &              'than 4.')
      ELSE IF (MOD(NODE,2) .EQ. 0) THEN
         CALL E1STI (1, NODE)
         CALL E1MES (5, 3, 'NODE = %(I1) must be an odd integer '//
     &              'greater than 4.')
      END IF
      IF (ALPHA .LE. 0.0) THEN
         CALL E1STR (1, ALPHA)
         CALL E1MES (5, 4, 'ALPHA = %(R1).  The penalty weighting '//
     &              'factor which controls smoothness, ALPHA, must '//
     &              'be greater than 0.')
      END IF
      IF (MAXIT .LE. 0.0) THEN
         CALL E1STI (1, MAXIT)
         CALL E1MES (5, 5, 'MAXIT = %(I1).  The maximum number '//
     &              'of iterations, MAXIT, must be greater than 0.')
      END IF
      IF (BNDS(1) .GT. BNDS(2)) THEN
         CALL E1STR (1, BNDS(1))
         CALL E1STR (2, BNDS(2))
         CALL E1MES (5, 6, 'BNDS(1) = %(R1) and BNDS(2) = '//
     &              '%(R2).  The minimum value for X, BNDS(1), must '//
     &              'be less than or equal to the maximum value for '//
     &              'X, BNDS(2).')
      END IF
      IF (INIT .NE. 0) THEN
         IF (DENS(1).NE.0 .OR. DENS(NODE).NE.0) THEN
            CALL E1STR (1, DENS(1))
            CALL E1STR (2, DENS(NODE))
            CALL E1STI (1, NODE)
            CALL E1MES (5, 7, 'DENS(1) = %(R1) and DENS(NODE=%(I1)) '//
     &                 '= %(R2).  The beginning and ending initial '//
     &                 'estimates of the density must be zero.')
         END IF
         IF (DENS(ISMIN(NODE,DENS,1)) .LT. 0) THEN
            CALL E1MES (5, 8, 'The initial estimates of the '//
     &                 'density, DENS, must be greater than or '//
     &                 'equal to 0.')
```

```
         END IF
      END IF
      NOB1 = 0
      DO 10  I=1, NOBS
         IF (X(I).LT.BNDS(1) .OR. X(I).GT.BNDS(2)) THEN
            NOB1 = NOB1 + 1
         END IF
   10 CONTINUE
      IF (NOB1 .EQ. NOBS) THEN
         CALL E1MES (5, 9, 'All elements in X lie outside the '//
     &                 'interval BNDS(1) to BNDS(2).  At least one '//
     &                 'element of X must lie in this interval.')
      END IF
      IF (EPS .LE. 0.0) THEN
         EPS1 = 1.0E-4
      ELSE
         EPS1 = EPS
      END IF
      IF (N1RCD(0) .NE. 0) GO TO 9000
C                                     Initialization
      IMPTR = 0
C                                     Set initial densities
      IF (INIT .EQ. 0) THEN
         DENS(1) = 0.0
         DENS(2) = 2.0/(BNDS(2)-BNDS(1))
         DENS(3) = 0.0
         M = 3
      ELSE
         M = NODE
      END IF
C                                     Refine mesh
   20 IF (INIT .EQ. 0) THEN
         MOLD = M
         IMPTR = IMPTR + 1
         M = MIN0(NODE,MINCR(IMPTR))
      END IF
C                                     Get mesh interval width
      H = (BNDS(2)-BNDS(1))/(M-1)
      H2 = H*H
      H3 = H2*H
C                                     Make initial DENS integrate to 1.
      IF (INIT .NE. 0) THEN
         CALL SSCAL (NODE, 1.0/(H*SSUM(NODE,DENS,1)), DENS, 1)
      END IF
C                                     Set mesh nodes
      B(1) = BNDS(1)
      DO 30  I=2, M
         B(I) = B(I-1) + H
   30 CONTINUE
C                                     Set B indices for interpolating X
      IPTR = 0
   40 IPTR = IPTR + 1
      IF (X(IPTR) .LT. BNDS(1)) GO TO 40
      DO 60  K=1, M - 1
         ILOHI(K,1) = IPTR
         ILOHI(K,2) = IPTR - 1
         IF (IPTR .LE. NOBS) THEN
   50       IF (X(IPTR) .LT. B(K+1)) THEN
               ILOHI(K,2) = ILOHI(K,2) + 1
               IPTR = IPTR + 1
               IF (IPTR .LE. NOBS) GO TO 50
            END IF
         END IF
   60 CONTINUE
```

```
      FACTOR = 2.0*ALPHA/M3
C                                     Initialize mesh node densities
      IF (INIT .EQ. 0) THEN
C                                     Via DESPT
         CALL D2SPT (M-2, B(2), 1, MOLD, BNDS, DENS, DENEST, WK, WK,
     &               WK)
         TEMP = 1.0/(M*M*H)
         DO 80  I=2, M - 1
            DENS(I) = AMAX1(TEMP,SQRT(DENEST(I-1,1)))
   80    CONTINUE
      ELSE
C                                     Via the initial estimates
         DO 90  I=2, M - 1
            DENS(I) = SQRT(DENS(I))
   90    CONTINUE
      END IF
      DENS(M) = 0.0
C                                     Maximize
      DO 140  ITER=1, MAXIT
C                                     Get Hessian - Lagrangian
         HESS(1,1) = 0.0
         HESS(1,2) = 0.0
         HESS(2,1) = 0.0
         BSMALL = 0.0
         SUM = 0.0
C                                     CK** are true estimates = FK**2
         DO 120  K=2, M - 1
            KM1 = K - 1
            KM2 = MAX0(1,K-2)
            KP1 = K + 1
            KP2 = MIN0(M,K+2)
            FK = DENS(K)
            FKM1 = DENS(KM1)
            FKM2 = DENS(KM2)
            CKM2 = FKM2**2
            CKM1 = FKM1**2
            CK = FK**2
            CKP1 = DENS(KP1)**2
            CKP2 = DENS(KP2)**2
            BK = B(K)
            BKM1 = B(KM1)
            SUM = SUM + CK
            IF (K .GE. 4) HESS(1,KM1) = 4.0*FK*FKM2*FACTOR
            SUM1 = 0.0D0
            SUM2 = 0.0D0
            SUM3 = 0.0D0
            DO 100  I=ILOHI(K,1), ILOHI(K,2)
               TEMP = (X(I)-BK)/H
               CONS = (1.0-TEMP)/(CK+(CKP1-CK)*TEMP)
               SUM1 = SUM1 - CONS
               SUM2 = SUM2 + CONS*CONS
  100       CONTINUE
            CKMCM1 = CK - CKM1
            DO 110  I=ILOHI(KM1,1), ILOHI(KM1,2)
               CONS = (X(I)-BKM1)/H
               TEMP = CKM1 + CKMCM1*CONS
               SUM1 = SUM1 - CONS/TEMP
               TEMP = TEMP*TEMP
               SUM2 = SUM2 + (CONS*CONS)/TEMP
               SUM3 = SUM3 + CONS*(1.0-CONS)/TEMP
  110       CONTINUE
            TEMP = FACTOR*(CKM2+CKP2-4.0*(CKM1+CKP1)+6.0*CK) + SUM1
            TEMP = 2.0*TEMP
            BSMALL = BSMALL + 2.0*CK*TEMP
```

```
            HESS(3,KM1) = TEMP + 4.0*FK*(3.0*FACTOR+SUM2)
            IF (K .NE. 2) HESS(2,KM1) = 4.0*FK*FKM1*(-4.0*FACTOR+SUM3)
            DENEST(KM1,1) = FK*TEMP
            DENEST(KM1,2) = -2.0*FK
  120    CONTINUE
         BSMALL = 1.0/H - SUM + BSMALL
C                                  Save portion of DENEST
         CALL SCOPY (M-2, DENEST(1,2), 1, DENEST(1,3), 1)
C                                  Finish with the hessian
         CALL SADD (M-2, -BSMALL/(2.0*SUM), HESS(3,1), LDHESS)
C                                  Fill out symmetric band structure
         CALL SCOPY (M-4, HESS(1,3), LDHESS, HESS(5,1), LDHESS)
         HESS(5,M-3) = 0.0
         HESS(5,M-2) = 0.0
         CALL SCOPY (M-3, HESS(2,2), LDHESS, HESS(4,1), LDHESS)
         HESS(4,M-2) = 0.0
C                                  Solve symmetric band linear system
         CALL L2TRB (M-2, HESS, LDHESS, 2, 2, HESS, LDHESS, IPVT, WK2)
         CALL LFSRB (M-2, HESS, LDHESS, 2, 2, IPVT, DENEST(1,2), 1,
     &               DENEST(1,2))
         IF (N1RCD(1) .NE. 0) GO TO 9000
C                                  Compute the constant
         CONS = SDOT(M-2,DENEST(1,3),1,DENEST(1,2),1)
         CONS = (1.0/H-SUM-SDOT(M-2,DENEST(1,3),1,DENEST(1,1),1))/CONS
C                                  Update the gradient
         CALL SAXPY (M-2, CONS, DENEST(1,2), 1, DENEST(1,1), 1)
C                                  Parameter updates
         CALL SAXPY (M-2, -1.0, DENEST(1,1), 1, DENS(2), 1)
C                                  Check the conversence criterion
         TEMP = SNRM2(M-2,DENS(2),1)
         IF (SNRM2(M-2,DENEST,1) .LT. EPS1*TEMP) GO TO 150
C                                  Ad hoc projection to plus quadrant
         TEMP = TEMP*1.0E-4/SQRT(M-2.0)
         DO 130  I=2, M - 1
            DENS(I) = AMAX1(TEMP,DENS(I))
  130    CONTINUE
  140 CONTINUE
      CALL E1STI (1, MAXIT)
      CALL E1MES (3, 1, 'The maximum number of iterations '//
     &            '(MAXIT=%(I1)) was exceeded.')
C                                  Replace DENS(*) with squares
  150 CALL SHPROD (M-2, DENS(2), 1, DENS(2), 1, DENS(2), 1)
      IF (M .NE. NODE) GO TO 20
C                                  Evaluate log likelihood and penalty
      SUM1 = 0.0
C                                  Penalty
      DO 160  K=1, M
         KM1 = MAX0(K-1,1)
         KP1 = MIN0(K+1,M)
         SUM1 = SUM1 + (DENS(KM1)-2.0*DENS(K)+DENS(KP1))**2
  160 CONTINUE
      STAT(2) = -0.5*FACTOR*SUM1
      SUM2 = 0.0
C                                  Log-likelihood
      DO 170  I=1, NOBS
         IF (X(I).GE.BNDS(1) .AND. X(I).LE.BNDS(2)) THEN
            CALL D2SPT (1, X(I), 1, NODE, BNDS, DENS, DENEST, WK, WK,
     &                  WK)
            SUM2 = SUM2 + ALOG(DENEST(1,1))
         END IF
  170 CONTINUE
      STAT(1) = SUM2
C                                  Evaluate M.L.P.E. mean and variance
```

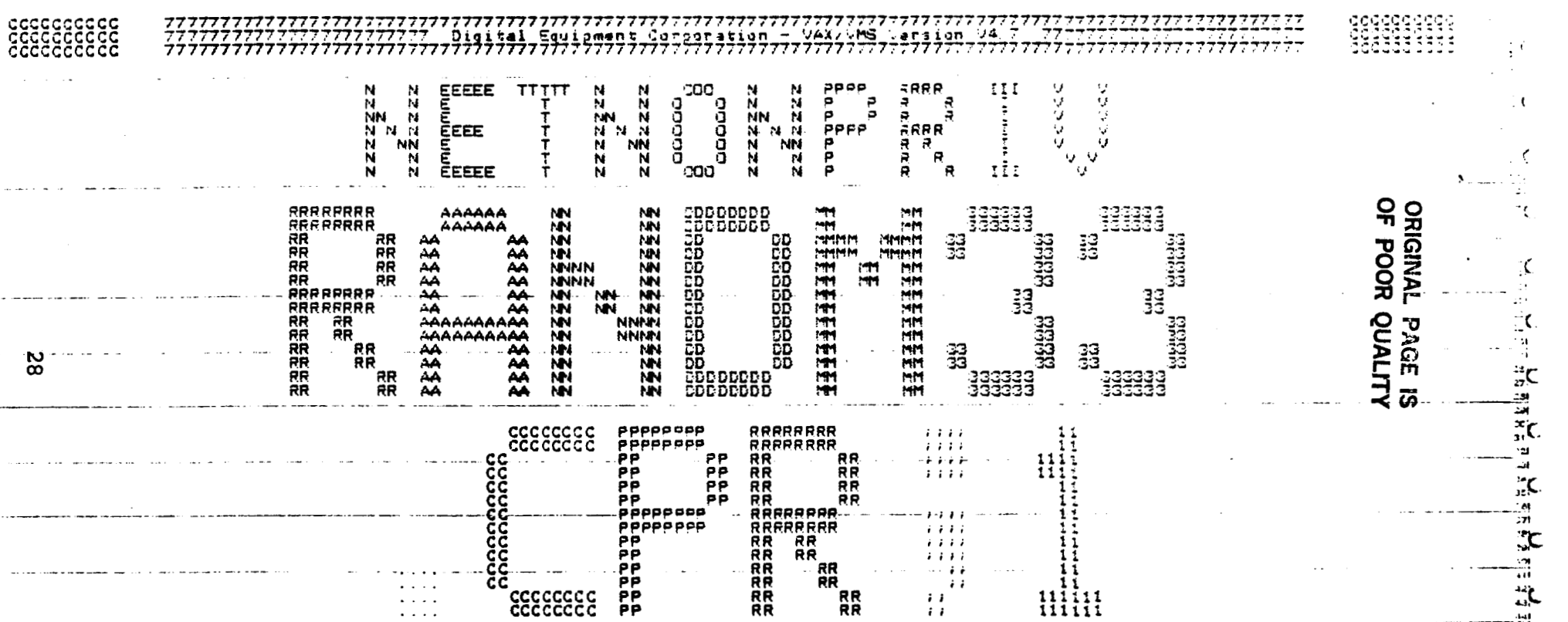
```
      SUM1 = 0.0
      SUM2 = 0.0
      DO 130  K=1, M - 1
         FK = DENS(K)
         FKP1 = DENS(K+1)
         BK = B(K)
         CONS = FK + FKP1
         TEMP = CONS + FKP1
         SUM1 = SUM1 + H2*TEMP/6.0 + 0.5*H*BK*CONS
         SUM2 = SUM2 + H3*(TEMP+FKP1)/12.0 + H2*BK*TEMP/3.0 +
     &          0.5*H*BK*BK*CONS
  130 CONTINUE
      STAT(3) = SUM1
      STAT(4) = SUM2 - SUM1*SUM1
C                                  Exit section
 9000 CALL E1POP ('D3SPL ')
      RETURN
      END
/EOF
```



File _DUA0:[]NORMAL.INP;41 (252,111,0), last revised on 22-DEC-1988 13:01, is a 1 block sequential file owned by UIC [DECNET]. The records are variable length with implied (CR) carriage control. The longest record is 39 bytes.

Job NORMAL (129) queued to TERM$LA120A on 22-DEC-1988 13:01 by user DECNET, UIC [DECNET], under account DECNET at priority 100, started on printer LTA4: on 22-DEC-1988 13:03 from queue TERM$LA120A.

	900.0000	45.0000			
	8.0000	0.3000			
	500.0000	25.0000			
	7.0000	0.7000			
	250.0000	12.5000			
	20.0000	1.0000			
	150.0000	7.5000			
	0.5000	0.0150			
	1500.0000	75.0000			
	20.0000	0.6000			
	950.0000	35.5000			
21	0	20.00	1.0E-05	30	
22					



ORIGINAL PAGE IS
OF POOR QUALITY

File _DBA0:[]RANDM33.CPR;1 (383,943,0), last revised on 23-NOV-1988 11:26, is a 33 block sequential file owned by UIC [11,11]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 120 bytes.

Job RANDM33 (689) queued to SYS$SPRT on 23-NOV-1988 11:27 by user NETNONPRIV, UIC [11,11], under account 20100ADD at priority 100, started on printer _TTF7: on 23-NOV-1988 11:27 from queue TTF7.

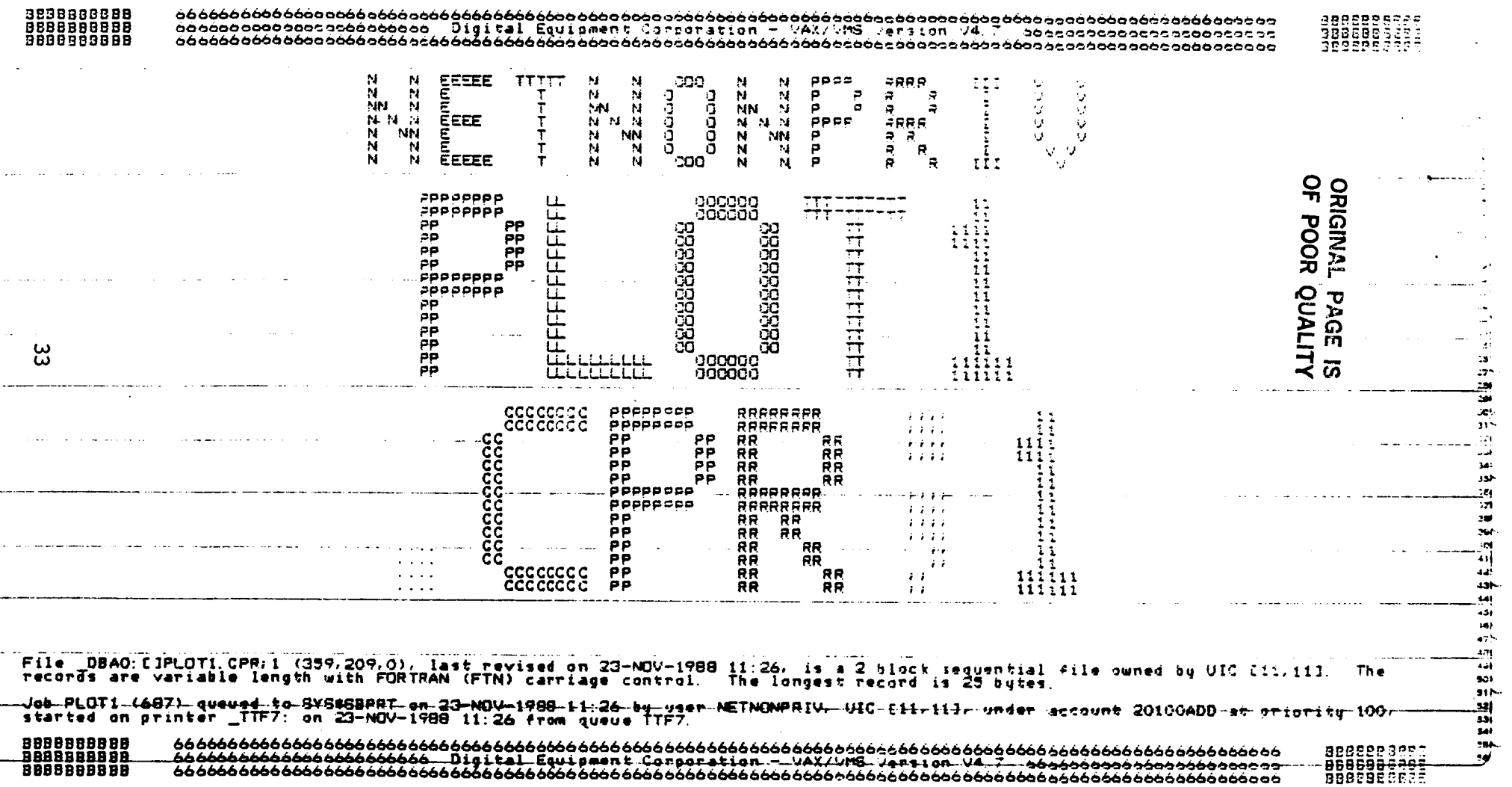
Digital Equipment Corporation - VAX/VMS Version V4.7

```
 1  40
  0.9000E+03   0.4500E+02
 LOGNORMAL SF
  0.8165E+03   0.9306E+03   0.8813E+03   0.8726E+03   0.9717E+03
  0.8942E+03   0.8963E+03   0.8590E+03   0.8760E+03   0.8505E+03
  0.9148E+03   0.9603E+03   0.9484E+03   0.8751E+03   0.9161E+03
  0.8845E+03   0.9782E+03   0.9993E+03   0.9872E+03   0.9657E+03
  0.1010E+04   0.8801E+03   0.9605E+03   0.8738E+03   0.9309E+03
  0.8546E+03   0.8574E+03   0.9446E+03   0.9070E+03   0.9341E+03
  0.8816E+03   0.9039E+03   0.8584E+03   0.8706E+03   0.8554E+03
  0.9232E+03   0.9712E+03   0.8188E+03   0.9656E+03   0.9446E+03
 1  40
  0.8000E+01   0.8000E+00
 LOGNORMAL XLNMF
  0.8570E+01   0.8531E+01   0.7652E+01   0.7503E+01   0.8299E+01
  0.7878E+01   0.7916E+01   0.7271E+01   0.7560E+01   0.7128E+01
  0.8245E+01   0.9083E+01   0.8859E+01   0.7946E+01   0.8066E+01
  0.7708E+01   0.7599E+01   0.7967E+01   0.9599E+01   0.7084E+01
  0.1005E+02   0.7632E+01   0.9088E+01   0.7524E+01   0.8591E+01
  0.7197E+01   0.7243E+01   0.8789E+01   0.8104E+01   0.8594E+01
  0.7658E+01   0.8049E+01   0.7261E+01   0.7469E+01   0.7209E+01
  0.9397E+01   0.9290E+01   0.8608E+01   0.9164E+01   0.8790E+01
 1  40
  0.5000E+03   0.2500E+02
 LOGNORMAL SO
  0.4536E+03   0.5170E+03   0.4896E+03   0.4848E+03   0.5398E+03
  0.4968E+03   0.4980E+03   0.4772E+03   0.4866E+03   0.4729E+03
  0.5082E+03   0.5335E+03   0.5269E+03   0.4862E+03   0.5090E+03
  0.4914E+03   0.4879E+03   0.4996E+03   0.5485E+03   0.4809E+03
  0.5613E+03   0.4890E+03   0.5336E+03   0.4855E+03   0.5188E+03
  0.4748E+03   0.4763E+03   0.5248E+03   0.5039E+03   0.5190E+03
  0.4898E+03   0.5022E+03   0.4769E+03   0.4837E+03   0.4752E+03
  0.5129E+03   0.5396E+03   0.4549E+03   0.5365E+03   0.5248E+03
 1  40
  0.7000E+01   0.7000E+00
 LOGNORMAL XLNMO
  0.5749E+01   0.7465E+01   0.6696E+01   0.6565E+01   0.8136E+01
  0.6894E+01   0.6926E+01   0.6362E+01   0.6615E+01   0.6237E+01
  0.7214E+01   0.7947E+01   0.7752E+01   0.6602E+01   0.7235E+01
  0.6744E+01   0.6649E+01   0.6971E+01   0.8399E+01   0.6461E+01
  0.8797E+01   0.6678E+01   0.7952E+01   0.6583E+01   0.7517E+01
  0.6297E+01   0.6338E+01   0.7690E+01   0.7091E+01   0.7521E+01
  0.6701E+01   0.7043E+01   0.6353E+01   0.6535E+01   0.6308E+01
  0.7347E+01   0.8129E+01   0.5782E+01   0.8036E+01   0.7691E+01
 1  40
  0.2500E+03   0.1250E+02
 LOGNORMAL S
  0.2268E+03   0.2585E+03   0.2448E+03   0.2424E+03   0.2699E+03
  0.2484E+03   0.2490E+03   0.2386E+03   0.2433E+03   0.2363E+03
  0.2541E+03   0.2667E+03   0.2634E+03   0.2431E+03   0.2545E+03
  0.2457E+03   0.2439E+03   0.2498E+03   0.2742E+03   0.2405E+03
  0.2807E+03   0.2445E+03   0.2668E+03   0.2427E+03   0.2594E+03
  0.2374E+03   0.2382E+03   0.2624E+03   0.2519E+03   0.2595E+03
  0.2449E+03   0.2511E+03   0.2385E+03   0.2418E+03   0.2376E+03
  0.2565E+03   0.2698E+03   0.2279E+03   0.2682E+03   0.2624E+03
 1  40
  0.2000E+02   0.1000E+01
 LOGNORMAL SIGO
 -0.1814E+02  -0.2068E+02  -0.1958E+02  -0.1939E+02  -0.2159E+02
 -0.1987E+02  -0.1992E+02  -0.1909E+02  -0.1947E+02  -0.1890E+02
```

```
-0.2033E+02 -0.2134E+02 -0.2107E+02 -0.1945E+02 -0.2036E+02
-0.1965E+02 -0.1952E+02 -0.1998E+02 -0.2194E+02 -0.1924E+02
-0.2245E+02 -0.1956E+02 -0.2135E+02 -0.1942E+02 -0.2079E+02
-0.1899E+02 -0.1905E+02 -0.2099E+02 -0.2015E+02 -0.2076E+02
-0.1959E+02 -0.2009E+02 -0.1908E+02 -0.1935E+02 -0.1901E+02
-0.2052E+02 -0.2158E+02 -0.1820E+02 -0.2146E+02 -0.2099E+02
1  40
  0.1500E+03  0.7500E+01
LOGNORMAL SIG
  0.1361E+03  0.1551E+03  0.1469E+03  0.1454E+03  0.1619E+03
  0.1490E+03  0.1494E+03  0.1432E+03  0.1460E+03  0.1418E+03
  0.1525E+03  0.1600E+03  0.1581E+03  0.1459E+03  0.1527E+03
  0.1474E+03  0.1464E+03  0.1489E+03  0.1645E+03  0.1443E+03
  0.1684E+03  0.1467E+03  0.1601E+03  0.1456E+03  0.1986E+03
  0.1424E+03  0.1429E+03  0.1574E+03  0.1512E+03  0.1557E+03
  0.1469E+03  0.1507E+03  0.1431E+03  0.1451E+03  0.1466E+03
  0.1539E+03  0.1619E+03  0.1365E+03  0.1609E+03  0.1574E+03
1  40
  0.5000E+00  0.1500E-01
NORMAL XXN
  0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
  0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
  0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
  0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
  0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
  0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
  0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
  0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
NORMAL XXM
  0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
  0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
  0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
  0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
  0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
  0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
  0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
  0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
NORMAL XXG
  0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
  0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
  0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
  0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
  0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
  0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
  0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
  0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
  0.1500E+04  0.7500E+02
NORMAL TF
  0.1176E+04  0.1416E+04  0.1552E+04  0.1492E+04  0.1506E+04
  0.1442E+04  0.1374E+04  0.1535E+04  0.1535E+04  0.1613E+04
  0.1478E+04  0.1504E+04  0.1572E+04  0.1364E+04  0.1379E+04
  0.1506E+04  0.1533E+04  0.1318E+04  0.1478E+04  0.1388E+04
  0.1484E+04  0.1537E+04  0.1517E+04  0.1611E+04  0.1577E+04
  0.1505E+04  0.1400E+04  0.1530E+04  0.1484E+04  0.1540E+04
  0.1601E+04  0.1554E+04  0.1452E+04  0.1375E+04  0.1547E+04
  0.1467E+04  0.1525E+04  0.1552E+04  0.1678E+04  0.1474E+04
1  40
```

```
 0.2000E+02   0.6000E+00
NORMAL TO
 0.1741E+02   0.1933E+02   0.2042E+02   0.1994E+02   0.2005E+02
 0.1953E+02   0.1900E+02   0.2028E+02   0.2028E+02   0.2091E+02
 0.1982E+02   0.2003E+02   0.2057E+02   0.1891E+02   0.1903E+02
 0.2004E+02   0.2027E+02   0.1959E+02   0.1992E+02   0.1910E+02
 0.1986E+02   0.2029E+02   0.2013E+02   0.2089E+02   0.2061E+02
 0.2004E+02   0.1920E+02   0.2024E+02   0.1987E+02   0.2032E+02
 0.2081E+02   0.1943E+02   0.1962E+02   0.1900E+02   0.2038E+02
 0.1973E+02   0.2020E+02   0.2042E+02   0.2142E+02   0.1979E+02
1   40
 0.8500E+03   0.2550E+02
NORMAL T
 0.7999E+03   0.8215E+03   0.8677E+03   0.8474E+03   0.8521E+03
 0.8302E+03   0.8073E+03   0.8219E+03   0.8619E+03   0.8885E+03
 0.8424E+03   0.8512E+03   0.8744E+03   0.8027E+03   0.8068E+03
 0.8519E+03   0.8613E+03   0.7882E+03   0.8424E+03   0.8118E+03
 0.8447E+03   0.8624E+03   0.8557E+03   0.8877E+03   0.8760E+03
 0.8517E+03   0.8161E+03   0.8601E+03   0.8446E+03   0.8635E+03
 0.8842E+03   0.8682E+03   0.8338E+03   0.8074E+03   0.8661E+03
 0.8387E+03   0.8586E+03   0.8677E+03   0.9104E+03   0.8412E+03
LOG OF CYCLES TO REACH MEAN FATIGUE STR =
50 MPA
 0.6026E+01   0.7799E+01   0.6977E+01   0.6849E+01   0.8486E+01
 0.7199E+01   0.7242E+01   0.6632E+01   0.6896E+01   0.6491E+01
 0.7529E+01   0.8289E+01   0.8074E+01   0.6904E+01   0.7564E+01
 0.7034E+01   0.6931E+01   0.7295E+01   0.8765E+01   0.6754E+01
 0.9179E+01   0.6961E+01   0.8292E+01   0.6851E+01   0.7828E+01
 0.6568E+01   0.6623E+01   0.8017E+01   0.7399E+01   0.7839E+01
 0.6975E+01   0.7339E+01   0.6634E+01   0.6833E+01   0.6574E+01
 0.7669E+01   0.8475E+01   0.6025E+01   0.8351E+01   0.8027E+01
SORTED LOG OF CYCLES
 0.6025E+01   0.6026E+01   0.6491E+01   0.6568E+01   0.6574E+01
 0.6623E+01   0.6632E+01   0.6634E+01   0.6754E+01   0.6833E+01
 0.6849E+01   0.6851E+01   0.6896E+01   0.6904E+01   0.6931E+01
 0.6961E+01   0.6975E+01   0.6977E+01   0.7034E+01   0.7199E+01
 0.7242E+01   0.7295E+01   0.7339E+01   0.7399E+01   0.7529E+01
 0.7564E+01   0.7669E+01   0.7799E+01   0.7828E+01   0.7839E+01
 0.8017E+01   0.8027E+01   0.8074E+01   0.8289E+01   0.8292E+01
 0.8351E+01   0.8475E+01   0.8486E+01   0.8765E+01   0.9179E+01
DESPL PARAMETERS
21     0     0.2000E+02   0.1000E-04    30
BNDS(1), BNDS(2)=   0.5723E+01    0.9638E+01
PDF OF LOG OF CURRENT CYCLES, LOG XNM, Y AXIS OF PDF PLOT
 0.0000E+00  -0.3553E-01  -0.9400E-01   0.1649E+00   0.2402E+00
 0.3114E+00   0.3720E+00   0.4176E+00   0.4468E+00   0.4591E+00
 0.4550E+00   0.4355E+00   0.4018E+00   0.3560E+00   0.3007E+00
 0.2394E+00   0.1765E+00   0.1164E+00   0.6368E-01   0.2296E-01
 0.0000E+00
OUTPUT STATISTICS
-0.4547E+02  -0.1308E+02   0.7603E+01   0.5956E+00
NUMBER OF MISSING VALUES
 0
LOG OF CURRENT CYCLES, LOG XNM
 0.5723E+01   0.9638E+01   0.5919E+01   0.6115E+01   0.6311E+01
 0.6506E+01   0.6702E+01   0.6898E+01   0.7094E+01   0.7289E+01
 0.7485E+01   0.7681E+01   0.7876E+01   0.8072E+01   0.8268E+01
 0.8464E+01   0.8659E+01   0.8855E+01   0.9051E+01   0.9246E+01
 0.9442E+01
ORDERED LOG OF CURRENT CYCLES, LOG, XNM,                    X AXIS PDF, CDF PLOT
```

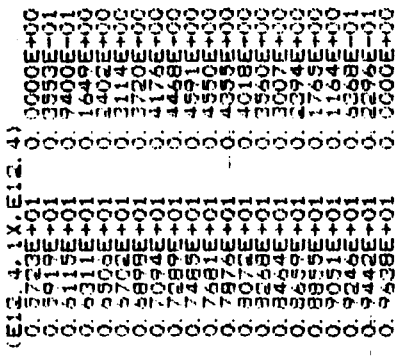
ORIGINAL PAGE IS
OF POOR QUALITY

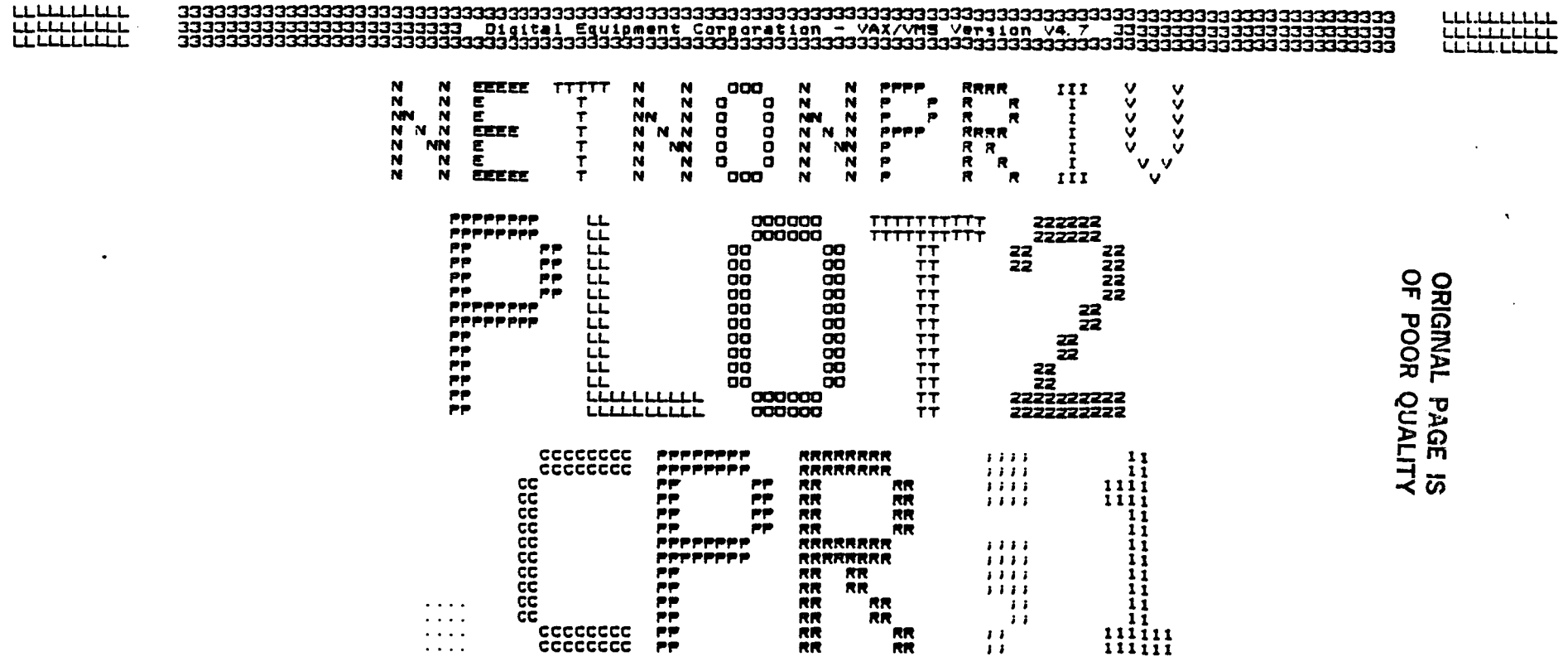


File _DBA0:[]PLOT1.CPR;1 (359,209,0), last revised on 23-NOV-1988 11:26, is a 2 block sequential file owned by UIC [11,11]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 25 bytes.

Job PLOT1 (687) queued to SYS$SPRT on 23-NOV-1988 11:26 by user NETNONPRIV, UIC [11,11], under account 2010GADD at priority 100, started on printer _TTF7: on 23-NOV-1988 11:26 from queue TTF7.

ORIGINAL PAGE IS
OF POOR QUALITY





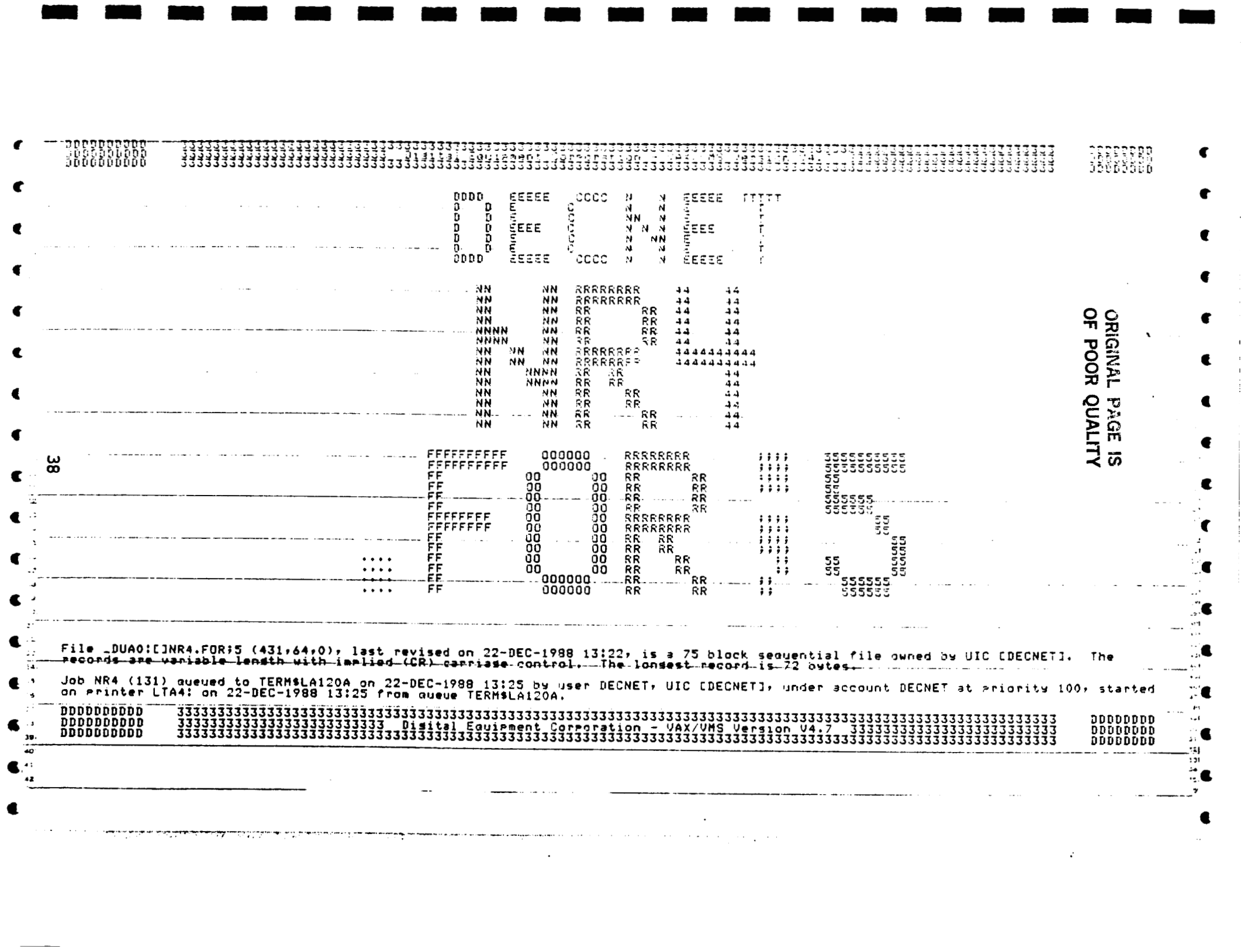
File _DBA0:[JPLOTZ.CPR;1 (363,178,0), last revised on 23-NOV-1988 11:26, is a 2 block sequential file owned by UIC [11,11]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 25 bytes.

Job PLOTZ (688) queued to SYS$SPRT on 23-NOV-1988 11:26 by user NETNONPRIV, UIC [11,11], under account 20100ADD at priority 100, started on printer _TTF6: on 23-NOV-1988 11:26 from queue TTF6.

```
(E12.4,1X,E12.4)
 0.5723E+01    0.0000E+00
 0.5917E+01    0.3477E-02
 0.6113E+01    0.1613E-01
 0.6311E+01    0.4147E-01
 0.6506E+01    0.8113E-01
 0.6702E+01    0.1391E+00
 0.6898E+01    0.2020E+00
 0.7094E+01    0.2793E+00
 0.7289E+01    0.3638E+00
 0.7485E+01    0.4525E+00
 0.7681E+01    0.5417E+00
 0.7876E+01    0.6291E+00
 0.8072E+01    0.7110E+00
 0.8268E+01    0.7852E+00
 0.8464E+01    0.8474E+00
 0.8659E+01    0.9023E+00
 0.8855E+01    0.9430E+00
 0.9051E+01    0.9716E+00
 0.9246E+01    0.9873E+00
 0.9442E+01    0.9978E+00
 0.9638E+01    0.1000E+01
```

8.0 APPENDIX C

RANDOM4 SAMPLE PROBLEM: SOURCE, INPUT AND OUTPUTFILES



File _DUA0:[]NR4.FOR;5 (431,64,0), last revised on 22-DEC-1988 13:22, is a 75 block sequential file owned by UIC [DECNET]. The records are variable length with implied (CR) carriage control. The longest record is 72 bytes.

Job NR4 (131) queued to TERM$LA120A on 22-DEC-1988 13:25 by user DECNET, UIC [DECNET], under account DECNET at priority 100, started on printer LTA4: on 22-DEC-1988 13:25 from queue TERM$LA120A.

DDDDDDDDDD 3333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333 DDDDDDDD
DDDDDDDDDD 33333333333333333333333 Digital Equipment Corporation - VAX/VMS Version V4.7 333333333333333333333333 DDDDDDDD
DDDDDDDDDD 3333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333 DDDDDDDD

```
JOB,JN=COMP*,US=USAQ530,RT=30,T=30,MFL=2000000.
ACCOUNT,UPW=LOLAB.
DELETE,PDN=NR4BLD,ID=SMBOYCE.
CFT77.
REWIND,DN=$BLD.
BUILD,I=0,OBL=0.
SAVE,DN=$NBL,PDN=NR4BLD,ID=SMBOYCE.
DELETE,PDN=NR4BLD,ID=SMBOYCE,ED=-1.
/EOF
C CHAMIS MICROMECHANICS CONSTITUTIVE EQUATIONS;
C RANDOMIZED AND APPLIED TO FATIGUE STRENGTH
      INTEGER NTOT,ISEED,M,INIT,NMISS,MAXIT,NODE
      REAL XM,XS,YM,YS,EPS,P,RWKSP(9999),ALPHA
      DIMENSION SF(10000),XLNMF(10000),SO(10000)
      DIMENSION XLNMO(10000),S(10000)
      DIMENSION SIGO(10000),SIG(10000)
      DIMENSION XXN(10000),XXM(10000),XXO(10000)
      DIMENSION XNM(10000),BNDSX(10000),DISTX(10000)
      DIMENSION TF(10000),TO(10000),T(10000)
      DIMENSION STAT(9999)
      DIMENSION BNDS(999),DENS(999)
      DIMENSION C(999),PP(999)
      DIMENSION SM(10)
      DIMENSION XP(1),CUM(1)
      DIMENSION AL(12)
C     COMMON/MEP1/KPRINT,TOL,MAXFN
C     KPRINT=1
C     TOL=1.0E-06
C     MAXFN=50
 1001 FORMAT(5E12.4)
 1002 FORMAT(E12.4,2X,I4)
 1003 FORMAT(I4,I4)
 1004 FORMAT(I4)
 1005 FORMAT(I12,I12)
 1006 FORMAT(2E12.4)
C LOGNORMAL ULTIMATE TENSILE STRENGTH, SF
      READ (5,1005) ISEED,NTOT
      WRITE(6,1005) ISEED,NTOT
C      XM=900.
C      XS=45.
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
      YS = SQRT(LOG(1.0+(XS/XM)**2.))
      YM = LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SF)
      WRITE(18,1001)(SF(I),I=1,NTOT)
      WRITE(6,2020)
 2020 FORMAT('LOGNORMAL SF')
      WRITE(6,1001)(SF(I),I=1,NTOT)
C LOGNORMAL LOG OF FINAL CYCLE, XLNMF
      WRITE(6,1005) ISEED,NTOT
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
C      XM  = 8.
C      XS  = 0.8
      YS = SQRT( LOG(1.0+(XS/XM)**2) )
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,XLNMF)
      WRITE(19,1001) (XLNMF(I),I=1,NTOT)
      WRITE(6,2021)
 2021 FORMAT(' LOGNORMAL XLNMF')
      WRITE(6,1001) (XLNMF(I),I=1,NTOT)
```

```
C LOGNORMAL FATIGUE STRENGTH AT REFERENCE CONDITIONS, SO
      WRITE(6,1005) ISEED,NTOT
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
C      XM  = 500.
C      XS  = .25
      YS = SQRT( LOG(1.0+(XS/XM)**2) )
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SO)
      WRITE(20,1001) (SO(I),I=1,NTOT)
      WRITE(6,2022)
 2022 FORMAT(' LOGNORMAL SO')
      WRITE( 6,1001) (SO(I),I=1,NTOT)
C LOGNORMAL LOG OF REFERENCE CYCLES, XLNMO
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
C      XM = 7.
C      XS = 0.7
      YS = SQRT(LOG(1.0+(XS/YM)**2))
      YM = LOG(XM) - 0.5*YS**2
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,XLNMO)
      WRITE (21,1001)(XLNMO(I),I=1,NTOT)
      WRITE (6,2023)
 2023 FORMAT (' LOGNORMAL XLNMO')
      WRITE (6,1001)(XLNMO(I),I=1,NTOT)
C LOGNORMAL FATIGUE STRENGTH AT CURRENT CONDITIONS, S
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
C      XM = 250.
C      XS = 12.5
      YS = SQRT(LOG(1.0+(XS/XM)**2.))
      YM = LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,S)
      WRITE(22,1001)(S(I),I=1,NTOT)
      WRITE (6,2024)
 2024 FORMAT(' LOGNORMAL S')
      WRITE(6,1001)(S(I),I=1,NTOT)
C DEFINE RANDOM STRESSES
C LOGNORMAL REFERENCE STRESS, SIGO
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
C      XM = 20.
C      XS = 1.
      YS=SQRT(LOG(1.0+(XS/XM)**2.))
      YM=LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SIGO)
C CHANGE SIGO TO NEGATIVE VALUES FOR COMPRESSIVE
C RESIDUAL STRESSES
      DO 401 I = 1,NTOT
      SIGO(I)=-SIGO(I)
  401 CONTINUE
      WRITE(26,1001)(SIGO(I),I=1,NTOT)
      WRITE(6,2036)
 2036 FORMAT(' LOGNORMAL SIGO')
      WRITE(6,1001)(SIGO(I),I=1,NTOT)
C LOGNORMAL CURRENT STRESS, SIG
      WRITE(6,1005)ISEED,NTOT
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
      READ(5,1006)XM,XS
      WRITE(6,1006)XM,XS
      XM= 150.
      XS= 7.5
      YS=SQRT(LOG(1.0+(XS/XM)**2.))
      YM=LOG(XM) - 0.5*YS**2.
      CALL RNSET(ISEED)
      CALL RNLNL(NTOT,YM,YS,SIG)
      WRITE(27,1001)(SIG(I),I=1,NTOT)
      WRITE(6,2037)
 2037 FORMAT(' LOGNORMAL SIG')
      WRITE(6,1001)(SIG(I),I=1,NTOT)
      WRITE(6,*) ' NORMAL EXPONENTS', XXN,XXM,XXQ
      YM = 0.5
      YS = 0.015
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)YM,YS
      WRITE(6,1006)YM,YS
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,XXN)
      DO 101 I=1,NTOT
      XXN(I)=YS*XXN(I)+YM
  101 CONTINUE
      WRITE(23,1001)(XXN(I),I=1,NTOT)
      WRITE(6,2025)
 2025 FORMAT(' NORMAL XXN')
      WRITE(6,1001)(XXN(I),I=1,NTOT)
      CALL RNSET(ISEED,NTOT
      CALL RNNOR(NTOT,XXM)
      DO 201 I=1,NTOT
      XXM(I)=YS*XXM(I)+YM
  201 CONTINUE
      WRITE(24,1001)(XXM(I),I=1,NTOT)
      WRITE(6,2026)
 2026 FORMAT(' NORMAL XXM')
      WRITE(6,1001)(XXM(I),I=1,NTOT)
      WRITE(6,1005)ISEED,NTOT
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,XXQ)
      DO 301 I=1,NTOT
      XXQ(I)=YS*XXQ(I)+YM
  301 CONTINUE
      WRITE(25,1001)(XXQ(I),I=1,NTOT)
      WRITE(6,2027)
 2027 FORMAT(' NORMAL XXQ')
      WRITE(6,1001)(XXQ(I),I=1,NTOT)
C     DEFINE DETERMINISTIC TEMPERATURES
      TF=1500.
      TO=20.
      I=850.
C     NORMAL TEMPERATURES,TF,TO,T
C     NORMAL FINAL (MELTING)TEMPERATURE,TF
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)YM,YS
      WRITE(6,1006)YM,YS
      YM=1500.
      YS=75.
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,TF)
      DO 405 I=1,NTOT
      TF(I)=YS*TF(I)+YM
  405 CONTINUE
      WRITE(6,2046)
```

```
 2046 FORMAT(' NORMAL TF')
      WRITE(6,1001)(TF(I),I=1,NTOT)
C NORMAL REFERENCE TEMPERATURE,TO
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)YM,YS
      WRITE(6,1006)YM,YS
C      YM=20.
C      YS=0.6
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,TO)
      DO 406 I=1,NTOT
      TO(I)=YS*TO(I)+YM
  406 CONTINUE
      WRITE(6,2047)
 2047 FORMAT(' NORMAL TO')
      WRITE(6,1001)(TO(I),I=1,NTOT)
C NORMAL CURRENT TEMPERATURE,T
      WRITE(6,1005)ISEED,NTOT
      READ(5,1006)YM,YS
      WRITE(6,1006)YM,YS
C      YM= 850.
C      YS= 42.5
      CALL RNSET(ISEED)
      CALL RNNOR(NTOT,T)
      DO 407 I=1,NTOT
      T(I)=YS*T(I)+YM
  407 CONTINUE
      WRITE(6,2048)
 2048 FORMAT(' NORMAL T')
      WRITE(6,1001)(T(I),I=1,NTOT)
C CALCULATE CURRENT LOG OF CYCLES, LOG XNM
      DO 102 I=1,NTOT
      RS=((SF(I)-SIG(I))/(SF(I)-SIGO(I)))**XXM(I)
C     WRITE(6,6876)RS
C6876 FORMAT(' RS=',E12.4)
C     TEMP=((TF-T)/(TF-TO))**XXN(I)
      TEMP=((TF(I)-T(I))/(TF(I)-TO(I)))**XXN(I)
C     WRITE(6,7876)TEMP
C7876 FORMAT(' TEMP=',E12.4)
C     SS=S(I)
C     SSO=SO(I)
C     XXXQ=XXQ(I)
C     WRITE(6,1001)SS
C     WRITE(6,1001)SSO
C     WRITE(6,1001)XXXQ
      XNM1=(S(I)/(SO(I)*TEMP*RS))**(1./XXQ(I))
C     WRITE(6,8876) XNM1
C8876 FORMAT(' XNM1=',E12.4)
      XNM2=(XLNMF(I)-((XLNMF(I)-XLNMO(I))*XNM1))
C     WRITE(6,8875)XNM2
C8875 FORMAT(' XNM2=',E12.4)
      IF(XNM2.LT.0.0)XNM2=0.0
      XNM(I)=XNM2
C     XNM(I)=10.**XNM2
  102 CONTINUE
      WRITE(28,1001)(XNM(I),I=1,NTOT)
      WRITE(6,2028)
 2028 FORMAT (' LOG OF CYCLES TO REACH MEAN FATIGUE STR =  ',/,
     1'250 MPA')
      WRITE(6,1001)(XNM(I),I=1,NTOT)
C SORT LOG OF CYCLES
      CALL SORT(XNM,NTOT)
      WRITE(29,1001)(XNM(I),I=1,NTOT)
      WRITE (6,2029)
```

```
 2029 FORMAT (' SORTED LOG OF CYCLES')
      WRITE (6,1001)(XNM(I),I=1,NTOT)
C CALCULATE PDF OF LOG OF CURRENT CYCLES, LOG XNM
C USING THE MAXIMUM ENTROPY METHOD
C CALCULATE SAMPLE MOMENTS,SM
C NUMBER OF MOMENTS, MMM
      MMM=4
      CALL SMOM(XNM,MMM,NTOT,SM)
      WRITE(30,1001)(SM(I),I=1,MMM)
      WRITE(6,2038)
 2038 FORMAT(' SAMPLE MOMENTS')
      WRITE(6,1001)(SM(I),I=1,MMM)
C OBTAIN MAXIMUM ENTROPY DISTRIBUTION
      KSTART=1
      KDATA=1
C CALCULATE MAX AND MIN ORDINATES FOR PDF (AND CDF)
      BNDS(1) = XNM(1) - 0.05*XNM(1)
      BNDS(2) = XNM(NTOT) + 0.05*XNM(NTOT)
      WRITE(66,8877) BNDS(1),BNDS(2)
      WRITE (6,8877) BNDS(1),BNDS(2)
 8877 FORMAT (' BNDS(1),BNDS(2)=',E12.4,1X,E12.4)
      CALL MEP1(MMM,SM,BNDS(1),BNDS(2),0,XP,KSTART,KDATA,AL,CUM)
      WRITE(31,1001)(AL(I),I=1,MMM+1)
      WRITE(6,2039)
 2039 FORMAT(' LAGRANGIAN MULTIPLIERS')
      WRITE(6,1001)(AL(I),I=1,MMM+1)
C CALCULATE VALUES OF ORDINATES FOR PDF (AND CDF)
CCC NUMBER OF ORDINATES USED
C CALCULATE WINDOW WIDTH, HH
      NODE=21
      HH=(BNDS(2)-BNDS(1))/(NODE-1)
CCC
CCC CALCULATE VALUES OF LOG OF CURRENT CYCLES AT WHICH PDF IS ESTIMATED;
CCC ALSO CALLED 'NODE' VALUES
CCC
      DO 6001,I=1,NODE-2
      BNDS(I+2)=BNDS(1) + (I*HH)
 6001 CONTINUE
      WRITE(6,983)
  983 FORMAT(' LOG OF CURRENT CYCLES, LOG XNM')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
C
CC REORDER BNDS FOR PLOTTING
CC
      SAVE1 = BNDS(2)
      SAVE2 = BNDS(NODE)
      BNDS(NODE)=BNDS(2)
      DO 6002, I=1, NODE-2
      BNDS(I+1)=BNDS(I+2)
 6002 CONTINUE
      BNDS(NODE-1)=SAVE2
      BNDS(NODE)=SAVE1
      WRITE(6,984)
  984 FORMAT(' ORDERED LOG OF CURRENT CYCLES, LOG,XNM,
     1X AXIS PDF, CDF PLOT')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
C CALCULATE VALUES OF THE PDF AT EACH ORDINATE
      DO 108 I=1,NODE
C FOR 4 MOMENTS THERE ARE 5 LAGRANGIAN MULTIPLIERS
      DENS(I)=EXP(AL(1)+AL(2)*BNDS(I)+AL(3)*BNDS(I)**2
     1+AL(4)*BNDS(I)**3+AL(5)*BNDS(I)**4)
  108 CONTINUE
C WRITE LOG OF CURRENT CYCLES AND PDF OF LOG OF CURRENT CYCLES,
```

```
C  LOG XNM TO PLOT FILES
      WRITE(34,990)
  990 FORMAT(' (E12.4,1X,E12.4)')
      WRITE(34,991)(BNDS(J),DENS(J),J=1,NODE)
  991 FORMAT(E12.4,1X,E12.4)
C
C CALCULATE CDF OF LOG OF CURRENT CYCLES
      IOPT=2
C
C      READ(3,1004)IOPT
      WRITE(6,992)
  992 FORMAT(' GCDF PARAMETERS')
      WRITE(6,1004)IOPT
      XO=BNDS(1)
      DO 6003,I=1,NODE
      P=GCDF(XO,IOPT,NODE,BNDS,DENS)
      BNDSX(I)=XO
      XO=XO+HH
      DISTX(I)=P
 6003 CONTINUE
      WRITE(6,994)
  994 FORMAT(' CDF OF LOG OF CURRENT CYCLES, LOG XNM,
     1Y AXIS OF PDF, CDF PLOT')
      WRITE(6,1001)(DISTX(I),I=1,NODE)
C
      WRITE(6,993)
  993 FORMAT(' ORDERED LOG OF CURRENT CYCLES, LOG XNM,
     1X AXIS OF PDF, CDF PLOT')
      WRITE(6,1001)(BNDS(I),I=1,NODE)
      WRITE(6,1001)(BNDSX(I),I=1,NODE)
C
C WRITE LOG OF CURRENT CYCLES AND CDF OF LOG OF CURRENT
C TO THE PLOT FILES
C
      WRITE(35,990)
      WRITE(35,991)(BNDS(J),DISTX(J),J=1,NODE)
      STOP
      END
C
      SUBROUTINE SORT (Y,N)
      DIMENSION Y(10000)
C Y IS THE ARRAY TO BE SORTED
C AT COMPLETION Y(1) IS SMALLEST VALUE
C AT COMPLETION Y(N) IS LARGEST VALUE
      N1 = N - 1
      DO 1 I=1,N1
      J = I + 1
      DO 2 K=J,N
      IF (Y(I).LT.Y(K))GO TO 2
      TEMP = Y(I)
      Y(I) = Y(K)
      Y(K) = TEMP
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE SMOM(X,M,NSAMP,SM)
C CALCULATES SAMPLE CENTRAL MOMENTS
C X(I) = SAMPLE VALUES, DIMENSION NSAMP
C M = NUMBER OF MOMENTS DESIRED
C NSAMP = SAMPLE SIZE
C SM = VALUE OF MOMENTS, DIMENSION M
      DIMENSION X(10000),SM(10)
```

```
C CALCULATE MEAN
      SUM=0.0
      DO 1 I=1,NSAMP
    1 SUM=SUM+X(I)
      SM(1)=SUM/FLOAT(NSAMP)
      IF(M.LT.2)RETURN
C CALCULATE VARIANCE
      SUM=0.0
      DO 2 I=1,NSAMP
    2 SUM=SUM+(X(I)-SM(1))**2
      SM(2)=SUM/(FLOAT(NSAMP-1))
      IF(M.LT.3)RETURN
C CALCULATE HIGHER MOMENTS
      DO 4 I=3,M
      SUM=0.0
      DO 3 J=1,NSAMP
    3 SUM=SUM+(X(J)-SM(1))**I
      SM(I)=SUM/FLOAT(NSAMP)
    4 CONTINUE
      RETURN
      END

      SUBROUTINE MEP1(N,CM,XMIN,XMAX,NXP,XP,KSTART,KDATA,AL,CUM)
C     IMPLICIT REAL*8 (A-H,O-Z)
C.... EXECUTIVE PROGRAM FOR USING MAXIMUM ENTROPY METHOD CONSTRAINED BY
C     MOMENTS TO GENERATE A DENSITY FUNCTION
C
      DIMENSION AL(*), CM(*), ETA(4), XP(*),CUM(*),CC(8),AL8(10)
      COMMON /FAIL/ NFAIL
      COMMON/HELP/S(101),XX(16,101),C(8),M
C.... ABOVE LINE DIFFERENT FROM TEXT
C     COMMON/MEP1/KPRINT,TOL,MAXFN
      DATA KPRINT,TOL,MAXFN/1,1.E-6,70/
      IF (N.EQ.1) KSTART=2
C
C     WRITE THE INPUT DATA
C
      IF (KDATA.EQ.0) GO TO 1
      WRITE (6,24)
      WRITE (6,25) KDATA
      WRITE (6,26) KPRINT
      WRITE (6,28) N
      WRITE (6,29) XMAX
      WRITE (6,30) XMIN
      WRITE (6,31) (CM(I),I=1,4)
      IF (N.GT.4) WRITE (6,21) (CM(I),I=5,N)
      IF(ABS(CM(1)).LT.1.E-4)GO TO 48
      WRITE (6,32) TOL
      WRITE (6,33) NXP
    1 CONTINUE
      NFAIL=0
      M=31
      X2MIN=0.0
      X2MAX=1.
C     SAVE CM
      DO 100 I=1,N
  100 CC(I)=CM(I)
C
C     CALCULATE THE MOMENTS AT THE MODIFIED LIMITS
C
      CALL TRN1 (XMAX,XMIN,CC,X2MAX,X2MIN,N)
C
C     CALCULATE THE MOMENTS ABOUT THE ORIGIN FOR THE MODIFIED LIMITS ,
C     STORE THEM IN COMMON IN C
```

```
C
      CALL CONVER(CC,N)
C
C     GENERATE THE SIMPSON MULTIPLIERS AND STORE THEM IN HELP COMMON
C
      CALL SIMSON
C
C     GENERATE THE X,S POWER FOR SUBROUTINE FUNCT, STORE THEM IN HELP
C     COMMON ARRAY
C
      CALL MULTI(X2MAX,X2MIN,N)
C
C     DEFINE THE INPUT DATA FOR SUBROUTINE MPOPT
C
      ETA(1)=1.D-12
      ETA(2)=TOL
      ETA(3)=1.D-24
      ETA(4)=1.D-24
      MODE=1
      UMIN=0.0
C
C     WRITE THE INTERMEDIATE RESULTS YOU HAVE OBTAINED SO FAR
C
      IF (KPRINT.EQ.0) GO TO 2
      WRITE (6,34)
      WRITE (6,35) M
      WRITE (6,36) X2MAX,X2MIN
      WRITE (6,37) (CC(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (CC(I),I=5,N)
      WRITE (6,38) (C(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (C(I),I=5,N)
      WRITE (6,39) (ETA(I),I=1,4)
    2 CONTINUE
C
C     FIND A STARTING POINT FOR SUBROUTINE MPOPT TO START THE OPTIMIZAT-
C     ION ALGORITHM
C
      IF(KSTART.EQ.0)GO TO 16
      IF(KSTART.EQ.4)WRITE(6,44)
      CALL START (X2MAX,X2MIN,AL,KSTART,CC,N,KPRINT,UMIN,MODE,MAXFN,ETA)
      IF (NFAIL.EQ.1) GO TO 9
C
C     PRINT THE STARTING VALUES
C
      IF (KPRINT.EQ.0) GO TO 7
      GO TO (3,4,5,6), KSTART
    3 WRITE (6,40)
      WRITE (6,41) (AL(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (AL(I),I=5,N)
      GO TO 7
    4 WRITE (6,42)
      WRITE (6,41) (AL(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (AL(I),I=5,N)
      GO TO 7
    5 WRITE (6,43)
      WRITE (6,41) (AL(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (AL(I),I=5,N)
      GO TO 7
    6 WRITE (6,41) (AL(I),I=1,4)
      IF (N.GT.4) WRITE (6,22) (AL(I),I=5,N)
      GO TO 7
   16 CONTINUE
      RANGE=XMAX-XMIN
C.... CHANGE STARTING VALUES TO 0-1 DOMAIN FOR KSTART=0
```

```
C.... THIS ALGORITHM IS SIMILAR TO IRN2 BUT APPEARS TO GIVE BETTER
C     NUMERICAL RESULTS
      NPL=N+1
      IF(ABS(XMIN).LT.1.E-10)GO TO 19
      DO 17 I=2,NPL
      ALS(I)=0.0
      I1=I-1
      DO 18 J=I1,N
      ALS(I)=ALS(I)+FACTO(J)*XMIN**(J-I1)*RANGE**I1*AL(J+1)/FACTO(I1)
     1/FACTO(J-I1)
18    CONTINUE
17    CONTINUE
      GO TO 50
19    DO 20 I=2,NPL
20    ALS(I)=RANGE**(I-1)*AL(I)
C.... PUT AL(I) IN PROPER LOCATIONS
50    DO 51 I=1,N
51    AL(I)=ALS(I+1)

7     CONTINUE
      NFAIL=0
      IF (KPRINT.EQ.0) GO TO 8
      WRITE (6,45)
8     CONTINUE
      AL(N+1)=2.
      AL(N+2)=0.0
      CALL MPOPT (AL,N,ETA,UMIN,MAXFN,MODE,KPRINT)
      IF (NFAIL.EQ.0) GO TO 10
      IF (KSTART.EQ.4) GO TO 9
C
C     THE PROGRAM HAS FAILED SO FAR , TRY ANOTHER STARTING POINT AND TRY
C     AGAIN
C
      KSTART=KSTART+1
      IF (KSTART.EQ.4.AND.N.LE.2) GO TO 9
      GO TO 2
9     CONTINUE
      WRITE (6,46)
      CALL EXIT
10    CONTINUE
C
C     CALCULATE THE ZEROTH LAGRANGIAN MULTIPLIER
C
      SUM=0.0
      DO 12 I=1,M
      SZ=0.0
      DO 11 K=1,N
      SZ=SZ+AL(K)*XX(K,I)
11    CONTINUE
      SUM=SUM+S(I)*EXP(SZ)
12    CONTINUE
      NPL=N+1
      DO 13 I=1,N
      K=N+2-I
      AL(K)=AL(K-1)
13    CONTINUE
      DELTA=(X2MAX-X2MIN)/FLOAT(M-1)
      AL(1)=-ALOG(SUM*DELTA/3.)
      WRITE(6,101)UMIN
101   FORMAT(26H SUM OF RESIDUALS SQUARED=,E12.5)
      IF (KPRINT.EQ.0) GO TO 14
      WRITE (6,47) (AL(I),I=1,NPL)
14    CONTINUE
C.... RESET KSTART TO ZERO
```

```
      KSTART=0
C
C     CALCULATE THE LAGRANGIAN MULTIPLIERS FOR THE ORIGINAL LIMITS
C
      CALL TRN2 (XMAX,XMIN,AL,X2MAX,X2MIN,N)
C
C     CALCULATE THE CUMULATIVE DISTRIBUTION FUNCTION VALUE AT THE GIVEN
C     POINT
      IF(NXP.EQ.0)RETURN
      DO 15 I=1,NXP
      CUM(I)=CDF(XMIN,XMAX,XP(I),AL,NPL)
   15 CONTINUE
      RETURN
C
   21 FORMAT (57X,4E18.9,//)
   22 FORMAT (57X,4E18.9,//)
   24 FORMAT (1H1,///,20X,'INPUT DATA FOR SUBROUTINE   MEP1',/,20X,33('-'
     1),//)
   25 FORMAT (' INPUT DATA IS PRINTED OUT FOR KDATA =1 ONLY . . .KDATA =
     1',I18,/)
   26 FORMAT (' INTERMEDIATE OUTPUT EVERY KPRINT(TH) CYCLE  . . KPRINT =
     1',I18,/)
   28 FORMAT (' NUMBER OF KNOWN FIRST MOMENTS . . . . . . . . . . . N=
     1',I18,/)
   29 FORMAT (' HIGHER LIMIT  . . . . . . . . . . . . . . . . . XMAX =
     1',E18.9,/)
   30 FORMAT (' LOWER LIMIT . . . . . . . . . . . . . . . . . . XMIN =
     1',E18.9,/)
   31 FORMAT (' FIRST MOMENTS . . . . . . . . . . . . . . . . . CC(I) =
     1',4E18.9,/)
   32 FORMAT (' THE ALLOWED TOLERANCE IN LAGRANGIAN EQUATIONS . . .TOL =
     1',E18.9,/)
   33 FORMAT (' THE CUMULATIVE DISTRIBUTION REQUIRED AT NXP POINTS.NXP =
     1',I18,/)
   34 FORMAT (1H1,///,20X,'INTERMEDIATE RESULTS FOR SUBROUTINE   MEP1',/,2
     10X,41('-'),///)
   35 FORMAT (' NUMBER OF INTEGRATION STATION . . . . . . . . . .  M =
     1',I18,/)
   36 FORMAT (' MODIFIED MAXIMUM AND MINIMUM LIMITS . . . X2MAX , X2MIN =
     1',2E18.9,/)
   37 FORMAT (' MODIFIED MOMENTS ABOUT THE EXPECTED VALUE . . . .CC(I) =
     1',4E18.9,/)
   38 FORMAT (' MODIFIED MOMENTS ABOUT THE ORIGIN . . . . . . . . C(I) =
     1',4E18.9,/)
   39 FORMAT (' SUBROUTINE MPOPT TOLERANCES . . . . . . . . . . ETA(I) =
     1',4E18.9,/)
   40 FORMAT (///,' NORMAL ASSUMPTION STARTING METHOD'/34('-'),/)
   41 FORMAT (' STARTING VALUES   . . . . . . . . . . . . . . .  AL(I) =
     1',4E18.9,/)
   42 FORMAT (///,' UNIFORM ASSUMPTION STARTING METHOD'/35('-'),/)
   43 FORMAT (///,' N POINTS STARTING METHOD'/25('-'),/)
   44 FORMAT (///,' STEP BY STEP STARTING METHOD'/29('-'),/)
   45 FORMAT (///,'  CYC NUMF   NORMGRAD          TOTAL',24X,'VARIABLES',40
     1X,'RESIDUALS',/,'  NO.',10X,'RESIDUALS       X(1)          X(2)
     2        X(3)          X(4)          R(1)          R(2)          R(3)          R
     3(4)',//)
   46 FORMAT (' THE PROGRAM HAS FAILED')
   47 FORMAT (' THE MODIFIED LAGRANGIAN MULTIPLIERS ARE . . . . . . . .
     1',4E18.9/57X,4E18.9)
   48 WRITE(6,49)
   49 FORMAT(53H WARNING - MEAN IS NEARLY ZERO AND MEP1 WILL NOT WORK/12
     1H TRANSFORM X)
      END
```

```
      SUBROUTINE MPOPT (X,NDIM,ETA,EST,MAX,MODE,IPRINT)
C     IMPLICIT REAL*8 (A-H,O-Z)
C     REAL* KTB,IPRINT
      COMMON /FAIL/ NFAIL
      DIMENSION X(*), X1(10), X2(10), G1(10), G2(10), ALFA(10), H(10), P
     1(10,10), Y(10), PY(10), PE(10), ETA(*), BIGV(10), RR(8)
      EXTERNAL FUNCT
      KRST=0
      KTB=0
      IFLAG=0
      M=0
      N2=NDIM+1
      N1=NDIM+2
      NUMF=0
      IER=0
      DO 1 I=1,N1
      X1(I)=X(I)
1     CONTINUE
      CALL FUNCT (NDIM,X1,F1,G1,RR)
      NUMF=NUMF+1
      DO 2 I=1,NDIM
      X2(I)=X1(I)
      G2(I)=G1(I)
      H(I)=-G1(I)
2     CONTINUE
      F2=F1
      X2(N2)=X1(N2)
      X2(N1)=X1(N1)
3     CONTINUE
      KOUNT=0
      EPS=ETA(4)
49    CALL LINES (FUNCT,X2,H,RO,NDIM,F2,G2,NUMF,IER,EPS,EST,RR)
      IF (NFAIL.EQ.1) RETURN
      IF (IER.NE.0) GO TO 30
      DO 4 I=1,N1
      BIGV(I)=X2(I)
      ALFA(I)=X2(I)
4     CONTINUE
      RO=-RO
      GG=0.
      DO 5 I=1,NDIM
      GG=GG+G2(I)*G2(I)
5     CONTINUE
      GG=SQRT(GG)
      IF (IPRINT.EQ.0) GO TO 7
      IF (MOD(KTB,IPRINT).NE.0) GO TO 6
      CALL OUTP (X2,F2,M,NDIM,GG,NUMF,RR)
6     KTB=KTB+1
7     DO 9 I=1,N1
      DO 8 J=1,N1
      P(I,J)=0.
8     CONTINUE
      P(I,I)=1.
9     CONTINUE
10    CONTINUE
      PRINT*,KOUNT
      KOUNT=0
      KOUNT=KOUNT+1
11    DO 12 I=1,NDIM
      Y(I)=G2(I)
C     PRINT*,'GOT BY A1'
12    CONTINUE
      Y(N2)=F2
```

```
      Y(N1)=ETA(1)
      V=0.
      DO 13 I=1,NDIM
      V=V+X2(I)*G2(I)
C     PRINT*,'GOT BY A2'
13    CONTINUE
      YA=0.
      DO 14 I=1,N1
      YA=YA+Y(I)*ALFA(I)
C     PRINT*,'GOT BY A3'
14    CONTINUE
      VYA=V-YA
      BIGV(KOUNT)=V
      DO 15 I=1,N1
      PY(I)=0.
      PE(I)=P(I,KOUNT)
      DO 15 J=1,N1
15    PY(I)=PY(I)+P(J,I)*Y(J)
C     PRINT*,'GOT BY A4'
      EPY=PY(KOUNT)
      IF (ABS(EPY).LT.ETA(3)) GO TO 31
      PY(KOUNT)=PY(KOUNT)-1.
      DO 16 I=1,N1
      DO 16 J=1,N1
16    P(I,J)=P(I,J)-PE(I)*PY(J)/EPY
C     PRINT*,'GOT BY A5'
      DO 17 I=1,N1
      ALFA(I)=0.
      DO 17 J=1,N1
17    ALFA(I)=ALFA(I)+P(I,J)*BIGV(J)
C     PRINT*,'GOT BY A6'
      DEL=0.
      DO 18 I=1,NDIM
      DEL=DEL+G2(I)*(X2(I)-ALFA(I))
C     PRINT*,'GOT BY A7'
18    CONTINUE
      IF (ABS(DEL).GT.ETA(4)) GO TO 19
      IF (IFLAG.EQ.1) RETURN
      IFLAG=1
      GO TO 31
19    IFLAG=0
      DO 20 I=1,N1
      H(I)=X2(I)-ALFA(I)
      IF (DEL.GT.0) H(I)=-H(I)
C     PRINT*,'GOT BY A8'
20    CONTINUE
      DO 21 I=1,NDIM
      X1(I)=X2(I)
      G1(I)=G2(I)
C     PRINT*,'GOT BY A9'
21    CONTINUE
      F1=F2
      X1(N2)=X2(N2)
      X1(N1)=X2(N1)
      X2(N2)=ALFA(N2)
      X2(N1)=ALFA(N1)
C     PRINT*,'GOT BY A10'
      CALL LINES (FUNCT,X2,H,RO,NDIM,F2,G2,NUMF,IER,EPS,EST,RR)
C     PRINT*,'GOT BY A11'
      IF (NFAIL.EQ.1) RETURN
C     PRINT*,'GOT BY A12'
      IF (IER.NE.0) GO TO 30
C     PRINT*,'GOT BY A13'
      IF (DEL.GT.0) RO=-RO
```

```
      PRINT*,'GOT BY A14'
      GG=0.
      DO 22 I=1,NDIM
      GG=GG+G2(I)*G2(I)
C     PRINT*,'GOT BY A15'
22    CONTINUE
      GG=SQRT(GG)
      KOUNT=KOUNT+1
      M=M+1
      IF (IPRINT.EQ.0) GO TO 23
      IF (MOD(KTB,IPRINT).NE.0) GO TO 23
C     PRINT*,'GOT BY G'
      CALL OUTP (X2,F2,M,NDIM,GG,NUMF,RR)
C     PRINT*,'GOT BY H'
23    CONTINUE
      KTB=KTB+1
      IF (MODE.EQ.2) GO TO 25
C     PRINT*,'GOT BY HA'
      IF (M.GT.MAX) GO TO 30
C     PRINT*,'GOT BY HB'
      NSOL=0
      DO 24 I=1,NDIM
      IF (ABS(RR(I)).GT.ETA(2)) NSOL=1
C     PRINT*,'GOT BY HC'
24    CONTINUE
C     PRINT*,'GOT BY HD'
      IF (NSOL.EQ.0) GO TO 26
C     PRINT*,'GOT BY HE'
      GO TO 29
C     PRINT*,'GOT BY HF'
25    IF ((GG.LT.ETA(1)).OR.(M.GT.MAX)) GO TO 26
C     PRINT*,'GOT BY HG'
      GO TO 29
C     PRINT*,'GOT BY HH'
26    CONTINUE
C     PRINT*,'GOT BY HI'
      IF (IPRINT.EQ.0) GO TO 27
C     PRINT*,'GOT BY HJ'
      WRITE (6,33)
C     PRINT*,'GOT BY I'
      CALL OUTP (X2,F2,M,NDIM,GG,NUMF,RR)
C     PRINT*,'GOT BY J'
27    DO 28 I=1,NDIM
      X(I)=X2(I)
28    CONTINUE
      EST=F2
      NFAIL=0
      RETURN
29    CONTINUE
C     PRINT*,KOUNT
C     PRINT*,'GOT BY JA'
      IF (KOUNT.LE.N1) GO TO 11
C     PRINT*,'GOT BY JB'
      GO TO 10
C     PRINT*,'GOT BY JC'
30    PRINT 34, IER
      NFAIL=1
      RETURN
31    KRST=KRST+1
      IF (KRST.GT.10) NFAIL=1
      IF (NFAIL.EQ.1) RETURN
      DO 32 I=1,NDIM
      X1(I)=X2(I)
      G1(I)=G2(I)
```

```
      H(I)=-01*I
   32 CONTINUE
      F1=F2
      X1(N2)=X(N2)
      X1(N1)=X(N1)
      X2(N2)=X(N2)
      X2(N1)=X(N1)
      GO TO 3
C
C
   33 FORMAT (' SOLUTION FOUND')
   34 FORMAT (///,1X,'  THE PROGRAM HAS FAILED---IER = ',I2)
      END

      SUBROUTINE OUTP (XNEW,FQ,KOUNT,N1,GG,NUMF,R)
C     IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XNEW(*), R(*)
      WRITE (6,6) KOUNT,NUMF,GG,FQ,(XNEW(I),I=1,4),(R(I),I=1,4)
      IF (N1.LT.4) RETURN
      NN=N1-3
      GO TO (1,2,3,4,5), NN
    1 RETURN
    2 WRITE (6,7) XNEW(5),R(5)
      RETURN
    3 WRITE (6,8) (XNEW(I),I=5,6),(R(I),I=5,6)
      RETURN
    4 WRITE (6,9) (XNEW(I),I=5,7),(R(I),I=5,7)
      RETURN
    5 WRITE (6,10) (XNEW(I),I=5,8),(R(I),I=5,8)
      RETURN
C
C
    6 FORMAT (1X,I3,I4,6E14.5,4E10.3)
    7 FORMAT (36X,E14.5,42X,E10.3)
    8 FORMAT (36X,2E14.5,28X,2E10.3)
    9 FORMAT (36X,3E14.5,14X,3E10.3)
   10 FORMAT (36X,4E14.5,4E10.3)
      END

      SUBROUTINE LINES (FUNCT,X,H,AMBDA,N,F,G,NUMF,IER,EPS,EST,RR)
C     IMPLICIT REAL*8 (A-H,O-Z)
C     REAL*8 Z,DX, DY
      COMMON /FAIL/ NFAIL
      DIMENSION H(*), X(*), G(*), RR(*)
      IER=0
      DY=0.
      HNRM=0.
      GNRM=0.
      DO 1 J=1,N
      HNRM=HNRM+ABS(H(J))
      GNRM=GNRM+ABS(G(J))
      DY=DY+H(J)*G(J)
C     PRINT*,'GOT BY B1'
    1 CONTINUE
      IF (DY) 2,31,31
C     PRINT*,'GOT BY B2'
    2 IF (HNRM/GNRM-EPS) 31,31,3
C     PRINT*,'GOT BY B3'
    3 FY=F
      ALFA=2.*(EST-F)/DY
      IF (X(N+1).GT.0.) ALFA=X(N+1)*ALFA/2.
C     PRINT*,'GOT BY B4'
```

```
      AMBDA=1.
      IF (ALFA) 5,6,4
C     PRINT*,'GOT BY B5'
4     IF (ALFA-AMBDA) 5,6,6
C     PRINT*,'GOT BY B6'
5     AMBDA=ALFA
6     ALFA=0.
      DO 8 I=1,N
      X(I)=X(I)+AMBDA*H(I)
C     PRINT*,'GOT BY B7'
8     CONTINUE
7     FX=FY
      DX=DY
C     PRINT*,'GOT BY B8'
      CALL FUNCT (N,X,F,G,RR)
C     PRINT*,'GOT BY B9'
      IF (NFAIL.EQ.1) RETURN
C     PRINT*,'GOT BY B10'
      NUMF=NUMF+1
      IF (F.LT.FX) RETURN
C     PRINT*,'GOT BY B11'
      FY=F
      DY=0.
      DO 9 I=1,N
      DY=DY+G(I)*H(I)
C     PRINT*,'GOT BY B12'
9     CONTINUE
C     PRINT*,'GOT BY B13'
      IF (DY) 10,30,13
C     PRINT*,'GOT BY B14'
10    IF (FY-FX) 11,13,13
C     PRINT*,'GOT BY B15'
11    AMBDA=AMBDA+ALFA
      ALFA=AMBDA
      IF (HNRM*AMBDA-1.E10) 7,7,12
C     PRINT*,'GOT BY B16'
12    IER=2
      GO TO 31
C     PRINT*,'GOT BY B17'
13    T=0.
14    IF (AMBDA) 15,30,15
C     PRINT*,'GOT BY B18'
15    Z=3.*(FX-FY)/AMBDA+DX+DY
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
      IF (DALFA) 31,16,16
C     PRINT*,'GOT BY B19'
16    W=ALFA*SQRT(DALFA)
      ALFA=DY-DX+W+W
      IF (ALFA) 17,18,17
C     PRINT*,'GOT BY B20'
17    ALFA=(DY-Z+W)/ALFA
      GO TO 19
C     PRINT*,'GOT BY B21'
18    ALFA=(Z+DY-W)/(Z+DX+Z+DY)
19    ALFA=ALFA*AMBDA
      DO 20 I=1,N
      X(I)=X(I)+(T-ALFA)*H(I)
20    CONTINUE
      CALL FUNCT (N,X,F,G,RR)
      IF (NFAIL.EQ.1) RETURN
      NUMF=NUMF+1
      IF (F.LT.FX) GO TO 30
```

```
      IF (F-FX) 21,21,22
   21 IF (F-FY) 30,30,22
   22 DALFA=0.
      DO 23 I=1,N
      DALFA=DALFA+G(I)*H(I)
   23 CONTINUE
      IF (DALFA) 24,27,27
   24 IF (F-FX) 26,25,27
   25 IF (DX-DALFA) 26,30,26
   26 FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 14
   27 IF (FY-F) 29,28,29
   28 IF (DY-DALFA) 29,30,29
   29 FY=F
      DY=DALFA
      AMBDA=AMBDA-ALFA
      GO TO 13
   30 AMBDA=AMBDA-ALFA
      RETURN
   31 CONTINUE
      IF (DY.GE.0.) IER=-2
      IF (GNRM.LE.1.E-10) GO TO 32
      IF (HNRM/GNRM.LE.EPS) IER=-3
   32 CONTINUE
      IF (DALFA.LT.0.) IER=-1
      NFAIL=1
      WRITE(6,33)
   33 FORMAT(///,1X,' THE PROGRAM HAS FAILED')
      RETURN
C
C
      END
      SUBROUTINE FUNCT (N,AL,U,GRAD,RR)
C     IMPLICIT REAL*8 (A-H,O-Z)
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE OPTIMIZATION AND THE
C     GRADIENT AT ANY GIVEN POINT FOR SUBROUTINE POPT
C
      DIMENSION AL(*), GRAD(*), SUM(17), RR(*)
      COMMON /FAIL/ NFAIL
      COMMON/HELP/S(101),XX(16,101),C(8),M
C.... ABOVE LINE CHANGED FROM TEXT

      N21=2*N+1
      ZERO=0.0
      DO 1 I=1,N21
      SUM(I)=0.0
C     PRINT*,'GOT BY C1'
    1 CONTINUE
    2 CONTINUE
      DO 4 I=1,M
      SZ=ZERO
      DO 3 K=1,N
      SZ=SZ+AL(K)*XX(K,I)
C     PRINT*,'GOT BY C2'
    3 CONTINUE
      IF (SZ.GT.74.) GO TO 9
C     PRINT*,'GOT BY C3'
      SS=EXP(SZ)*S(I)
      SUM(1)=SUM(1)+SS
```

```
      DO 4 J=2,N21
      SUM(J)=SUM(J)+XX(J-1,I)*SS
C     PRINT*,'GOT BY C4'
4     CONTINUE
      DO 5 I=2,N21
      SUM(I)=SUM(I)/SUM(1)
C     PRINT*,'GOT BY C5'
5     CONTINUE
      U=0.0
      DO 6 I=1,N
      RR(I)=(SUM(I+1)-C(I))/C(I)
      U=U+RR(I)*RR(I)
C     PRINT*,'GOT BY C6'
6     CONTINUE
      DO 8 K=1,N
      GRAD(K)=0.0
      DO 7 J=1,N
      GRAD(K)=GRAD(K)+(SUM(J+K+1)-SUM(J+1)*SUM(K+1))*RR(J)/C(J)
C     PRINT*,'GOT BY C7'
7     CONTINUE
      GRAD(K)=GRAD(K)*2.
C     PRINT*,'GOT BY C8'
8     CONTINUE
C     PRINT*,'GOT BY C9'
      RETURN
C     PRINT*,'GOT BY C10'
9     CONTINUE
      AA=SZ-32.
      ZERO=ZERO-AA
      GO TO 2
C
CC    PRINT*,'GOT BY C11'
      END

      SUBROUTINE START (XMAX,XMIN,ALAMDA,KSTART,CC,NL,IPRINT,UMIN,MODE,M
     1AXFN,ETA)
C     IMPLICIT REAL*8 (A-H,O-Z)
CC
CC    THIS SUBROUTINE IS USED TO FIND A REASONABLE STARTING POINT FOR
CC    SUBROUTINE  MPOPT
CC
      DIMENSION R(11)
      DIMENSION CC(*), ETA(*)
      DIMENSION ALAMDA(*), X(10), Y(10), W(10,10)
      COMMON/HELP/S(101),XX(16,101),C(8),M
C     ABOVE LINE CHANGED FROM TEXT
      COMMON /FAIL/ NFAIL
      GO TO (3,1,5,26), KSTART
1     CONTINUE
      NFAIL=0
      DO 2 I=1,NL
      ALAMDA(I)=0.0
2     CONTINUE
      RETURN
3     CONTINUE
      NFAIL=0
      ALAMDA(1)=CC(1)/CC(2)
      ALAMDA(2)=-.5/CC(2)
      DO 4 I=3,NL
      ALAMDA(I)=0.0
4     CONTINUE
      RETURN
5     CONTINUE
```

```
      NFAIL=0
      NNN=NL/2
      NNN=NNN*2
      NP1=NL+1
      DELTA=(XMAX-XMIN)/FLOAT(NL)
      DO 6 I=1,NP1
      X(I)=XMIN+FLOAT(I-1)*DELTA
    6 CONTINUE
      IF (NNN.NE.NL) GO TO 19
      W(1,1)=1.
      W(1,NP1)=1.
      DO 7 I=2,NL,2
      W(1,I)=4.
    7 CONTINUE
      IF (NL.EQ.2) GO TO 9
      NM1=NL-1
      DO 8 I=3,NM1,2
      W(1,I)=2.
    8 CONTINUE
    9 CONTINUE
      DO 10 J=1,NP1
      DO 10 I=2,NP1
   10 W(I,J)=W(I-1,J)*X(J)
      Y(1)=3./DELTA
      DO 11 I=1,NL
      Y(I+1)=C(I)*Y(1)
   11 CONTINUE
      CALL SOLVE (W,Y,XID,NP1,10)
   12 CONTINUE
      DO 13 I=1,NP1
      DO 13 J=1,NP1
   13 W(I,J)=.0
      DO 14 I=1,NP1
      IF (Y(I).LE.0.0) Y(I)=.0002
   14 CONTINUE
      DO 15 I=1,NP1
      Y(I)=ALOG(Y(I))
   15 CONTINUE
      DO 16 I=1,NP1
      W(I,1)=1.
   16 CONTINUE
      DO 17 I=2,NP1
      DO 17 J=1,NP1
   17 W(J,I)=W(J,I-1)*X(J)
      CALL SOLVE (W,Y,XID,NP1,10)
      DO 18 I=1,NL
      ALAMDA(I)=Y(I+1)
   18 CONTINUE
      RETURN
   19 CONTINUE
      R(1)=3./8.
      R(4)=3./8.
      R(2)=9./8.
      R(3)=9./8.
      IF (NL.EQ.3) GO TO 22
      R(NL+1)=1./3.
      R(4)=R(4)+1./3.
      DO 20 I=5,NL,2
      R(I)=4./3.
   20 CONTINUE
      IF (NL.EQ.5) GO TO 22
      NS=NL-1
      DO 21 I=6,NS,2
      R(I)=2./3.
```

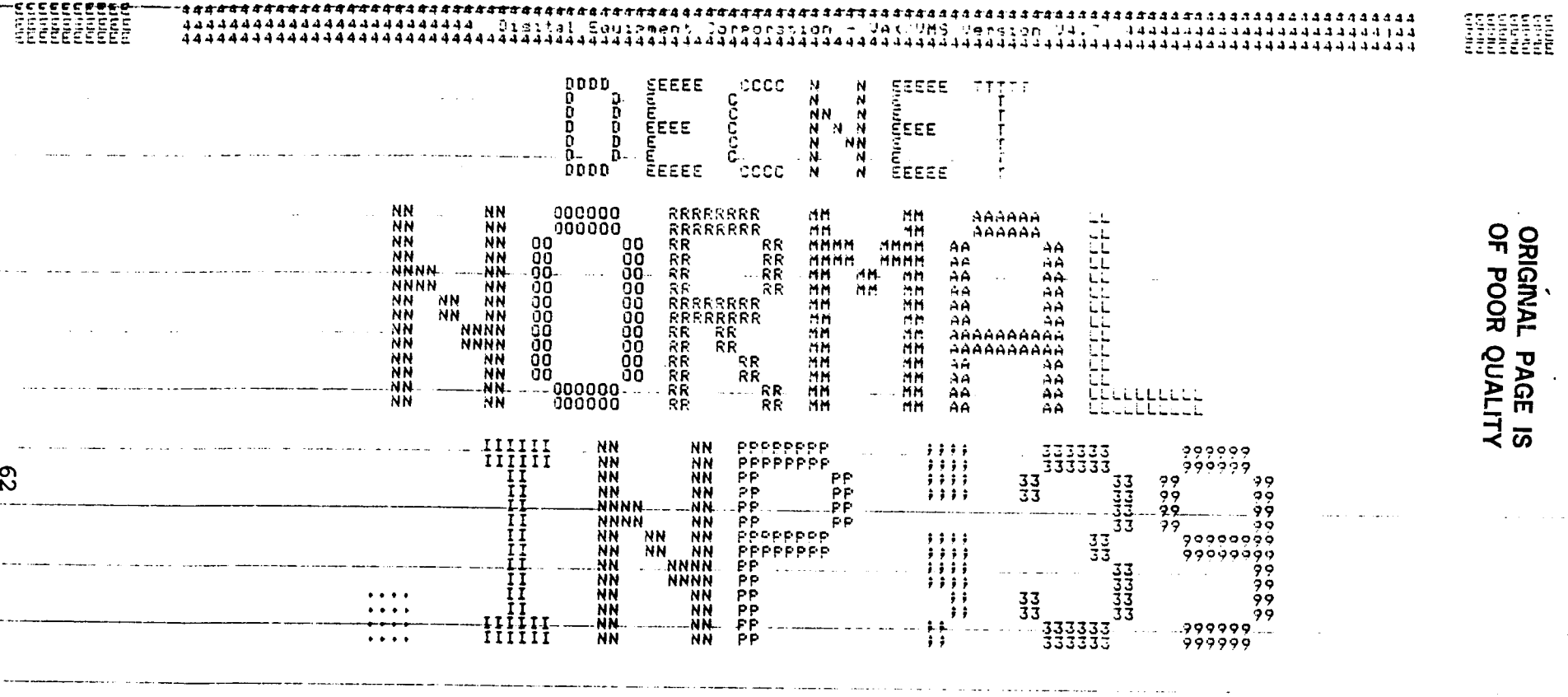
```
 21     CONTINUE
 22     CONTINUE
        DO 23 I=1,NP1
        W(1,I)=R(I)
 23     CONTINUE
        DO 24 J=1,NP1
        DO 24 I=2,NP1
 24     W(I,J)=W(I-1,J)*X(J)
        Y(1)=1./DELTA
        DO 25 I=1,NL
        Y(I+1)=C(I)*Y(1)
 25     CONTINUE
        CALL SOLVE (W,Y,XID,NP1,10)
        GO TO 12
 26     CONTINUE
        N=2
        ALAMDA(2)=-.5/CC(2)
        ALAMDA(1)=CC(1)/CC(2)
        NFAIL=0
 27     CONTINUE
        ALAMDA(N+1)=2.0
        ALAMDA(N+2)=0.0
 C      PRINT*,'GOT BY A'
        CALL MPOPT (ALAMDA,N,ETA,UMIN,MAXFN,MODE,IPRINT)
 C      PRINT*,'GOT BY B'
        IF (NFAIL.EQ.1) RETURN
        IF (N.EQ.NL) RETURN
        ALAMDA(N+1)=0.0
        N=N+1
        GO TO 27
        END


 C      SUBROUTINE SOLVE (A,X,XID,N,NA)
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION A(NA,*), X(*)
        D=0.
        DATA DIV/.693147181/
        DO 6 I=1,N
        AA=0.
        DO 1 J=I,N
        AB=ABS(A(J,I))
        IF (AB.LE.AA) GO TO 1
        K=J
        AA=AB
 1      CONTINUE
        D=D+ALOG(AA)
        IF (I.EQ.N) GO TO 7
        IF (K.EQ.I) GO TO 3
        DO 2 J=I,N
        AB=A(I,J)
        A(I,J)=A(K,J)
        A(K,J)=AB
 2      CONTINUE
        AB=X(I)
        X(I)=X(K)
        X(K)=AB
 3      I1=I+1
        DO 5 J=I1,N
        AA=-A(J,I)/A(I,I)
        A(J,I)=0.
        DO 4 K=I1,N
        A(J,K)=A(J,K)+AA*A(I,K)
 4      CONTINUE
```

```
      X(J)=X(J)+AA*X(I)
    5 CONTINUE
    3 CONTINUE
    7 XID=0/DIV
      X(N)=X(N)/A(N,N)
      DO 9 II=2,N
      I=N+1-II
      I1=I+1
      AA=0.
      DO 8 J=I1,N
      AA=AA+A(I,J)*X(J)
    8 CONTINUE
      X(I)=(X(I)-AA)/A(I,I)
    9 CONTINUE
      RETURN
      END

      SUBROUTINE SIMSON
C     IMPLICIT REAL*8 (A-H,O-Z)
C
C     THIS SUBROUTINE IS TO CALCULATE THE SIMPSON MULTIPLIERS
CC
      COMMON/HELP/S(101),XX(16,101),C(8),M
C.... ABOVE LINE CHANGED FROM TEXT
      S(1)=1.
      S(M)=1.
      N=M-1
      DO 1 I=2,N,2
      S(I)=4.
    1 CONTINUE
      N=N-1
      DO 2 I=3,N,2
      S(I)=2.
    2 CONTINUE
      RETURN
      END

      SUBROUTINE MULTI (XMAX,XMIN,N)
C     IMPLICIT REAL*8 (A-H,O-Z)
CC
CC    THIS SUBROUTINE IS USED TO GENERATE THE X,S POWER FOR SUBROUTINE
CC    FUNCT
CC
      COMMON/HELP/S(101),XX(16,101),C(8),M
C.... ABOVE LINE CHANGED FROM TEXT
      DELTA=(XMAX-XMIN)/FLOAT(M-1)
      DO 1 I=1,M
      XX(1,I)=XMIN+FLOAT(I-1)*DELTA
      NN=2*N
      DO 1 J=2,NN
      XX(J,I)=XX(J-1,I)*XX(1,I)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE CONVER (CM,NL)
C     IMPLICIT REAL*8 (A-H,O-Z)
CCC
CCC   THIS SUBROUTINE IS TO CALCULATE THE MOMENTS ABOUT THE ORIGIN
      DIMENSION CM(*)
```

```
      COMMON/HELP/S(101),X(10,10),C(8),M
C.... ABOVE LINE CHANGED FROM TEXT
      C(1)=CM(1)
      IF (NL.EQ.1) RETURN
      DO 2 J=2,NL
      C(J)=CM(J)-C(1)**J*(-1.)**J
      N=J-1
      DO 1 K=1,N
      C(J)=C(J)-(-1.)**K*FACTO(J)/(FACTO(K)*FACTO(J-K))*C(1)**(K)*C(J-K)
1     CONTINUE
2     CONTINUE
      RETURN
      END

      SUBROUTINE TRN1 (X1MAX,X1MIN,C,X2MAX,X2MIN,NL)
C     IMPLICIT REAL*8 (A-H,O-Z)
CCC
C     THIS SUBROUTINE IS USED TO CALCULATE THE MOMENTS FOR THE MODIFIED
C     LIMITS
C
      DIMENSION C(1)
      SCL=(X1MAX-X1MIN)/(X2MAX-X2MIN)
      C(1)=C(1)/SCL-X1MIN/SCL+X2MIN
      IF (NL.EQ.1) RETURN
      DO 1 I=2,NL
      C(I)=C(I)/SCL**(FLOAT(I))
1     CONTINUE
      RETURN
      END

C      SUBROUTINE TRN2(X1MAX,X1MIN,X,X2MAX,X2MIN,N)
C      IMPLICIT REAL*8 (A-H,O-Z)
C.... THIS SUBROUTINE IS AN ALTERNATIVE TO TRN2 (BELOW)
C.....
C.... CALCULATES THE LAGRANGIAN MULTIPLIERS FOR A DIFFERENT INTERVAL
C.... DOUBLE PRECISION VERSION
C      DOUBLE PRECISION S,A,DX(10),FAC,DX1MAX,DX1MIN,DX2MAX,DX2MIN
C      DIMENSION X(*)
C      DX1MAX=X1MAX
C      DX1MIN=X1MIN
C      DX2MAX=X2MAX
C      DX2MIN=X2MIN
C      NP1=N+1
C      DO 10 I=1,NP1
C10    DX(I)=X(I)
C      S=(DX1MAX-DX1MIN)/(DX2MAX-DX2MIN)
C      A=DX2MIN-DX1MIN/S
C      DX(1)=DX(1)-ALOG(S)
C      DO 1 I=1,N
C      DX(1)=DX(1)+DX(I+1)*A**I
C1     CONTINUE
C      IF(N.EQ.1)GO TO 6
C      DO 5 J=2,N
C      DO 3 I=J,N
C      FAC=1.
C      KK=I-J+2
C      DO 2 K=KK,I
C      FAC=FAC*DBLE(FLOAT(K))
C2     CONTINUE
C      DX(J)=DX(J)+FAC/DBLE(FACTO(J-1))*A**(I-J+1)*DX(I+1)
C3     CONTINUE
C4     DX(J)=DX(J)/S**(J-1)
```

```
      5     CONTINUE
      6     CONTINUE
            DX(N+1)=DX(N+1)/S**N
            DO 11 I=1,NP1
     11     X(I)=DX(I)
            RETURN
            END

            SUBROUTINE TRN2 (X1MAX,X1MIN,X,X2MAX,X2MIN,N)
      C     IMPLICIT REAL*8 (A-H,O-Z)
      C
      CC    THIS SUBROUTINE IS USED TO CALCULATE THE LAGRANGIAN MULTIPLIERS
      CC    AT  THE ORIGINAL LIMITS
      C
            DIMENSION X(1)
            S=(X1MAX-X1MIN)/(X2MAX-X2MIN)
            A=X2MIN-X1MIN/S
            X(1)=X(1)-ALOG(S)
            DO 1 I=1,N
            X(1)=X(1)+X(I+1)*A**I
      1     CONTINUE
            IF (N.EQ.1) GO TO 6
            DO 5 J=2,N
            DO 3 I=J,N
            FAC=1.
            KK=I-J+2
            DO 2 K=KK,I
            FAC=FAC*FLOAT(K)
      2     CONTINUE
            X(J)=X(J)+FAC/FACTO(J-1)*A**(I-J+1)*X(I+1)
      3     CONTINUE
      4     X(J)=X(J)/S**(J-1)
      5     CONTINUE
      6     CONTINUE
            X(N+1)=X(N+1)/S**N
            RETURN
            END

            FUNCTION CDF (XMIN,XMAX,XP,AL,N)
      C     IMPLICIT REAL*8 (A-H,O-Z)
      C     THIS FUNCTION SUBROUTINE IS TO CALCULATE THE CUMMULATIVE DISTRIBU-
      C     TION FUNCTION AT A GIVEN POINT
      C.... INPUT
      C       XMIN = LOWER BOUND
      C       XMAX = UPPER BOUND
      C       XP = SPECIFIED POINT
      C       AL(I) = ARRAY OF PARAMETERS, DIMENSION N
      C       N = NUMBER OF PARAMETERS
            DIMENSION AL(*)
            IF (XP.LE.XMIN) GO TO 3
            IF (XP.GE.XMAX) GO TO 4
            RANGE=XMAX-XMIN
            RANGEN=XP-XMIN
            SS=RANGEN/RANGE*51.
            JSS=SS
            JSS=(JSS/2)*2+5
            AREA=0.0
            JSM1=JSS-1
            DELTA=RANGEN/FLOAT(JSM1)
            DO 1 I=2,JSM1,2
            X=XMIN+FLOAT(I-1)*DELTA
            AREA=AREA+4.*ENTRPF(AL,N,X)
```

```
    1 CONTINUE
      JSM1=JSM1-1
      DO 2 I=3,JSM1,2
      X=XMIN+FLOAT(I-1)*DELTA
      AREA=AREA+2.*ENTRPF(AL,N,X)
    2 CONTINUE
      AREA=AREA+ENTRPF(AL,N,XMIN)+ENTRPF(AL,N,XP)
      AREA=AREA*DELTA/3.
      CDF=AREA
      GO TO 5
    3 CDF=0.0
      GO TO 5
    4 CDF=1.
    5 CONTINUE
      RETURN
      END

      FUNCTION ENTRPF (AL,NPL,X)
C     IMPLICIT REAL*8 (A-H,O-Z)
C
C     FUNCTION TO EVALUATE THE ENTROPY DENSITY FUNCTION AT A GIVEN POINT
C.... INPUT
C       AL(I) = ARRAY CONTAINING PARAMETERS, DIMENSION NPL
C       NPL = NUMBER OF PARAMETERS
C       X = GIVEN VALUE
C
      DIMENSION AL(*)
      S=AL(1)
      DO 1 I=2,NPL
      S=S+AL(I)*X**(I-1)
    1 CONTINUE
      ENTRPF=EXP(S)
      RETURN
      END

      FUNCTION FACTO (M)
C     IMPLICIT REAL*8 (A-H,O-Z)
C.... CALCULATES FACTORIAL OF M
      FACTO=1.
      IF (M.EQ.0) RETURN
      DO 1 I=1,M
      FACTO=FACTO*FLOAT(I)
    1 CONTINUE
      RETURN
      END
```

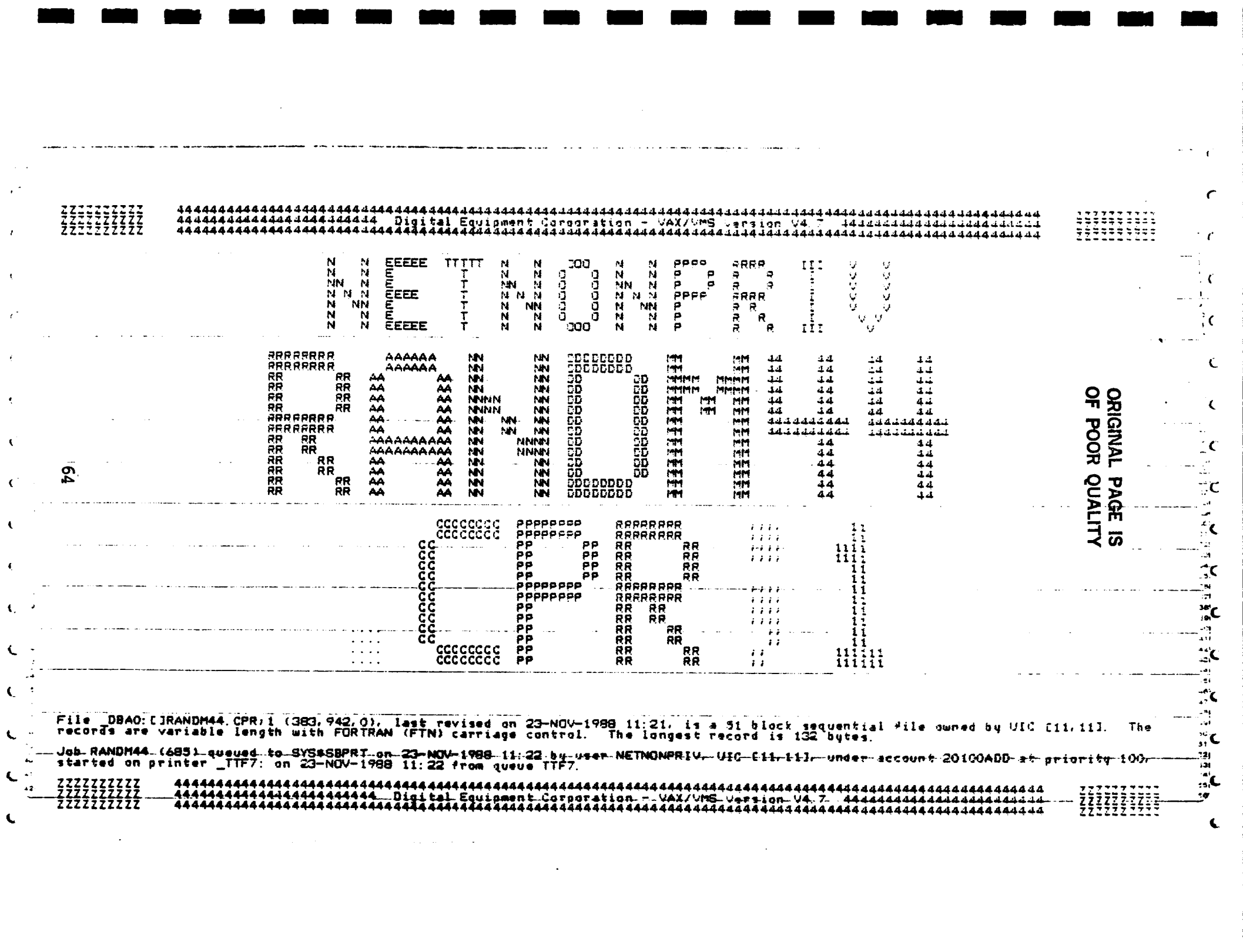


File _DUA0:[]NORMAL.INP;39 (445,121,0), last revised on 22-DEC-1988 13:25, is a 1 block sequential file owned by UIC [DECNET]. The records are variable length with implied (CR) carriage control. The longest record is 24 bytes.

Job NORMAL (132) queued to TERM$LA120A on 22-DEC-1988 13:26 by user DECNET, UIC [DECNET], under account DECNET at priority 100, started on printer LTA4: on 22-DEC-1988 13:30 from queue TERM$LA120A.

Digital Equipment Corporation - VAX/VMS Version V4.7

900.0000	45.0000
8.0000	0.8000
500.0000	25.0000
7.0000	0.7000
250.0000	12.5000
20.0000	1.0000
150.0000	7.5000
0.5000	0.0150
1500.0000	75.0000
20.0000	1.6000
850.0000	25.5000



File _DBA0:[JRANDM44.CPR;1 (383,942,0), last revised on 23-NOV-1988 11:21, is a 51 block sequential file owned by UIC [11,11]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 132 bytes.

Job RANDM44 (685) queued to SYS$SBPRT on 23-NOV-1988 11:22 by user NETNONPRIV, UIC [11,11], under account 20100ADD at priority 100, started on printer _TTF7: on 23-NOV-1988 11:22 from queue TTF7.

Digital Equipment Corporation - VAX/VMS Version V4.7

```
1   40
 0.9000E+03   0.4500E+02
OGNORMAL SF
 0.9165E+03   0.9306E+03   0.8813E+03   0.8726E+03   0.9717E+03
 0.8942E+03   0.8963E+03   0.8590E+03   0.8760E+03   0.8505E+03
 0.9148E+03   0.9603E+03   0.9484E+03   0.8751E+03   0.9161E+03
 0.8845E+03   0.8782E+03   0.8993E+03   0.9872E+03   0.8637E+03
 0.1010E+04   0.9801E+03   0.9609E+03   0.8738E+03   0.9339E+03
 0.9546E+03   0.8574E+03   0.9446E+03   0.9070E+03   0.9341E+03
 0.8816E+03   0.9039E+03   0.8584E+03   0.9706E+03   0.8554E+03
 0.9232E+03   0.9712E+03   0.8158E+03   0.9656E+03   0.9446E+03
1   40
 0.8000E+01   0.8000E+00
LOGNORMAL XLNMF
 0.6570E+01   0.8531E+01   0.7652E+01   0.7503E+01   0.9299E+01
 0.7878E+01   0.7916E+01   0.7271E+01   0.7560E+01   0.7128E+01
 0.8245E+01   0.9085E+01   0.8859E+01   0.7946E+01   0.8268E+01
 0.7708E+01   0.7599E+01   0.7967E+01   0.9599E+01   0.7384E+01
 0.1005E+02   0.7632E+01   0.9088E+01   0.7524E+01   0.8591E+01
 0.7197E+01   0.7243E+01   0.8789E+01   0.8104E+01   0.8596E+01
 0.7638E+01   0.8049E+01   0.7261E+01   0.7469E+01   0.7209E+01
 0.8397E+01   0.9290E+01   0.6608E+01   0.9184E+01   0.8790E+01
1   40
 0.5000E+03   0.2500E+02
LOGNORMAL SO
 0.4536E+03   0.5170E+03   0.4896E+03   0.4848E+03   0.5398E+03
 0.4968E+03   0.4980E+03   0.4772E+03   0.4866E+03   0.4725E+03
 0.5082E+03   0.5335E+03   0.5269E+03   0.4862E+03   0.5090E+03
 0.4914E+03   0.4879E+03   0.4996E+03   0.5485E+03   0.4809E+03
 0.5613E+03   0.4890E+03   0.5336E+03   0.4855E+03   0.5188E+03
 0.4748E+03   0.4763E+03   0.5248E+03   0.5039E+03   0.5190E+03
 0.4898E+03   0.5022E+03   0.4769E+03   0.4837E+03   0.4752E+03
 0.5129E+03   0.5396E+03   0.4549E+03   0.5365E+03   0.5248E+03
1   40
 0.7000E+01   0.7000E+00
LOGNORMAL XLNMO
 0.5749E+01   0.7465E+01   0.6696E+01   0.6565E+01   0.8136E+01
 0.6894E+01   0.6926E+01   0.6362E+01   0.6615E+01   0.6237E+01
 0.7214E+01   0.7947E+01   0.7752E+01   0.6602E+01   0.7235E+01
 0.6744E+01   0.6649E+01   0.6971E+01   0.8399E+01   0.6461E+01
 0.8797E+01   0.6678E+01   0.7952E+01   0.6583E+01   0.7517E+01
 0.6297E+01   0.6338E+01   0.7690E+01   0.7091E+01   0.7521E+01
 0.6701E+01   0.7043E+01   0.6353E+01   0.6535E+01   0.6308E+01
 0.7347E+01   0.8129E+01   0.5782E+01   0.8036E+01   0.7691E+01
1   40
 0.2500E+03   0.1250E+02
LOGNORMAL S
 0.2268E+03   0.2585E+03   0.2448E+03   0.2424E+03   0.2699E+03
 0.2484E+03   0.2490E+03   0.2386E+03   0.2433E+03   0.2363E+03
 0.2541E+03   0.2667E+03   0.2634E+03   0.2431E+03   0.2545E+03
 0.2457E+03   0.2439E+03   0.2498E+03   0.2742E+03   0.2405E+03
 0.2807E+03   0.2445E+03   0.2668E+03   0.2427E+03   0.2594E+03
 0.2374E+03   0.2382E+03   0.2624E+03   0.2519E+03   0.2595E+03
 0.2449E+03   0.2511E+03   0.2385E+03   0.2418E+03   0.2376E+03
 0.2565E+03   0.2698E+03   0.2275E+03   0.2682E+03   0.2624E+03
1   40
 0.2000E+02   0.1000E+01
LOGNORMAL SIGO
-0.1814E+02  -0.2068E+02  -0.1958E+02  -0.1939E+02  -0.2159E+02
-0.1987E+02  -0.1992E+02  -0.1909E+02  -0.1947E+02  -0.1890E+02
```

```
-0.2033E+02 -0.2134E+02 -0.2107E+02 -0.1945E+02 -0.2036E+02
-0.1965E+02 -0.1952E+02 -0.1988E+02 -0.2194E+02 -0.1934E+02
-0.2245E+02 -0.1956E+02 -0.2135E+02 -0.1942E+02 -0.2075E+02
-0.1899E+02 -0.1905E+02 -0.2099E+02 -0.2015E+02 -0.2075E+02
-0.1959E+02 -0.2009E+02 -0.1908E+02 -0.1935E+02 -0.1901E+02
-0.2052E+02 -0.2158E+02 -0.1820E+02 -0.2146E+02 -0.2099E+02
1  40
 0.1500E+03  0.7500E+01
LOGNORMAL SIG
 0.1361E+03  0.1551E+03  0.1469E+03  0.1454E+03  0.1619E+03
 0.1490E+03  0.1494E+03  0.1432E+03  0.1460E+03  0.1419E+03
 0.1525E+03  0.1600E+03  0.1581E+03  0.1459E+03  0.1527E+03
 0.1474E+03  0.1464E+03  0.1499E+03  0.1645E+03  0.1443E+03
 0.1684E+03  0.1467E+03  0.1601E+03  0.1456E+03  0.1558E+03
 0.1424E+03  0.1429E+03  0.1574E+03  0.1512E+03  0.1597E+03
 0.1469E+03  0.1507E+03  0.1431E+03  0.1451E+03  0.1426E+03
 0.1539E+03  0.1619E+03  0.1365E+03  0.1609E+03  0.1574E+03
1  40
 0.5000E+00  0.1500E-01
NORMAL XXN
 0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
 0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
 0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
 0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
 0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
 0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
 0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
 0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
NORMAL XXM
 0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
 0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
 0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
 0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
 0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
 0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
 0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
 0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
NORMAL XXG
 0.4352E+00  0.4832E+00  0.5104E+00  0.4984E+00  0.5012E+00
 0.4884E+00  0.4749E+00  0.5070E+00  0.5070E+00  0.5227E+00
 0.4956E+00  0.5007E+00  0.5144E+00  0.4727E+00  0.4758E+00
 0.5011E+00  0.5066E+00  0.4637E+00  0.4956E+00  0.4775E+00
 0.4969E+00  0.5073E+00  0.5034E+00  0.5222E+00  0.5153E+00
 0.5010E+00  0.4801E+00  0.5059E+00  0.4968E+00  0.5079E+00
 0.5201E+00  0.5107E+00  0.4905E+00  0.4750E+00  0.5095E+00
 0.4933E+00  0.5051E+00  0.5104E+00  0.5355E+00  0.4948E+00
1  40
 0.1500E+04  0.7500E+02
NORMAL TF
 0.1176E+04  0.1416E+04  0.1552E+04  0.1492E+04  0.1506E+04
 0.1442E+04  0.1374E+04  0.1535E+04  0.1535E+04  0.1613E+04
 0.1478E+04  0.1504E+04  0.1572E+04  0.1364E+04  0.1379E+04
 0.1506E+04  0.1533E+04  0.1318E+04  0.1478E+04  0.1388E+04
 0.1484E+04  0.1537E+04  0.1517E+04  0.1611E+04  0.1577E+04
 0.1505E+04  0.1400E+04  0.1530E+04  0.1484E+04  0.1540E+04
 0.1601E+04  0.1554E+04  0.1452E+04  0.1375E+04  0.1547E+04
 0.1467E+04  0.1525E+04  0.1552E+04  0.1678E+04  0.1474E+04
1  40
```

```
  0.2000E+02   0.6000E+00
NORMAL TO
  0.1741E+02   0.1933E+02   0.2042E+02   0.1994E+02   0.2005E+02
  0.1953E+02   0.1900E+02   0.2028E+02   0.2028E+02   0.2091E+02
  0.1982E+02   0.2003E+02   0.2057E+02   0.1891E+02   0.1903E+02
  0.2004E+02   0.2027E+02   0.1855E+02   0.1982E+02   0.1910E+02
  0.1988E+02   0.2029E+02   0.2013E+02   0.2089E+02   0.2061E+02
  0.2004E+02   0.1920E+02   0.2024E+02   0.1987E+02   0.2032E+02
  0.2081E+02   0.2043E+02   0.1962E+02   0.1900E+02   0.2038E+02
  0.1973E+02   0.2020E+02   0.2042E+02   0.2142E+02   0.1979E+02
1
 40
  0.8500E+03   0.2550E+02
NORMAL T
  0.7399E+03   0.8215E+03   0.8677E+03   0.8474E+03   0.8531E+03
  0.8302E+03   0.8073E+03   0.8618E+03   0.8619E+03   0.8885E+03
  0.8424E+03   0.8512E+03   0.8744E+03   0.8037E+03   0.8089E+03
  0.8519E+03   0.8613E+03   0.7982E+03   0.8424E+03   0.8118E+03
  0.8447E+03   0.8624E+03   0.8557E+03   0.8877E+03   0.8760E+03
  0.8517E+03   0.8161E+03   0.8601E+03   0.8446E+03   0.8635E+03
  0.8842E+03   0.8682E+03   0.8338E+03   0.8074E+03   0.8661E+03
  0.8387E+03   0.8586E+03   0.8677E+03   0.9104E+03   0.8412E+03
LOG OF CYCLES TO REACH MEAN FATIGUE STR =
50 MPA
  0.6026E+01   0.7799E+01   0.6977E+01   0.6849E+01   0.6486E+01
  0.7199E+01   0.7242E+01   0.6632E+01   0.6896E+01   0.6491E+01
  0.7529E+01   0.8289E+01   0.8074E+01   0.6904E+01   0.7564E+01
  0.7034E+01   0.6931E+01   0.7295E+01   0.8765E+01   0.6754E+01
  0.9179E+01   0.6961E+01   0.8292E+01   0.6851E+01   0.7828E+01
  0.6568E+01   0.6623E+01   0.8017E+01   0.7399E+01   0.7839E+01
  0.6975E+01   0.7339E+01   0.6634E+01   0.6833E+01   0.6974E+01
  0.7669E+01   0.8479E+01   0.6025E+01   0.8351E+01   0.8027E+01
SORTED LOG OF CYCLES
  0.6025E+01   0.6026E+01   0.6491E+01   0.6568E+01   0.6574E+01
  0.6623E+01   0.6632E+01   0.6634E+01   0.6754E+01   0.6833E+01
  0.6849E+01   0.6851E+01   0.6896E+01   0.6904E+01   0.6931E+01
  0.6961E+01   0.6975E+01   0.6977E+01   0.7034E+01   0.7199E+01
  0.7242E+01   0.7295E+01   0.7339E+01   0.7399E+01   0.7529E+01
  0.7564E+01   0.7669E+01   0.7799E+01   0.7828E+01   0.7839E+01
  0.8017E+01   0.8027E+01   0.8074E+01   0.8289E+01   0.8292E+01
  0.8351E+01   0.8479E+01   0.8486E+01   0.8765E+01   0.9179E+01
SAMPLE MOMENTS
  0.7355E+01   0.5703E+00   0.1782E+00   0.7624E+00
BNDS(1), BNDS(2)=   0.5723E+01    0.9638E+01
```

INPUT DATA FOR SUBROUTINE MEP1

INPUT DATA IS PRINTED OUT FOR KDATA =1 ONLY . . .	KDATA =	1			
INTERMEDIATE OUTPUT EVERY KPRINT(TH) CYCLE . .	KPRINT =	1			
NUMBER OF KNOWN FIRST MOMENTS	N=	4			
HIGHER LIMIT	XMAX =	0.963779301E+01			
LOWER LIMIT	XMIN =	0.572349819E+01			
FIRST MOMENTS	CC(I) =	0.735481628E+01	0.570334345E+00	0.178168963E+00	0.762378196E+00
THE ALLOWED TOLERANCE IN LAGRANGIAN EQUATIONS . .	TOL =	0.100000000E-05			
THE CUMULATIVE DISTRIBUTION REQUIRED AT NXP POINTS.	NXP =	0			

INTERMEDIATE RESULTS FOR SUBROUTINE MEP

NUMBER OF INTEGRATION STATION . . . . . M = 31

MODIFIED MAXIMUM AND MINIMUM LIMITS . . X2MAX , X2MIN = 0.10000000E+01 0.00000000E+00

MODIFIED MOMENTS ABOUT THE EXPECTED VALUE . . . . CC(I) = 0.41675912E+00 0.37223950E-01 0.2970786e1E-02 0.3247553e3E-02

MODIFIED MOMENTS ABOUT THE ORIGIN . . . . . C(I) = 0.41675912E+00 0.21091211E+00 0.12189717E+00 0.77159701E-01

SUBROUTINE MPOPT TOLERANCES . . . . . ETA(I) = 0.10000000E-11 0.10000000E-05 0.10000000E-22 0.10000000E-22

NORMAL ASSUMPTION STARTING METHOD

STARTING VALUES . . . . . AL(I) = 0.11199994E+02 -0.13432212E+02 0.00000000E+00 0.00000000E+00

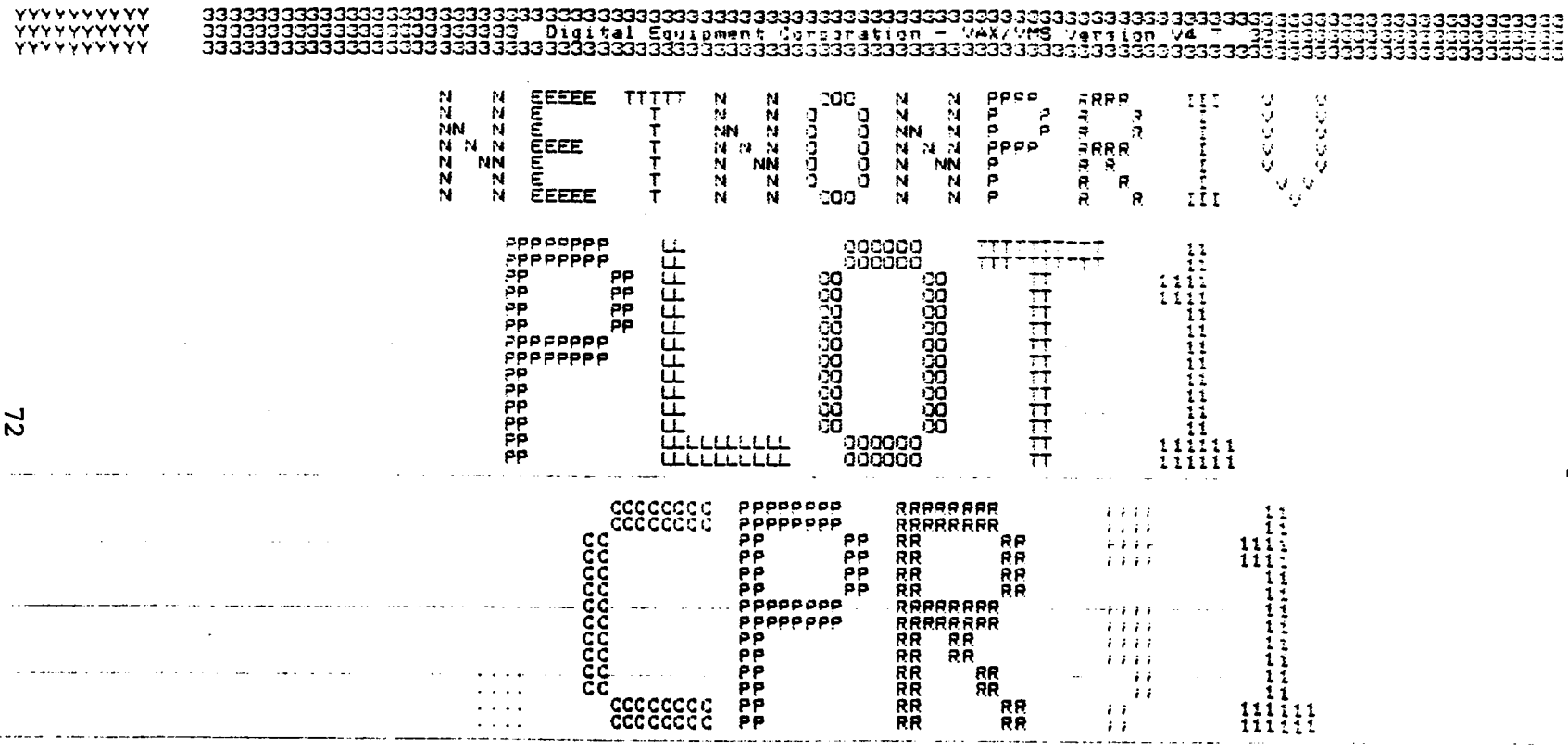
CYC NO.	NUMF	NORMGRAD RESIDUALS	TOTAL X(1)	VARIABLES X(2)	X(3)	X(4)	RESIDUALS R(1)	R(2)	R(3)	R(4)
0	2	0.17606E-01	0.23715E-02	0.11222E+02	-0.13405E+02	0.23839E-01	0.19798E-01	0.224E-01	0.210E-01	-0.417E-02 -0.376E-01
0										
1	4	0.32974E-02	0.22064E-02	0.11262E+02	-0.13405E+02	0.23839E-01	0.19798E-01	0.256E-01	0.265E-01	0.309E-02 -0.290E-01
2	5	0.51891E-02	0.97080E-04	0.80751E+01	-0.10197E+02	0.03839E-01	0.19798E-01	-0.309E-02	0.106E-02	0.334E-02 0.441E-01
3	7	0.19418E-02	0.92570E-04	0.77949E+01	-0.95481E+01	-0.41358E+00	0.19798E-01	-0.894E-02	0.215E-03	0.333E-02 0.273E-01
4	8	0.42977E-02	0.31381E-04	0.21444E+02	-0.62921E+02	0.79107E+02	-0.39801E+02	-0.382E-02	0.172E-02	0.391E-02 0.231E-02
5	10	0.27787E-02	0.23060E-04	0.17639E+02	-0.47370E+02	0.55444E+02	-0.27847E+02	-0.280E-02	0.294E-02	0.358E-02 0.363E-04
6	11	0.24290E-02	0.18863E-04	0.20061E+02	-0.56621E+02	0.69056E+02	-0.34621E+02	-0.200E-02	0.304E-02	0.233E-02 -0.444E-03
7										
7	13	0.10508E-02	0.10587E-04	0.22892E+02	-0.67284E+02	0.84353E+02	-0.42016E+02	-0.254E-02	0.126E-02	0.569E-03 -0.150E-02
8	15	0.13076E-02	0.10539E-04	0.23238E+02	-0.68597E+02	0.86281E+02	-0.42975E+02	-0.249E-02	0.120E-02	0.493E-03 -0.167E-02
9	16	0.35516E-03	0.36451E-05	0.26844E+02	-0.79546E+02	0.98742E+02	-0.47577E+02	-0.164E-02	0.391E-04	-0.180E-03 0.966E-03
10	19	0.12140E-03	0.36089E-05	0.27017E+02	-0.80287E+02	0.99926E+02	-0.48205E+02	-0.169E-02	-0.380E-04	-0.258E-03 0.831E-03
11	20	0.27919E-03	0.25729E-05	0.26117E+02	-0.77343E+02	0.96306E+02	-0.46736E+02	-0.144E-02	0.659E-03	0.144E-03 0.242E-03
12	21	0.34299E-03	0.12052E-05	0.27481E+02	-0.81924E+02	0.10237E+03	-0.49498E+02	-0.697E-03	0.843E-03	-0.107E-04 0.929E-04
7										
13	23	0.14400E-03	0.72443E-06	0.28685E+02	-0.85797E+02	0.10725E+03	-0.51594E+02	-0.224E-03	0.689E-03	-0.429E-03 -0.128E-03
14	25	0.14845E-03	0.71197E-06	0.28916E+02	-0.86679E+02	0.10855E+03	-0.52246E+02	-0.192E-03	0.685E-03	-0.432E-03 -0.138E-03
15	27	0.43064E-03	0.47545E-06	0.36093E+02	-0.11405E+03	0.14890E+03	-0.72381E+02	0.224E-03	0.293E-03	-0.331E-03 0.501E-03
16	29	0.34003E-03	0.40703E-06	0.34528E+02	-0.10818E+03	0.14037E+03	-0.68171E+02	0.306E-03	0.445E-03	-0.299E-03 0.478E-03
17	31	0.63374E-04	0.38902E-06	0.34313E+02	-0.10719E+03	0.13870E+03	-0.67247E+02	0.137E-03	0.305E-03	-0.471E-03 0.223E-03
18	32	0.14503E-03	0.30074E-06	0.35067E+02	-0.11034E+03	0.14369E+03	-0.69884E+02	0.854E-04	0.246E-03	-0.478E-03 0.673E-04
7										
19	34	0.59585E-04	0.26184E-06	0.34631E+02	-0.10878E+03	0.14149E+03	-0.68817E+02	0.670E-05	0.279E-03	-0.374E-03 0.206E-03
20	37	0.91125E-04	0.26054E-06	0.35094E+02	-0.11054E+03	0.14407E+03	-0.70110E+02	0.437E-04	0.275E-03	-0.367E-03 0.219E-03
21	39	0.21647E-03	0.20172E-06	0.35750E+02	-0.11341E+03	0.14878E+03	-0.72662E+02	0.733E-05	0.354E-03	-0.209E-03 0.186E-03
22	41	0.24074E-03	0.20096E-06	0.35869E+02	-0.11388E+03	0.14949E+03	-0.73022E+02	0.408E-05	0.347E-03	-0.194E-03 0.207E-03
23	42	0.18256E-03	0.18373E-06	0.36052E+02	-0.11456E+03	0.15048E+03	-0.73509E+02	-0.419E-05	0.303E-03	-0.227E-03 0.201E-03
24	43	0.15020E-03	0.17917E-06	0.36163E+02	-0.11496E+03	0.15103E+03	-0.73773E+02	-0.434E-05	0.279E-03	-0.251E-03 0.302E-03
7										
25	44	0.19623E-03	0.13847E-06	0.37770E+02	-0.12126E+03	0.16057E+03	-0.78647E+02	0.379E-04	0.227E-03	-0.194E-03 0.220E-03

ORIGINAL PAGE IS
OF POOR QUALITY

```
 26  45  0.20238E-03  0.12726E-06  0.37821E+02  -0.14146E+03  0.16088E+03  -0.78814E+02  0.35ZE-04  0.
 27  46  0.21288E-03  0.12440E-07  0.37860E+02  -0.14113E+03  0.16144E+03  -0.79141E+02  0.35ZE-04  0.
 28  47  0.20564E-03  0.12980E-07  0.38740E+02  -0.14445E+03  0.16558E+03  -0.82455E+02  0.37ZE-04  0.
 29  48  0.16682E-03  0.91745E-07  0.38489E+02  -0.14386E+03  0.16555E+03  -0.83450E+02  0.37ZE-04  0.
 30  49  0.10854E-03  0.62807E-07  0.39089E+02  -0.14686E+03  0.16855E+03  -0.83450E+02  0.37ZE-04  0.
 7
 31  50  0.23574E-04  0.13258E-07  0.39594E+02  -0.14865E+03  0.17205E+03  -0.84645E+02  0.39ZE-04  0.
 32  51  0.31018E-04  0.14388E-07  0.39993E+02  -0.15043E+03  0.17444E+03  -0.85870E+02  0.39ZE-04  0.
 33  52  0.33159E-04  0.14145E-07  0.40410E+02  -0.15184E+03  0.17686E+03  -0.87103E+02  0.40ZE-04  0.
 34  53  0.33159E-04  0.14145E-07  0.40906E+02  -0.15380E+03  0.17886E+03  -0.88644E+02  0.40ZE-04  0.
 35  54  0.10450E-03  0.20629E-07  0.41496E+02  -0.15618E+03  0.18355E+03  -0.91315E+02  0.42ZE-04  0.
 36  55  0.98119E-04  0.20219E-07  0.41744E+02  -0.15713E+03  0.18499E+03  -0.91315E+02  0.42ZE-04  0.
 7
 37  56  0.10435E-03  0.18385E-07  0.41758E+02  -0.15705E+03  0.18523E+03  -0.91448E+02  0.42ZE-04  0.
 38  57  0.10549E-03  0.18012E-07  0.41849E+02  -0.15738E+03  0.18557E+03  -0.91448E+02  0.42ZE-04  0.
 39  58  0.74156E-04  0.19012E-07  0.42038E+02  -0.15803E+03  0.18644E+03  -0.92044E+02  0.42ZE-04  0.
 40  59  0.74206E-04  0.19012E-07  0.42421E+02  -0.15951E+03  0.18814E+03  -0.93044E+02  0.43ZE-04  0.
 41  60  0.57316E-05  0.45590E-08  0.43522E+02  -0.16401E+03  0.19348E+03  -0.96612E+02  0.44ZE-04  0.
 42  61  0.22994E-04  0.43590E-08  0.43444E+02  -0.16388E+03  0.19534E+03  -0.96832E+02  0.44ZE-04  0.
 7
 43  62  0.19892E-05  0.33545E-08  0.43947E+02  -0.16579E+03  0.19813E+03  -0.98203E+02  0.45ZE-04  0.
 44  63  0.30781E-04  0.19981E-08  0.43767E+02  -0.16510E+03  0.19774E+03  -0.98203E+02  0.45ZE-04  0.
 45  64  0.20087E-05  0.11673E-08  0.43923E+02  -0.16574E+03  0.19814E+03  -0.98391E+02  0.45ZE-04  0.
 46  65  0.12797E-04  0.58200E-09  0.43980E+02  -0.16596E+03  0.19848E+03  -0.98612E+02  0.45ZE-04  0.
 47  66  0.16871E-04  0.55845E-09  0.44083E+02  -0.16637E+03  0.19910E+03  -0.98612E+02  0.45ZE-04  0.
 48  67  0.13342E-04  0.34397E-09  0.44083E+02  -0.16637E+03  0.19910E+03  -0.98612E+02  0.45ZE-04  0.
 7
 49  68  0.91758E-05  0.13009E-09  0.44281E+02  -0.14713E+03  0.20026E+03  -0.99203E+02  -0.384E-05  0.434E-05  -0.
 50  69  0.73374E-05  0.85527E-10  0.44349E+02  -0.14739E+03  0.20063E+03  -0.99391E+02  -0.384E-05  0.434E-05  -0.
 51  70  0.43006E-05  0.89728E-10  0.44312E+02  -0.14725E+03  0.20044E+03  -0.99284E+02  -0.384E-05  0.434E-05  -0.
 52  71  0.16525E-05  0.77633E-11  0.44401E+02  -0.14760E+03  0.20096E+03  -0.99568E+02  -0.384E-05  0.434E-05  -0.
 53  72  0.54000E-06  0.87324E-11  0.44401E+02  -0.14760E+03  0.20096E+03  -0.99568E+02  -0.384E-05  0.434E-05  -0.
 54  73  0.38198E-06  0.87947E-12  0.44473E+02  -0.14789E+03  0.20140E+03  -0.99795E+02  0.171E-06-0.398E-06  0.521E-06-0.671E-06
 SOLUTION FOUND  0.38198E-06  0.87947E-12  0.44473E+02  -0.14789E+03  0.20140E+03  -0.99795E+02  0.171E-06-0.398E-06  0.521E-06-0.671E-06
 54  86
 SUM OF RESIDUALS SQUARED= 0.87947E-12  -0.39044040E+01  -0.44473114ZE+02  -0.14788617E+03  0.20140271ZE+03
 THE MODIFIED LAGRANGIAN MULTIPLIERS ARE  -0.39795108E+02
 LAGRANGIAN MULTIPLIERS
 -0.14726+04  0.77077E+03  -0.15096+03  0.13096+02  -0.42516+00
 LOG OF CURRENT CYCLES, LOG XNM
 0.57236+01  0.96386+01  0.59196+01  0.61136+01  0.63116+01
 0.65066+01  0.67026+01  0.68986+01  0.70946+01  0.72896+01
 0.74856+01  0.76816+01  0.78766+01  0.80726+01  0.82686+01
 0.84646+01  0.86596+01  0.88556+01  0.90516+01  0.92466+01
 ORDERED LOG OF CURRENT CYCLES, LOG XNM,  X AXIS OF PDF, CDF PLOT
 0.57236+01  0.59196+01  0.61136+01  0.63116+01  0.65066+01
 0.67026+01  0.68986+01  0.70946+01  0.72896+01  0.74856+01
 0.76816+01  0.78766+01  0.80726+01  0.82686+01  0.84646+01
 0.86596+01  0.88556+01  0.90516+01  0.92466+01  0.94426+01
 CDF PARAMETERS  Y AXIS OF PDF, CDF PLOT
 2
 CDF OF LOG OF CURRENT CYCLES, LOG XNM,  0.3763E-01  0.1267E+00
 0.0000E+00  0.3801E-02  0.1898E-01  0.4288E+00  0.3219E+00  0.6038E+00
 0.2208E+00  0.3259E+00  0.7398E+00  0.7981E+00  0.8515E+00  0.8992E+00
 0.9391E+00  0.9689E+00  0.9874E+00  0.9963E+00  0.1000E+01
```

ORDERED LOG OF CURRENT CYCLES, LOG XNM, X AXIS OF PDF, CDF PLOT

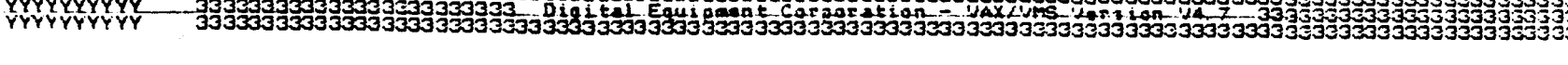
0.5723E+01	0.5919E+01	0.6115E+01	0.6311E+01	0.6506E+01
0.6702E+01	0.6898E+01	0.7094E+01	0.7289E+01	0.7485E+01
0.7681E+01	0.7876E+01	0.8072E+01	0.8268E+01	0.8464E+01
0.8659E+01	0.8855E+01	0.9051E+01	0.9246E+01	0.9442E+01
0.9638E+01				
0.5723E+01	0.5919E+01	0.6115E+01	0.6311E+01	0.6506E+01
0.6702E+01	0.6898E+01	0.7094E+01	0.7289E+01	0.7485E+01
0.7681E+01	0.7876E+01	0.8072E+01	0.8268E+01	0.8464E+01
0.8659E+01	0.8855E+01	0.9051E+01	0.9246E+01	0.9442E+01
0.9638E+01				



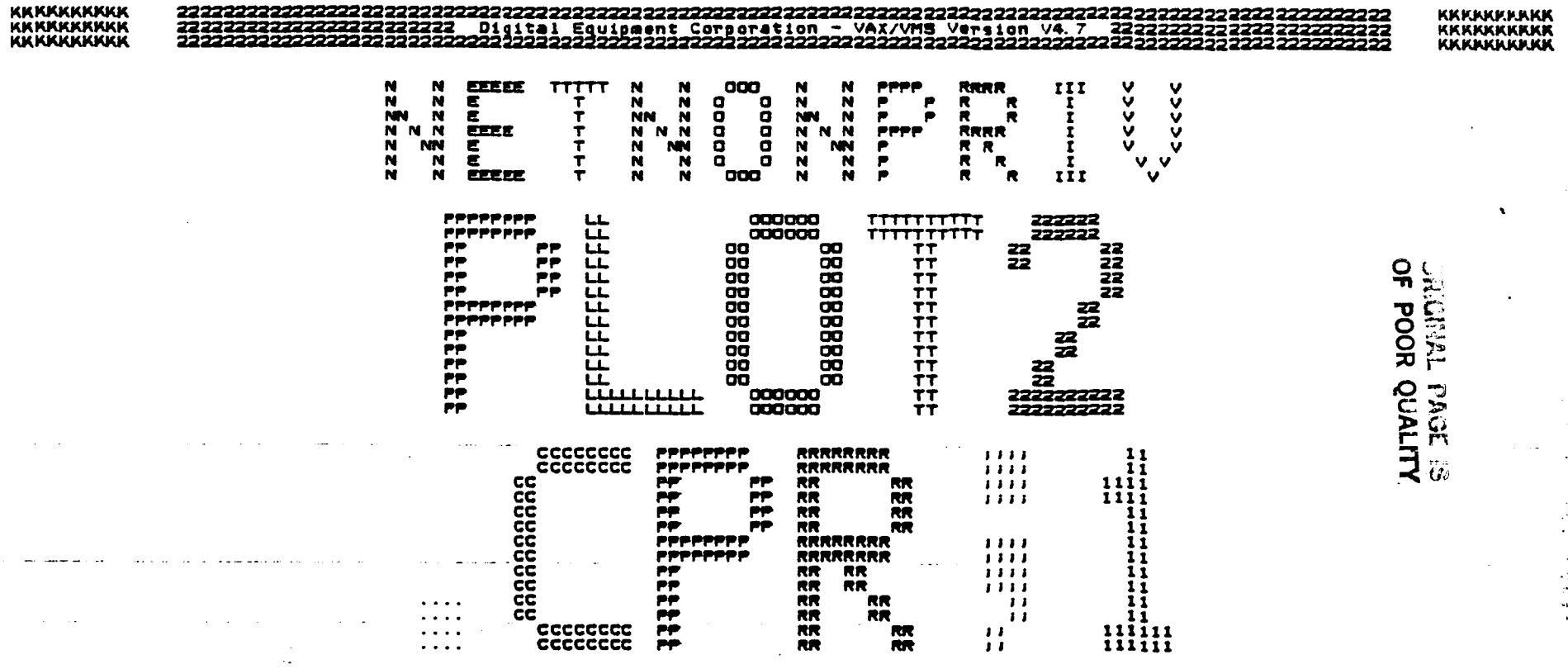
ORIGINAL PAGE IS
OF POOR QUALITY

File _DBA0:[]PLOT1.CPR;1 (359,204,0), last revised on 23-NOV-1988 11:20, is a 2 block sequential file owned by UIC [11,11]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 25 bytes.

Job PLOT1 (683) queued to SYS$SBPRT on 23-NOV-1988 11:20 by user NETNONPRIV, UIC [11,11], under account 20100ADD at priority 100, started on printer _TTF7: on 23-NOV-1988 11:20 from queue TTF7.



```
(E12.4,1X,E12.4)
 0.5723E+01    0.5149E-02
 0.5919E+01    0.3369E-01
 0.6115E+01    0.1213E+00
 0.6311E+01    0.2735E+00
 0.6506E+01    0.4324E+00
 0.6702E+01    0.5291E+00
 0.6898E+01    0.5448E+00
 0.7094E+01    0.5056E+00
 0.7289E+01    0.4464E+00
 0.7485E+01    0.3897E+00
 0.7681E+01    0.3444E+00
 0.7876E+01    0.3110E+00
 0.8072E+01    0.2852E+00
 0.8268E+01    0.2600E+00
 0.8464E+01    0.2272E+00
 0.8659E+01    0.1810E+00
 0.8855E+01    0.1230E+00
 0.9051E+01    0.6575E-01
 0.9246E+01    0.2512E-01
 0.9442E+01    0.6140E-02
 0.9638E+01    0.8464E-03
```



File _DBAO:[]PLOT2.CPR;1 (363,197,0), last revised on 23-NOV-1988 11:21, is a 2 block sequential file owned by UIC [11,111]. The records are variable length with FORTRAN (FTN) carriage control. The longest record is 25 bytes.

Job PLOT2 (684) queued to SYS$SPRT on 23-NOV-1988 11:21 by user NETNONPRIV, UIC [11,111], under account 20100ADD at priority 100, started on printer _TTF6: on 23-NOV-1988 11:21 from queue TTF6.

Digital Equipment Corporation - VAX/VMS Version V4.7

```
(E12.4,1X,E12.4)
 0.5723E+01    0.0000E+00
 0.5919E+01    0.3801E-02
 0.6115E+01    0.1878E-01
 0.6311E+01    0.3763E-01
 0.6506E+01    0.1267E+00
 0.6702E+01    0.2208E+00
 0.6898E+01    0.3257E+00
 0.7094E+01    0.4288E+00
 0.7289E+01    0.5219E+00
 0.7485E+01    0.6038E+00
 0.7681E+01    0.6736E+00
 0.7876E+01    0.7378E+00
 0.8072E+01    0.7781E+00
 0.8268E+01    0.8515E+00
 0.8464E+01    0.8772E+00
 0.8659E+01    0.9371E+00
 0.8855E+01    0.9687E+00
 0.9051E+01    0.9874E+00
 0.9246E+01    0.9963E+00
 0.9442E+01    0.9993E+00
 0.9638E+01    0.1000E+01
```

9.0 APPENDIX D

IMSL SUBROUTINE CALLS FROM RANDOM3 AND RANDOM4

RANDOM3

1. RNSET - Initializes a random seed for use in the IMSL random number generators.
2. RNNOR - Generates pseudorandom numbers from a standard normal distribution using an inverse CDF method.
3. RNLNL - Generates pseudorandom numbers from a lognormal distribution.
4. DESPL - Performs nonparametric probability density function estimation by the penalized likelihood method.
5. GCDF - Evaluates a general continuous cumulative distribution function given the ordinates of the density.

RANDOM4

1. RNSET - Initializes a random seed for use in the IMSL random number generators.
2. RNNOR - Generates pseudorandom numbers from a standard normal distribution using an inverse CDF method.
3. RNLNL - Generates pseudorandom numbers from a lognormal distribution.

10.0 APPENDIX E

SAMPLE SAS/GRAPH PROGRAM FOR RANDOM3 AND RANDOM4

```
data a;
INFILE 'PLOT1.CPR' FIRSTOBS=2;input x y;
GOPTIONS DEVICE=HP7470;
proc gplot;
   axis1 label=(h=1 f=simplex 'LOG OF CYCLES')
         value=(h=1 f=simplex);
   axis2 value=(h=1 f=simplex) label=none;
   plot y*x / haxis=axis1 vaxis=axis2;
   TITLE H=1 A=90 F=SIMPLEX 'PROBABILITY DENSITY FUNCTION';
   symbol i=spline v=square;
data B;
INFILE 'PLOT2.CPR' FIRSTOBS=2;input x y;
proc gplot;
   axis1 label=(h=1 f=simplex 'LOG OF CYCLES')
         value=(h=1 f=simplex);
   axis2 value=(h=1 f=simplex) label=none;
   plot y*x / haxis=axis1 vaxis=axis2;
   TITLE H=1 A=90 F=SIMPLEX 'CUMULATIVE DISTRIBUTION FUNCTION';
   symbol i=spline v=square;
```