EXPLOITING SYMMETRIES IN THE MODELING
AND ANALYSIS OF TIRES

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INTRODUCTION

In recent years nonlinear analysis of static and dynamic problems has become the focus of intense research efforts. This endeavor has prompted the development of versatile and powerful finite-element discretization methods as well as of improved numerical methods and software systems for nonlinear static and dynamic analysis of structures and solids. One of the most challenging applications of computational structural mechanics is the numerical simulation of the response of aircraft tires during taxi, takeoff and landing operations. The commonly used models for predicting the tire response are reviewed in Refs. 1 to 3. Figure 1 lists some of the difficulties encountered in the modeling and analysis of tires and their implications.

First, the tire is a composite structure composed of rubber and textile constituents which exhibit anisotropic nonhomogeneous material properties. The laminated carcass of the aircraft tire is thick enough to allow significant transverse shear deformation. *Second*, the tire geometry is complicated and due to the presence of unavoidable imperfections, the cross section is unsymmetric; and *third*, the tire is subjected to inflation pressure and to a variety of unsymmetric mechanical and thermal loads which can result in large structural rotations and deformation, as well as to a variation in the characteristics of the tire constituents. Moreover, the detailed stress and temperature distribution in the tire may require the use of three-dimensional finite elements in certain regions of the tire.

The aforementioned difficulties make the computational expense of the numerical simulation of the tire response prohibitive. Hence, the need for the development of modeling techniques and analysis methods to reduce this expense. Among the modeling strategies which show promise in reducing the cost is the exploitation of symmetries and *quasi-symmetries* exhibited by the tire response.

DIFFICULTY	IMPLICATION

o <u>TIRE SHAPE</u>
  - COMPLEX GEOMETRY
  - IMPERFECTIONS	

o <u>TIRE MATERIAL AND CONSTRUCTION</u>
  - ANISOTROPY
  - NONHOMOGENEITY
	o LARGE MODEL SIZE
	
	o COMPUTATIONALLY EXPENSIVE

o <u>TIRE LOADS</u> - UNSYMMETRY	

Figure 1

The objectives of this paper are listed in Figure 2. They are

1) To review the different types of symmetry exhibited by the tire response

2) To present simple and efficient computational procedures for reducing the size of the analysis model of tires

3) To discuss the potential of the proposed techniques and their application to practical, quasi-symmetric tire problems

To sharpen the focus of the study, discussion is limited to two-dimensional shell models of the tire, with elliptic cross-section and linear material response. The analytical formulation is based on a Sanders-Budiansky-type shell theory with transverse shear deformation, anisotropic material behavior, and geometric nonlinearities (moderate rotations) included (Refs. 4 and 5). Displacement finite-element models are used for the discretization. However, the procedure presented herein is expected to be particularly useful for the analysis of three-dimensional tire models.

OBJECTIVES

- REVIEW SYMMETRIES PRESENT IN TIRES

- PRESENT TECHNIQUES FOR MODEL-SIZE REDUCTIONS IN QUASI-SYMMETRIC PROBLEMS

- DISCUSS POTENTIAL APPLICATIONS OF TECHNIQUES

SCOPE

- TWO-DIMENSIONAL SHELL MODELS

- LINEAR MATERIAL RESPONSE

- ELLIPTIC CROSS SECTION

- DISPLACEMENT F. E. MODELS

Figure 2

SYMMETRIES EXHIBITED BY TIRE RESPONSE

The three types of symmetry commonly exhibited by the tire response are shown in Figs. 3 and 4. Also, the difference between the symmetries of orthotropic and anisotropic tires are illustrated.

The first type of symmetry is the *axial symmetry* exhibited by tires whose geometric, material characteristics, and loading are independent of the circumferential coordinate, i.e., axisymmetric. The response of these tires will also be axisymmetric. An example of this situation is that of a tire subjected to uniform inflation pressure. For an orthotropic tire the generalized displacements u, w and ϕ_1 are axisymmetric and v and ϕ_2 are zero. In contradistinction, the five generalized displacements are nonzero for an anisotropic tire.

Reflection (or mirror) symmetry with respect to coordinate planes is exhibited by the response of orthotropic tires when subjected to loadings that exhibit the same type of symmetry. Anisotropic tires, on the other hand, exhibit *rotational (or inversion) symmetry* with respect to the center of symmetry shown in Fig. 4. If these symmetries are exploited in the finite-element analysis, the size of analysis model for an anisotropic tire is twice that of the corresponding orthotropic tire (see, for example, Refs. 6 and 7).

When the external loading exhibits *periodic (or translational) symmetry*, the tire response also exhibits periodic symmetry. In orthotropic tires this is demonstrated by the presence of more than two planes of reflection symmetry. In anisotropic tires periodic symmetry is demonstrated by the presence of more than two centers of rotational (inversion) symmetry. Again, the size of the analysis model for an anisotropic tire with periodic symmetry is twice that of the corresponding orthotropic tire (see Fig. 4).
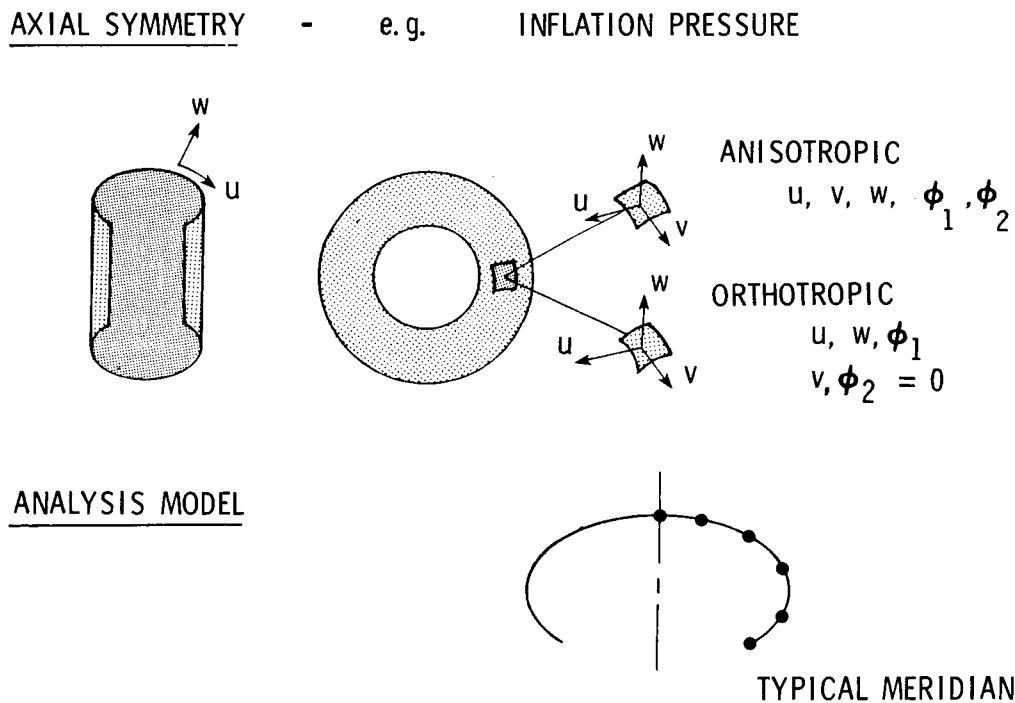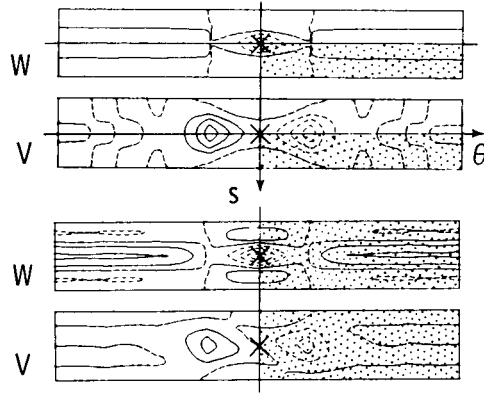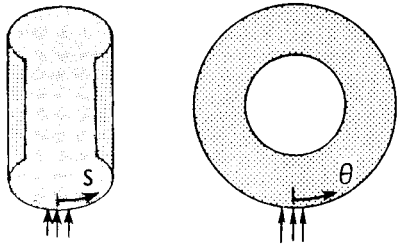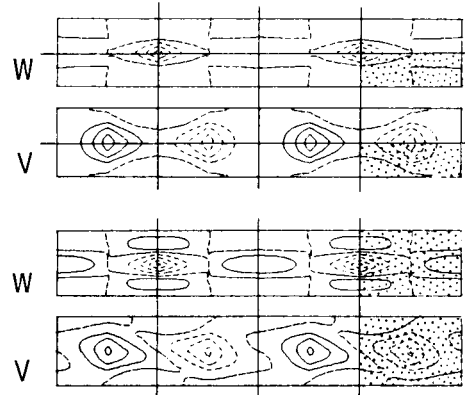
AXIAL SYMMETRY - e.g. INFLATION PRESSURE

ANISOTROPIC

u, v, w, ϕ_1, ϕ_2

ORTHOTROPIC

u, w, ϕ_1

v, ϕ_2 = 0

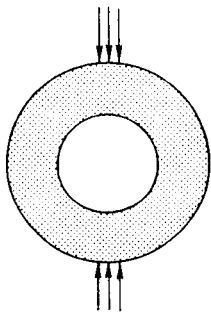ANALYSIS MODEL

TYPICAL MERIDIAN

Figure 3

REFLECTION AND
ROTATIONAL SYMMETRIES

W

V

S

θ

ORTHOTROPIC
(REFLECTION AND
ROTATIONAL SYMMETRIES)

W

V

ANISOTROPIC
(ROTATIONAL SYMMETRY
ONLY)

PERIODIC SYMMETRY

W

V

ORTHOTROPIC

W

V

ANISOTROPIC

Note: Shading shows analysis region.

Figure 4

Figure 5 lists a number of quasi-symmetric problems. The three basic quasi-symmetric problems are the ones for which either the tire material, loading, or geometry are not symmetric, but the other two are symmetric. Here material anisotropy is considered to be a source of reflection unsymmetry. The unsymmetry in geometry can be caused by the presence of unsymmetric imperfections in the tire. In a practical situation combination of the three basic types of unsymmetry can exist. In the present study, a computational procedure is presented for reducing the size of the analysis models for quasi-symmetric problems of tires to those of the corresponding symmetric problems.

CASE	MATERIAL	LOADING	GEOMETRY
I	ANISOTROPIC	SYMMETRIC	AXIALLY SYMMETRIC
II	ORTHOTROPIC	UNSYMMETRIC	AXIALLY SYMMETRIC
III	ORTHOTROPIC	SYMMETRIC	UNSYMMETRIC DUE TO IMPERFECTIONS

Figure 5

Figure 6 lists the two key elements of the model reduction technique when applied to the finite-element analysis of anisotropic tires with symmetric geometry subjected to symmetric loading. The two elements are: a) decomposition of the stiffness matrix into the sum of an orthotropic and nonorthotropic (anisotropic) parts; and b) successive application of the finite-element method and the classical Rayleigh-Ritz technique. The finite-element method is first used to generate few global approximation vectors (or modes). Then the amplitudes of these modes are computed by using the Rayleigh-Ritz technique.

ANISOTROPIC MATERIALS

- DECOMPOSITION OF MATRICES IN GOVERNING FINITE-ELEMENT EQUATIONS INTO ORTHOTROPIC AND NONORTHOTROPIC PARTS

- SUCCESSIVE APPLICATION OF

    - FINITE-ELEMENT ANALYSIS TO GENERATE A FEW GLOBAL APPROXIMATION VECTORS USING SAME SIZE MODEL AS FOR THE ORTHOTROPIC CASE

    - CLASSICAL RAYLEIGH-RITZ TECHNIQUE TO COMPUTE AMPLITUDES OF APPROXIMATION VECTORS

Figure 6

MATHEMATICAL FORMULATION

Figure 7 outlines the mathematical formulation for the proposed model-reduction technique when applied to linear problems of anisotropic tires. The global stiffness matrix [K] is decomposed into orthotropic and nonorthotropic matrices $[K]_0$ and $[K]_a$, respectively. The nonorthotropic matrix $[K]_a$ is multiplied by a tracing parameter λ which identifies *all* the nonorthotropic material coefficients. The original finite-element equations correspond to the case $\lambda=1$.

The global approximation vectors are selected to be the solution corresponding to $\lambda=0$ (zero nonorthotropic matrix) and its various-order derivatives with respect to λ (path derivatives). The path derivatives are obtained by successive differentiation of the governing finite-element equations with respect to λ. Note that the coefficient matrix appearing on the left-hand sides of the recursion formulas is $[K]_0$, and that the size of the analysis region used in evaluating each of the global approximation vectors is the same as that for the orthotropic case ($\lambda=0$).

The vector $\{X\}_0$ and its path derivatives are now chosen as approximation vectors, and the vector of nodal displacements for the anisotropic tire, $\{X\}$, is expressed as a linear combination of these vectors. A Rayleigh-Ritz technique is used to replace the original finite-element equations by a reduced system of equations in the unknown parameters, $\{\psi\}$, which represent the amplitudes of the global approximation vectors.

GOVERNING FINITE-ELEMENT EQUATIONS

$$[K]\{X\} = \{P\}$$
$$\text{LET } [K] = [K]_0 + \lambda[K]_a$$
$$\lambda = \text{TRACING PARAMETER}$$

BASIS REDUCTION

$$\{X\} = [\Gamma]\{\psi\}$$

WHERE

$$[\Gamma] = \left[\{X\}_0 , \left\{\frac{\partial X}{\partial \lambda}\right\}\Big|_0 \cdots \right]$$

$$\{\psi\} = \text{VECTOR OF AMPLITUDES OF MODES}$$

GLOBAL APPROXIMATION VECTORS

$$\left.\begin{array}{l} [K]_0 \{X\}_0 = \{P\} \\[2mm] [K]_0 \left\{\frac{\partial X}{\partial \lambda}\right\}_0 = -[K]_a\{X\}_0 \\ \vdots \end{array}\right\}$$ OBTAINED FROM SAME SIZE MODEL AS FOR THE ORTHOTROPIC CASE

REDUCED EQUATIONS (VIA RAYLEIGH-RITZ TECHNIQUE)

$$[\Gamma]^T [K][\Gamma]\{\psi\} = [\Gamma]^T \{P\}$$

Figure 7

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As a first application of the proposed model-reduction technique consider the anisotropic tire subjected to the symmetric localized loading shown in Figure 8. As shown in Figure 4, the tire response exhibits rotational (inversion) symmetry. It does not exhibit reflection symmetry, and therefore, the analysis model consists of half the tire. As can be seen from the contour plots of Figure 8, all the global approximation vectors exhibit reflection symmetry (and antisymmetry) and therefore, they can be obtained by analyzing only one quadrant of the tire (same size model as that used for analyzing the corresponding orthotropic tire).
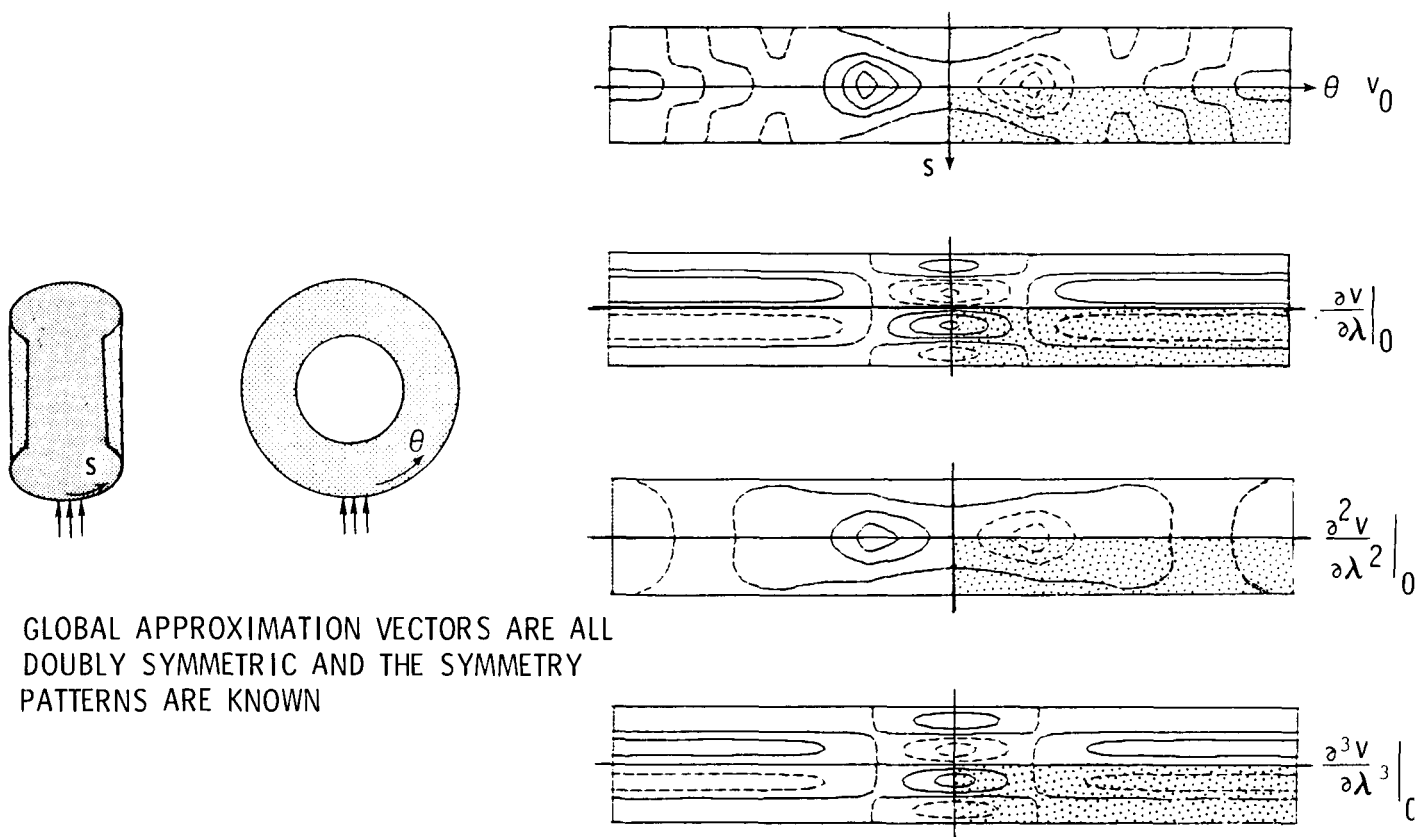
GLOBAL APPROXIMATION VECTORS ARE ALL
DOUBLY SYMMETRIC AND THE SYMMETRY
PATTERNS ARE KNOWN

Figure 8

ACCURACY AND CONVERGENCE OF SOLUTIONS OBTAINED
BY PROPOSED TECHNIQUE

An indication of the accuracy and convergence of the solutions obtained by the proposed model reduction technique is given in Fig. 9. The standard of comparison is taken to be the direct finite-element solution of the anisotropic tire. As can be seen from Fig. 9, the solutions obtained by the proposed technique are highly accurate even when a small number of approximation vectors are used. Numerical experiments have shown that for highly anisotropic tires no more than five approximation vectors are needed.
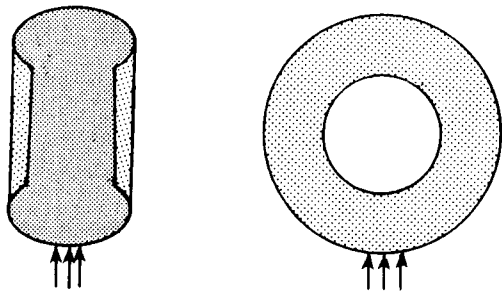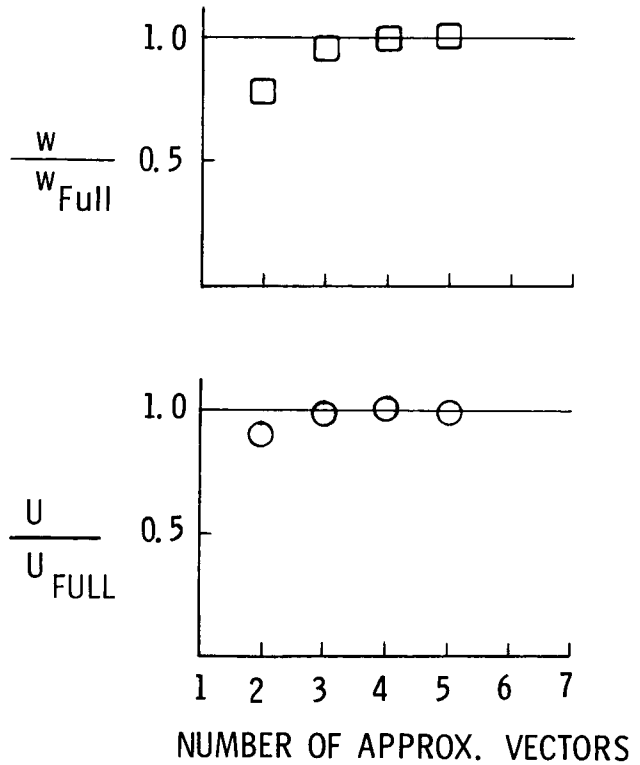
U = TOTAL STRAIN ENERGY

Figure 9

As another application of the proposed model-reduction technique, consider the orthotropic tire subjected to the localized loading shown in Fig. 10. The analysis model consists of one quadrant of the tire. For linear problems, the decomposition of the loading into symmetric and antisymmetric components and the consequent reduction of the size of the analysis model to one-octant of the tire are well known. However, it is generally assumed that such a decomposition is not useful for nonlinear problems in which the principle of superposition is not applicable.

The foregoing model-reduction technique can be used to reduce the size of the analysis model to one octant of the tire. This is accomplished by: 1) decomposition of the given loading into symmetric and antisymmetric components with load parameters p_1 and p_2, respectively; and 2) use of the multiple-parameter reduction technique described in Refs. 8 and 9. The vector of nodal displacements of the tire is approximated by a linear combination of few global vectors or modes. These vectors are selected to be the various-order derivatives of the displacment vector with respect to load parameters p_1 and p_2. Each of these vectors is evaluated at $p_1 = p_2 = 0$. Figure 10 shows that the global approximation vectors exhibit periodic symmetries (and/or antisymmetries), and therefore, each vector can be evaluated using only one octant of the tire.
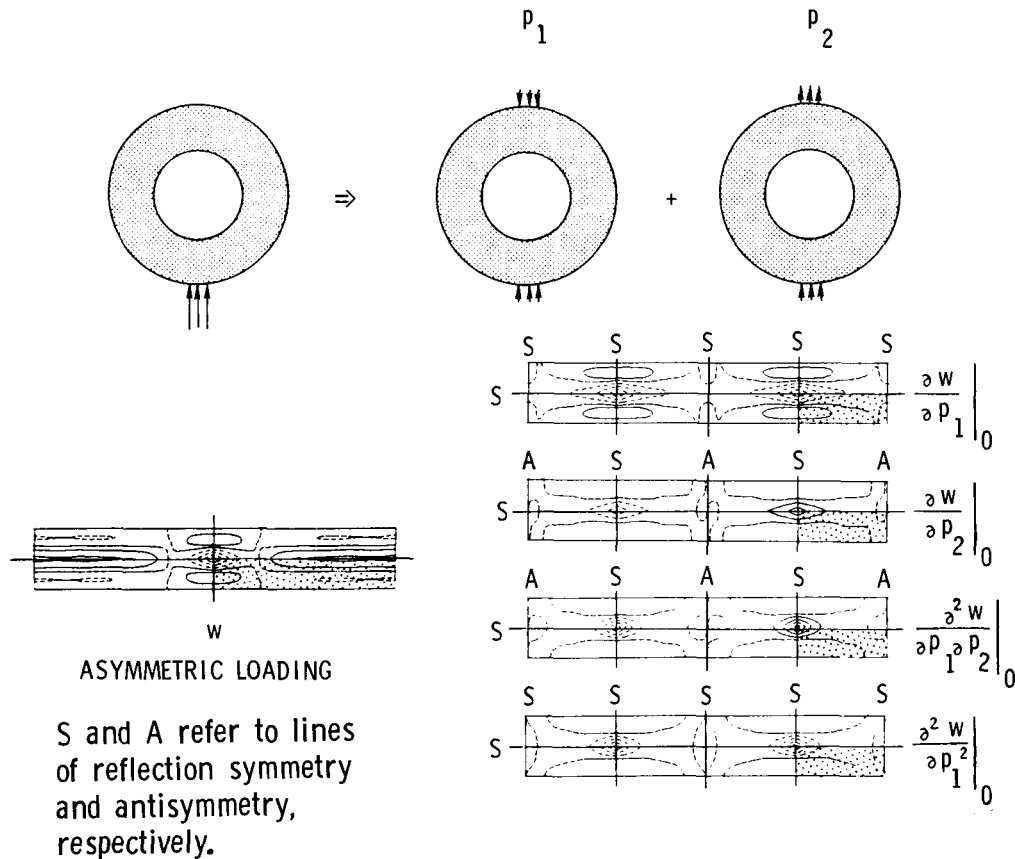

ASYMMETRIC LOADING

S and A refer to lines
of reflection symmetry
and antisymmetry,
respectively.

Figure 10

POTENTIAL OF PROPOSED MODEL-REDUCTION TECHNIQUE

The proposed model reduction technique appears to have high potential for analysis of practical tire problems. In particular, in the presence of combinations of unsymmetries in the material (viz. anisotropy), the geometry or loading on the tire, the same size model can be used as for symmetric material (viz., orthotropic), geometry and loading (Fig. 11). This is accomplished by the introduction of a tracing parameter for each of these unsymmetric effects; and the successive application of a reduction method with each of these parameters. The global approximation vectors are selected to be the various-order derivatives with respect to the tracing parameters.

● ASYMMETRIC LOADING AND ANISOTROPY

● ANISOTROPY AND INITIAL UNSYMMETRIC IMPERFECTIONS

● ASYMMETRIC LOADING AND INITIAL UNSYMMETRIC IMPERFECTIONS

● THREE-DIMENSIONAL MODELS

● MIXED FINITE-ELEMENT MODELS

Figure 11

SUMMARY

In summary, a computational procedure is presented for reducing the size of the analysis models of tires having unsymmetric material, geometry and/or loading. The two key elements of the procedure when applied to anisotropic tires are: a) decomposition of the stiffness matrix into the sum of an orthotropic and nonorthotropic parts; and b) successive application of the finite-element method and the classical Rayleigh-Ritz technique. The finite-element method is first used to generate few global approximation vectors (or modes). Then the amplitudes of these modes are computed by using the Rayleigh-Ritz technique.

The proposed technique has high potential for handling practical tire problems with anisotropic materials, unsymmetric imperfections and asymmetric loading. It is also particularly useful for use with three-dimensional finite-element models of tires.

● MODEL-SIZE REDUCTION PROCEDURE PRESENTED FOR ANALYSIS OF TIRES
  BASED ON

  - DECOMPOSITION OF MATRICES

  - SUCCESSIVE APPLICATIONS OF FINITE ELEMENTS AND
    RAYLEIGH-RITZ TECHNIQUES

● PROPOSED TECHNIQUE HAS HIGH POTENTIAL FOR HANDLING

  - ASYMMETRIC LOADING

  - ANISOTROPIC MATERIALS

  - UNSYMMETRIC IMPERFECTIONS

  - THREE DIMENSIONAL MODELS

Figure 12

REFERENCES

1. Clark, S. K. (ed.), "Mechanics of Pneumatic Tires," U.S. Dept. of Transportation, National Highway Traffic Safety Administration, Washington, D.C., 1981.

2. Tanner, J. A. (compiler), "Tire Modeling," NASA CP-2264, 1983.

3. Noor, A. K. and Tanner, J. A., "Advances and Trends in the Development of Computational Models for Tires," Computers and Structures, 1985.

4. Sanders, J. L., "Nonlinear Theories for Thin Shells," Quarterly of Applied Mathematics, Vol. 21, April 1963, pp. 21-36.

5. Budiansky, B., "Notes on Nonlinear Shell Theory," Journal of Applied Mechanics, Vol. 35, June 1968, pp. 393-401.

6. Noor, A. K. and Camin, R. A., "Symmetry Considerations for Anisotropic Shells," Computer Methods in Applied Mechanics and Engineering, Vol. 9, 1976, pp. 317-335.

7. Noor, A. K., Mathers, M. D. and Anderson, M. S., "Exploiting Symmetries for Efficient Postbuckling Analysis of Composite Plates," AIAA Journal, Vol. 15, January 1977, pp. 24-32.

8. Noor, A. K. and Peters, J. M., "Recent Advances in Reduction Methods for Instability Analysis of Structures," Computers and Structures, Vol. 16, 1983, pp. 67-80.

9. Noor, A. K., Andersen, C. M. and Tanner, J. A., "Mixed Models and Reduction Techniques for Large Rotation, Nonlinear Analysis of Shells of Revolution with Application to Tires," NASA TP-2343, October 1984.