AN INTEGRATED APPROACH TO THE OPTIMUM DESIGN OF ACTIVELY CONTROLLED COMPOSITE WINGS

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INTRODUCTION AND OUTLINE OF THE PRESENTATION

The importance of interactions among the various disciplines in airplane wing design has been recognized for quite some time. With the introduction of high gain, high authority control systems and the design of thin, flexible, lightweight composite wings, the integrated treatment of control systems, flight mechanics and dynamic aeroelasticity became a necessity. A research program is underway now aimed at extending structural synthesis (Ref. 1) concepts and methods to the integrated synthesis of lifting surfaces, spanning the disciplines of structures, aerodynamics and control for both analysis and design. Mathematical modeling techniques are carefully selected to be accurate enough for preliminary design purposes of the "complicated, built-up lifting surfaces of real aircraft with their multiple design criteria and tight constraints" (Ref. 2, p.17). The presentation opens with some observations on the multidisciplinary nature of wing design. A brief review of some available state of the art practical wing optimization programs and a brief review of current research effort in the field serve to illuminate the motivation and support the direction taken in our research.(These reviews are not exhaustive, and the interested reader is referred to the review papers, Refs. 3-8.) The goals of this research effort will be presented next, followed by a description of the analysis and behavior sensitivity techniques used. The presentation will conclude with a status report and some forecast of upcoming progress (Figure 1.).

* BRIEF REVIEW OF CURRENT WING OPTIMIZATION CAPABILITIES AND RESEARCH ACTIVITY, SOME OBSERVATIONS

* GOALS FOR MULTIDISCIPLINARY WING SYNTHESIS RESEARCH AT UCLA

* DESCRIPTION OF ANALYSIS TECHNIQUES CHOSEN

* STATUS REPORT ON THE SYNTHESIS CAPABILITY UNDER DEVELOPMENT

Figure 1

THE MULTIDISCIPLINARY NATURE OF WING DESIGN

Figure 2 describes the multidisciplinary nature of wing design. Discussion is limited to wings operating in the subsonic to low supersonic flight speeds, so that thermal effects can be neglected. It is instructive to unite the sets of Preassigned Parameters and Design Variables (Ref. 1) into the set of "Design Parameters", whose elements define a particular wing design. Which of the parameters will be preassigned and which will be used as design variables depends on the level of application for optimization techniques in the hierarchy described in Ref. 1, namely, whether the design space includes sizing, configuration (geometry) or topological design variables. The set of behavior functions, from which constraints and objectives will be selected, can be divided into two categories. Primary (system level) Behavior Functions are those performance measures which determine the overall quality and competitiveness of the wing. Secondary (sub-system level) Behavior Functions are the behavior functions which must be taken into account during the design to guarantee the prevention of failure in all possible failure modes and introduce known constraints on subsystem performance. They are usually not the real design objectives although sometimes there is high correlation between a secondary behavior and a primary behavior function (e.g. mass and airplane performance).

DISCIPLINE	STRUCTURES/ STRUCTURAL DYNAMICS	AERODYNAMICS	CONTROL
SIZING DESIGN PARAMETERS : (DESIGN VARIABLES OR PREASSIGNED) PARAMETERS	ELEMENT SIZE (AREA, THICKNESS)	CAMBER, LE/TE CONTROL SURFACE DEFLECTION	GAINS, TRANSFER FUNCTION COEFFICIENTS
CONFIGURATION DESIGN PARAMETERS	PLANFORM SHAPE (SWEEP,AR, TAPER RATIO) AIRFOIL CROSS SECTION, PLY ANGLE	PLANFORM SHAPE (SWEEP,AR TAPER RATIO) AIRFOIL CROSS SECTION	ORDER OF TRANSFER FUNCTIONS
TOPOLOGICAL DESIGN PARAMETERS :	NUMBER OF RIBS, SPARS, PLIES	NUMBER OF LE, TE CONTROL DEVICES	CONTROL SYSTEM STRUCTURE AND CONNECTIVITY
SECONDARY BEHAVIOR FUNCTIONS :	DEFLECTION/SLOPE, STRESS,BUCKLING, NATURAL FREQ'S, FATIGUE, LOAD FACTOR, MASS, CONTROL EFFECTIVENESS	DRAG, LIFT,CLMAX, DRAG POLAR, CONTROL EFFECTIVENESS	DYNAMIC STABILITY (INCLUDING FLUTTER, BODY FREEDOM FLUTTER), HINGE MOMENTS, CONTROL ACTIVITY
PRIMARY BEHAVIOR FUNCTIONS:		AIRPLANE PERFORMANCE : (ROLL RATE, TURN RATE, ENDURANCE, RANGE, COST...)	

Figure 2

SOME EXISTING PROGRAMS FOR PRACTICAL WING OPTIMIZATION

Several approaches, with a varying degree of multidisciplinary capability, aimed at the synthesis of practical composite wings were developed during the seventies (Refs. 3-8). In addition to the constraints on stress, displacement and aeroelastic stability, performance constraints in terms of induced drag or drag polar specification were added in the TSO computer code (Refs. 9,10) and to WIDOWAC (Refs. 11-13). It was reported recently that a rudimentary servoaeroelastic analysis capability was about to be inserted into the ASTROS computer code (Ref. 14). It should be noticed that except for the TSO code, the design space in the programs contains only structural design variables, thus they are really multidisciplinary in analysis only. The TSO code makes it possible to include some configuration design variables (the fiber orientation of cover skin layers) and some aerodynamic constraints in the form of wing twist or camber distribution under load (Figure 3.).

PROGRAMS SURVEYED :
• TSO, FASTOP, WIDOWAC, ELFINI, ASTROS

ANALYSIS PROBLEM

DISCIPLINES :
• STRUCTURES, AERODYNAMICS, AEROELASTICITY.

MODELING :

STRUCTURAL :
• EQUIVALENT PLATE (TSO)
• FINITE ELEMENTS (FASTOP,WIDOWAC,ELFINI,ASTROS)

AERODYNAMIC :
STEADY AERODYNAMICS :
• LINEAR POTENTIAL PANEL METHODS (e.g. Woodward in TSO)
UNSTEADY AERODYNAMICS :
• DOUBLET LATTICE ($M < 1$) (TSO, FASTOP, ASTROS)
• KERNEL FUNCTION ($M < 1$) (WIDOWAC)
• MACHBOX ($M > 1$) (FASTOP)
• POTENTIAL GRADIENT METHOD ($M > 1$) (ASTROS)
• PISTON THEORY ($M >> 1$) (WIDOWAC)

BEHAVIOR SENSITIVITY
• ANALYTIC (FASTOP, WIDOWAC, ELFINI, ASTROS)
• FINITE DIFFERENCES (TSO)

SYNTHESIS PROBLEM

PREASSIGNED PARAMETERS :
• PLANFORM SHAPE, CROSS SECTION, STRUCTURAL TOPOLOGY, MATERIALS

DESIGN VARIABLES :
• STRUCTURAL SIZING (TSO - ALSO FIBER ORIENTATION)

CONSTRAINTS :
• DEFLECTION, STRESS, FLUTTER, DIVERGENCE, CONTROL EFFECTIVENESS
• DRAG (TSO, WIDOWAC)
• BUCKLING (ELFINI, ADDED TO WIDOWAC IN A SIMPLIFIED FORM)

OBJECTIVE FUNCTIONS :
• MASS, DRAG, CONTROL EFFECTIVENESS

OPTIMIZATION
• MATH PROGRAMMING (TSO)
• MATH PROGRAMMING + APPROXIMATION CONCEPTS (WIDOWAC, ELFINI, ASTROS)
• OPTIMALITY CRITERIA (FASTOP)

Figure 3

THE NEED FOR MULTIDISCIPLINARY WING OPTIMIZATION

During the last decade structural synthesis has matured. Realistic designs described by a large number of design variables and subject to a variety of load conditions can now be efficiently treated. However, it is still quite common to find fixes and modifications being introduced late in the development stage of fighter aircraft, when aeroservoelastic effects, rigid body- elastic mode coupling or static aeroelastic effects have not been properly accounted for in the design process (Refs. 15-20). At the same time, following almost twenty years of progress in active flutter suppression and gust alleviation (Refs. 22-26), there is a growing recognition that multidisciplinary interactions might be harnessed to benefit modern, complex wing designs. However, a review of the literature reveals that the application of modern optimization methods to wing design problems involving multiple objective functions and a diverse mix of constraints based on analyses from several discipline areas (e.g. structures, structural dynamics, controls, aerodynamics and performance) has not yet been treated in a comprehensive and realistic manner. To overcome the inherent complexity and address the computationally intensive nature of this problem two approaches have been suggested in the literature. The first approach is based on the application of multi-level decomposition techniques combined with existing tools for detailed analysis and sensitivity analysis for each of the disciplines (Refs. 27,28). The second approach attempts to gain some insight into the nature of the problem by using highly simplified mathematical models or simple airplane configurations for structural, aerodynamic and control system analysis (Refs. 29-35). Research is now under way in several research centers and universities in two main directions :

a) the addition of control system sizing type design variables to a design space spanning design variables for structures and control (control augmented aeroelastic optimization) (Refs. 31,36)

b) expanding the wing design space by adding configuration design variables (structural and aerodynamic shape) (Refs. 37-39) (Figure 4.).

MOTIVATION :

PROBLEMS :

- SERVOAERELASTIC INTERACTIONS
- INSUFFICIENT CONTROL EFFECTIVENESS
- BODY FREEDOM FLUTTER

HARNESS MULTIDISCIPLINARY INTERACTIONS FOR ACHIEVING BETTER DESIGNS

CURRENT DIRECTIONS IN RESEARCH :

OPENING UP A SIZING DESIGN SPACE TO INCLUDE SIZING TYPE CONTROL SYSTEM DESIGN VARIABLES

OPENING UP THE DESIGN SPACE TO INCLUDE STRUCTURAL AND AERODYNAMIC SHAPE DESIGN VARIABLES

PROBLEMS :

- HEAVY COMPUTATIONAL COST OF ANALYSIS
- LACK OF INTUITION AND EXPERIENCE TO GUIDE IN CONSTRUCTING ROBUST APPROXIMATIONS
- ABSENCE OF STANDARD TERMINOLOGY, CRITERIA, MODELING AND ANALYSIS METHODS WIDELY ACCEPTED IN ALL DISCIPLINES

APPROACHES TO THE PROBLEM :

- SIMPLIFIED MODELING (BEAM,STRIP THEORY) OR STUDIES INVOLVING SIMPLE CONFIGURATIONS (SAILPLANES) -
  ONE LEVEL OPTIMIZATION
  GAIN INSIGHT
- MULTILEVEL DECOMPOSITION BASED ON DETAILED MODELING AND SENSITIVITY ANALYSIS

Figure 4

RESEARCH GOALS

In Ref. 2 Ashley writes :" In the absence of experience when new technology is being tried for the first time, the search for extremas can produce unanticipated, surprising and often very satisfactory discoveries". But he adds a word of caution : "Yet the counterintuitive may also be counterproductive and even ridiculous. Very undesirable consequences can result from omission or careless handling of constraints".

It is one of the major goals of the present research to begin to bridge the gap between over idealized modeling and detailed structural and aerodynamic modeling by introducing balanced design and analysis models that capture the essential behavior characteristics, without making the integrated multidisciplinary design optimization task intractable. This balanced approach combines high quality, approximate, but computationally efficient analyses for the structural, aerodynamic and aeroservoelastic behavior of realistic composite wings. Thus, the entire optimization problem may be treated at one level without the need for multilevel decomposition. A rich variety of constraints makes it possible to study the effect of multidisciplinary interactions on synthesis as well as on analysis (Figure 5.).

OBJECTIVES :

DEVELOP MULTIDISCIPLINARY WING SYNTHESIS CAPABILITY WITH AN EMPHASIS ON STRUCTURE/CONTROL/UNSTEADY AERODYNAMICS INTERACTION

BRIDGE THE GAP IN MODELING DETAIL BETWEEN THE VERY SIMPLE AND DETAILED ANALYSIS TECHNIQUES SO AS TO ENABLE MULTIDISCIPLINARY SYNTHESIS OF REAL WINGS FOR PRELIMINARY DESIGN

STUDY THE CONSTRUCTION OF ROBUST APPROXIMATIONS TO BEHAVIOR FUNCTIONS

PROVIDE A TEST CASE FOR ASSESSING DECOMPOSITION TECHNIQUES

SELECTED APPROACH

ANALYSIS :

CAREFUL SELECTION OF ANALYSIS TECHNIQUES -
GOOD ACCURACY
HIGH COMPUTATIONAL SPEED
BALANCED APPROACH

BEHAVIOR SENSITIVITY :

ANALYTIC

SYNTHESIS PROBLEM :

SIZING :
STRUCTURAL,AERODYNAMIC AND CONTROL D.V.'s PLUS Θ_i's

PREASSIGNED : SHAPE, TOPOLOGY

CONSTRAINTS : δ , σ, SERVOAEROELASIC STABILITY, CONTROL POWER

ALTERNATIVE OBJECTIVE FUNCTIONS : MASS, PERFORMANCE MEASURES

OPTIMIZATION STRATEGY :

MATH PROGRAMMING + APPROXIMATION CONCEPTS
(10 ANALYSES PER OPTIMIZATION - GOAL)

Figure 5

ANALYSIS METHODS SELECTED

The integrated optimum design capability outlined here is based on approximate analysis techniques for the required disciplines, which are consistent with each other in terms of accuracy and efficiency and lead to a balanced treatment. In the structures area, an equivalent plate analysis, as incorporated in the TSO computer code (Ref. 10) and further generalized by Giles (Refs. 37-39), is used. Although the equivalent plate approach for structural modeling of low aspect ratio wings has been known for many years, it was Giles who recently showed that, using present day computers, a single high order power series can be used for approximating displacements over wing planforms made of several trapezoidal segments to obtain accurate stress as well as displacement information. The simplicity of manipulating simple power series leads to analytic rather than numerical integration for the mass and stiffness expressions. With the careful organization of computer storage space and ordering of calculations, major savings can be achieved in terms of computation times and core storage requirements. The extended equivalent plate approach is integrated with the PCKFM (Piecewise Continuous Kernel Function Method) of Nissim and Lottati for lifting surface unsteady aerodynamics (Refs. 40-43). This method combines the power of the doublet lattice method in dealing with pressure singularities with the accuracy and speed of the kernel function method. Extensive numerical experimentation has demonstrated (Ref. 40) that the PCKFM method is highly accurate and converges rapidly. For configurations involving control surfaces, it is faster and considerably more accurate than the doublet lattice method. Thus, it is especially suited for calculating the generalized unsteady air loads (on lifting surfaces made up of wing and control surface elements) that are needed for active flutter suppression and gust alleviation studies.

For the finite state modeling of the unsteady air loads, the Minimum State Method of Karpel (Ref. 44) is used to generate accurate approximations to unsteady generalized aerodynamic forces with addition of only a small number of augmented states to the mathematical model of the aeroservoelastic system. In comparison with other finite state modeling techniques, the number of added states needed in the minimum state method can be smaller for the same overall accuracy of approximation (Ref. 45). This leads to a state space model of lower order, thus reducing memory requirements and computation times considerably. The integrated servoaeroelastic system is modeled as a Linear Time Invariant (LTI) system and its stability is examined by computing the eigenvalues of a generalized eigenvalue problem (Figure 6.).

STRUCTURE :

EQUIVALENT PLATE

AERODYNAMICS :

SUBSONIC/SUPERSONIC LIFTING SURFACE PIECEWISE CONTINUOUS KERNEL FUNCTION METHOD (PCKFM) (NISSIM/LOTTATI)

UNSTEADY AERODYNAMIC FINITE STATE MODELING :

MINIMUM STATE APPROXIMATION

CONTROL :

STATE SPACE LTI SYSTEM MODELING

Figure 6

EQUIVALENT PLATE MODELLING OF AIRPLANE/ WING/ CONTROL SURFACE ASSEMBLIES BY THE PRESENT CAPABILITY

Figure 7 shows an airplane modeled as an assembly of flexible lifting surfaces. Each lifting surface is modeled as an equivalent plate whose stiffness is controlled by contribution from thin cover skins (fiber composite laminates). and the internal structure (spar and rib caps). Plate sections are connected to each other via stiff springs (to impose displacement compatibility at attach points) and flexible springs (representing the stiffness of actuators and their backup structure). Each wing section can be made of several trapezoidal parts continuously connected to each other. Concentrated masses are used to model nonstructural items and balance masses.

The present equivalent plate modeling capability makes it possible to efficiently analyze combined wing box/control surface configurations. A wing assembly and a canard or horizontal tail may be attached to a fuselage (modeled as a flexible beam or a flexible plate) to simulate complete airplane configurations. Modeling detail of all plate sections can be identical. Thus the degree of detail in modeling control surfaces for analysis and synthesis is not limited, as is the case in the TSO code.

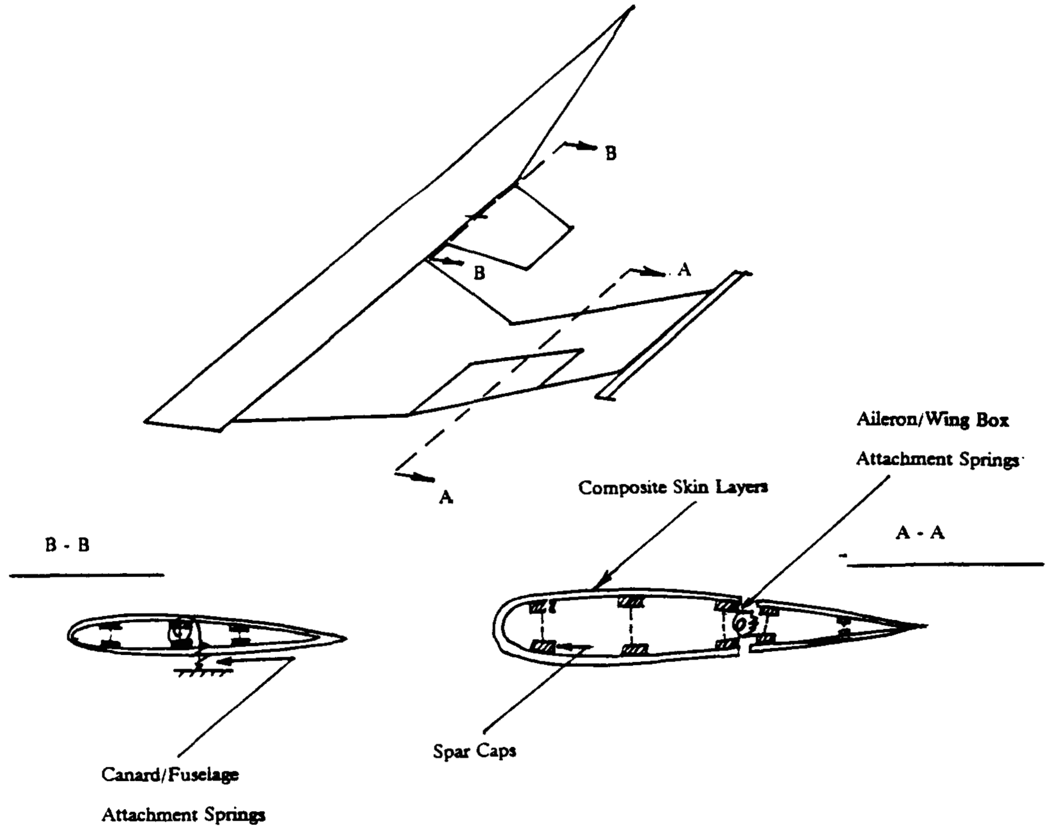

Figure 7

SOME ANALYTICAL ASPECTS OF THE EQUIVALENT PLATE APPROACH

It is a well known fact in the numerical solution of partial differential equations that the use of a simple polynomial series to approximate the solution in a Ritz or Galerkin analysis leads to ill conditioning of the problem matrices when it is of an order higher than a certain degree. However, Giles (Refs. 37,38) has shown that when a simple polynomial series is used in a Ritz solution of anisotropic plate static and dynamic problems, accurate displacements, stresses and natural frequencies can be obtained for practical wings before ill conditioning appear. His results were obtained on a CDC Cyber 173 (60 bit words). Our results obtained on an IBM 3090 computer in extended precision and on a SUN 3/280 computer using double precision support his findings. When the depth of the wing and the thickness distribution of skin layers are also expressed as power series, it can be shown that the stiffness and mass matrices are expressed as linear combinations of certain area and line integrals and polynomial terms calculated at points where wing section are connected or where concentrated masses are placed. These integrals and polynomial tables are fixed once a planform shape is given. Thus they are evaluated only once at the beginning of an optimization task. This leads to major computation time savings along with the fact that the relatively small number of generalized coordinates needed to accurately approximate displacement and stresses in a wing section (about 21-30) result in small mass and stiffness matrices(although fully populated) compared with finite element analysis (Figure 8.).

POLYNOMIAL FUNCTIONS :

THE SHAPE FUNCTIONS : $f_i(x,y) = x^m y^n \qquad (m = m(i),\ n = n(i))$

A TYPICAL SKIN LAYER THICKNESS DISTRIBUTION : $t(x,y) = \sum_{i=1}^{i=It} T_i \ x^k \ y^l$
(k,l DEPEND ON i)

WING DEPTH : $h(x,y) = \sum_{i=1}^{i=Ih} H_i \ x^r \ y^s$ (r,s DEPEND ON i)

SPAR/RIB CROSS SECTIONAL AREA : $A(x) = A_0 + A_1 x$

FUNDAMENTAL INTEGRALS :

AREA INTEGRAL OVER A SKIN TRAPEZOIDAL SECTION : $I_{mn} = \int\int x^m y^n \, dxdy$

LINE INTEGRAL OVER THE LENGTH OF A RIB : $I_m^{Rib} = \int_{x_F}^{x_A} x^m \, dx$

LINE INTEGRAL OVER THE LENGTH OF A SPAR : $I_{mn}^{Spar} = \int_{y=y_L}^{y=y_R} x(y)^m \ y^n \, dy$

ASSEMBLY :

MASS AND STIFFNESS MATRICES ARE LINEAR COMBINATIONS OF THESE INTEGRAL TERMS WITH VARYING INDICES : (m,n)

DISPLACEMENT RITZ SERIES FOR TWO WING SECTIONS CONNECTED TO EACH OTHER VIA SPRINGS :

$w_1 = [\ldots\ldots (x1)^{m1} (y1)^{n1} \ldots\ldots] \{q_1\} \qquad w_2 = [\ldots\ldots (x2)^{m2} (y2)^{n2} \ldots\ldots] \{q_2\}$

FUNDAMENTAL POLYNOMIAL TERM TABLES :

$x_1^m y_1^n \ , \ x_2^m y_2^n \ , \ x_1^m y_2^n \ , \ x_2^m y_1^n$

ASSEMBLY :
ATTACHMENT CONTRIBUTION TO STIFFNESS MATRIX IS A LINEAR COMBINATION OF POLYNOMIAL TERMS TAKEN FROM THE FUNDAMENTAL TABLES.

Figure 8

FEATURES OF THE PRESENT EQUIVALENT PLATE MODELLING

In order to structurally analyze (statics and dynamics) wing/control surface/ canard or tail configurations and to accelerate the generation of approximate problems for synthesis, the equivalent plate approach of Giles was further extended to include multi-element wing box/control surfaces plus analytic behavior sensitivity derivatives with respect to structural design variables. Stiffness and mass matrices can now be generated using analytic integration for wing structures made of composite skins, spars and ribs, concentrated masses and equivalent springs which connect plate sections to each other (Figure 9.).

CONFIGURATIONS MODELLED INCLUDE :

WING/ CONTROL SURFACE/ CANARD/ FUSELAGE ASSEMBLIES

FUSELAGE AND MISSILES CAN BE MODELLED AS EQUIVALENT BEAMS

DESIGN VARIABLES INCLUDE :

SKIN LAYER THICKNESS DISTRIBUTION POLYNOMIAL COEFFICIENTS,

SPAR/ RIB CAP AREA DISTRIBUTION (LINEAR ALONG SPAR/RIB LINE)

CONCENTRATED MASSES

LINEAR AND ROTATIONAL SPRING STIFFNESSES

ANALYSIS CAPABILITY :

FAST STIFFNESS,MASS MATRIX GENERATION

STATIC SOLUTION FOR DISPLACEMENTS AND STRESSES UNDER GIVEN LOADS

CALCULATION OF NATURAL FREQUENCIES AND MODE SHAPES

SENSITIVITY :

ANALYTIC BEHAVIOR SENSITIVITY ANALYSIS FOR DISPLACEMENTS, SLOPE, QUADRATIC FAILURE CRITERIA FOR STRESSES IN SKINS, STRESSES IN SPAR/RIB CAPS

ADJOINT OR DIRECT METHOD - OPTIONAL

Figure 9

NUMERICAL TESTING

Extensive numerical tests were carried out to study the accuracy of the present equivalent plate modeling and assess its computational efficiency. Several wings of different construction, aspect ratio and thickness were used. Displacements, stresses in skins and spar caps as well as natural frequencies and mode shapes were compared to finite element results and to test results where available. As an example, Figure 10 includes a comparison between YF16 wing natural frequencies calculated using a detailed finite element analysis, the TSO program and our present structural module. The YF16 wing configuration includes a wing box plus a leading edge flap and a flaperon. The results demonstrate the accuracy of the new multi-element equivalent plate modeling capability in analyzing wing/ control surface configurations. Some ground vibration test results available in Ref. 46 made it possible to check the accuracy of the present code when a fuselage,wing,control surfaces and tip missile configuration is analyzed. Although the first bending frequency of the cantilevered wing as calculated here is 6.5% below the reference result, it is somewhat sensitive to the modeling of root structure and a better correlation can be achieved by tuning the springs representing root and wing-fuselage attachment flexibility. Overall the correlation is good, and further refinement of the model seems unnecessary at this stage.

EQUIVALENT PLATE CAPABILITY TESTING

NATURAL FREQUENCIES (HZ) OF THE YF16

CANTILEVERED WING/LE FLAP/ FLAPERON ASSEMBLY				F-F A/C WITH WING TIP MISSILES (ANTI-SYMMETRIC)		
No.	F.E.M (REF.10)	TSO (REF.10)	PRESENT CODE	No.	GVT (REF.46)	PRESENT CODE
1	10.67	10.74	9.98	1	6.5	6.30
2	33.92	35.05	34.98	(Missile		
3	35.78	42.75	36.48	Pitch)		
4	56.45	64.24	54.02	2	8.0	7.99
5	62.47	73.43	65.28	(Wing 1st		
6	67.96	95.31	73.57	Bending)		

Figure 10

NUMERICAL TESTING (CONCLUDED)

The first six mode shapes for the cantilevered YF16 example (without tip missile), generated by the new multi-element equivalent plate analysis, are shown in Fig. 11. These mode shapes correlate well with finite element results reported in Ref. 10. The quality of this correlation can be attributed to the high order of control surface displacement representation and better modeling of elastic point attachment of the control surfaces to the wing box.

A typical computation time for the static analysis of the wing of Ref. 37 (including the calculation of 384 displacement, slope and stress constraints and their sensitivities with respect to inner and outer panel skin thicknesses at an array of points over the wing) is 12.6 cpu seconds on the UCLA IBM 3090. Analysis and constraint generation for YF16 six static load cases and natural modes take 18.9 seconds. These relatively short computation times are essential to the construction of an efficient multidisciplinary synthesis capability.

MODE SHAPES OF THE YF16 CANTILEVERED WING

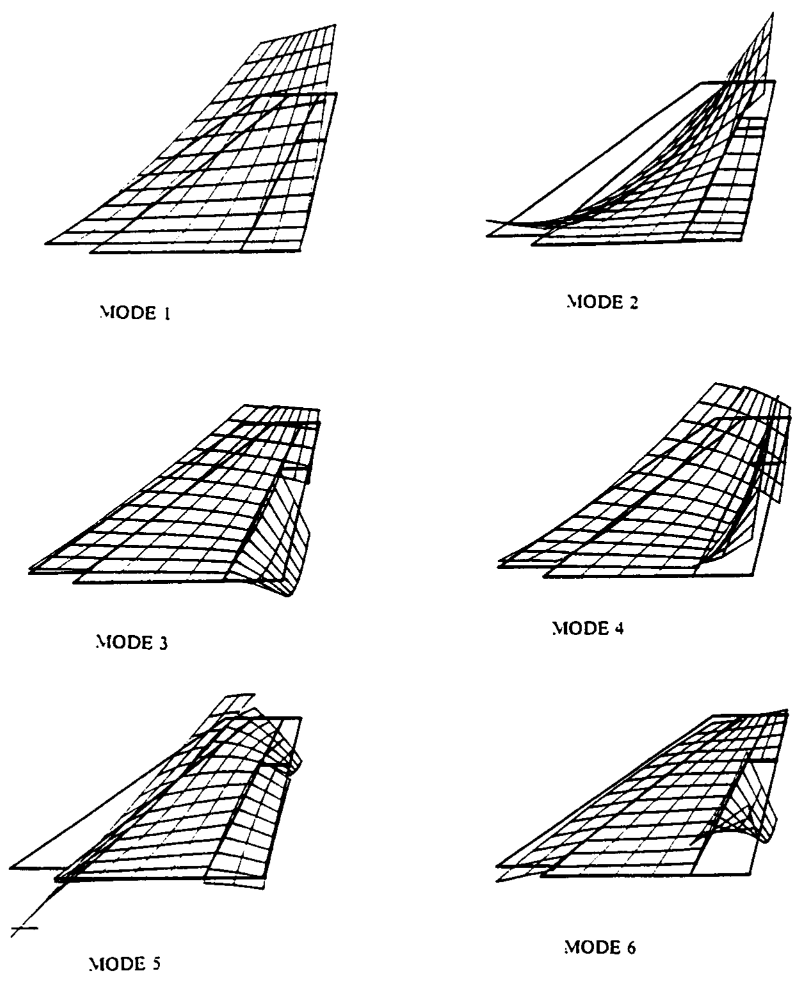

Figure 11

LIFTING SURFACE UNSTEADY AERODYNAMICS :THE PCKF METHOD

Along the line of improving the mathematical modeling of the servoaeroelastic wing dynamic system, the use of lifting surface theory (Refs. 47,48) for the calculation of the unsteady aerodynamic loads is considered a definite step forward compared with strip theories. Lifting surface aerodynamics are still widely accepted in the aerospace industry for the flutter and gust response analysis of airplanes in the subsonic and supersonic speed regimes. Thus including lifting surface modeling in the analysis part of a multidisciplinary wing synthesis is important if the synthesis of real wings is sought.

In the PCKF method for the solution of the integral equation relating downwash and pressure distribution over a lifting surface (Refs. 40-43) an assembly of lifting surfaces is divided into a group of trapezoidal boxes, as shown in Fig. 12 for a subsonic case.

MODELING A CONFIGURATION BY AN ASSEMBLY OF TRAPEZOIDAL BOXES : (SUBSONIC)

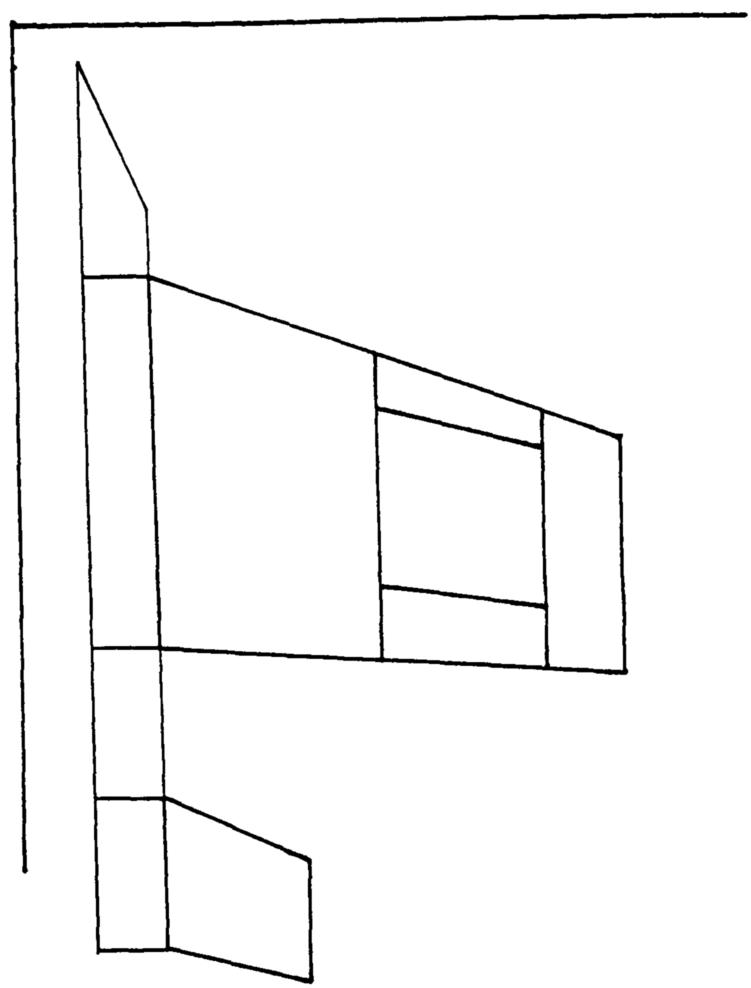

Figure 12

THE PCKF METHOD : SOME ANALYTICAL ASPECTS

The pressure distribution on each box is approximated by weighting functions representing the known pressure singularities along the box edges multiplied by a series of polynomials orthogonal to these weighting functions. Collocation points over the planform are chosen so as to minimize the error in the pressure integrals needed to calculate generalized aerodynamic forces. The PCKF method is fast, accurate and especially suited to handle wing/control surface configurations. It is more accurate than the vortex lattice method especially when leading edge flaps or controls with gaps around them are considered (Ref. 40). This is due to the inability of lattice methods to impose the pressure singularities along the different boundaries of the wing. In the present application it is integrated with the equivalent plate structural analysis to generate a set of generalized loads for the same generalized polynomial coordinates used for structural analysis. The number of collocation points per box and the number of integration points used are carefully selected to be compatible with the order of displacement polynomials used (Figure 13.).

POLYNOMIAL SERIES APPROXIMATION FOR PRESSURE OVER A BOX :

$$\frac{\Delta p(\xi,\eta)}{q_D} = \sum_{j=1}^{nspan} \sum_{i=1}^{nchord} A_m\, W(\eta)\, p_j(\eta)\, w(\xi)\, p_i(\xi)$$

where $m = (j-1)nchord + i$

$W(\eta), w(\xi)$:
WEIGHT FUNCTIONS REPRESENTING KNOWN PRESSURE SINGULARITY ALONG BOX EDGES

COLLOCATION POINT PLACEMENT : OPTIMAL SO AS TO MINIMIZE ERROR IN PRESSURE INTEGRALS (GENERALIZED AERODYNAMIC FORCES)

ADVANTAGES :

SUBSONIC / SUPERSONIC

GENERAL NON PLANAR WING/CONTROL SURFACE CONFIGURATIONS

FAST CONVERGENCE OF GENERALIZED LOADS WITH INCREASED NUMBER OF POLYNOMIALS - HIGH COMPUTATIONAL SPEED

GOOD ACCURACY OF CONTROL SURFACE HINGE MOMENT AND CONTROL SURFACE DERIVATIVES (VORTEX LATTICE METHOD OVERPREDICTS HINGE MOMENTS) - IMPORTANT FOR SERVOAEROELASTIC MODELING

EXTENSIVE NUMERICAL TESTING BY THE DEVELOPERS FOLLOWED BY ACCURATE RESULTS IN THE FLUTTER AND SERVOAEROELASTIC ANALYSIS OF THE F16 IN AN INDUSTRY ENVIRONMENT

A DEFINITE IMPROVEMENT OVER STRIP THEORIES

Figure 13

UNSTEADY AERODYNAMICS FINITE STATE MODELING

The generalized aerodynamic loads in the Laplace transformed small perturbation equations of motion (for a steady level flight) given below are transcendental functions of the Laplace variable s. The flight dynamic pressure and flight speed are q_D, U_∞ respectively; M,C,K are the mass,damping and stiffness matrices; $Q(s,Mach)$ is the matrix of generalized aerodynamic forces in the Laplace domain; $w_G(s)$ is the Laplace transformed vertical gust velocity ; S is a reference area and $\{q(s)\}$ is the vector of Laplace transformed generalized displacements.

To use modern control system analysis and design techniques, it is necessary to cast them in Linear Time Invariant (LTI) state space form. The common practice is to match rational function approximations to generalized aerodynamic loads calculated for harmonic motion at a set of reduced frequencies (Ref. 45). There is a resulting increase in the order of the LTI state space model due to the addition of aerodynamic states. This increase in size can be quite significant. With n generalized displacements, each lag term in the commonly used Roger approximation (see Ref. 45 for further detail) adds n states to the model order. Since four lag terms are usually needed for a reasonable approximation in this method, 4n states are added to the system. This makes it computationally expensive to carry out any control system analysis and behavior sensitivity analysis using state space techniques. In the Minimum State Method of Karpel (Ref. 44), the functional dependence of the generalized aerodynamic force matrix on the Laplace variable, is approximated by a rational expression of a special form so as to reduce the number of added states needed to achieve given quality of fit.

Given the generalized aero forces in simple harmonic motion for a number of reduced frequencies, it is possible to match the approximation exactly to the data for k = 0 and one other reduced frequency. This determines the matrices P_1, P_2, P_3. Choosing R to be a diagonal matrix with negative elements, the matrices D and E are determined in an iterative process so that the approximation fits the rest of the data in a least- squares manner (Figure 14.).

THE SMALL PERTURBATIONS LAPLACE TRANSFORMED EQ. OF MOTION OF AN ELASTIC AIRPLANE IN LEVEL FLIGHT :

$$\{[M]s^2 + [C]s + [K]\}\{q(s)\} - q_D S\,[Q(s,Mach)]\{q(s)\} = q_D S\,\{Q_G(s,Mach)\}\frac{w_G(s)}{U_\infty}$$

PURPOSE OF FINITE STATE MODELING : CAST EQ. OF MOTION IN LINEAR TIME INVARIANT STATE SPACE FORM

PRINCIPLE : RATIONAL FUNCTION APPROXIMATIONS OF UNSTEADY AIRLOADS IN TERMS OF LAPLACE VARIABLE

PRICE : ADDED STATES

MINIMUM STATE APPROXIMATION FORM :

$$[Q_{(s)}] = [P_1]s^2 + [P_2]s + [P_3] + [D][sI - R]^{-1}[E]s$$

MATCHING PROCESS :

- GENERALIZED AERO FORCES ARE GIVEN FOR HARMONIC MOTION AT A SET OF REDUCED FREQUENCIES
- A SET OF AERODYNAMIC LAG TERMS IS CHOSEN : R_{ii}
- P_3 IS EQUATED TO $Q(k = 0)$
- P_1, P_2 ARE EXPRESSED IN TERMS OF D,E SO AS TO ENSURE PERFECT FIT AT A SELECTED REDUCED FREQUENCY k_f
- D,E ARE DETERMINED IN AN ITERATIVE LEAST-SQUARES PROCESS TO FIT THE REST OF THE DATA

ADVANTAGE :
MINIMAL INCREASE IN MODEL ORDER

PROBLEMS :

ITERATIVE PROCESS IS TIME CONSUMING

RELATIVELY LITTLE EXPERIENCE WITH REAL CONFIGURATIONS

Figure 14

SOME PRELIMINARY MINIMUM STATE FITS TO A LARGE MATRIX OF UNSTEADY AERODYNAMIC GENERALIZED FORCES

Order reduction of the state space model used for servoaeroelastic stability and control analysis is essential for synthesis purposes in order to make the analysis cycle as computationally fast as possible. This motivates the choice of the Minimum State Approximation for finite state unsteady aerodynamic modeling in the current research. Preliminary tests of the quality of approximation achieved when applied to a large matrix of generalized aerodynamic forces show promising results. A 44 x 44 matrix of generalized aero forces for the YF16 airplane with tip missiles is approximated using only 22 lag terms. Comparison with a one lag term Roger approximation (which will add 44 aerodynamic states to the model) shows an advantage of the minimum state approach (Figure 15.).

SOME RECENT EXAMPLES OF QUALITY OF FIT FOR A YF16 COMPLETE A/C CONFIGURATION :

(44 POLYNOMIAL GENERALIZED COORDINATES)

SOME LOW ORDER SHAPE FUNCTIONS FOR THE WING BOX $f_1(x,y) = 1$, $f_2(x,y) = x$, $f_4(x,y) = x^2$

A HIGHER ORDER SHAPE FUNCTION : $f_{11}(x,y) = x^4$

ROGER APPROXIMATION BASED ON 1 LAG (44 ADDED STATES)

MINIMUM STATE BASED ON 22 LAGS (22 ADDED STATES)

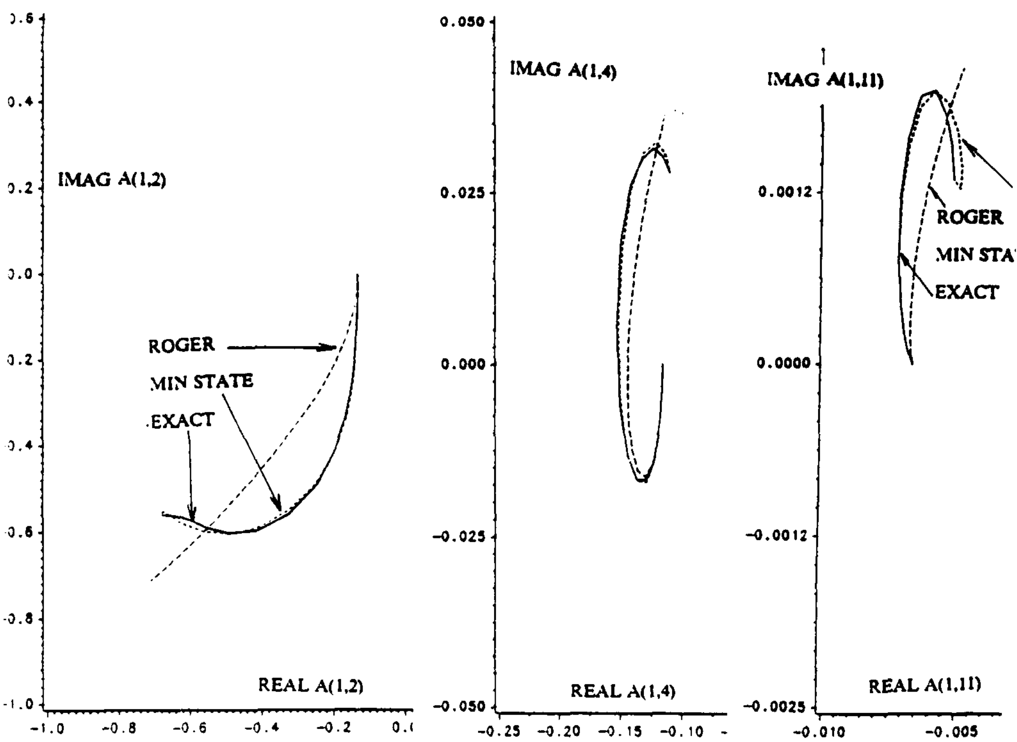

Figure 15

CONTROL SYSTEM MODELING

A block diagram of the actively controlled servoaeroelastic system is shown in figure 16. Airplane motions (acceleration and angular rates) are sensed by a set of sensors placed at different points on the structure. The resulting signals are used as inputs to the control law block which commands control surface actuators. The control surface motions guarantee stability and desirable dynamic response of the complete system.

For the control system, only sizing type design variables are considered at present to keep the balance in our approach, and these are the coefficients of numerator and denominator polynomials in the control law transfer functions. Control surface locations, sensor locations, the structure of the control system and order of transfer functions are preassigned. It is assumed that sensor and actuator transfer function are given, although the formulation is general enough to allow treating their elements as design variables as well.

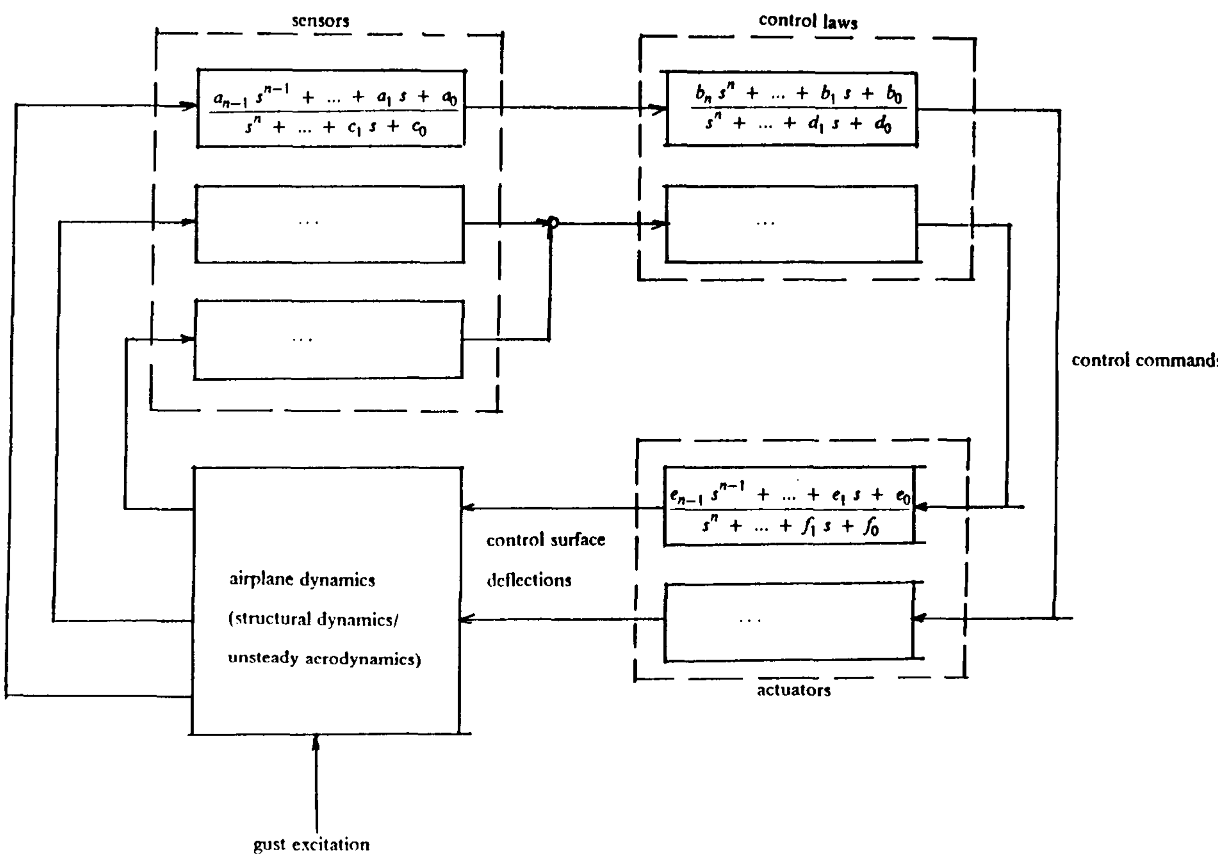

Figure 16

LTI STATE SPACE MODEL AND STABILITY ANALYSIS

Formulations of the state space control augmented servoaeroelastic equations of motions can be found in many works on active flutter suppression (e.g. Ref. 49). A transfer function model of an element of the control system (whether sensor, actuator or a control law) can be transformed into a state space model, where the A, B, C and D matrices are explicitly expressed in terms of the transfer function numerator and denominator polynomial coefficients. Assembly of the sensor, actuator, control law, structural dynamics, gust and unsteady aerodynamics state space models leads to the system matrices U, V and W in a LTI state space model of the whole system. These matrices are functions of the structural design variables through their dependence on the stiffness and mass matrices. They depend on the control system design variables through their dependence on the state space models of the control elements.

For given flight conditions (Mach number and altitude) the stability of the system is determined by the real part of the eigenvalues of a generalized eigenvalue problem. Sensitivity of a critical eigenvalue with respect to any design variable, p , is calculated using standard eigenvalue sensitivity analysis based on the derivatives : $\partial U/\partial p$, $\partial V/\partial p$ and the left and right corresponding eigenvectors $\{\psi\}$, $\{\phi\}$. It is planned to use the original Ritz functions directly as generalized coordinates. This approach leads to an increased order model but avoids natural mode calculation and aerodynamic force updates associated with natural mode reduced models. Computation times and accuracy will determine whether there is a need to resort to natural modes. Alternative approximations to system eigenvalues in terms of structural and control system design variables will be studied (Figure 17.).

STATE SPACE MODELS OF ACTUATORS, SENSORS AND GUST FILTER :

$$s\{x_i\} = [A_i]\{x_i\} + [B_i]\{u_i\}$$

$$\{y_i\} = [C_i]\{x_i\}$$

$i = ACT$ FOR ACTUATORS
$i = SEN$ FOR SENSORS
$i = G$ FOR GUST

STATE SPACE MODEL OF THE CONTROL BLOCK :

$$s\{x_{LAW}\} = [A_{LAW}]\{x_{LAW}\} + [B_{LAW}]\{u_{LAW}\}$$

$$\{y_{LAW}\} = [C_{LAW}]\{x_{LAW}\} + [D_{LAW}]\{u_{LAW}\}$$

THE A,B,C,D MATRICES ARE EXPLICITLY EXPRESSED AS A FUNCTION OF THE CONTROL SYSTEM DESIGN VARIABLES.

SYSTEM STATE VECTOR $\{x\} = \{x_1\, x_2 \,..\, x_8\}$ CONTAINS :

STRUCTURAL STATES ; ACTUATOR STATES ; SENSOR STATES ; CONTROL LAW STATES ; GUST STATES ; AERODYNAMIC STATES ASSOCIATED WITH GENERALIZED AERO MATRIX ; AERODYNAMIC STATES ASSOCIATED WITH GUST VECTOR .

THE CLOSED LOOP STATE SPACE EQUATIONS OF THE COMPLETE SYSTEM :

$$s[U]\{x(s)\} = [V]\{x(s)\} + \{W\}\, u_G(s)$$

STABILITY BY EIGENVALUE ANALYSIS :

$$\lambda\, [U(p)]\{\phi\} = [V(p)]\{\phi\}$$

EIGENVALUE SENSITIVITY WITH RESPECT TO DESIGN VARIABLE p :

$$\frac{\partial \lambda}{\partial p} = \frac{\psi^T [\lambda \frac{\partial U}{\partial p} - \frac{\partial V}{\partial p}]\phi}{\psi^T U \phi}$$

Figure 17

STATUS OF MULTIDISCIPLINARY ANALYSIS AND BEHAVIOR SENSITIVITY

Figure 18 presents status of research activities associated with the development of the analysis and sensitivity capabilities for the multidisciplinary synthesis of wings. It is expected that based on these capabilities, it will be practical to synthesize on a preliminary design level realistic representations of control augmented wings. The generality of the approximation concepts based mathematical programming aproach to synthesis and the realism in modeling are expected to be of major importance in coping with complicated multidisciplinary interaction, where little experience exists and intuition is often misleading.

ANALYSIS AND BEHAVIOR SENSITIVITY STATUS

	STRUCTURE	AERODYNAMICS #	CONTROL
FORMULATION : ANALYSIS	+	+	+
SENSITIVITY	+	+	+
ANALYSIS IMPLEMENTATION :	+	+	+
ANALYSIS TESTING :	+	+	in progress
SENSITIVITY IMPLEMENTATION :	+	in progress	in progress
SENSITIVITY TESTING :	+	-	-
APPROXIMATION CONCEPTS PERFORMANCE ASSESSED :	+	-	-

AERODYNAMICS INCLUDE :
• UNSTEADY AERODYNAMICS FOR SERVOAEROELASTIC ANALYSIS
• STEADY TRIM AND DRAG CALCULATIONS

Figure 18

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