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ISOTHERMAL LIFE PREDICTION OF COMPOSITE LAMINA USING A DAMAGE MECHANICS APPROACH

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SUMMARY

A method for predicting isothermal plastic fatigue life of a composite lamina is presented in which both fibers and matrix are isotropic materials. In general, the fatigue resistances of the matrix, fibers, and interfacial material must be known in order to predict composite fatigue life. Composite fatigue life in this paper is predicted using only the matrix fatigue resistance due to inelasticity micromechanisms. The effect of the fiber orientation on loading direction is accounted for while predicting composite life. The application is currently limited to isothermal cases where the internal thermal stresses that might arise from thermal strain mismatch between fibers and matrix are negligible. The theory is formulated to predict the fatigue life of a composite lamina under either load or strain control. It is applied currently to predict the life of tungsten-copper composite lamina at 260 °C under tension-tension load control. The calculated life of the lamina is in good agreement with available composite low cycle fatigue data.

INTRODUCTION

Fatigue failure of a metal matrix composite (MMC) is a complex process. Failure modes can depend on the applied load, the properties of the matrix, fibers and interface, fiber volume fraction and orientation, and the service temperature and environment. Depending on these factors, the active failure mode can be matrix dominated, fiber dominated, fiber/matrix interfacial failure, or self-similar fatigue damage (ref. 1). A fatigue life prediction method has to consider the most active mode of failure to obtain a good estimate for life.

In this work a tungsten reinforced copper composite containing unidirectional continuous fibers was studied. For specimens whose fibers were oriented parallel to the load axis, the failure mode was matrix dominated. Therefore, for the ensuing analysis this mode of failure was assumed to dominate for all angles of fiber orientations.

Further, it is assumed, for simplicity, that the composite fatigue failure can be considered as a sequence of two events where complete fatigue failure of the matrix is followed by immediate complete failure of the fibers. Matrix cracking introduces additional cyclic axial and shear stresses to the fibers. The localized nature of these stresses severely reduces the fiber residual

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life; therefore, its contribution to composite fatigue life can be neglected. The analysis in this paper is limited to composites with a strong fiber-to-matrix bond and no interfacial phase. In tungsten-copper composites the bonding is excellent, and the two constituents are mutually insoluble.

CONSTITUTIVE RELATIONSHIP

The state variable constitutive relationship used in this paper is derived in (ref. 2) and is represented by the following two equations for inelastic strain rate and the rate of evolution of the state variable X, referred to herein as "resistance to flow,"

$$\dot{\mathbf{e}}_{ij}^{i} = AO \left(J_{2}/X^{2}\right)^{r} \mathbf{s}_{ij} \tag{1}$$

$$\dot{x} = F(X) J_2^{r+1} = \pm \exp(CX+D) J_2^{r+1}$$
 (2)

where $\mathbf{e_{ij}}$ is the inelastic strain tensor and where AO, C, D and r are material constants. Values of the constants at 260 °C are given in table I. J₂ is the second invariant of the deviatoric stress tensor $\mathbf{s_{ij}}$. The sign of the rate of evolution is positive if the material hardens and negative if it softens or is damaged. The superscript i refers to inelastic strain, and the dot represents the first derivative with respect to time. The function F(X) is a material state function that controls response and determines life. The state variable increases for hardening materials and then achieves a stabilized or shake down value, X_S , after which the material resistance decreases until catastrophic failure occurs, (fig. 1). It follows for the continuously applied cycles that the integration of J_2^{r+1} over the applied cycle increases with the increase of X_S and decreases with its decrease. This integration is called the loading function L shown as a function of the number of applied cycles. The value of this integration is assumed constant and equal to its value at shake down L_S . Using this assumption and integrating equation (2) over the entire life, it follows that the relationship between the number of cycles to failure, N_f , under continuously applied cycles is related to the initial resistance X_O and shake down resistances as follows:

$$N_{f} L_{s} = \frac{[2exp(-CX_{s}-D) - exp(-CX_{o}-D)-exp(-D)]}{-C}$$
 (3)

where the resistance at failure equals zero. Equation (3) is used to predict fatigue life whenever $L_{\rm S}$, $X_{\rm S}$ and $X_{\rm O}$ are known.

SHAKE DOWN RESISTANCE

The evaluation of the shake down resistance X_S of the composite constituents requires an experimental relationship between this resistance and the applied stress under load control or the applied strain under strain control. The life prediction method used in this paper requires the evaluation of the shake down resistance X_S and load function L_S . These values are employed in equation (3) to calculate the number of cycles to failure. The shake down resistance is calculated iteratively. A value for the shake down resistance

is first assumed and used in the constitutive relationship to calculate the strain or stress response under load or strain control, respectively. The calculated response is applied to the experimental relationship between the resistance X_S and the applied stress or strain to update X_S . The updated resistance is then used to update the calculated response. This process continues until the change in the calculated shake down resistance over an iteration is negligible. The load function for a constituent is calculated from its constitutive relations taking the resistance constant and equal to its value at shake down.

LOCAL STRESSES AND STRAINS IN COMPOSITE LAMINA

Strain Control

In this section the local stresses and strains in the constituents of a composite lamina with perfectly bonded continuous fibers are considered. An element of the composite lamina is shown in figure 2(a). For convenience this element is assumed to be composed of uniformly spaced square fibers and matrix strips and is subjected to an in-plane stress that is produced by a biaxial strain control. It is required to evaluate the stress and strain at any point in the lamina. The evaluation begins by introducing a continuum displacement field, which represents the longitudinal displacements along the centerline of fibers/and matrix strips. The continuum longitudinal strain obtained by differentiating the displacement field is assumed equal to that of fibers and matrix at any time. Therefore, the continuum longitudinal strain rate equals the fiber and matrix longitudinal strain rate. The fiber and matrix shear and strain in the lateral direction of the fibers are not equal to the continuum strain obtained by differentiating the displacement field. The equality of such strains contradicts the equilibrium at fiber surface due to the difference between fiber and matrix mechanical properties. The shear and lateral continuum strain are smaller than those of the matrix and larger than those of fibers. Let us assume a linear displacement field, and consider the problem shown in figure 2(c) where the composite is subjected to biaxial strain rates. It can be shown that the continuum longitudinal, lateral, and shear strain rates in axes parallel and perpendicular to the fibers are given by

$$\dot{\mathbf{e}}_{11c} = \dot{\mathbf{e}}_{\mathbf{x}} \cos^2 \theta + \dot{\mathbf{e}}_{\mathbf{y}} \sin^2 \theta$$

$$\dot{\mathbf{e}}_{22c} = \dot{\mathbf{e}}_{\mathbf{x}} \sin^2 \theta + \dot{\mathbf{e}}_{\mathbf{y}} \cos^2 \theta$$

$$\dot{\mathbf{e}}_{12c} = (\dot{\mathbf{e}}_{\mathbf{x}} - \dot{\mathbf{e}}_{\mathbf{y}}) \sin \theta \cos \theta$$
(4)

where \dot{e}_x and \dot{e}_y are the applied strain rates in the x and y directions, respectively; where e_{11c} , e_{22c} and e_{12c} , are the continuum axial strains measured along the fiber direction, perpendicular to the fiber direction, and the shear strain in the plane of these two directions respectively; and where θ is the angle between the x axis and fiber direction. The matrix lateral and shear strain rates are obtained from the continuum strain rates by assuming that the displacement along the centerlines of the fibers and the matrix strips is equal to those obtained from the displacement field. It follows from figure 2(a) that the longitudinal, lateral, and shear total strain rates

in the matrix strip are represented in terms of the continuum strain rate and fiber lateral and shear strain rates, as follows:

$$\dot{\theta}_{11m} = \dot{\theta}_{11c}$$

$$\dot{\theta}_{22m} = \dot{\theta}_{22c} (1 + v) - \dot{\theta}_{22f} v$$

$$\dot{\theta}_{12m} = \dot{\theta}_{12c} (1 + v) - \dot{\theta}_{12} f v$$
(5)

where v is the ratio of volume fractions, equal to v_f/v_m (vf is the volume fraction of the fibers and v_m is the volume fraction of the matrix) and where the matrix strains e_{11m} , e_{22m} , and e_{12m} and the fiber strains e_{11f} , e_{22f} , and e_{12f} (fig. 2(b)) are measured in directions parallel to the corresponding continuum strain components. The lateral and shear fiber strains have to satisfy the lateral and shear equilibrium on the fiber surfaces. For nonlinear constitutive fiber and matrix response, these stresses are obtained by integrating the equations of stress rates simultaneously with the constituents constitutive relationships. The stress tensor in both fibers and matrix is derived from their elastic strain tensor. Therefore, the stress rate tensor of fibers and matrix are derived from the corresponding elastic strain rate tensor, which is the difference between the total and inelastic strain rate tensors. It can be shown that the components of the stress rate tensors of the matrix and the fibers are represented by the following differential equations:

$$\dot{S}_{11m} = K_{m}[\dot{e}_{11c} - \dot{e}_{11m}^{i} + u_{m} (1 + v) \dot{e}_{22c} - u_{m} \dot{e}_{22m}^{i} - vu_{m} \dot{e}_{22m}^{i}] - K_{m}vu_{m}\dot{e}_{22f}^{e}$$

$$= A1 + B1 \dot{e}_{22f}$$

$$\dot{S}_{22m} = K_{m} [(1 + v) \dot{e}_{22c} - \dot{e}_{22m}^{i} + u_{m} \dot{e}_{11c} - u_{m} \dot{e}_{11m}^{i} - v \dot{e}_{22f}^{i}] - K_{m} v \dot{e}_{22f}^{e}$$

$$= A2 + B2 \dot{e}_{22f}^{e}$$

$$\dot{S}_{12m} = G_{m} [(1 + v) \dot{e}_{12c} - \dot{e}_{12m}^{i} - v \dot{e}_{12f}^{i}] - G_{m} v \dot{e}_{12f}^{e}$$

$$= A3 + B3 \dot{e}_{12f}$$

$$\dot{S}_{11f} = K_{f} [\dot{e}_{11c} - \dot{e}_{11f}^{i}] + K_{f} u_{f} \dot{e}_{22f}^{e}$$

$$= A4 + B4 \dot{e}_{22f}^{e}$$

$$\dot{S}_{22f} = \dot{S}_{22m}$$

$$\dot{S}_{12f} = \dot{S}_{12m}$$
(6)

where $K_m = E_m/(1-u_m)$ and, $K_f = E_f/((1-u_f))$ and E_m , G_m , u_m , E_f , G_f and u_f are the modulus, shear modulus, and poisson ratio for the matrix and fibers, respectively. The superscripts, e and e, refer to elastic and inelastic strain rates, respectively. The matrix stress components e0 and e1 are parallel to the matrix and fiber strain components e1 and e1 are parallel to the matrix and fiber strain components having the same indices. The rate of elastic lateral and shear strain in the fibers is obtained from the lateral and shear equilibrium between the matrix and fibers and can be written as follows:

The inelastic strain rates and the rate of evolution of the resistance both in the matrix and fibers are obtained from equations (1) and (2) upon using the appropriate material constants. The evaluation of local matrix and fiber stress state under strain control requires the simultaneous solution of eight differential equations; the first four equations of the set of equations (6), equation (7), and two evolution equations of the constituents state variables. These equations reduce to five if the fibers are elastic.

Load Control

The internal in plane stress state of the composite lamina subjected to biaxial stress control shown in figure 2(a) can be obtained by assuming a continuum stress field for the composite. In this paper the field is assumed uniform; therefore, the longitudinal, lateral, and shear continuum stress rates under the biaxial stress rates shown in figure 2 are given by

$$\dot{\mathbf{S}}_{11c} = \dot{\mathbf{S}}_{x} \cos^{2} \theta + \dot{\mathbf{S}}_{y} \sin^{2} \theta$$

$$\dot{\mathbf{S}}_{22c} = \dot{\mathbf{S}}_{x} \sin^{2} \theta + \dot{\mathbf{S}}_{y} \cos^{2} \theta$$

$$\dot{\mathbf{S}}_{12c} = -(\dot{\mathbf{S}}_{x} - \dot{\mathbf{S}}_{y}) \sin \theta \cos \theta$$
(8)

where \dot{S}_X and \dot{S}_y are the applied stress rates in the x and y directions. The continuum stresses S_{11c} , S_{22c} and S_{12c} are parallel to the continuum strains having the same indices. The lateral and shear stress in the fibers and matrix are equal to satisfy the equilibrium at fiber surfaces and are therefore assumed equal to the continuum lateral and shear stresses at any time. The longitudinal fiber stress is different from that of the matrix. The continuum longitudinal stress is an average stress that provides the same

longitudinal traction in the composite. Therefore, the longitudinal stresses of fibers and matrix are related according to the rule of mixtures, $S_{11m} = (1 + v) S_{11c} - v S_{11f}$. Notice that the longitudinal fiber and matrix total strain rates are equal. Therefore it follows that the fiber longitudinal stress rate and the stress state in fibers and matrix are given by

$$\dot{S}_{11f} = H_1 \dot{S}_{11c} + H_2 \dot{S}_{22c} + H_3 (e_{11m}^i - e_{11f}^i)$$

$$S_{11m} = (1 + v) S_{11c} - v S_{11f}$$

$$S_{22m} = S_{22f} = S_{22c}$$

$$S_{12m} = S_{12f} = S_{12c}$$
(9)

where, $H_1 = E_f[(1 + v)/(E_m + vE_f)]$, $H_2 = [(u_fE_m - u_mE_f)/(E_m + vE_f)]$, and $H_3 = [E_mE_f/(E_m + vE_f)]$. Thus the evaluation of the stress state under stress control requires the simultaneous solution of three differential equations. The first one of the set of equation (9) and two evolution equations for the state variables of the fibers and the matrix. For elastic fibers, the three equations reduce to two.

EXPERIMENTAL PROGRAM

The material studied was a 4 ply, unidirectional tungsten-fiber reinforced copper matrix composite. The volume fraction of the 0.008 in.-diameter tungsten fiber was 9 percent, and the orientation of the fibers was parallel to the load axis (i.e., θ = 0°). Load-controlled fatigue experiments were performed at 260 °C in vacuum (<1.0E - 5 torr). Five specimens were tested to failure and two other tests were interrupted.

Figure 3(a) shows the typical cyclic behavior of the composite under cyclic tension-tension load control. The composite specimen ratchets continuously up to failure. Also, the range of cyclic strain decreases continuously with cycling, and the hysteresis loops become nearly elastic at failure. Figure 3(b) is a plot of the maximum cyclic strain of six specimens as a function of the applied number of cycles. The ratcheting behavior is somewhat analogous to typical creep behavior of monolithic materials. The ratcheting rate is high at the beginning of the test and decreases to approach a steady state ratcheting rate after about 50 to 100 cycles. This steady state ratcheting regime spans over a quarter of the life for the lowest load level and up to about three-quarters of the life for the highest load level. The ratcheting rate then increases as failure approaches. The failure strain increased with increasing maximum cyclic stress, ranging from 4.7 percent for a maximum stress of 35 ksi to 12.7 percent for a maximum stress of 41.1 ksi.

Examination of the failed specimens revealed that fatigue cracks nucleated in the copper matrix via grain boundary cavitation. The cavities linked together to form cracks that grew around the fibers. However, some fibers broke before the composite specimens failed. No oxidation of the specimens was observed.

APPLICATION AND DISCUSSION

The material constants C and D should be evaluated from the fatigue data at the temperature of application. Such data were not available for tungsten and are not needed for the current application. The copper fatigue data at 260 °C were not available either. However, room temperature high cycle fatigue data of copper was reported in (ref. 3). The values of C and D at 260 °C are assumed to be equal to those at room temperature (table I). The initial values of resistance XO for both copper and tungsten were obtained to provide a calculated transient tensile response close to the observed one. These values also are given in table I.

The fatigue shake down resistance of the insitu composite constituents may be different from the resistance of the individual constituents. Therefore, the shake down resistance of the matrix is evaluated from the composite fatigue data. Specimens 3, 4, and 5 were used to develop a correlation between the matrix shake down resistance and applied stress. To develop such correlation, the tungsten fiber resistance is assumed to be constant and equal to its initial value. The matrix shake down resistance is obtained iteratively. Initially, the matrix is assumed linear elastic, and equation (9) is used to obtain an initial estimate for L_S of the matrix. This estimate is used in equation (3) to obtain an initial estimate for X_S of the matrix. Equation (9) is then used to update Ls, which is subsequently applied to equation (3) to update X_s . This process continues until the values of the L_s and X_S for the matrix converge. An effective stress is defined as the square root of the maximum value of 3*J2 for the local applied cyclic stress in the matrix. Upon correlating the shake down resistance of specimens 3, 4, and 5 to their effective stress, it can be shown that the copper shake down resistance at 260 °C is almost constant and equal to 69.0 ksi. This value for shake down is applied to predict the lives of the specimens 1, 2, 6, and 7.

The calculated lives of these specimens are given in table II. Good agreement is evident between the observed and predicted low cycle fatigue specimens 3, 4, 5, 6, and 7. The agreement is not good for the high cycle fatigue specimens 1 and 2. This can be attributed to the fact that the estimated value of $X_S = 69.0$ is obtained from composite low cycle fatigue results, which may not be appropriate for high cycle fatigue.

The variation of predicted life with fiber orientation is shown in figure 4 for the same stresses as had been applied to specimen 7. This prediction is based on the assumption that composite fatigue failure remains matrix controlled and that the matrix fails due to inelasticity micromechanisms. Notice that the effective stress and $L_{\rm S}$ in the matrix decreases as the angle of fiber orientation increases up to an angle of approximately to 33°, after which they monotonically increase as shown in figure 4. The decrease in $L_{\rm S}$ is followed by a predicted increase in matrix life. The composite life at any given angle of fiber orientation, however, could be fiber-controlled or fiber/matrix interface-controlled. An appropriate life prediction requires the predicted lives of each potential contributor on figure 4. The composite life at any orientation would be the lowest of the predicted constituent lives.

FUTURE RESEARCH

Three recommendations for future research are proposed. First, the analysis should be developed to account for the effect of inelasticity due to thermal strain. Such a step is mandatory if the model is to be applied to predict thermomechanical fatigue life. Second, it is required to conduct higher cyclic life composite fatigue experiments to improve the representation of the shake down resistance for high cycle fatigue. Third, composite fatigue experiments should be conducted at different angles of orientation to verify the validity of the proposed life prediction method.

REFERENCES

- 1. Johnson, W.S.: Fatigue Testing and Damage Development in Continuous Fiber Reinforced Metal Matrix Composites. NASA TM-100628, 1988.
- 2. Abuelfoutouh, N.M.: Prediction of Isothermal Low Cycle Fatigue Response And Associated Life of High Temperature Isotropic Materials. Submitted for publication in the 'Symposium of Plasticity 89', Tsu, Japan, July 1989.
- 3. Lukas, P.; and Kunz, L.: Effect of Grain Size on The High Cycle Fatigue Behavior of Polycrystalline Copper. Mater. Sci. Eng., vol. 85, no. 1-2, 1987, pp. 67-75.

TABLE I. - MATERIAL CONSTRUCTION

Cons	Units	Cu	W
Α0	1/sec ksi	10 000	10 000 15.00
C		2526	
×0	ksi	-37.2 23.0	800.0

TABLE II. - FATIGUE RESULTS
[θ - 0°]

Number	Maximum stress, ksi	Minimum stress, ksi	N _f , exp	N _f , theo
1	33.5	1.42	a30 419	28 430
2	34.4	1.53	a62 403	18 510
3	35.0	1.48	b12 588	12 370
4	37.2	1.53	b3 117	4 648
5	38.6	1.52	b2 727	2 464
6	40.2	1.49	1 864	1 250
7	41.1	1.51	933	8 855

aTest interrupted prior to failure

bTest used to calculate material constants

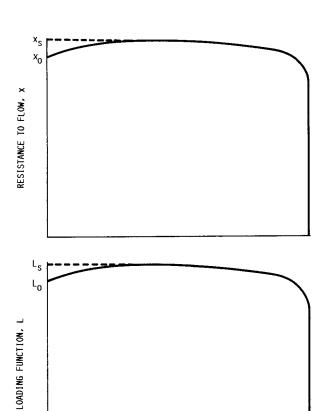
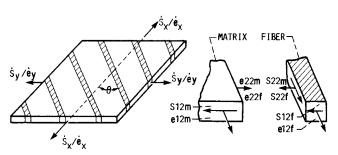


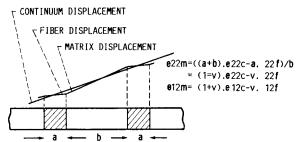
FIGURE 1. - SCHEMATIC VARATIONS IN THE RESISTANCE OF MATERIAL TO FLOW AND IN LOADING FUNCTION WITH THE NUMBER OF APPLIED FATIGUE CYCLES.

NUMBER OF CYCLES.



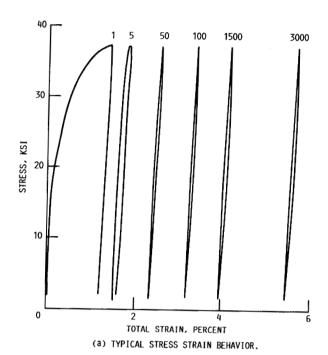
(a) APPLIED BIAXIAL STRESS AND STRAIN RATES.

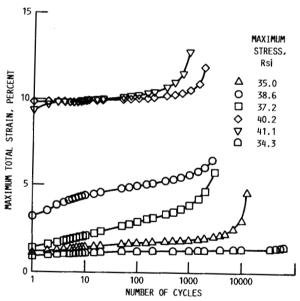
(b) FIBER AND MATRIX IN PLANS STRESSES AND STRAINS.



— a ← b → a ← (c) FIBER, MATRIX, AND CONTINUUM DISPLACEMENTS AND STRAINS IN A DIRECTION PERPENDICULAR TO FIBERS.

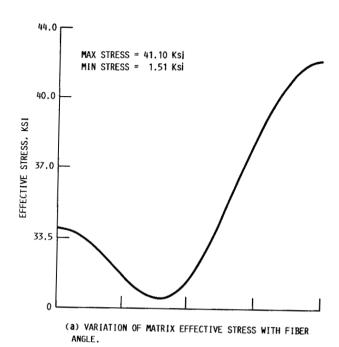
FIGURE 2. - ELEMENT OF COMPOSITE LAMINA.





(b) MAXIMUM CYCLIC STRAIN VERSUS NUMBER OF CYCLES AT SIX LEVELS OF LOAD-CONTROLLED FATIGUE.

FIGURE 3. - EXPERIMENTAL RESULTS FOR 9% W REINFORCED CU UNDER LOAD-CONTTROLLED FATIGUE AT 260 $^{\rm O}{\rm C}$.



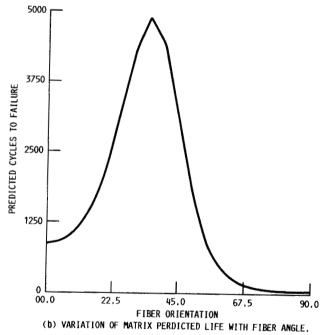


FIGURE 4. - CACULATED RESULTS.

	National Aeronautics and Space Administration	eport Docume	entation Page)		
1.	Report No. NASA TM-102032	2. Government Acces	sion No.	3. Recipient's Catalog	j No.	
4.	Title and Subtitle Isothermal Life Prediction of Composi	5. Report Date				
	Damage Mechanics Approach			6. Performing Organization Code		
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12.	Sponsoring Agency Name and Address			Technical Memorandum		
	National Aeronautics and Space Admin Washington, D.C. 20546-0001		14. Sponsoring Agency Code			
15	Supplementary Notes					
16.	Abstract					
	A method for predicting isothermal plasmatrix are isotropic materials. In generous the known in order to predict components to be known in order to predict components to the matrix fatigue resistance due to loading direction is accounted for while cases where the internal thermal stress are negligible. The theory is formulate strain control. It is applied currently to tension-tension load control. The calculate cycle fatigue data.	ral, the fatigue resis imposite fatigue life. It to inelasticity micros is predicting composit es that might arise f d to predict the fatig o predict the life of the	cances of the matrix Composite fatigue li- mechanisms. The ef- e life. The application from thermal strain is use life of a compositing sten-copper con	ife in this paper is prefect of the fiber original on is currently limite mismatch between fixite lamina under einposite lamina at 26	cial material predicted using entation on ed to isothermal libers and matrix ther load or 0 °C under	
17.	Key Words (Suggested by Author(s)) Fatigue metal life; Tungsten-copper; Moreon composite; High temperature; Constitute the prediction		18. Distribution Statem Unclassified - Subject Categ	-Unlimited		
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