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A numerical study of bifurcations in a barotropic shear flow

By P. Huerre[1], L. R. Keefe[2], G. Meunier[1], M. M. Rogers[3], L. G. Redekopp[1], and P. R. Spalart[3]

1. Introduction

In the last few years, more and more evidence has emerged suggesting that transition to turbulence may be viewed as a succession of bifurcations to deterministic chaos. Most experimental and numerical observations have, for the most part, been restricted to Rayleigh-Benard convection and Taylor-Couette flow between concentric cylinders. The goal of the present study is to accurately describe the bifurcation sequence leading to chaos in a two-dimensional temporal free shear layer on the β-plane.

The β-plane is a locally-Cartesian reduction of the equations describing the dynamics of a shallow layer of fluid on a rotating spherical planet. It is a valid model for large-scale flows of interest in meteorology and oceanography. The mathematical formulation involves the following vorticity equation

$$\left(\frac{\partial}{\partial t}+U(y)\frac{\partial}{\partial x}\right)\nabla^2\Psi+J(\nabla^2\Psi,\Psi)+\left(\beta-U''(y)\right)\Psi_x=\frac{1}{Re}\nabla^4\Psi,$$

where $U(y)$ is the basic hyperbolic-tangent velocity profile, Ψ is the perturbation stream function, and Re is the Reynolds number. The symbol β denotes the gradient of the planetary vorticity. Inviscid linearized stability analyses (Dickinson and Clare 1973) reveal that the flow is unstable within a neutral curve in the $\beta-k$ plane as sketched in Figure 1 (from Burns & Maslowe 1983), k denoting the streamwise wavenumber. Thus β is a convenient control parameter with which one can bring about qualitative changes in the attractor as a function of the supercriticality $|\beta-\beta_c|$. The limit of the classical mixing layer is approached as β goes to zero.

2. Numerical approach

The three-dimensional code developed by M. Rogers has been adapted to the present problem by adding the appropriate β terms in the equations. The basic velocity profile has been changed to $U(y)=\tanh(y)$ (a body force is applied to hold that velocity profile against viscous diffusion). We recall that periodic boundary conditions are chosen in the streamwise direction and that a Cain mapping is used in

1 Department of Aerospace Engineering, University of Southern California
2 Center for Turbulence Research
3 NASA/Ames Research Center

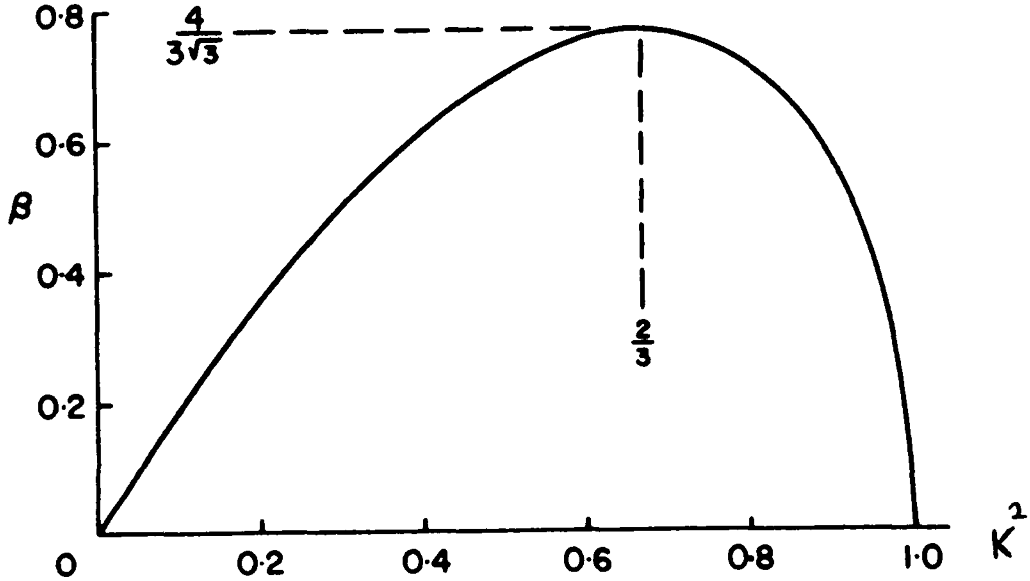

FIGURE 1. Neutral curve of linear theory in inviscid limit.

the cross-stream direction. The exponential tails in y for the perturbation variables are known to decay more slowly than in the $\beta = 0$ case and the value of the B stretching factor in y had to be increased up to 10 (typical values are around 2) to properly account for this phenomenon. The numerical code is well-behaved with 16 by 96 grids, but the execution is slow (8 Mflops on the Cray 2) due to the small length of the vectors.

3. The first Hopf bifurcation

Numerical simulations were conducted at $Re = 1000$ and a streamwise box length corresponding to the critical wavenumber $k_c = \sqrt{2/3}$. The critical value $\beta_c = 4/(3\sqrt{3})$ was well approximated and linear growth rates were also found to be consistent with inviscid stability theory.

As β decreases below β_c, the basic flow bifurcates to a finite-amplitude *traveling-wave state* which takes the form of a vortex as shown on Figure 2. As a result of the β effect, the Kelvin's cat's-eye pattern is shifted off the axis $y = 0$. The traveling wave is characterized by a circular frequency ω and normalized amplitude

$$|A| \equiv \max_y \; \widehat{v}(y, k_c).$$

The normal form pertaining to a supercritical Hopf bifurcation is known to be

$$\frac{dA}{dt} = -i\left[\omega_c + \omega_\beta(\beta - \beta_c)\right]A - \Gamma|A|^2 A, \tag{1}$$

where A is the complex amplitude of the wave and ω_c the frequency at $k = k_c$, $\beta = \beta_c$. Other quantities appearing in (1) are $\omega_\beta \equiv \partial\omega/\partial\beta|_c$ and the Landau

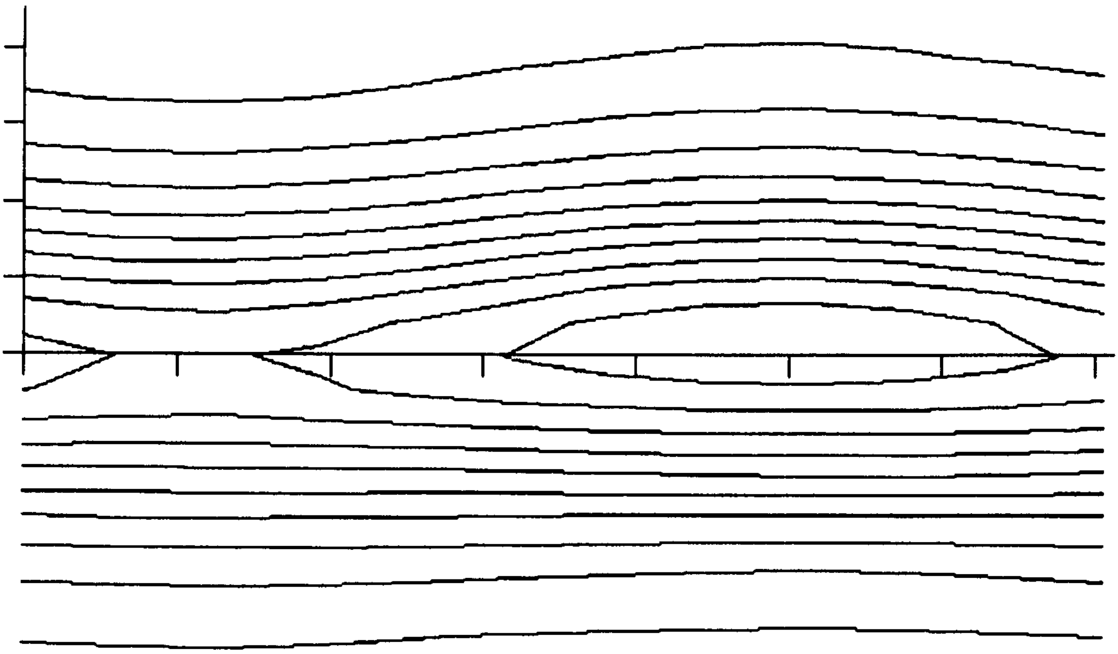

FIGURE 2. Isovorticity contours at $\beta = 0.7$, $Re = 1000$, $k = k_c$.

constant $\Gamma = \Gamma_r + i\Gamma_i$. An elementary analysis of (1) indicates that as β decreases below β_c, the solution of (1) bifurcates from the basic flow $A = 0$ to a limit cycle of amplitude

$$|A|^2 \propto \beta_c - \beta \tag{2}$$

and frequency

$$\omega - \omega_c \propto \beta_c - \beta. \tag{3}$$

The quantities $|A|^2$ and ω are represented as a function of β on Figures 3 and 4 respectively. The exact linear value of ω has also been plotted on Figure 4. One notes that $|A|^2$ and ω do vary linearly with decreasing β in the vicinity of β_c, in agreement with (2) and (3). The value of ω at $\beta = \beta_c$ is indeed equal to $\omega_c = \sqrt{2}/3$. Finally one notes a nonlinear frequency shift in the value of ω when compared to linear estimates.

We attempted to compare the values of the slopes of these curves with the analytical results obtained by Churilov (1988), Churilov and Shukhman (1986, 1987), and Lipps (1965). There were large discrepancies and we suspect that the critical-layer structure is not satisfactorily resolved with the present distribution of points in the y direction.

4. A preliminary look at pairing in the presence of dispersion

To test the validity of the code, numerical simulations of the evolution of 3 vortices were conducted in a computational domain of length $6\pi/k_c$ at $\beta = 0.2$. The

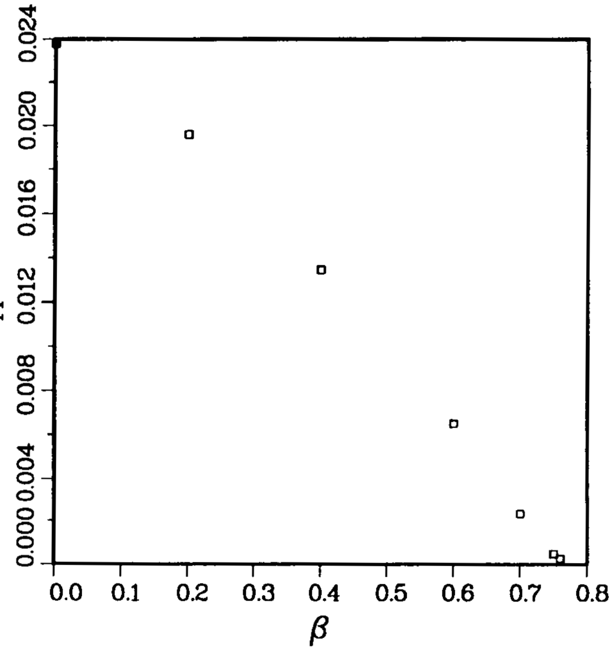

FIGURE 3. $|A|^2$ versus β at $Re = 1000$, $k = k_c$.

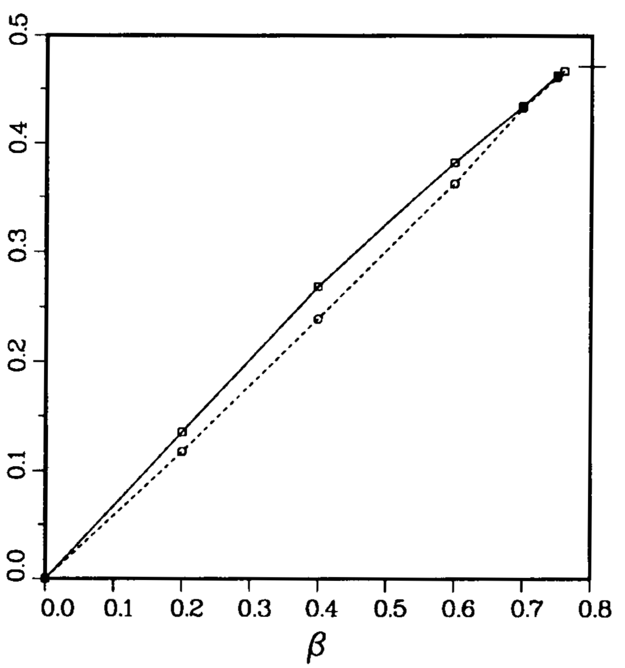

FIGURE 4. ω versus β at $Re = 1000$, $k = k_c$. —, nonlinear frequency; - - -, linear frequency.

fundamental k_c was given a finite energy level at $t = 0$, the 2 subharmonics at $k_c/3$ and $2k_c/3$ being initialized to low levels. Figures 5 a, b, c show the resulting evolution. One notes the nutation of the vortical structures in the course of time, indicated by a tilting motion. Pairing eventually takes place between 490 and 500 time steps with a corresponding energy transfer from the k_c to the $2k_c/3$ component. At $\beta = 0.2$, the $k_c/3$ component lies in the linearly stable domain and it is not triggered by nonlinear interactions. Thus, no further pairing events are observed and the asymptotic state consists of 2 vortices. This is distinctly different from the homogeneous mixing layer case where pairings take place until one observes a triple vortex condition in the computational domain.

5. Future work

The results obtained so far, namely the fact that a Hopf bifurcation takes place at $\beta = \beta_c$, suggest that nontrivial chaotic dynamics are likely to occur within subregions of the neutral curve. To back up this claim, we note that in the vicinity of β_c, the streamwise modulations of the complex amplitude $A(x,t)$ are governed by the p.d.e. equivalent of (1), i.e.:

$$\frac{\partial A}{\partial t} = -i\left[\omega_c + \omega_\beta(\beta - \beta_c)\right]A - \omega_k\frac{\partial A}{\partial x} + \frac{i}{2}\omega_{kk}\frac{\partial^2 A}{\partial x^2} - \Gamma|A^2|A.$$

This Ginzburg-Landau equation is known to give rise to chaos via a modulational instability mechanism [Keefe (1985)]. In the present context, we therefore suspect that amplitude and phase modulations applied to a row of vortices would induce a disordered evolution as the streamwise extent of the computational domain is increased.

To test this hypothesis, we need to considerably increase the reliability and speed of the numerical code for low-wavenumber simulations.

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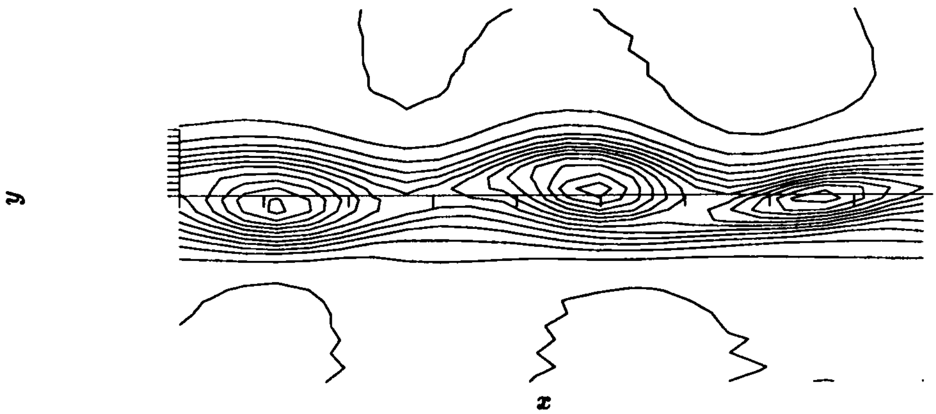
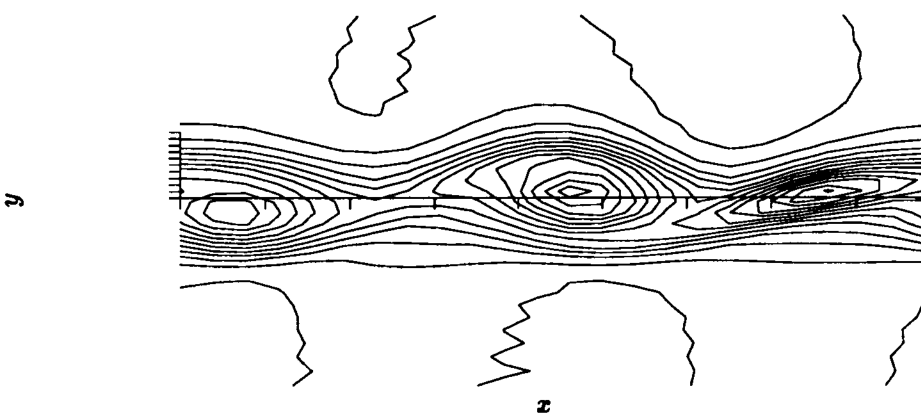
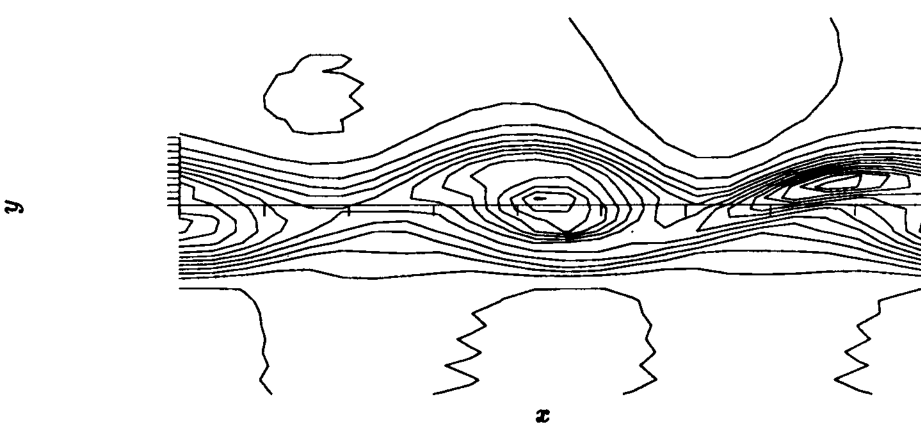


(A)



(B)



(C)

FIGURE 5A, B, C. Evolution of 3 vortices in a domain of size $6\pi/k_c$ at $\beta = 0.2$, $Re = 200$.

REFERENCES

BURNS, A. G. & MASLOWE, S. A. 1983 . *J. Atmos. Sci.* **40, 1,** 3.

CHURILOV, S. M. 1988 The nonlinear stabilization of a zonal shear flow instability, preprint.

CHURILOV, S. M. AND SHUKHMAN, I. G. 1986 . *Geophys. Astrophys. Fluid Dyn.* **36,** 31-52.

CHURILOV, S. M. AND SHUKHMAN, S. G. 1982 . *Geophys. Astrophys. Fluid Dyn.* **38,** 145-175.

DICKINSON, R. E. AND CLARE, F. J. 1973 . *J. Atm. Sci.* **30,** 1035-1049.

KEEFE, L. R. 1985 . *Stud. Appl. Math.* **73,** 91-153.

LIPPS, F. B. 1965 . *J. Fluid Mech.* **21,** 225-239.