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Further Developments in Modeling Digital Control Systems With MA-Prefiltered Measurements

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TECHNICAL PAPER

FURTHER DEVELOPMENTS IN MODELING DIGITAL CONTROL SYSTEMS WITH MA-PREFILTERED MEASUREMENTS

I. INTRODUCTION

Books on modern digital control systems usually treat the problem of controlling a linear time-invariant plant, driven by a zero-order-hold with a sampled output, as shown in Figure 1 (see, for example, Reference 1). The state vector $\underline{x}(t) \in R^n$; the control input vector $\underline{u}(kT) \in R^r$; the output or measurement vector $\underline{y}_I(kT) \in R^m$; the system matrix $F \in R^{nxn}$; the control matrix $G \in R^{nxr}$; the output matrix $C_I \in R^{mxn}$. It is well known that this system can be modeled at the sampling instants by the discrete state equations [1]

$$\underline{x}(\overline{k+1}T) = A\,\underline{x}(kT) + B\,\underline{u}(kT) \tag{1}$$

$$\underline{y}_I(kT) = C_I\,\underline{x}(kT) \tag{2}$$

where

$$\phi(t) = \mathcal{L}^{-1}[(sI-F)^{-1}] \in R^{nxn} \quad , \tag{3}$$

$$A = \phi(T) \in R^{nxn} \quad , \tag{4}$$

$$B = [\int_0^T \phi(\lambda)d\lambda]\, G \in R^{nxr} \quad . \tag{5}$$

In light of equation (2), $\underline{y}_I(kT)$ represents an instantaneous measure of the system at the sampling instant kT and, hence, can be thought of as being an instantaneous measurement vector.

Unfortunately, not all systems in the real world produce discrete measurements like this. For example, there exist systems where discrete measurements represent average measures of the system over the time interval T between outputs. Such systems can be found in the aerospace field wherever startrackers and some state-of-the-art rate gyroscopes annd accelerometers are used, to name a few [2]. In one system of this type, discrete measurements are generated every T/N seconds and averaged as in Figure 2. Every N measurements are averaged to produce the averaged measurement vector $\underline{y}_A(kT) \in R^h$, every T seconds. The system in Figure 2 also allows for the possibility of instantaneous measurements. Discrete state variable models for this system were derived in References 2 and 3.

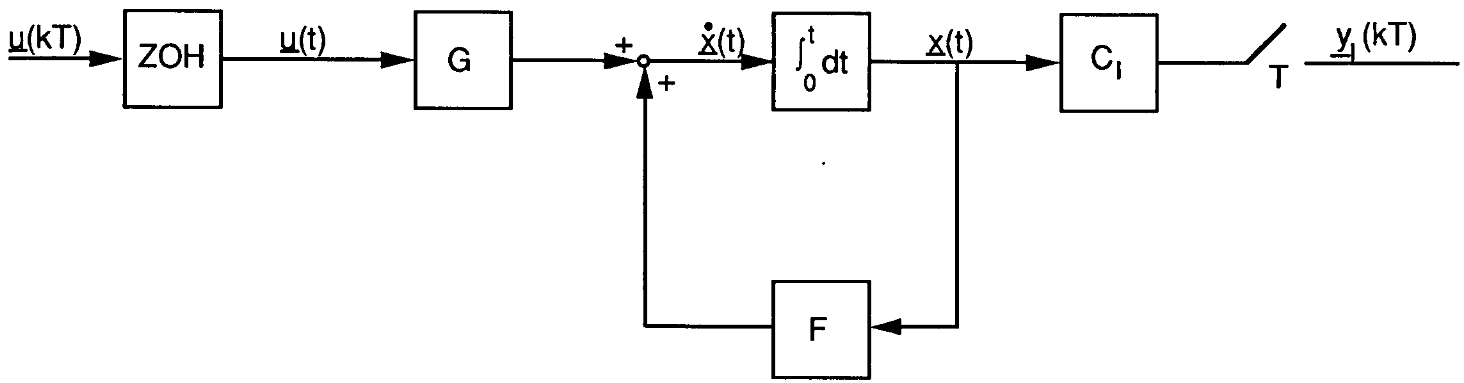

Figure 1. Linear time-invariant plant driven by a zero-order-hold with instantaneous measurements.

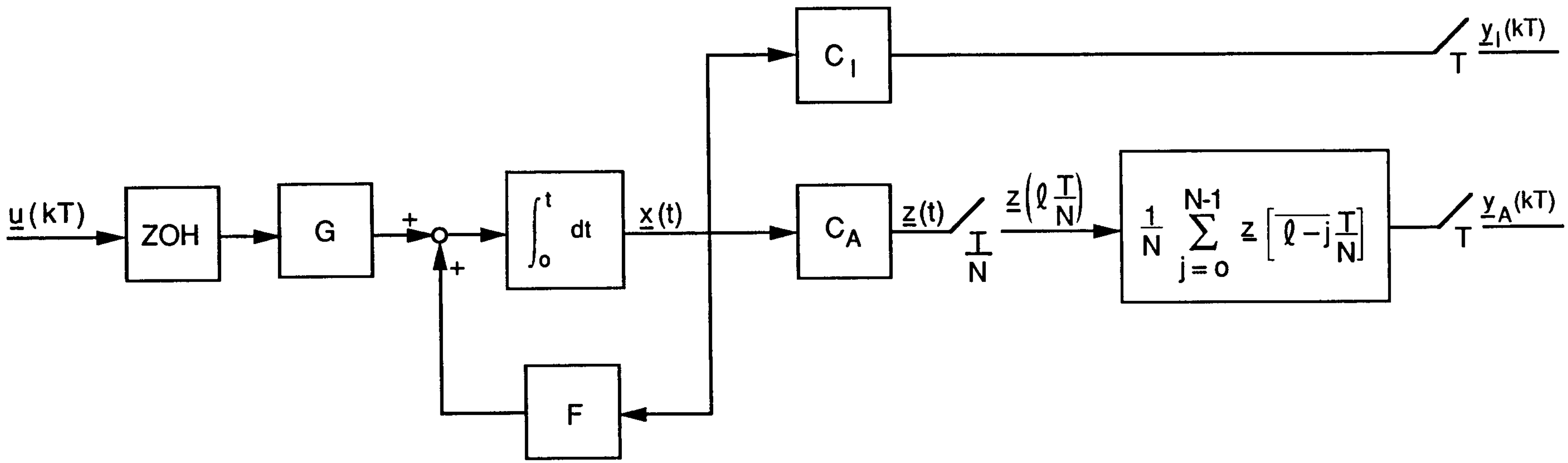

Figure 2. Linear time-invariant plant driven by a zero-order-hold with instantaneous and averaged measurements.

The system in Figure 2 was then generalized to that shown in Figure 3 and discrete state variable models for it were derived in Reference 4. In Figure 3, the output matrix $C_F \in R^{p \times n}$ and so the output vector $\underline{z}(t) \in R^p$. This is sampled every T/N seconds and multiplied by the weighting matrices $H_j \in R^{q \times p}$, $j=0,1,\ldots,N-1$. After N repetitions, the results are summed to produce the weighted-averaged measurement vector $\underline{y}_F(kT) \in R^q$ every T seconds. Functionally, this is equivalent to passing the measurements generated every T/N seconds through a multi-input/multi-output moving-average (MA) process with coefficient matrices H_j, $j=0,1,\ldots,N-1$ [5]. The output of the MA prefilter is sampled every T seconds to produce the output vector $\underline{y}_F(kT)$. Hence, $\underline{y}_F(kT)$ can also be thought of as an MA-prefiltered measurement vector. Observe that the system in Figure 2 is a special case of the one in Figure 3.

Reference 4 also derived discrete state equations for the system in Figure 4 when the matrix $E_- \in R^{q \times r}$ is uniquely chosen. The system in Figure 4 is the same as the one in Figure 3 except the MA-prefiltered measurement vector $\underline{y}_F(kT)$ is modified to yield the output vector $\underline{y}_F'(kT) \in R^q$. This system is the basis for a new type state reconstructor that is developed in References 6 to 10.

Enlightened by the work of Hagiwara and Araki [11], it became clear that the results for the systems in Figures 3 and 4 could be extended to the systems shown in Figures 5 and 6. In Figure 5, the sampling intervals on the measurements to be MA-prefiltered vary from T/2 to T/N with the length of the MA prefilters varying from 2 to N. For $i=2,3,\ldots,N$, the output matrix $C_{Fi} \in R^{p_i \times n}$; the output vector $\underline{z}_i(t) \in R^{p_i}$; the weighting matrix $H_{ij} \in R^{q_i \times p_i}$ where $j=0,1,\ldots,N-1$; the MA-prefiltered measurement vector $\underline{y}_{Fi}(kT) \in R^{q_i}$. Figure 6 is like Figure 5 except that each MA-prefilter measurement vector $\underline{y}_{Fi}(kT)$ is modified to produce the output vector $\underline{y}_{Fi}'(kT) \in R^{q_i}$. The matrix $E_{i-} \in R^{q_i \times r}$. Observe that Figures 3 and 4 are special cases of Figures 5 and 6, respectively.

This paper presents state variable representations for the systems in Figures 5 and 6. This is done in Section III by extending the results for Figures 3 and 4 that are summarized in Section II. Concluding remarks are made in Section IV.

II. PREVIOUS STATE VARIABLE REPRESENTATIONS FOR DIGITAL CONTROL SYSTEMS WITH MA-PREFILTERED MEASUREMENTS

In Reference 4, two discrete state variable representations are derived for the system in Figure 3. The first one is

$$\begin{bmatrix} \underline{x}(\overline{k+1}T) \\ \underline{\eta}(\overline{k+1}T) \end{bmatrix} = \begin{bmatrix} A & 0 \\ D_+ & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} + \begin{bmatrix} B \\ E_+ \end{bmatrix} \underline{u}(kT) \tag{6}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_F(kT) \end{bmatrix} = \begin{bmatrix} C_I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} \tag{7}$$

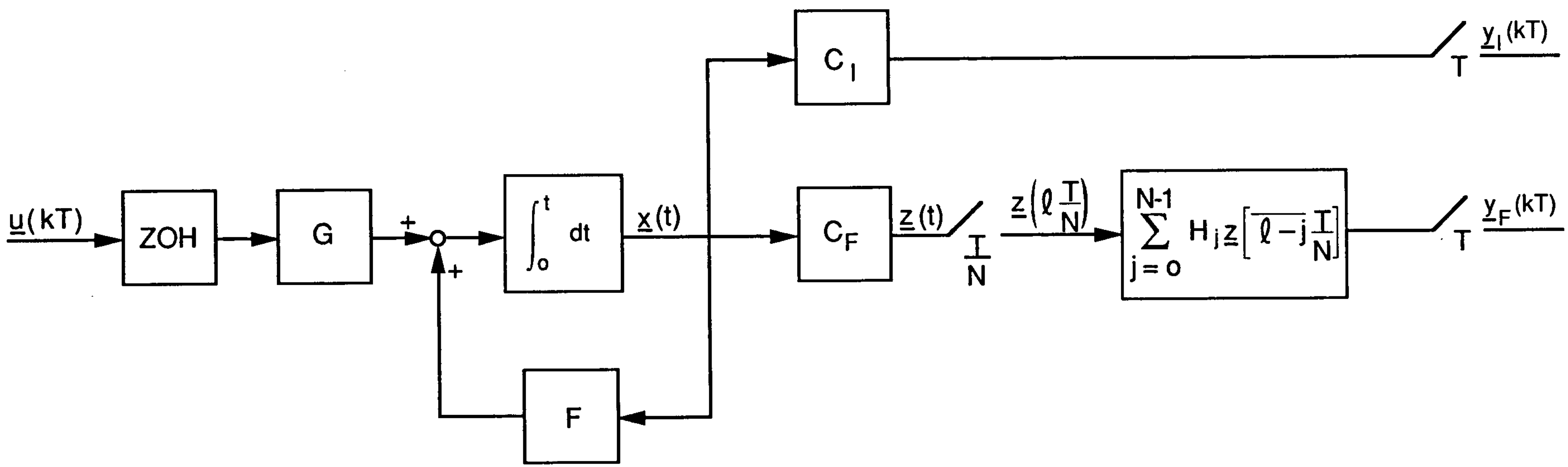

Figure 3. Linear time-invariant plant driven by a zero-order-hold with instantaneous and MA-prefiltered measurements.

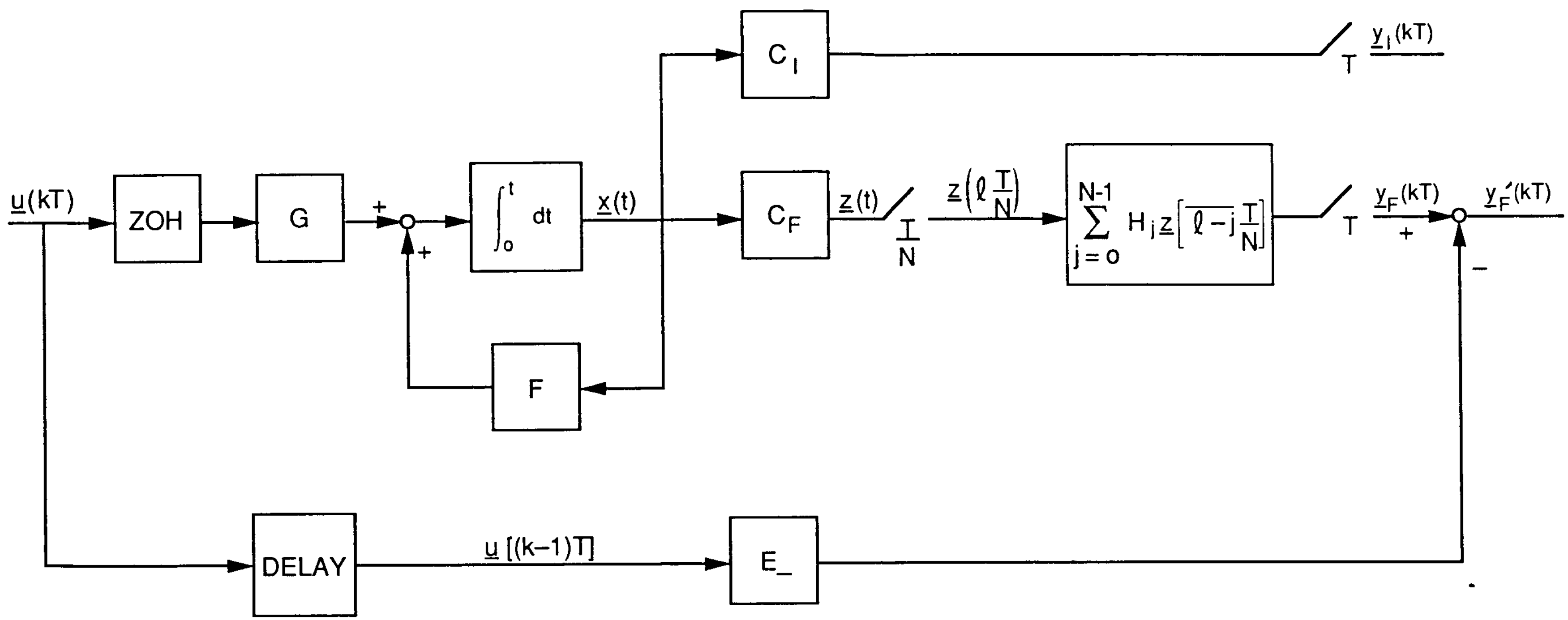

Figure 4. The system in Figure 3 with modified MA-prefiltered measurements.

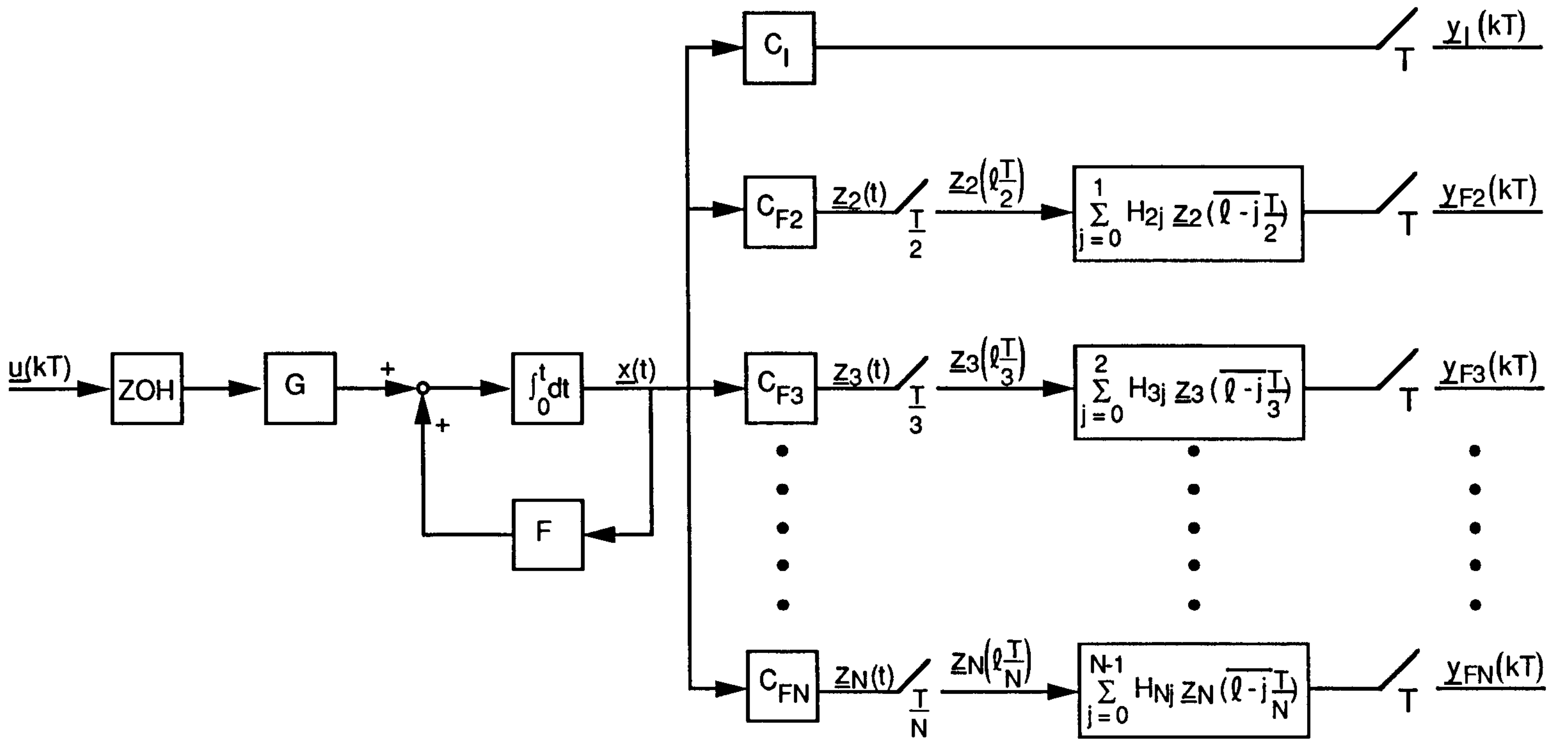

Figure 5. Linear time-invariant plant driven by a zero-order-hold with instantaneous and multirate-sampled/MA-prefiltered measurements.

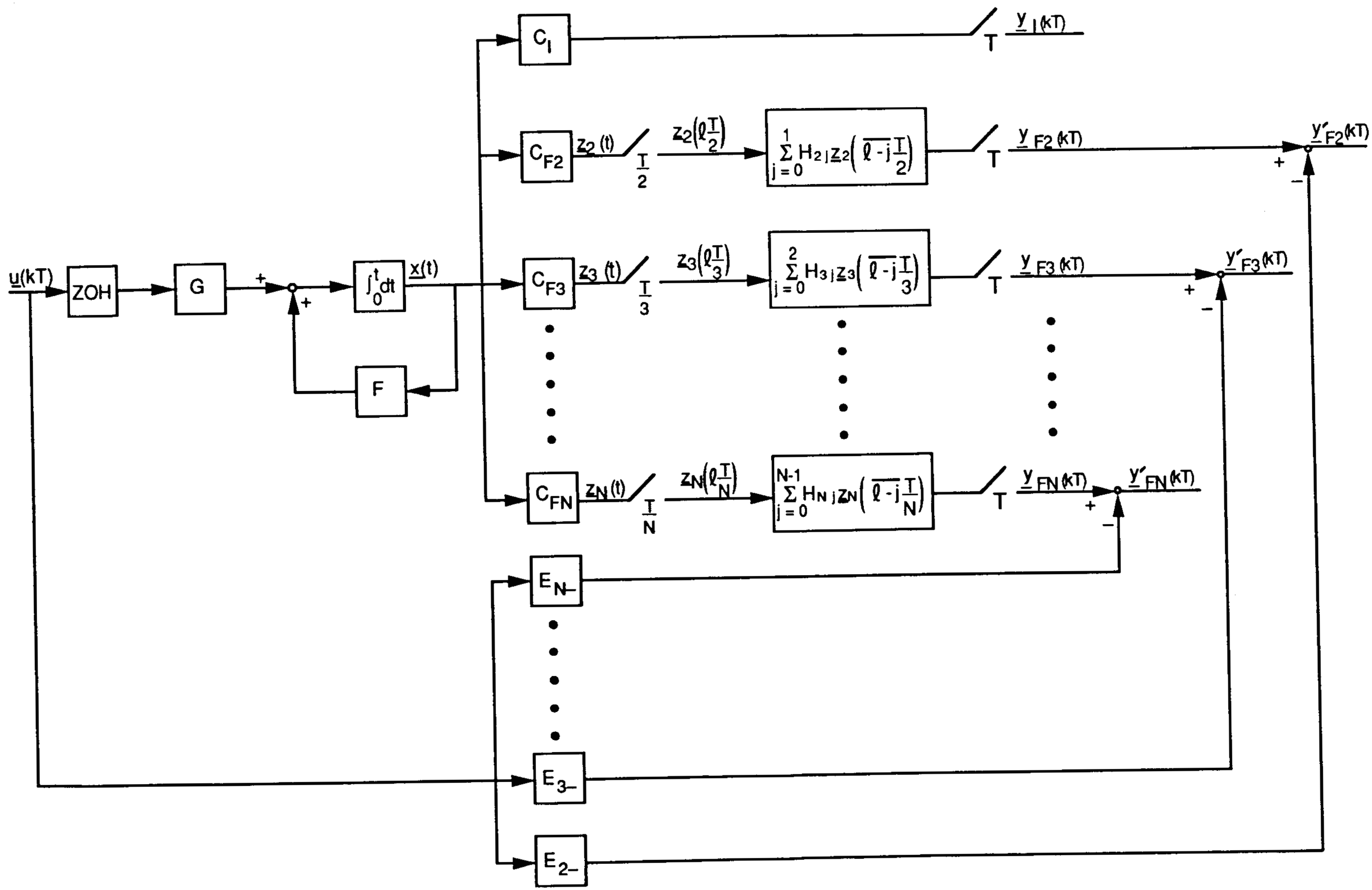

Figure 6. The system in Figure 5 with modified multirate-sampled/MA-prefiltered measurements.

where the matrices A and B are defined by equations (3) to (5), the vector $\underline{\eta}(kT) \in R^q$, and the matrix I is a qxq identity matrix. The matrix D_+ is defined by the relationship

$$D_+ = H\alpha_+ \in R^{qxn}$$

where

$$H = [H_0 \mid H_1 \mid \ldots \mid H_{N-1}] \in R^{qx(Np)}$$

and

$$\alpha_+ = \begin{bmatrix} C_F\ \phi(t) \\ C_F\ \phi[T-(T/N)] \\ \vdots \\ C_F\ \phi[T-(N-1)T/N] \end{bmatrix} \in R^{(Np)xn} .$$

The matrix E_+ is defined by the relationship

$$E_+ = H\ \beta_+ \in R^{qxr}$$

where

$$\beta_+ = \begin{bmatrix} C_F\ [\int_0^{T} \phi(\lambda)d\lambda]\ G \\ C_F\ [\int_0^{T-T/N} \phi(\lambda)d\lambda]\ G \\ \vdots \\ C_F\ [\int_0^{T-(N-1)T/N} \phi(\lambda)d\lambda]\ G \end{bmatrix} \in R^{(Np)xr} .$$

The second state variable representation for the system in Figure 3, derived in Reference 4, is

$$\begin{bmatrix} \underline{x}(\overline{k+1}T) \\ \underline{\eta}(\overline{k+1}T) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \underline{u}(kT) \tag{8}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_F(kT) \end{bmatrix} = \begin{bmatrix} C_I & 0 \\ D_- & E_- \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} \tag{9}$$

where $\underline{\eta}(kT) \in R^r$ and the matrix I is an rxr identity matrix. The matrix D_- is defined by the relationship

$$D_- = H\alpha_- \in R^{qxn}$$

where

$$\alpha_- = \begin{bmatrix} C_F\ \phi(0) \\ C_F\ \phi\ (-T/N) \\ \vdots \\ C_F\ \phi\ [-(N-1)T/N] \end{bmatrix} \in R^{(Np)xn} \quad .$$

The matrix E_- is defined by the relationship

$$E_- = H\beta_- \in R^{qxr}$$

where

$$\beta_- = \begin{bmatrix} C_F\ [\int_0^0 \phi(\lambda)d\lambda]\ G \\ C_F\ [\int_0^{-(T/N)} \phi(\lambda)d\lambda]\ G \\ \vdots \\ C_F\ [\int_0^{-(N-1)(T/N)} \phi(\lambda)d\lambda]\ G \end{bmatrix} \in R^{(Np)xr} \quad .$$

In Reference 4, the system in Figure 3 was modified to become that in Figure 4. A discrete state variable representation for it was shown to be

$$\underline{x}(\overline{k+1}T) = A\underline{x}(kT) + B\underline{u}(kT) \tag{10}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_F'(kT) \end{bmatrix} = \begin{bmatrix} C_I \\ D_{-} \end{bmatrix} \underline{x}(kT) \quad . \tag{11}$$

The system in Figure 4 and this discrete state variable representation for it are the basis for the new type state reconstructor developed in References 6 to 10.

III. NEW STATE VARIABLE REPRESENTATIONS FOR DIGITAL CONTROL SYSTEMS WITH MA-PREFILTERED MEASUREMENTS

Extending the results in Section II for the system in Figure 3, two discrete state variable representations can be defined for the system in Figure 5. From equations (6) and (7), the first one is

$$\begin{bmatrix} \underline{x}(\overline{k+1}T) \\ \underline{\eta}_2(\overline{k+1}T) \\ \vdots \\ \underline{\eta}_N(\overline{k+1}T) \end{bmatrix} = \begin{bmatrix} A & \\ D_{2+} & \\ \vdots & [0] \\ D_{N+} & \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}_2(kT) \\ \vdots \\ \underline{\eta}_N(kT) \end{bmatrix} + \begin{bmatrix} B \\ E_{2+} \\ \vdots \\ E_{N+} \end{bmatrix} \underline{u}(kT) \tag{12}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_{F2}(kT) \\ \vdots \\ \underline{y}_{FN}(kT) \end{bmatrix} = \begin{bmatrix} C_I & & [0] & \\ & I_2 & & \\ & [0] & \ddots & \\ & & & I_N \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}_2(kT) \\ \vdots \\ \underline{\eta}_N(kT) \end{bmatrix} \tag{13}$$

where $i = 2,3,\ldots,N$, the vector $\underline{\eta}_i(kT) \in R^{q_i}$, and the matrix I_i is a $q_i x q_i$ identity matrix. The matrix D_{i+} is defined by the relationship

$$D_{i+} = H_i \alpha_{i+} \in R^{q_i x n}$$

where

$$H_i = [H_{i0} \mid H_{i1} \mid \dots \mid H_{i(i-1)}] \in R^{q_i x (ip_i)}$$

and

$$\alpha_{i+} = \begin{bmatrix} C_{Fi}\ \phi(T) \\ C_{Fi}\ \phi(T - \frac{T}{i}) \\ \vdots \\ C_{Fi}\ \phi[T - (i-1)\frac{T}{i}] \end{bmatrix} \in R^{(ip_i)xn} \quad .$$

The matrix E_{i+} is defined by the relationship

$$E_{i+} = H_i \beta_{i+} \in R^{q_i x r}$$

where

$$\beta_{i+} = \begin{bmatrix} C_{Fi}\ [\int_0^{T} \phi(\lambda)d\lambda]\ G \\ C_{Fi}\ [\int_0^{T-T/i} \phi(\lambda)d\lambda]\ G \\ \vdots \\ C_{Fi}\ [\int_0^{T-(i-1)T/i} \phi(\lambda)d\lambda]\ G \end{bmatrix} \in R^{(ip_i)xr} \quad .$$

From equations (8) and (9), the second state variable representation for the system in Figure 5 is

$$\begin{bmatrix} \underline{x}(k+1T) \\ \underline{\eta}(k+1T) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} u(kT) \tag{14}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_{F2}(kT) \\ \vdots \\ \underline{y}_{FN}(kT) \end{bmatrix} = \begin{bmatrix} C_I & 0 \\ D_{2-} & E_{2-} \\ \vdots & \vdots \\ D_{N-} & E_{N-} \end{bmatrix} \begin{bmatrix} \underline{x}(kT) \\ \underline{\eta}(kT) \end{bmatrix} \tag{15}$$

where all vectors and matrices have been previously defined except for the following. For $i=2,\ldots,N$, the matrix D_{i-} is defined by the relationship

$$D_{i-} = H_i \alpha_{i-} \in R^{q_i x n}$$

where

$$\alpha_{i-} = \begin{bmatrix} C_{Fi}\ \phi(0) \\ C_{Fi}\ \phi(-T/i) \\ \vdots \\ C_{Fi}\ \phi[-(i-1)\ T/i] \end{bmatrix} \in R^{(ip_i)xn} \quad .$$

The matrix E_{i-} is defined by the relationship

$$E_{i-} = H_i \beta_{i-} \in R^{q_i x r}$$

where

$$\beta_{i-} = \begin{bmatrix} C_{Fi} \, [\int_0^{0} \phi(\lambda)d\lambda] \, G \\ C_{Fi} \, [\int_0^{-T/i} \phi(\lambda)d\lambda] \, G \\ \vdots \\ C_{Fi} \, [\int_0^{-(i-1)(T/i)} \phi(\lambda)d\lambda] \, G \end{bmatrix} \in R^{(ip_i)xr} \quad .$$

For the system in Figure 6, a discrete state variable representation can be formed by extending the results in Section II for the system in Figure 4. From equations (10) and (11), the result is

$$\underline{x}(\overline{k+1}T) = A\underline{x}(kT) + B\underline{u}(kT) \tag{16}$$

$$\begin{bmatrix} \underline{y}_I(kT) \\ \underline{y}_{F2}'(kT) \\ \vdots \\ \underline{y}_{FN}'(kT) \end{bmatrix} = \begin{bmatrix} C_I \\ D_{2-} \\ \vdots \\ D_{N-} \end{bmatrix} \underline{x}(kT) \quad , \tag{17}$$

where all vectors and matrices have been previously defined.

IV. CONCLUDING REMARKS

This paper presented new general state variable representations for digital control systems with MA-prefiltered measurements. The previous models, derived in Reference 4 for the systems in Figures 3 and 4, were extended to the systems in Figures 5 and 6.

For the system in Figure 5, two models were presented. One is defined by equations (12) and (13) while the other by equations (14) and (15). As to which one is best to use for a given system, the following advice is offered. Choose the one which yields the least number of states. The resulting discrete state equations are more likely to be observable and controllable.

For the system in Figure 6, the model presented is defined by equations (16) and (17). It is especially intriguing because it has the same plant equation as the state variable model given by equations (1) and (2) for the system in Figure 1.

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16. ABSTRACT

This paper presents new state variable representations for a continuous-time plant driven by a zero-order-hold with multirate-sampled measurements prefiltered by multi-input/multi-output moving average (MA) processes. These representations have broad application, but are known to be useful in the aerospace field for modeling systems with startrackers and some state-of-the-art rate-gyroscopes and accelerometers.

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