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A PENALTY APPROACH FOR NONLINEAR OPTIMIZATION WITH DISCRETE DESIGN VARIABLES

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INTRODUCTION

Most optimization techniques have been developed under the implicit assumption that the design variables are continuous-valued. For most practical optimization problems, however, the designer must choose the design variables from a list of commonly available values. The design variables such as cross-sectional areas of trusses, thicknesses of plates and membranes, fiber orientations and number of layers for laminated composite structures, fall into this category.

Although numerous algorithms for structural optimization problems have been developed, relatively little effort has been made for optimum structural design incorporating design variables with discrete values. The most common way of achieving a design with discrete-valued design variables is to round off the optimum values of the design variables, obtained by assuming them to be continuous variables, to the nearest acceptable discrete value. Although the idea is simple, for problems with a large number of design variables, selection of the discrete values without violating the design constraints can pose serious difficulties. More systematic methods are proposed in the research environment. Reinschmidt [1] used the Branch and Bound method for the plastic design of frames by posing the problem as an integer linear programming problem. Garfinkel and Nemhauser [2] showed the Branch and Bound method can also be used in solving convex nonlinear problems. This method forms new subproblems, called candidates, after obtaining the continuous optimum. These candidates exclude the infeasible (non-discrete) regions by branching, and bounds are used to rapidly discard many of the possible candidates without analyzing them due to the convexity of the problem. Schmit and Fleury [3] proposed a method in which approximation concepts and dual methods are extended to solve structural synthesis problems. In this technique, the structural optimization problem is converted into a sequence of explicit approximate primal problems of separable form. These problems are solved by constructing continuous explicit dual functions. Sizing type problems with discrete variables are solved successfully by this method. Another approach introduced by Olsen and Vanderplaats [4] used approximation techniques to develop subproblems suitable for linear mixed-integer programming methods. The solution is found by using two different softwares for continuous optimization and integer programming. Use of two different softwares, however, can be inconvenient for the practicing engineer. Also, the integer programming problems are difficult to handle.

This paper introduces a simple approach to minimization problems with discrete design variables by modifying the penalty function approach of converting the constrained problems into sequential unconstrained minimization technique (SUMT) problems [5]. It was discovered, during the course of the present work, that a similar idea was suggested by Marcal and Gellatly [6]. However, no further work has been encountered following Ref. [6]. In the following sections first, for the sake of clarity, a brief description of the SUMT is presented. Form of the penalty function for the discrete-valued design variables and strategy used for the implementation of the procedure are discussed next. Finally, several design examples are used to demonstrate the procedure, and results are compared with the ones available in the literature.

SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE

The SUMT algorithm transforms the constrained optimization problem into a sequence of unconstrained problems. The classical approach to using SUMT is to create a pseudo-objective function by combining the original objective function and the constraint equations. The constraints are added to the objective function in a way to penalize it if the constraint relations are not satisfied. That is, the constrained minimization problem,

$$\begin{aligned} &\text{Minimize :} && F(X) && (1)\\ &\text{Such that :} && g_j(X) \geq 0,\ j = 1,2,\ldots,n_g \\ &\text{where} && X = \{x_1, x_2, \ldots, x_n\}^T \\ & && n : \text{total number of design variables} \\ & && n_g : \text{number of constraints} \end{aligned}$$

is replaced by the following unconstrained one,

$$\text{Minimize :} \quad \Phi(X,r) = F(X) + r \sum_{j=1}^{n_g} y(g_j) \qquad (2)$$

where $y(g_j)$ takes different forms depending on the method of penalty introduction [7]. The positive multiplier, r, in Eq. (2) controls the contribution of the constraint penalty terms. For a given value of the penalty multiplier, r, Eq. (2) describes the bounds of the feasible design space, often referred to as the response surface. As the penalty multiplier is decreased, the contours of the response surface conform with the original objective function and the constraints more closely. Therefore, minimization of the unconstrained problem is performed repeatedly as the value of r is decreased until the minimum value of the pseudo-objective function coincides with the value of the original objective function. Several response surfaces generated by using the extended interior penalty technique [7] for a problem with one design variable and a single constraint are shown in Figure 1.

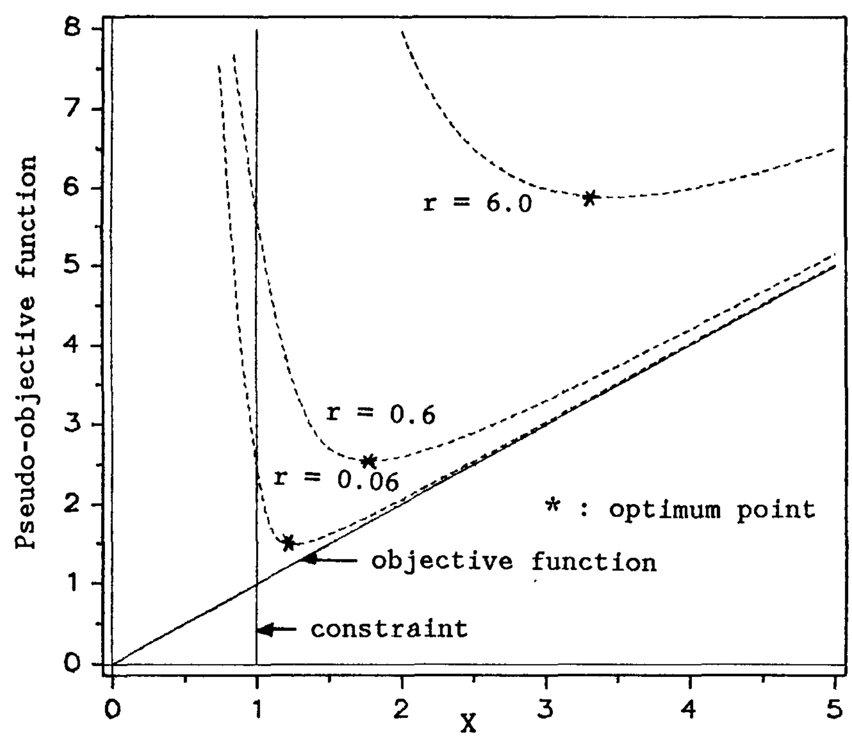


Fig. 1. Response surfaces for extended interior penalty method ($F = x$ subject to $g = x - 1.0 \geq 0$)

DISCRETE PENALTY FUNCTION

The basic idea behind the proposed method is to include additional penalty terms in the pseudo-objective function to reflect the requirement that the design variables take discrete values. The general formulation for a problem having discrete variables is presented below.

$$
\begin{array}{lll}
\text{Minimize} & : & F(X) \\
\text{Subject to} & : & g_j(X) \geq 0 \quad j = 1,2,\ldots,n_g \\
\text{where} & & x_i = \{d_{i1}, d_{i2}, \ldots, d_{iq}\}^T \\
 & & d_{ik} : \text{k-th discrete value of the i-th design variable} \\
 & & q : \text{number of discrete values for each design variable}
\end{array} \tag{3}
$$

In general, the number of available discrete values for each design variable may be different, or even in some cases continuous variation of some of the design variables may be allowed. The modified pseudo-objective function Ψ which includes the penalty terms due to constraints and the non-discrete values of the design variables is defined as

$$\Psi(X, r, s) = F(X) + r \sum_{j=1}^{m} p^{\times}(g_j) + s \sum_{i=1}^{p} \Phi_d^i(X) \tag{4}$$

where r is the penalty multiplier for constraints, $p^{\times}(g_j)$ is a quadratic extension function [8], s is the penalty multiplier for non-discrete values of the design variables, and $\Phi_d^i(X)$ denotes the penalty term for non-discrete values of the $i-$th design variable. Different forms for the discrete penalty function are possible. In the present study, the penalty terms $\Phi_d^i(X)$ are assumed to take the following sine-function form,

$$\Phi_d^i(X) = \frac{1}{2}\left(\sin \frac{2\pi[x_i - \frac{1}{4}(d_{ij+1} + 3d_{ij})]}{d_{ij+1} - d_{ij}} + 1\right) \tag{5}$$

The proposed functions $\Phi_d^i(X)$ penalize only non-discrete design variables and assure the continuity of the first derivatives of the modified pseudo-function at the discrete values of the design variables. The discrete penalty functions of the sine and elliptical forms are shown in Figure 2.

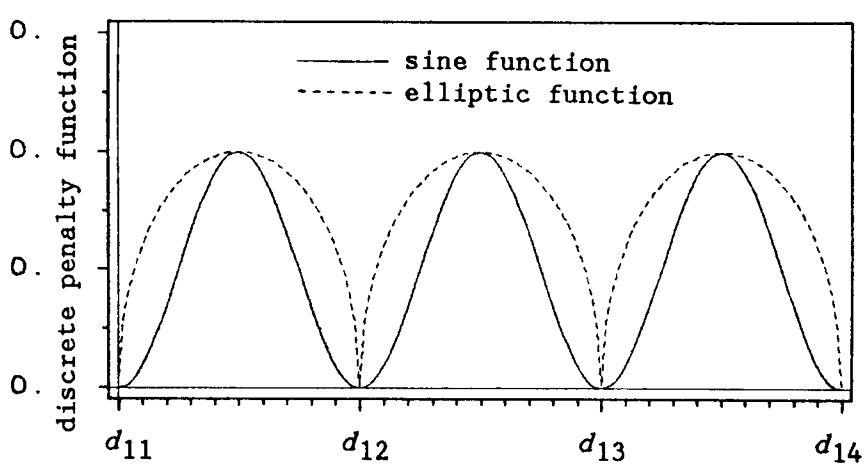


Fig. 2. Discrete penalty functions

The proposed method can be implemented with either exterior, interior, or extended interior penalty function approaches. The additional penalty terms for non-discrete design variables are incorporated to the optimization package NEWSUMT-A [9] which employs the extended interior penalty approach; hence, the following discussion is confined to the extended interior penalty technique. In equation (4), the response surfaces are determined according to the values of the penalty multipliers r and s since they control the amount of penalty for the constraints and for the non-discrete values, respectively. As opposed to the multiplier r, the value of the multiplier s is initially zero and is increased slowly from one response surface to another. One of the important factors in the application of the proposed method is to determine when to activate s, and how fast to increase it to obtain discrete optimum design. Clearly, if s is introduced too early in the design process, the design variables will be trapped by a local minimum resulting in a sub-optimal solution. To avoid this problem, the multiplier s has to be activated after several response surfaces which includes only constraint penalty terms. In fact, since the optimum design with discrete values is in the neighborhood of the continuous optimum, it may be desirable not to activate the penalty for the non-discrete design variables until a reasonable convergence to the continuous solution is achieved. This is especially true for problems with a large number of design variables and/or the intervals between discrete values are very close. The modified pseudo-objective function Ψ defined in equation (4) is shown in Figure 3 for a problem with one design variable and one constraint (See Example 1).

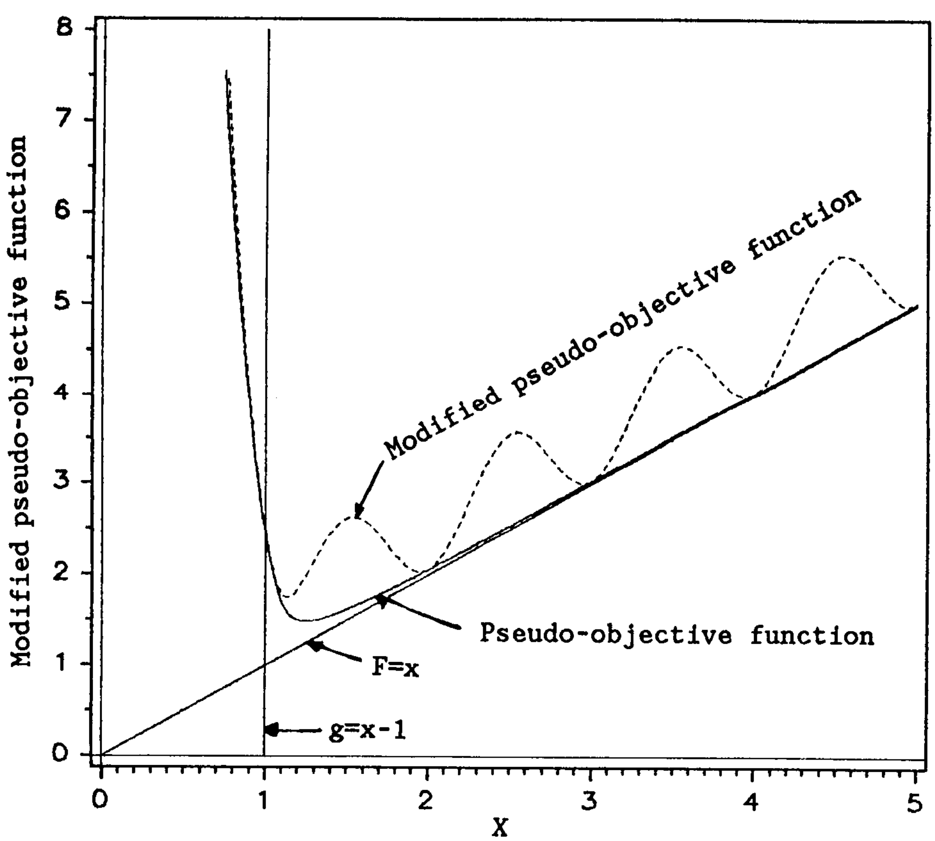


Fig. 3. Modified pseudo-objective function
($F = x$ subject to $g = x - 1.0 \geq 0$ and $x = 1, 2, 3, \ldots$)

DISCUSSION OF IMPLEMENTATION AND CONVERGENCE CRITERION

In using the SUMT, convergence is usually examined by comparing the total objective function with the corresponding value of the original objective function. A similar scheme is used to determine how close the design approaches to the continuous optimum before activating the discrete penalty. A criterion,

$$\varepsilon_c = \frac{\text{amount of constraint penalty}}{\text{objective function}} \tag{6}$$

is used, where ε_c denotes the tolerance to activate the discrete optimization process.

One of the important aspects of the present procedure is to determine how big a penalty term to apply at the first discrete response surface. If too large s is applied at the first few discrete iterations, the design can be trapped at a local minimum. To prevent the design from stalling, s must be applied slowly. The magnitude of the non-discrete penalty multiplier, s, at the first discrete iteration, is calculated such that the amount of penalty associated with the non-discrete design variables is equal to the constraint penalty. As the iteration for discrete optimization proceeds, the non-discrete penalty multiplier for the new iteration, $s^{(i+1)}$, is calculated by multiplying the $s^{(i)}$ by 5. Increasing $s^{(i)}$ implies that the discrete values for design variables are becoming more important than the constraint violation as the discrete optimization process continues.

Another important aspect of the proposed procedure is how to control the other penalty multiplier for the constraints, r, during the discrete optimization process. If r is decreased for each discrete optimization iteration as for the continuous optimization process, the design can be stalled due to too strict penalty on constraint violation. On the other hand, if r is increased, the design may move away from the optimum resulting in a sub-optimal solution. Thus, it is logical to freeze the penalty multiplier r at the end of the continuous optimization process. However, the nearest discrete solution at this response surface may not be a feasible design, in which case the design is forced to move away from the continuous optimum by moving back to the previous response surface. This was achieved by increasing the penalty multiplier, r, by a factor of 10.

The solution process for the discrete optimization is terminated if the design variables are sufficiently close to the prescribed discrete values. The convergence criterion for discrete optimization used in this effort is

$$\varepsilon_d = \sum_{i=1}^{n} \min\{|x_i - d_{ij}|,\ |x_i - d_{ij+1}|\} \tag{7}$$

where ε_d is the convergence tolerance.

During the discrete iteration process, it was experienced that some of the design variables were sometimes trapped at the middle of two discrete values, especially for a large value of penalty multiplier s. This is due to the vanishing nature of the first derivative of the sine function (5) at the mid-point. If it is detected that any one of the design variables is at the mid-point where the values of the first and second derivative of the sine function (5) approach 0 and -1, respectively, the trapping was avoided by removing the penalty terms for non-discrete values. This means only the original objective function and constraint penalty terms take part in the minimization process. The move direction is determined from the original response surface excluding the penalty terms due to non-discrete values. The flow chart for the proposed method combined with the extended interior approach is shown in Fig. 4.

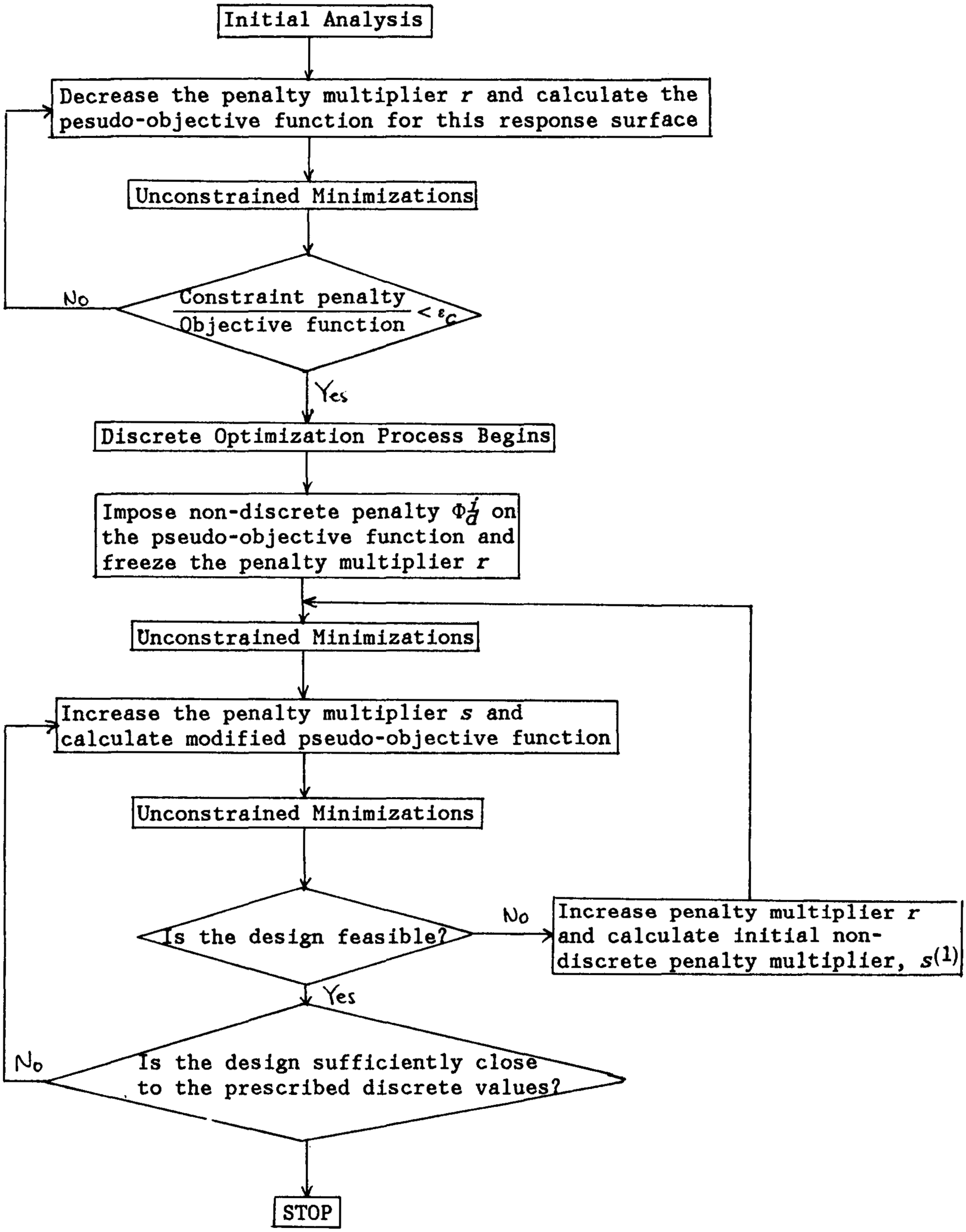


Fig. 4. Flow Chart for Continuous and Discrete Optimization Process

EXAMPLES

All the results presented in this section are generated by using NEWSUMT-A modified with the proposed technique.

1. One Design Variable Problem

For pictorial demonstration, the following simple problem with one design variable and one constraint is presented.

$$\begin{aligned} &\text{Minimize} : && F = x \\ &\text{Subject to} : && g = x - 1.0 \geq 0 \\ &\text{where} && x = \{1.0, 2.0, \ldots\} \end{aligned}$$

The process of the discrete optimization is shown in Fig. 5 for two different values of ε_C. For each discrete iteration, the amount of penalty on the non-discrete values is increased and the design converges to one of the discrete values. Fig. 5-(a) shows that the final design can be trapped at the local minimum if the discrete design process begins too early ($\varepsilon_C = 0.5$). For this example, correct discrete optimum was obtained if $\varepsilon_C < 0.5$.

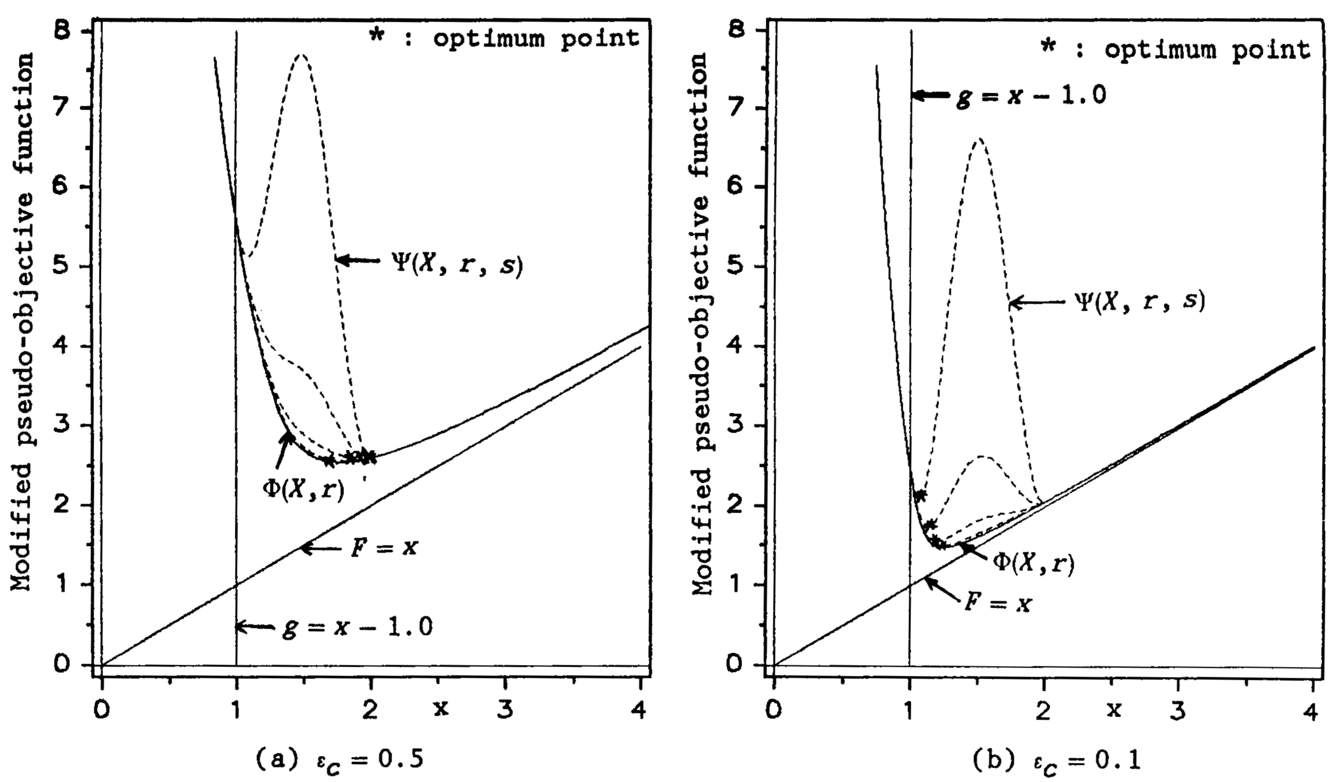


Fig. 5. Discrete optimization process for example 1

2. Three-Bar Truss Problem

The indeterminate three bar truss as shown in Fig. 6 is subject to vertical and horizontal forces. The structural weight, W, is minimized under the constraint that the stress in all members should be smaller than the allowable stress, σ_0, in absolute magnitude. After nondimensionalization of the objective function and design variables ($F = \frac{\sigma_0 W}{P \rho l}$ and $x_i = \frac{a_i \sigma_0}{P}$), the optimum problem can be stated as

Minimize : $F = 2x_1 + x_2 + \sqrt{2}\,x_3$

Subject to :

$$g_1 = 1 - \frac{\sqrt{3}\,x_2 + 1.932x_3}{1.5x_1x_2 + \sqrt{2}\,x_2x_3 + 1.319x_1x_3} \geq 0$$

$$g_2 = 1 - \frac{0.634x_1 + 2.828x_3}{1.5x_1x_2 + \sqrt{2}\,x_2x_3 + 1.319x_1x_3} \geq 0$$

$$g_3 = 1 - \frac{0.5x_1 - 2x_2}{1.5x_1x_2 + \sqrt{2}\,x_2x_3 + 1.319x_1x_3} \geq 0$$

$$g_4 = 1 + \frac{0.5x_1 - 2x_2}{1.5x_1x_2 + \sqrt{2}\,x_2x_3 + 1.319x_1x_3} \geq 0$$

$$x_i = \{0.1,\ 0.2,\ 0.3,\ 0.5,\ 0.8,\ 1.0,\ 1.2\},\ i=1,2,3$$

The continuous optimum design is $F = 2.7336$, $x_1 = 1.1549$, $x_2 = 0.4232$ and $x_3 = 0.0004$. The discrete solution by the proposed method was found to coincide with the actual discrete optimum, $F = 3.0414$, $x_1 = 1.2$, $x_2 = 0.5$ and $x_3 = 0.1$ when $\varepsilon_c \leq 0.1$. For $\varepsilon_c = 0.5$, the design was at a local minimum, $F = 3.1828$, $x_1 = 1.2$, $x_2 = 0.5$, and $x_3 = 0.2$. NEWSUMT-A program reached the continuous optimum in 7 response surfaces, whereas the discrete optimum was converged in 11, 11 and 13 response surfaces when $\varepsilon_c = 0.5$, 0.1 and 0.01, respectively. The computing time for discrete design can be saved if the discrete iteration begins early, although it can result in local optimum.

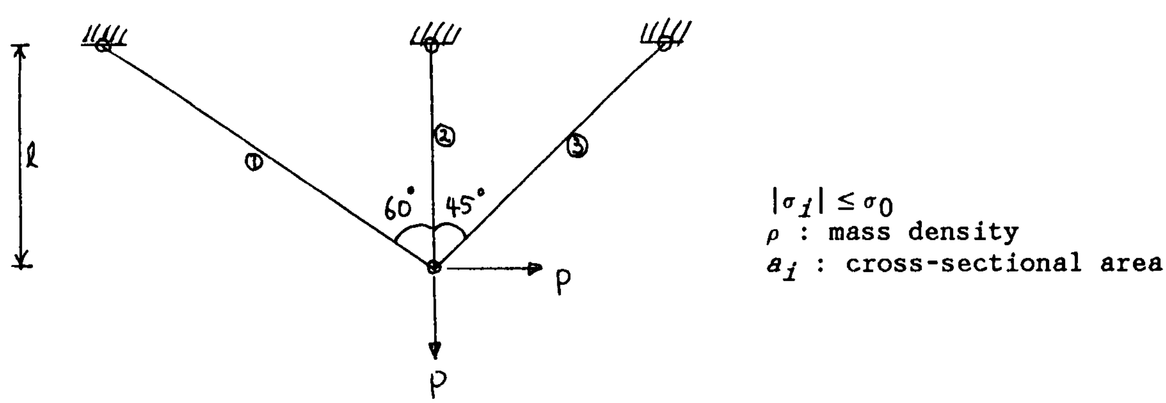
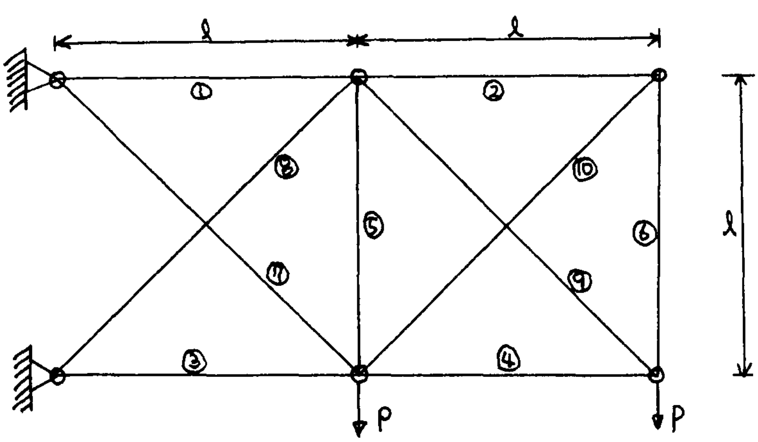


Fig. 6. Three bar truss for example 2

3. Ten-Bar Truss Problem

The classical 10 bar truss is shown in Fig. 7. The design variables are the cross-sectional areas of the 10 bar. The structural weight, W, is minimized subject to a maximum stress limit of 25,000 psi and maximum displacement limit of 2.0 in. All design variables are assumed to take the discrete values, x_i = { 0.1, 0.5, 1.0, 1.5, 2.0, 2.5,.... }. The discrete optimum solution obtained by proposed method is compared in Table 1 with other solutions by different techniques obtained from Reference [10]. The solution by the proposed method is different from all other solutions. The results by Branch & Bound, Ref. [3] and proposed method are slightly infeasible though negligible in engineering sense. It can be seen from Tab. 1 that the different starting point for discrete optimization, ε_c, can result in different discrete solutions. The continuous optimum was obtained in 7 response surfaces and discrete solutions were found in 10 and 13 response surfaces for ε_c =0.01 and 0.001, respectively. The continuous solution was obtained in 8 response surfaces and discrete solutions were found in 11 and 13 response surfaces for ε_c =0.01 and 0.001, respectively. With a relatively large number of design variables as this case, the continuous solution process has to be terminated in the close neighborhood of the continuous optimum.



$E = 10^7$ psi
$\rho = 0.1$ lbm/in^3
P = 100 kip
l = 360 in

Fig. 7. Ten bar truss (example 3)

	Branch & Bound	Generalized Lagrangian	Dynamic Round-off	Schmit & Fleury	Proposed Method(1)	Proposed Method(2)
W	5059.9	5067.3	5077.9	5059.9	5059.9	5103.3
x_1	30.5	30.0	30.0	30.5	30.5	31.0
x_2	0.1	0.1	0.1	0.1	0.1	0.1
x_3	23.0	23.5	23.5	23.0	23.5	23.5
x_4	15.5	15.0	15.5	15.5	15.0	15.0
x_5	0.1	0.1	0.1	0.1	0.1	0.1
x_6	0.5	0.5	1.0	0.5	0.5	0.5
x_7	7.5	7.5	7.5	7.5	7.5	7.5
x_8	21.0	21.0	21.0	21.0	21.0	21.5
x_9	21.5	22.0	21.5	21.5	21.5	21.5
x_{10}	0.1	0.1	0.1	0.1	0.1	0.1

(1) : $\varepsilon_c = 0.001$, (2) : $\varepsilon_c = 0.01$

Table 1. Discrete solutions for 10 bar truss ($\varepsilon_d = 0.001$)

CONCLUDING REMARKS

A simple penalty approach combined with the extended interior penalty function technique for the problems with discrete design variables was presented. Criteria for starting the discrete optimization and for convergence were proposed. The procedure was demonstrated on several numerical examples. It was found that the non-discrete penalty multiplier, s, has to be increased step-by-step and the continuous penalty multiplier, r, has to be relaxed in the discrete optimization process if the constraints are violated. If the problem has a large number of design variables and/or the intervals between prescribed discrete values are close, the penalty terms for non-discrete values have to be activated in the close neighborhood of the continuous optimum. While these preliminary results are encouraging, further numerical tests with more complex problems are required to use the proposed technique with confidence.

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