A Novel Implementation of Method of Optimality Criterion in Synthesizing Spacecraft Structures with Natural Frequency Constraints

B. P. Wang
Department of Mechanical Engineering
The University of Texas at Arlington
Arlington, Texas

F. H. Chu
General Electric
Astro Space Division
Princeton, New Jersey

INTRODUCTION

In the design of spacecraft structures, fine tuning the structure to achieve minimum weight with natural frequency constraints is a time consuming process. In this paper, a novel implementation of the method of optimality criterion (OC) is developed. In this new implementation of OC, the free vibration analysis results are used to compute the eigenvalue sensitivity data required for the formulation. Specifically, the modal elemental strain and kinetic energies are used. Additionally, normalized design parameters are introduced as a second level linking that allows design variables of different values to be linked together. With the use of this novel formulation, synthesis of structures with natural frequency constraint can be carried out manually using modal analysis results. Design examples are presented to illustrate this novel implementation of the optimality criterion method.

PROBLEM STATEMENT

The optimal design problem to be solved is determination of the values of design variables such that the structure weight is minimized while maintaining a specified fundamental natural frequency of the system. The design variables are sizing of structural members, e.g. cross-sectional areas of truss elements; area moment of inertia of beam elements; and thickness of plate elements. Bounds on design variables are also considered.

FIND $X \in R^n$

TO MINIMIZE

$$W = W(X) \quad (1)$$

SUBJECT TO THE CONSTRAINTS

$$f_1 \geqslant f_{1d} \quad (\text{OR } \lambda_1 \geqslant \lambda_{1d}) \quad (2)$$

$$\text{AND} \quad X_L \leqslant X \leqslant X_U \quad (3)$$

NOTE THAT $f_1 = \sqrt{\lambda_1} / 2\pi$

AND λ_1 IS RELATED TO THE DESIGN VARIABLE THROUGH THE EIGENVALUE PROBLEM

$$[K(X)]\{\phi_1\} = \lambda_1 [M(X)]\{\phi_1\}$$

OPTIMALITY CRITERION

The optimal design problem defined in the previous section can be solved by mathematical programming techniques. To derive a simpler approach, we will treat the frequency constraint defined in Eq. (2) as equality constraint. Additionally, the side constraints will be ignored for the time being. With these simplifications, the set of optimality criteria can be derived among the design variables by the Lagrange multiplier technique. The optimality criterion can be interpreted as:

> At the optimal design, the ratio of the eigenvalue sensitivity to weight sensitivity is a constant for all design variables.

FROM LAGRANGIAN:

$$L = W - \mu(\lambda_1 - \lambda_{1d})$$

THE OPTIMALITY CRITERION:

$$\frac{\partial L}{\partial X_i} = 0 \qquad (4)$$

LEADS TO

$$\frac{\partial W}{\partial X_i} - \mu \frac{\partial \lambda_1}{\partial X_i} = 0$$

OR

$$\frac{\partial \lambda_1 / \partial X_i}{\partial W / \partial X_i} = \frac{1}{\mu} = \text{CONSTANT} \qquad (5)$$

EQUATION (5) IS THE OPTIMALITY CRITERION AN OPTIMAL DESIGN MUST SATISFY.

BASIC REDESIGN ALGORITHM

Following the optimality criterion method suggested by Khot [1], linear recurrence relations can be developed based on Eq. (5). The Lagrange multiplier is computed by requiring that the updated design satisfy the frequency constraint. The basic redesign algorithm is summarized in Eqs. (6) and (7).

REDESIGN FORMULA:

$$(X_i)_{s+1} = (X_i)_s \left[\alpha + \mu(1-\alpha)\frac{G_i}{C_i}\right] \quad (6)$$

LAGRANGE MUTIPLIER:

$$\mu = \frac{\Delta\lambda + (1-\alpha)\sum G_i (X_i)_s}{(1-\alpha)\sum (G_i^2 \cdot X_i)_s / C_i} \quad (7)$$

$$\Delta\lambda = \lambda_{desired} - \lambda_{current}$$

APPROXIMATE EIGENVALUE SENTIVITY ANALYSIS

In the redesign algorithm, we need to know the derivatives of weight and eigenvalue with respect to the design variables. While the weight sensitivity is simple to calculate, the computation of eigenvalue sensitivity could be quite involved because of the need to know derivatives of element stiffness and mass matrices with respect to design variables. In this paper, we adopt an approximate approach for computing eigenvalue sensitivity which use elemental strain and kinetic energy in the vibration mode [2].

EIGENVALUE SENSITIVITY

GENERAL EQUATION:

$$\frac{\partial \lambda_1}{\partial X_i} = \frac{1}{M_1} \{\phi_1\}^T \left(\frac{\partial [K]}{\partial X_1} - \lambda_1 \frac{\partial [M]}{\partial X_1} \right) \{\phi_1\} \qquad (8)$$

SIMPLIFIED EQUATION [2]:

$$\frac{\partial \lambda_1}{\partial X_i} = \frac{2}{M_1} \left(\frac{\beta_i}{X_i} V_{1i1} - \frac{\gamma_i}{X_i} T_{1i1} \right) \qquad (9)$$

ASSUMPTIONS:

$$K_i = (X_i)^{\beta_i} K_i^* \qquad (10)$$

$$M_i = (X_i)^{\gamma_i} M_i^* \qquad (11)$$

SPECIAL CASE FOR:

1. TRUSS ELEMENTS
2. SYSTEM MASS MATRIX DOMINATED BY NON-STRUCTURE MASS

THEN

$$\frac{\partial \lambda_1}{\partial X_i} = \frac{2 V_{1i1}}{M_1 \cdot X_i} \qquad (12)$$

A NOVEL IMPLEMENTATION OF THE BASIC REDESIGN ALGORITHM

The redesign algorithm given by Eqs. (6) and (7) can be implemented easily for truss structure. For example, if one uses MSC/NASTRAN [3] for structural analysis, the strain energy and strain energy density can be obtained together with the modal analysis results. Using these data and assuming that a majority of the system weight is contributed from non-structural mass, the redesign algorithm can be implemented using the following procedures.

(1) PERFORM MODAL ANALYSIS WITH STRAIN ENERGY AND STRAIN ENERGY DENSITY CALCULATIONS.

(2) COMPUTE $C_i = \frac{\partial W}{\partial X_i}$ USING

$$C_i = \sum \rho_e (ESE)_e / (ESED_e \cdot X_e) \tag{13}$$

(3) COMPUTE $G_i = \frac{\partial \lambda_1}{\partial X_i}$ USING

$$G_i = 2 \sum (ESE_e / X_e M_1) \tag{14}$$

NOTE THAT THE SUBSCRIPT e REFERS TO ELEMENT NUMBER AND THE SUMMATIONS IN EQS. (13) AND (14) ARE OVER ALL THE ELEMENTS THAT ARE ASSIGNED AS DESIGN VARIABLE X_i.

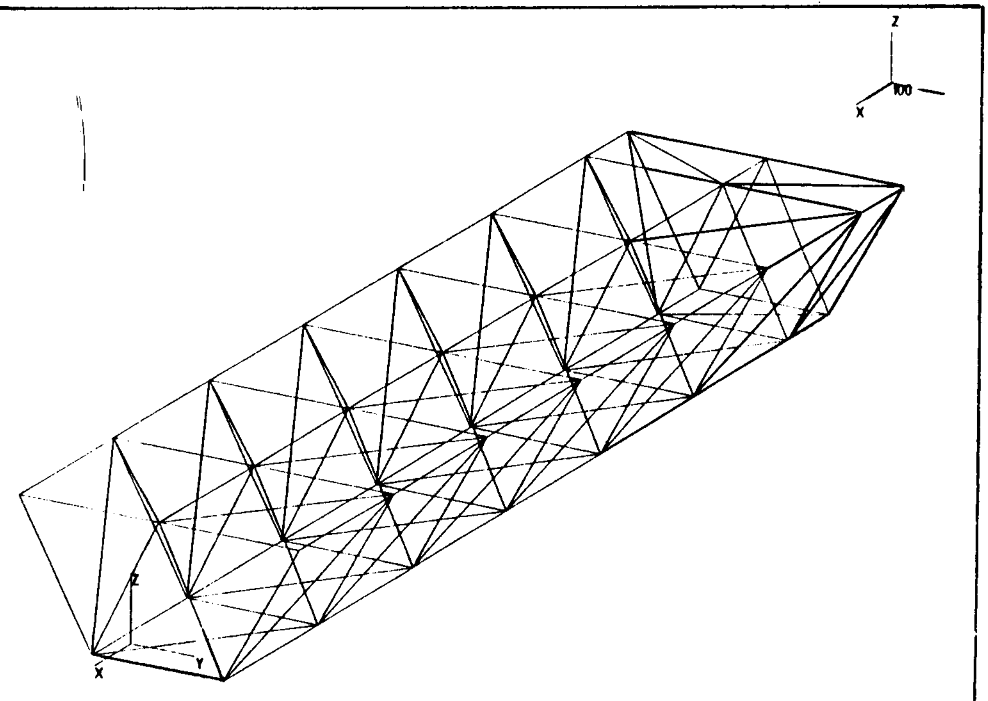
(4) COMPUTE LAGRANGE MULTIPLIER USING EQ. (7).

(5) UPDATE THE DESIGN VARIABLES USING EQ. (6).

DESIGN EXAMPLE

The design procedure described in this paper has been applied to a truss structure shown below. The objective is to find the minimum weight design of the truss structure to maintain a specified fundamental natural frequency of the system. Starting from a uniform truss structure that satisfied the 5.7 Hz constraint on the fundamental natural frequency, the design is manually optimized by typical trade studies. The optimality criterion algorithm is then applied to this manually optimized structure to obtain an additional 25 pound saving in the structural weight. The comparison of the truss structures weights is shown in the table below.

DESIGN CASE	TRUSS STRUCTURE WEIGHT	f
UNIFORM TRUSS SIZE	2102.0 lbs	5.7 Hz
MANUALLY OPTIMIZED TRUSS SIZE	1014.0 lbs	5.7 Hz
OPTIMIZED TRUSS SIZE USING METHOD OF OC	989.0 lbs	5.7 Hz



TRUSS STRUCTURE FOR OPTIMIZATION STUDY

CONCLUDING REMARKS

A method of optimality criterion was shown to be a powerful tool for minimum weight design of structures with constraint on fundamental natural frequency. With the new method of implementation presented in this paper, the design procedure can be carried out by simple calculations. The effectiveness of this approach has been demonstrated by a truss structure. This method can be extended to other types of structure elements using eigenvalue sensitivity formulation in Ref. [2].

REFERENCES

1. Khot, N., Optimality Criterion Methods in Structural Optimization. Chapter 5 of Foundations of Structural Optimazation: A Unified Approach, Edited by A.J. Morris, John Wiley and Sons, Ltd., 1982.

2. Wang B. P., On Computing Eigensolution Sensitivity Data Using Free Vibration Solutions. in Sensitivity Analysis in Engineering, NASA CP2457, 1987, pp. 223-245.

3. MSC/NASTRAN USER'S MANUAL, The MacNeal-Schwendler Co., 1982.

SYMBOLS AND ABBREVIATIONS

C_i = $\partial W / \partial X_i$ FOR CURRENT DESIGN
ESE_e = STRAIÑ ENERGY
$ESED_e$ = STRAIN ENERGY DENSITY
f_1 = FIRST NATURAL FREQUENCY
f_{1d} = DESIRED FIRST NATURAL FREQUENCY
G_i = $\partial \lambda_1 / \partial X_i$ FOR CURRENT DESIGN
$[K(X)]$ = GLOBAL STIFFNESS MATRIX
$[M(X)]$ = GLOBAL MASS MATRIX
M_1 = GENERALIZED MASS OF THE FIRST MODE
R = SPACE OF DESIGN VARIABLES
T_{1i1} = TOTAL KINETIC ENERGY OF ELEMENTS ASSOCIATED WITH DESIGN VARIABLE i FOR MODE 1
V_{1i1} = TOTAL STRAIN ENERGY OF ELEMENTS ASSOCIATED WITH DESIGN VARIABLE i FOR MODE 1
W = STRUCTURE WEIGHT
X = DESIGN VARIABLE VECTOR
X_L = LOWER BOUNDS OF X
X_U = UPPER BOUNDS OF X
X_i = CURRENT DESIGN VARIABLE
$(X_i)_s$ = CURRENT DESIGN
$(X_i)_{s+1}$ = UPDATED DESIGN
λ = EIGENVALUE
α = RELAXATION FACTOR
ρ_e = WEIGHT DENSITY