N89-25216

FUZZY SET APPLICATIONS IN ENGINEERING OPTIMIZATION

Multilevel Fuzzy Optimization

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INTRODUCTION

Since the landmark paper by Bellman and Zadeh in 1970 [1], fuzzy sets have been used to solve a variety of optimization problems. In mechanical and structural design, however, relatively few applications of fuzzy optimization have appeared in the literature, even though the formulations and algorithms developed in management science are readily applicable in engineering fields. In engineering design fuzzy optimization has most often been suggested as a mechanism to represent 'soft' constraints (e.g. [2]) and as an alternative formulation to solve multiobjective optimization problems (e.g. [3,4].) A formulation for multilevel optimization with fuzzy objective functions is presented in this paper.

With few exceptions, formulations for fuzzy optimization have dealt with a one- level problem in which the objective is the membership function of a fuzzy set formed by the fuzzy *intersection* of other sets. This concept dates back to the original work by Bellman and Zadeh in which objective and constraints are handled in identical fashion: if f(**x**) is the function to be minimized under constraints $g_i(\mathbf{x})\leq 0$, $i=1,2,...,m$, the fuzzy optimization problem maximizes the membership function μ_G of

$$G=G_0\cap G_1\cap,..,\cap G_m \quad \text{with} \quad \mu_G=\min(\mu_0,\mu_1,...,\mu_m)$$

The set G_0 has membership function μ_0 such that

$\mu_0(f)\rightarrow 1$ as f *decreases*

while, for other G_i's,

$\mu_j(g_j)\rightarrow 1$ as g_j *decreases* from 0, and

$\mu_j(g_j)\rightarrow 0$ as g_j *increases* from 0.

This model has been used extensively to relax ('fuzzify') the constraint set and to deal with multiple objectives.

A somewhat different problem is discussed here. First, the goal set G is defined in a more general way, using an aggregation operator H that allows arbitrary combinations of set operations (union, intersection, addition) on the individual sets G_i. This is a straight-forward extension of the standard form, but one that makes possible the modeling of interesting evaluation strategies. This feature has been discussed in detail elsewhere [3,4] but, for completeness, it will be briefly outlined in the next section.

A second, more important departure from the standard form will be the construction of a multilevel problem analogous to the design decomposition problem in optimization [5-8]. This arrangement facilitates the simulation of a system design process in which

- Different components of the system are designed by different teams.

- Different levels of design detail become relevant at different time stages in the process: global design features early, local features later in the process.

SINGLE -LEVEL PROBLEM

The optimization problem can be solved in a single level when a design alternative can be fully described and evaluated using local design variables and functions. Let $\mathbf{x} \in R^n$ represent the vector of design variables. The goal of the problem is to find the design $\mathbf{x}$ that

maximizes $\mu_G = h(\mu_1, \mu_2 \ldots, \mu_p)$,

the membership in the design goal

$G = H(G_1, G_2, .., G_p)$,

subject to *crisp* constraints

$\mathbf{x} \in X = \{\mathbf{x} \in R^n : g_i(\mathbf{x}) \leq 0,\ i = 1, 2, \ldots, m\}$.

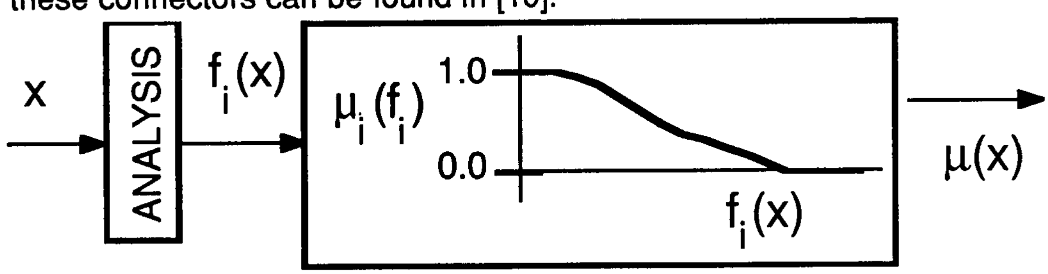
Each individual goal G_i has membership μ_i and represents a desirable design quality such as strength, low cost, or reliability. The mapping from $\mathbf{x}$ to each of the μ_i's depends on specific details of the design process and designer's preferences and it is assumed to be known. The function $h:[0,1]^p \rightarrow [0,1]$ is an acceptable connective associated with H, as defined in [3,9]. For instance, in a standard 3- objective fuzzy optimization problem

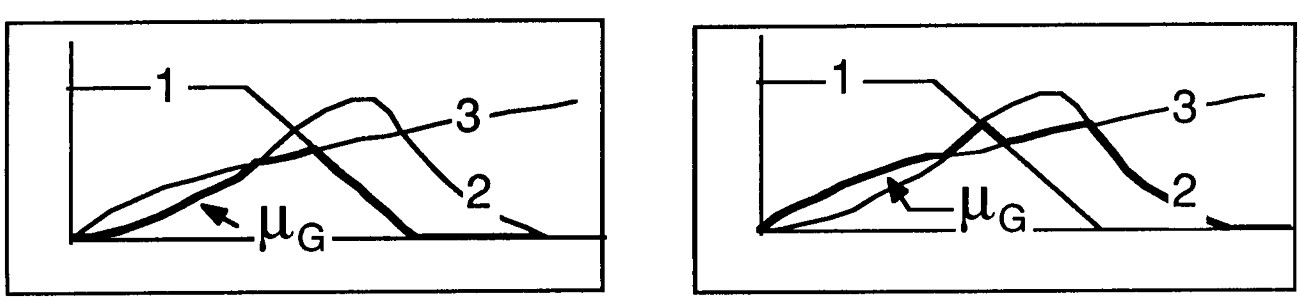
$H(G_1, G_2, G_3) = G_1 \cap G_2 \cap G_3$ and $h(\mu_1, \mu_2, \mu_3) = \min\{\mu_1, \mu_2, \mu_3\}$.

This is by no means the only possible or meaningful choice for H and h. For instance, the choice

$H(G_1, G_2, G_3) = (G_1 \cap G_2) \cup (G_1 \cap G_3) \cup (G_3 \cap G_2)$

indicates that the optimum solution needs to satisfy only 2 out of 3 objectives. The connective h can be built using different intersection and union operators. An extensive review of these connectors can be found in [10].





$G = G_1 \cap G_2 \cap G_3$ $\qquad$ $G = (G_1 \cap G_2) \cup (G_2 \cap G_3) \cup (G_1 \cap G_3)$

MULTILEVEL PROBLEM

In a multilevel approach the system being designed is decomposed into several sub-systems. This decomposition is often made along boundaries determined by the organization and expertise of design teams involved in the design task. Each team becomes responsible for the design of its own sub- system and is an expert only in this limited area. Sobieszczanski-Sobieski, J. [5,7,8], Haftka [6], and Parkinson et. al. [11], among others, have suggested different approaches to the optimization problem in this setting.

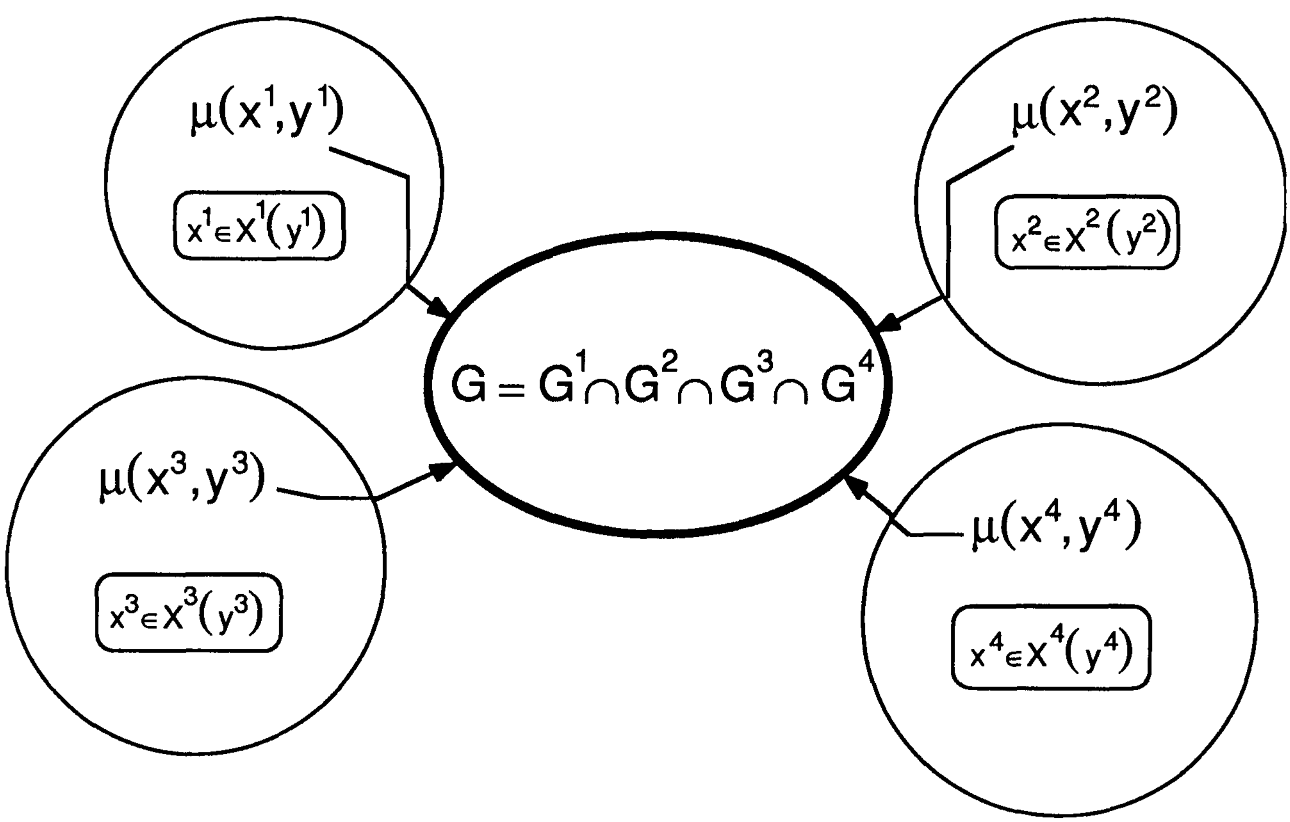
Two important questions arise in the multilevel optimization problem: what is a meaningful objective function to guide the optimization and how to maintain feasibility when only local constraints can be evaluated exactly. The fuzzy set approach offers an operationally useful answer to the first question. The second question will be addressed using convex approximations to non- local functions.

In the fuzzy multilevel problem each sub- problem (j) has its own variables $\mathbf{x}^{(j)}$ and goal $G^{(j)}$. The sub- problems contribute their goals to the overall design objective G, formed in a hierarchical fashion using set operations on the sub-problem goals. This arrangement:

- Introduces the flexibility of the fuzzy formulation at the sub- problem level. All of the tools from fuzzy optimization are available there, including the traditional 'crisp' optimization problem.

- Facilitates the comparison of dissimilar objectives arising in different environments: membership functions are dimensionless.

- Makes possible the construction of a design goal that varies as the design process unfolds. The vagueness that characterizes the early stages of the design can be easily modeled at the local level in the definition of the local goal.



GLOBAL PROBLEM

Let $\mathbf{x} \in R^N$ be the vector of all design variables. The solution to the global optimization problem

maximizes $\mu_G(\mathbf{x}) = h_G(\mu^{(1)}, \mu^{(2)}, \ldots, \mu^{(p)})$

under the constraints of the local problems defined below. The functions $\mu^{(j)}$ are membership functions of the sets $G^{(j)}$ corresponding to acceptable solutions to the j-th sub- problem and the function μ_G corresponds to the fuzzy set G that describes the optimum global solution,

$G = H_G(G^{(1)}, G^{(2)}, \ldots, G^{(p)})$

The operators h_G and H_G describe the way different sub- problems interact and, in general, change to reflect different stages in the design problem.

LOCAL PROBLEM $P^{(j)}$

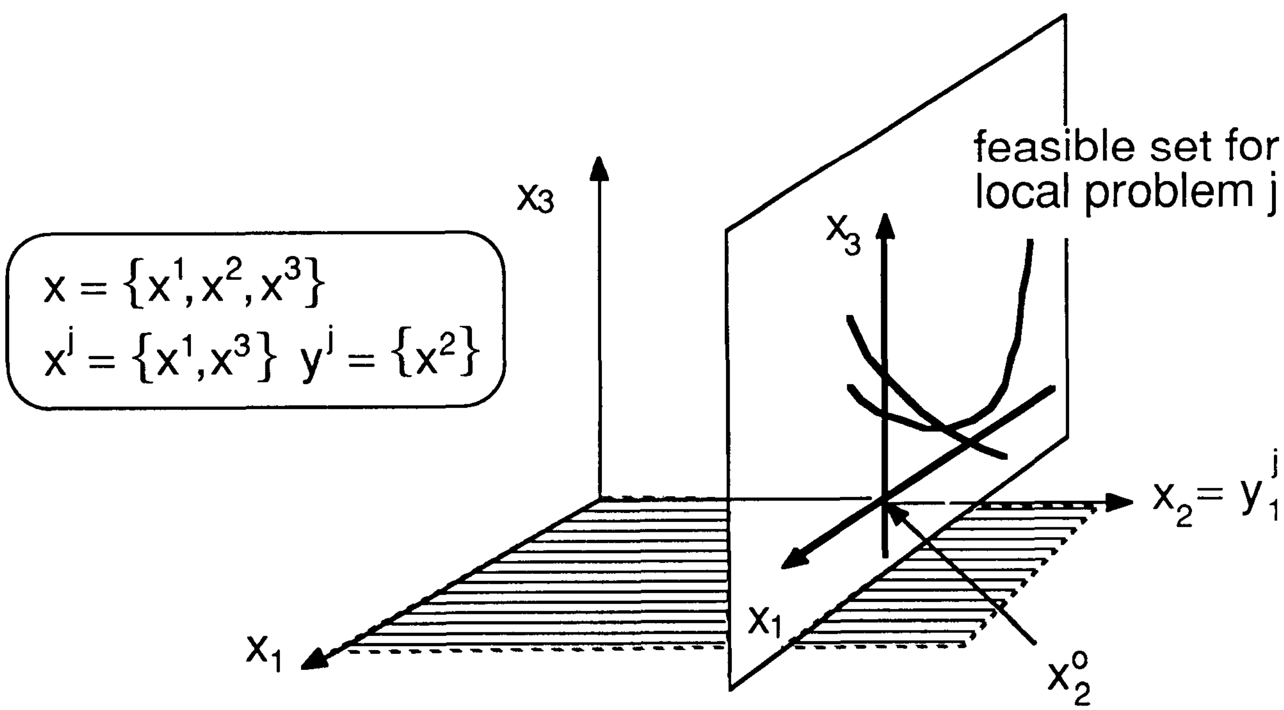
Let $\mathbf{x}^{(j)} \in R^{n(j)}$ represent the vector of local design variables and let $\mathbf{y}^{(j)} \in R^{N-n(j)}$ represent design variables outside the j-th problem, i.e., $\mathbf{y}^{(j)} = \mathbf{x} - \mathbf{x}^{(j)}$. Problems are assumed to be coupled: objectives and constraints in the j-th problem are functions of $\mathbf{y}^{(j)}$. However, while $\mathbf{x}^{(j)}$ varies in the j-th problem, $\mathbf{y}^{(j)}$ remains fixed. The goal of the local problem is to find the design $\mathbf{x}^{(j)}$ that

maximizes $\mu_G(\mathbf{x}^{(j)}; \mathbf{y}^{(j)}, \mathbf{f}^{(j)}(\mathbf{x}^{(j)}, \mathbf{y}^{(j)}))$

subject to *crisp* constraints

$\mathbf{x}^{(j)} \in X^{(j)} = \{\mathbf{x}^{(j)} \in R^{n(j)} : g^{(j)}_i (\mathbf{x}^{(j)}; \mathbf{y}^{(j)}, \mathbf{f}^{(j)}(\mathbf{x}^{(j)}, \mathbf{y}^{(j)})) \leq 0,\ i=1,2,\ldots,m(j)\}$

Local goals are coupled with other sub-problems via $\mathbf{y}^{(j)}$ and $\mathbf{f}^{(j)}$. The $f^{(j)}_i$'s are functions evaluated *outside* problem $P^{(j)}$. Convex approximations are used to evaluate these functions.



EVALUATING NON-LOCAL FUNCTIONS: Maintaining feasibility.

A challenging difficulty in the multilevel problem involves maintaining global feasibility while solving the sub-problems. The question is, how to insure that all variables, not just the local variables, remain feasible without actually evaluating non-local constraints. An answer to this question will be attempted using convex approximations of non-local functions.

Convex approximations were introduced by Starnes and Haftka [12] and used by Fleury and Braibant [13] to solve a range of structural optimization problems. If f(**x**) is a differentiable function in R^N, the convex approximation of f at $\mathbf{x}^o$ is the function

$$\tilde{f}(\mathbf{x}) \equiv \sum_{(+)} \left(\frac{\partial f}{\partial x_i}\right)_{x^o} (x_i - x_i^0) + \sum_{(-)} \left(\frac{\partial f}{\partial x_i}\right)_{x^o} \frac{x_i^0}{x_i}(x_i - x_i^0)$$

where the (+) sum is over positive derivatives while the (-) sum is over the negative derivatives of f. It is easy to show that for positive variables the convex approximation is the more 'conservative' approximation out of the linear, reciprocal, or concave approximations of f [12] . Indeed, the set f(x)≤0 is often contained in the set $\tilde{f}(x) \le 0$. This motivates the following variation of the problem $P^{(j)}$:

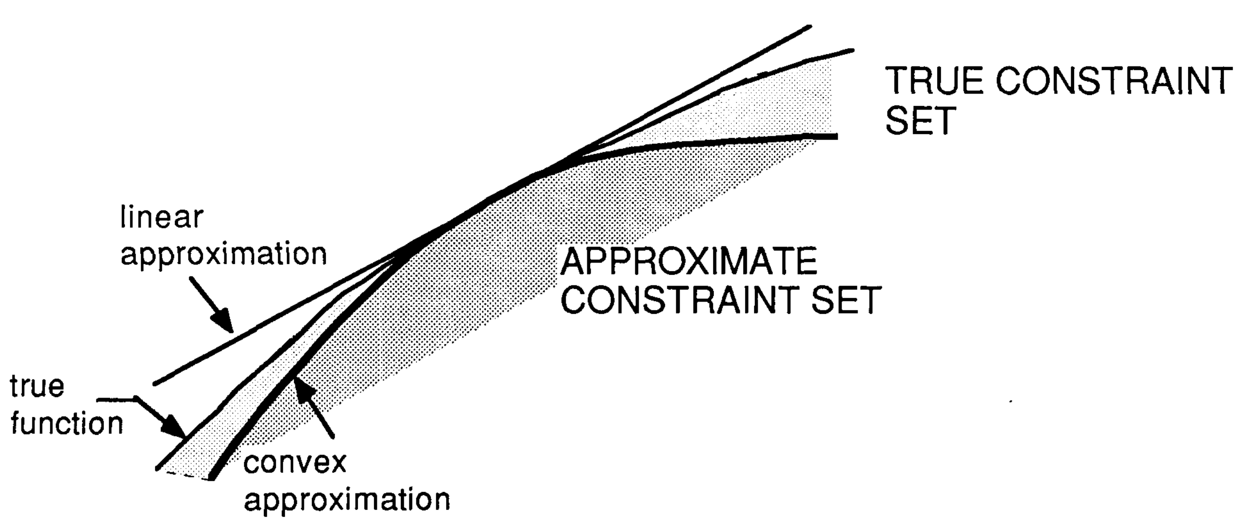
Find $\mathbf{x}^{(j)} \in R^{n(j)}$ that maximizes μ_G subject to the local constraints

$$\mathbf{x}^{(j)} \in \tilde{X}^{(j)} = \left\{ \mathbf{x}^{(j)} \in R^{(n(j))} : g_i^{(j)}\left(\mathbf{x}^{(j)}; \mathbf{y}^{(j)}, \tilde{\mathbf{f}}^{(j)}(\mathbf{x}^{(j)}, \mathbf{y}^{(j)})\right) \le 0,\ i = 1,..,m(j) \right\}$$

and the convex approximations of the non-local constraints at the starting point,

$$\mathbf{x}^{(j)} \in \tilde{X}^{(k)} = \left\{ \mathbf{x}^{(j)} \in R^{(n(j))} : \tilde{g}_i^{(k)}(\mathbf{x}^{(j)}) \le 0,\ i = 1,..,m(k) \right\}, k = 1,...,p; k \ne j$$

Along with a step-size restriction, the addition of these constraints is often enough to insure that $\mathbf{x}^{(j)}$ remains within all feasible sets, local and non-local, without need to evaluate the non-local constraints $\mathbf{g}^{(k)}$ and functions $\mathbf{f}^{(j)}$ exactly. The convex approximations are easy to compute and only require the kind of sensitivity information that is often available in many nonlinear programming algorithms. Efficient methods to compute these derivatives have been reported in the literature [14].



SEQUENCE OF SOLUTION

In the solution strategy of the multilevel problem one local problem, the *active problem* $P^{(i)}$, is solved at a time by searching only in the space of local variables $\mathbf{x}^{(i)}$. The choice of operator h_G determines the relative importance of each sub-problem and hence the order in which problems will be solved. Consider for example a non-associative connective such as

$$h_G(\mu^{(1)},\mu^{(2)})=\min\{\mu^{(1)},\mu^{(2)}\} \quad (G=G^{(1)}\cap G^{(2)})$$

If $\mu^{(1)}>\mu^{(2)}$ at a given point $\mathbf{x}$, the global problem is dominated by the sub-problem goal $G^{(2)}$ at that point and efforts should be directed to improve $\mu^{(2)}$. If, on the other hand,

$$h_G(\mu^{(1)},\mu^{(2)})=\mu^{(1)}\times\mu^{(2)}$$

some gain is possible even if only $\mu^{(1)}$ is improved. Indeed, sub-problem goal $G^{(1)}$ dominates with possibility $\mu^{(2)}$. In general, goal $G^{(i)}$ will dominate with possibility

$$m_j=\max\left\{\left(\frac{\partial h_G}{\partial \mu^{(j)}}\right)^{+},\left(\frac{\partial h_G}{\partial \mu^{(j)}}\right)^{-}\right\}, \qquad (+)=\text{right},\ (-)=\text{left derivatives}$$

To simplify the solution strategy only the 'min' operator will be used to connect sub-goals (any operator can be used *within* each problem). With this simplification m_j is always 1 for the dominant goals (0 otherwise) and the procedure is simplified.

In some steps, $\mu^{(i)}$ can be improved within $X^{(i)}$. It may happen, however, that an increase in a dominant $\mu^{(i)}$ is possible only after relaxing some constraint that depends on a non-local variable $\mathbf{y}^{(i)}$. When this happens it may be necessary to make another problem, say $P^{(k)}$, $k\neq j$, active and seek to relax the troublesome constraint in $X^{(k)}$ instead of improving its natural local objective. A robust strategy to select the next active problem and its objective function is essential to the success of the approach. Efforts are being directed toward this goal but more research is still required. An example that applies to the 'min' operator is outlined below.

At $\mathbf{x}^o$, after solving P^i, $M\equiv\{j: \mu_j=1\}$		$S^j_G=$ feasible descent direction for μ_G in X^j $S^j=$ feasible descent direction for μ^j in X^j
IF $S^j_G\neq 0,\ j\in M,$	THEN	$\mathrm{MAX}_{x^j}\ \mu^j_G$
IF $S^j_G\neq 0,$ any $j,$	THEN	$\mathrm{MAX}_{x^j}\ \mu^j_G$
IF $S^j\neq 0, j\in M, j\neq i$	THEN	Set relaxation: $r_k>0$, $k\neq j$, $k\in M$ $r_k=0$, otherwise. $\mathrm{MAX}_{x^j}\ \mathrm{MIN}_k\left\{\mu^{(k)}+r_k\right\}$
ELSE for $j\in M$, $k^*\equiv\left\{k\notin M: g^k_r(\mathbf{x})=0,\ \left\langle\nabla^k\mu^j,\nabla^k g^k_r\right\rangle\to\min_{k\notin M}\right\}$ $\mathrm{MIN}_{x^{k^*}}\ g^{k^*}_r,\ \mu_G(\mathbf{x})\geq\mu_G(\mathbf{x}^o)$		

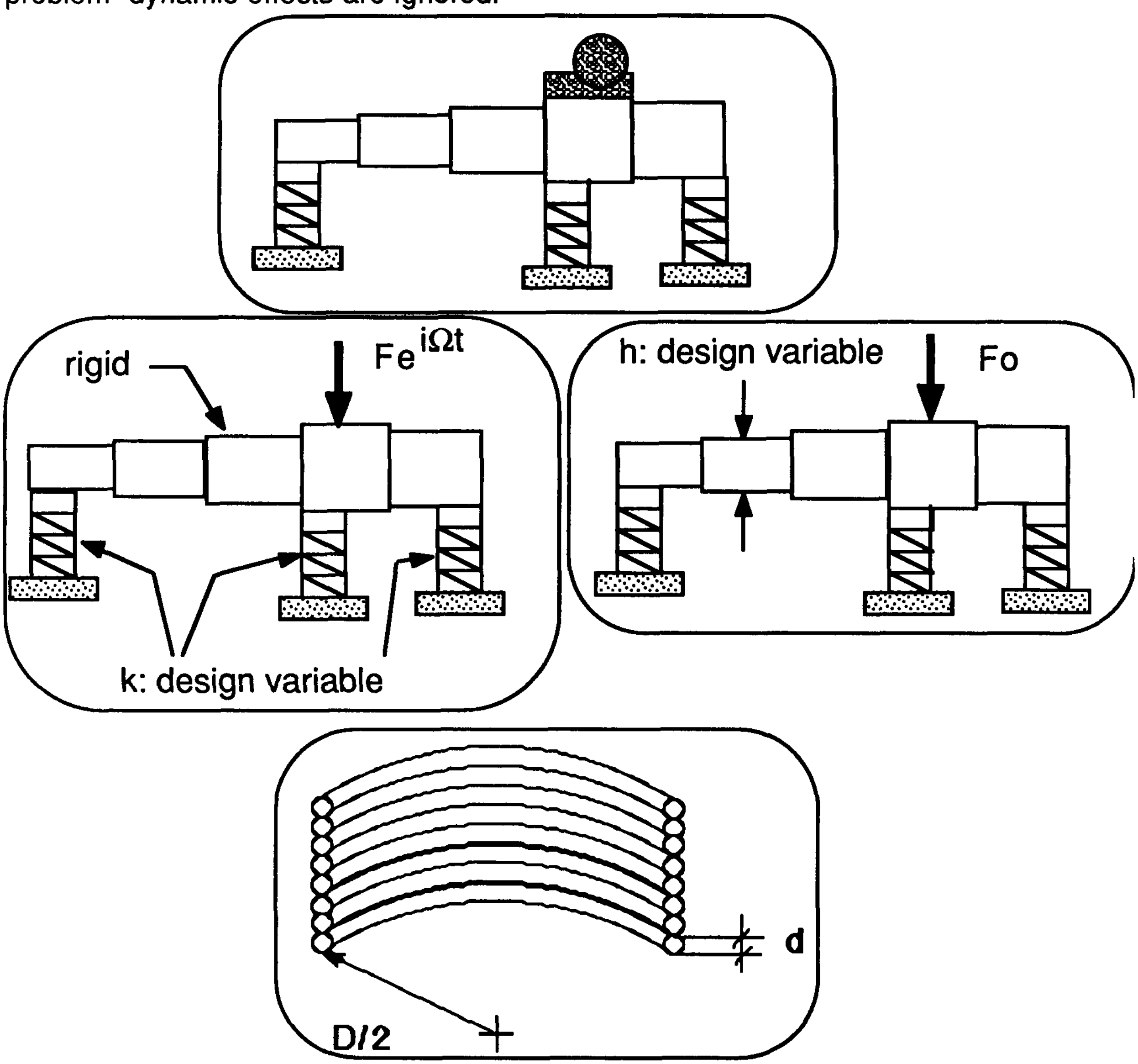
EXAMPLE

The multilevel problem with fuzzy objectives can be illustrated by the following simple model of support system for a heavy piece of machinery. The problem is modeled by a flexible beam supported by three spring- dashpot components. To illustrate the approach the device will be decomposed in two subsystems: the suspension (springs) and the structure (beam).

1. Suspension: Spring stiffnesses are to be selected to transmit a periodic load to the foundation. The amount of force transmitted and the maximum deflection are relevant performance measures. In this problem the flexibility of the structure is ignored.

1.1 At a detailed level , the springs themselves are to be designed. Stiffness, stress, clearance, and natural frequency are relevant measures. Coil and wire diameters are design variables.

2. Structure: Beam cross-sectional properties are to be selected to support a load. Amount of material, stresses and deflections are relevant performance measures. In this problem dynamic effects are ignored.



The 'min' intersection operator will be used to connect the sub-goals, i.e, the overall goal is

$G=G_{susp} \cap G_{struct} \cap G_{spring}$, $\mu_G=\min\{\mu_{susp}, \mu_{struct}, \mu_{spring}\}$

The objective in the suspension sub-problem (1) is the maximum force transmitted to the foundation by the three springs. The weight of the beam is the objective in the structure sub-problem. Although these are rather simple objectives for a fuzzy optimization problem, no significant loss of generality is introduced. Membership functions for these goals are shown below.

Problem 1.1 is special: it corresponds to the detailed design in the suspension design problem.The precise description of the spring (problem 1.1) becomes relevant only after some knowledge of the required stiffnesses is available (problem 1). This organization is present in typical design problems in which detailed design decisions are taken later in the design process. In this problem diameters D and d must be selected so that the spring stiffness matches the stiffness k prescribed by the problem (1) without exceeding limits on stress. The goal is set as

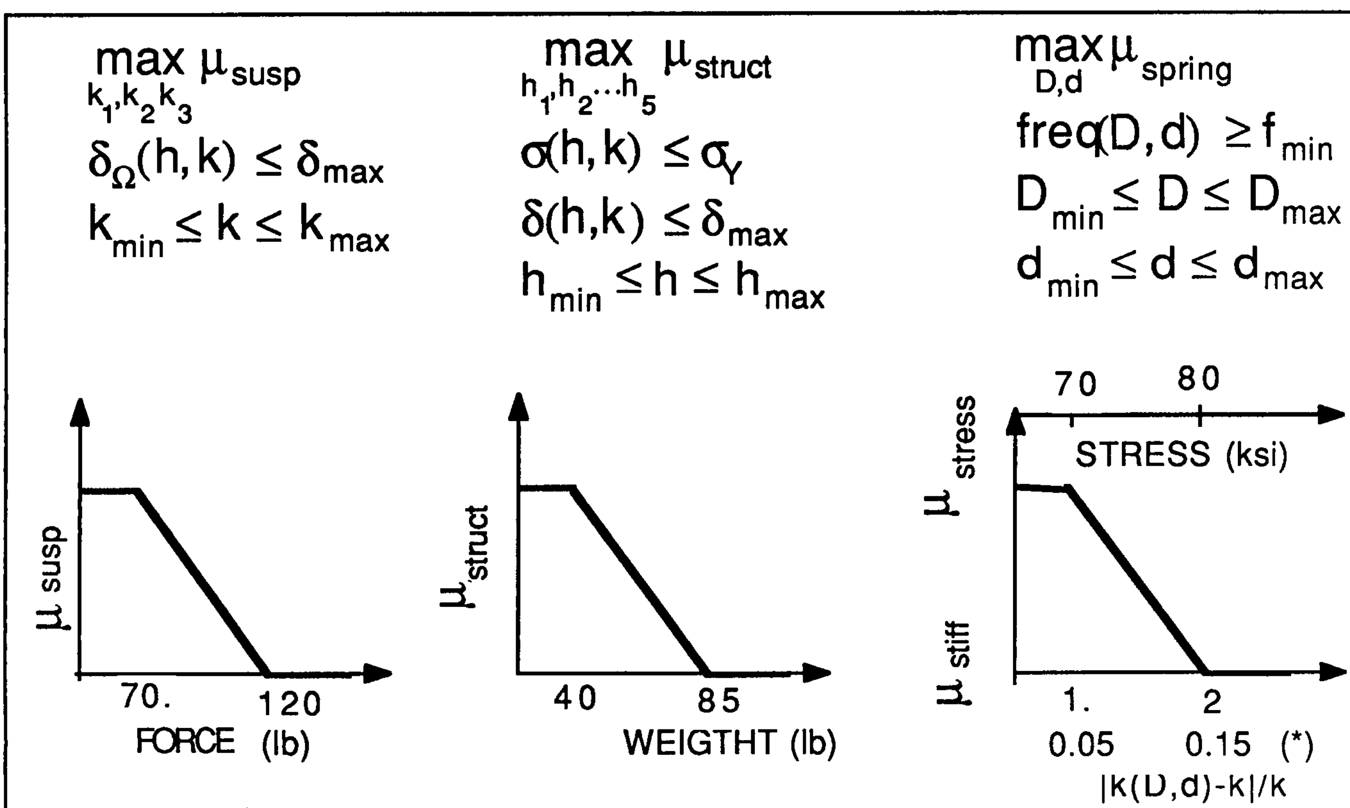
$G_{spring}=G_{stiff} \cap G_{stress}$, $\mu_{spring}=\min\{\mu_{stiff}, \mu_{stress}\}$

where

$G_{stiff}=\{(D,d): k(D,d)\approx k\}$

$G_{stress}=\{(D,d): \sigma(D,d) <\approx \sigma_Y\}$

Membership functions for these sets are shown below.



Solution history for the problem is shown in the figure below.

(0) μ_{susp} dominates. Problem 1 is solved with objective μ_G.

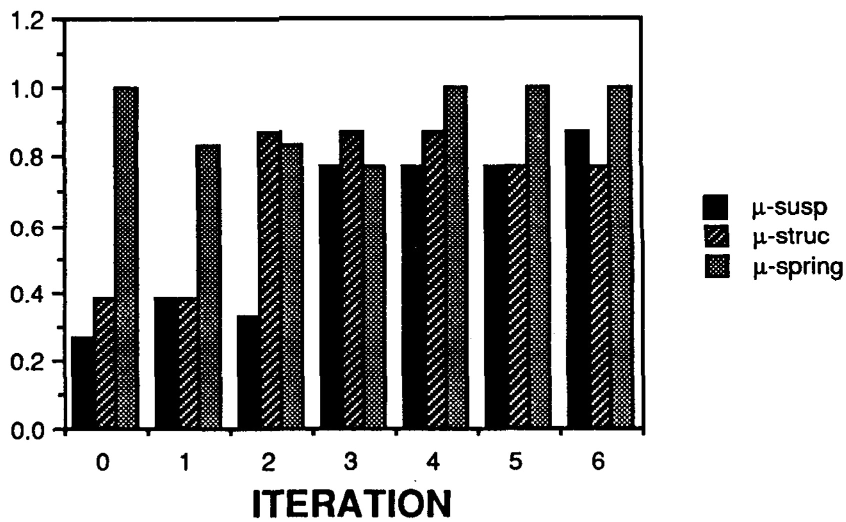
(1) $\mu_{susp} = \mu_{struct}$. Problem 2 is solved. Objective is μ_{struct} and μ_{susp} is not allowed to decrease (G_{susp} to worsen) more than 20%.

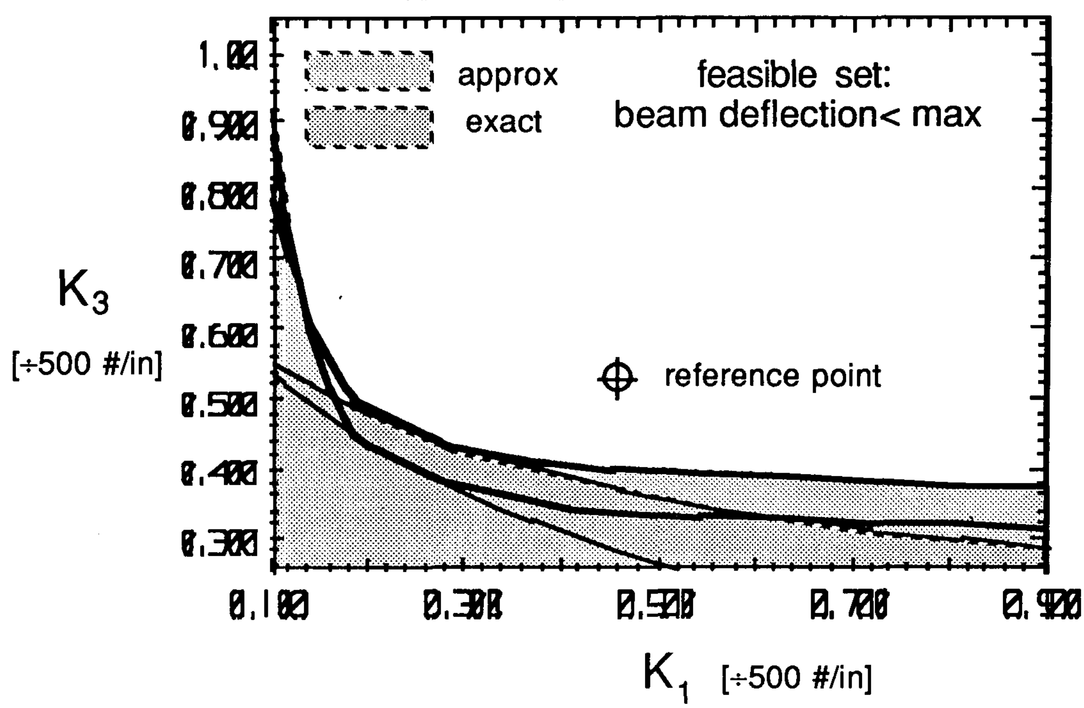
(2) $\mu_{susp} < \mu_{struct}$. Problem 1 i solved with objective μ_G.

(3) $\mu_{spring} = \mu_{susp}$. Problem 1.1 is solved with objective μ_{spring}. At the end of this step tighter restrictions on σ are imposed ((*) in membership function above).

(4) $\mu_{susp} < \mu_{struct}$ and a non-local constraint (maximum beam deflection) is active (figure). Problem 2 is solved to reduce this constraint. Notice that the convex approximation is effective to determine constraint activity.

(5) $\mu_{susp} = \mu_{struct}$. Problem 1 is solved with objective μ_G.





CONCLUSIONS

The introduction of fuzzy objective functions into the multilevel optimization problem adds a new dimension to the problem. Interactions among sub-problems can be represented as set operations on fuzzy sets, a feature that introduces flexibility and insight into the problem.

A more robust strategy to select the next active problem and objective needs to be developed. Heuristic approaches to determine constraint activity may prove useful here.

Some of the problems associated with keeping solutions feasible can be handled using convex approximations of non-local constraints. This is a promising approach, but one that needs further tests. More research is needed on issues such as how often sensitivity analyses must be performed to keep approximations accurate and how to determine appropriate step size limits.

ACKNOWLEDGEMENT

This material is based upon work supported by the National Science Foundation under Grant No. DME 8706902. This support is gratefully acknowledged.

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