EXPERIENCES IN APPLYING OPTIMIZATION TECHNIQUES TO CONFIGURATIONS FOR THE CONTROL OF FLEXIBLE STRUCTURES (COFS) PROGRAM

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INTRODUCTION

Over the last two decades an extensive amount of work has been done in developing and applying mathematical programming methods to the optimum design of structures (refs. 1-11). In the past, optimization techniques have been applied mainly in the conceptual (refs. 2 and 6) and preliminary design (refs. 7 and 8) phases with few applications to realistic problems such as those found in references 9-11. In reference 2 Ashley discusses the lack of applications of optimization techniques to realistic problems. Generally, transforming a realistic problem into a mathematical programming formulation is difficult and a high degree of engineering judgment and experience is needed. Also the choice of an objective function is not always obvious. Ashley offers three reasons why classically optimized structures are not being found in actual service: first, developmental engineers are sometimes reluctant to try "new and unfamiliar" methods; second, they sometimes find it difficult to translate realistic design or operational requirements into a mathematical programming formulation; and third, they sometimes find it easier to perform many finite element parametric analyses than learn optimization software. The latter is especially true when a designer is faced with time and schedule deadlines and will often choose to perform parametric studies rather than try formal optimization procedures.

This paper will address several of these issues - namely the objective function choice and the difficulty of translating realistic design requirements into a mathematical programming formulation. The paper will also show that optimization procedures can also be helpful later in the postdesign phase.

The purpose of the paper is to relate experiences gained in applying optimization procedures to design large flexible spacecraft for the Control of Flexible Structures (COFS) program. First some background and a brief discussion of the motivation behind the COFS work will be presented. Next the paper will discuss two studies using optimization techniques related to the COFS project which address the issue of objective function choice. In the first study an optimization procedure was developed for frequency spacing for a simple model of a COFS-II configuration. The next study involved an optimization procedure for a detailed model of the COFS-I configuration in connection with a buckling deficiency problem. The third study describes a redesign activity of the COFS-I mast in which optimization techniques were used to redesign the mast structure using the same design requirements as the contractor who originally designed the mast using parametric studies. Finally the paper will relate some experiences and insights gained in incorporating into a structural optimization procedure requirements that are realistic and which were continually being modified as the study was being conducted.

CONTROL OF FLEXIBLE STRUCTURES (COFS)

As spacecraft structural concepts increase in size, complexity, and flexibility, a need exists to develop and validate analytical methods to design and assess the performance of such spacecraft. The Control of Flexible Structures (COFS) research program shown in figure 1 was initiated by the NASA Office of Aeronautics and Space Technology (OAST) to develop a validated technology data base for understanding the structural response, pointing and shape control, suppression of inherent dynamic responses, and avoidance of undesirable interaction between flexible structures and controls. Information on the COFS program can be found in references 12-19. Shown in the figure are two projects in the COFS program. First the COFS-I Project was to involve a series of on-ground and in-flight tests to investigate the dynamics/control interactions utilizing a beam. Second the COFS-II project was to build on the control technology developed in COFS-I project to investigate three-dimensional dynamics/control interactions.

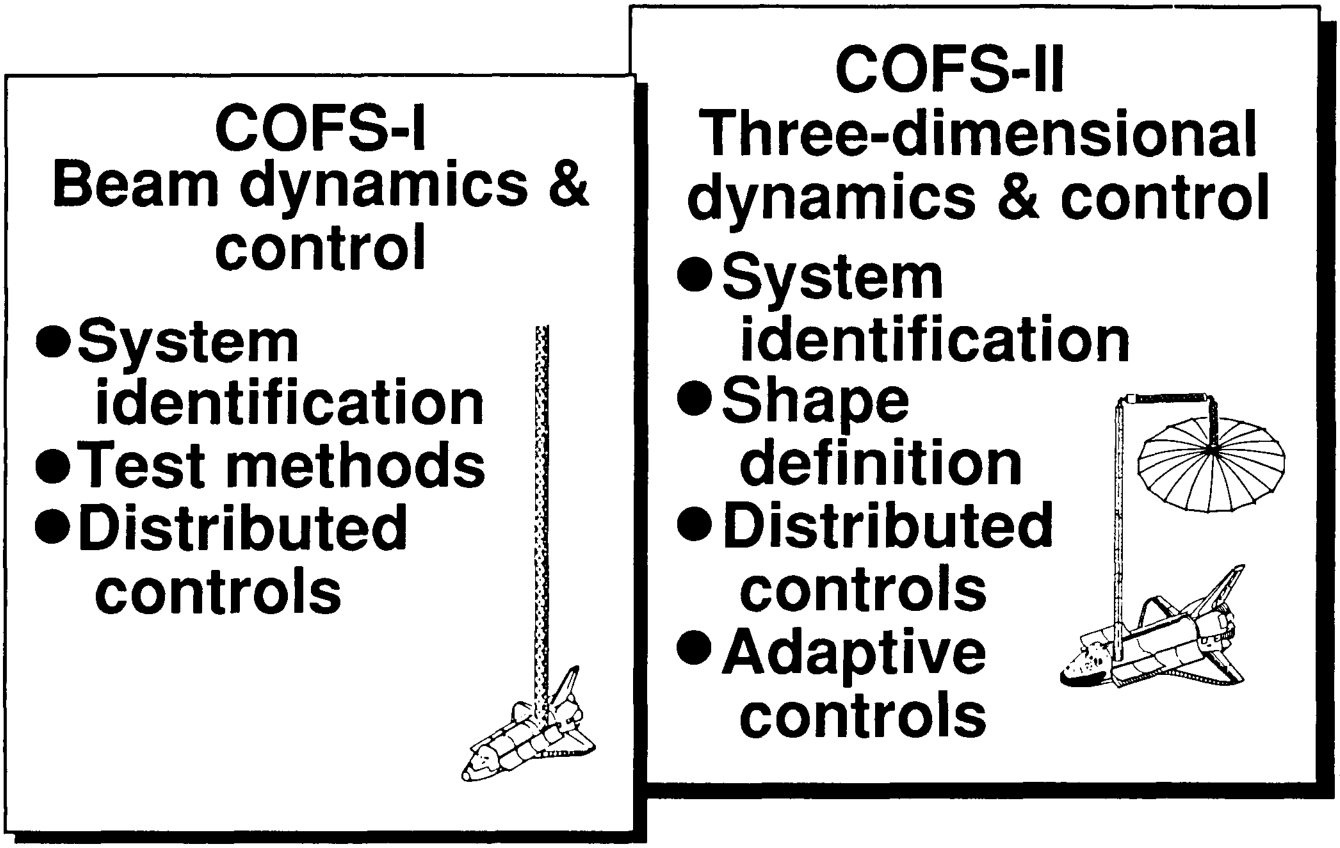


FIGURE 1

MOTIVATION FOR COFS OPTIMIZATION

The COFS structure is to be designed to have closely coupled vibration modes (fig. 2). This is contrary to the normal process in which the designer seeks widely spaced frequencies as he tries to control rigid body motions and avoid control/structures interactions. However, the COFS program requires a structure which has closely spaced frequencies in order to challenge control law and system identification methodology. The need for a method to systematically design large spacecraft with closely spaced frequencies was the motivation for the initial optimization work for frequency spacing of COFS.

- COFS required closely spaced frequencies to challenge:
  - Control law synthesis
  - System identification
- Need

Systematic method to design large space systems (LSS) for close-spacing of vibration frequencies

FIGURE 2

OVERVIEW OF OPTIMIZATION PROCEDURES FOR COFS STUDIES

An optimization procedure (fig. 3) was developed which systematically designed a large space system with closely spaced frequencies. The procedure uses the general-purpose finite-element analysis program Engineering Analysis Language (EAL, ref. 20) and a combination of the general-purpose optimization program CONMIN (ref. 21) and piecewise linear approximate analyses for the optimization. The eigenvalue analysis and constraint calculations are performed using EAL. The EAL system contains individual processors that communicate through a data base containing data sets. The data sets typically contain data describing the finite-element model of the structure (such as geometry) as well as response information that is accumulated during the execution of the processors. The processors can be executed in any appropriate sequence, and a sequence of processor executions is denoted as a "runstream". The EAL system also uses a set of flexible FORTRAN-like statements called executive control system (ECS) commands. These commands allow branching, testing data, looping, and calling runstreams (similar to calling FORTRAN subroutines). The EAL processors, with the appropriate ECS commands organized as runstreams are used to calculate the eigenvalues, eigenvectors, constraints, objective function and derivatives of these quantities. CONMIN is a general-purpose optimization program that performs constrained minimization using a usable-feasible directions search algorithm. In the search for new design variable values, CONMIN requires derivatives of the objective function and constraints. The user has the option of either letting CONMIN determine the derivative by finite differences or supplying derivatives to CONMIN. The latter method will be used herein. In the approximate analysis method, previously calculated derivatives of the objective function and constraint functions with respect to the design variables are used for linear extrapolation of these functions. The assumption of linearity is valid over a suitably small change in the design variable values and will not introduce a large error into the analysis provided the changes are small. This approximate analysis will be referred to as a "piecewise linear approximation." Errors which may be introduced by use of the piecewise linear approach are controlled by imposing "move limits" on each design variable during a design cycle. A move limit which is specified as a fractional change, δ, of each design variable value (for this work, $\delta=0.1$) is imposed as an upper and lower design variable bound on each cycle. These move limits must not exceed the absolute design variable values. Details of the algorithm are contained in reference 11.

- **Use formal mathematical programming techniques**
- **Combine EAL, CONMIN, and approximate analyses**
- **Free vibration eigenvalue problem**

$$\left([K] - \omega_j^2 [M]\right) \{\theta\}_j = 0$$

- **Eigenvalue derivative**

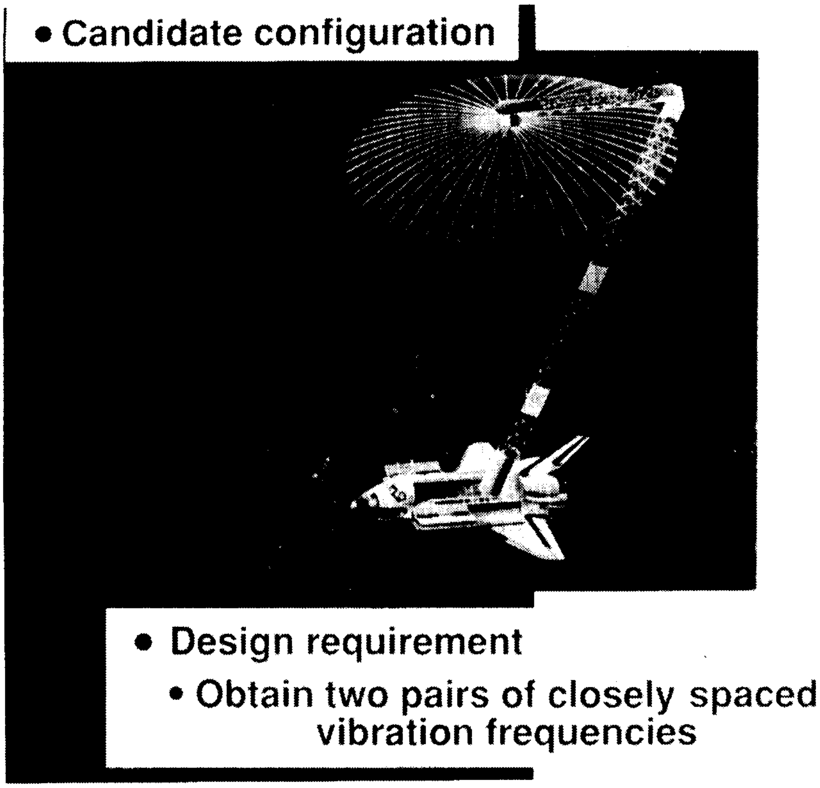
$$\frac{\partial \omega_j^2}{\partial V_K} = \{\theta\}_j^T \left(\frac{\partial [K]}{\partial V_K} - \omega_j^2 \frac{\partial [M]}{\partial V_K}\right) \{\theta\}_j$$

FIGURE 3

COFS-II FREQUENCY SPACING STUDY

The purpose of the COFS-II frequency study was to develop the methodology for systematically obtaining two pairs of closely-spaced frequencies. A conceptual design of a candidate COFS-II configuration is shown in figure 4. The configuration consisted of a mast, a boom, and a structure attached to the tip (such as an antenna).

Earlier unpublished parametric studies[*] using a simple model indicated the most suitable frequency pairs for close-spacing are the third frequency, f_3, with the fourth frequency, f_4, and the fifth frequency, f_5, with the sixth frequency, f_6. The third mode is characterized by bending and twisting of the mast and rigid body movement of the boom. The fourth mode is characterized by first in-plane bending of the mast and first in-plane bending of the boom. The fifth mode is characterized as second in-plane bending of the mast and second in-plane bending of the boom. The sixth mode is characterized by second out-of-plane bending coupled with torsion of the mast and first out-of-plane bending of the boom. These parametric studies verified the feasibility of closely spacing two pairs of frequencies and led to the development of an optimization procedure to systematically closely space pairs of frequencies. More details on the COFS-II frequency spacing optimization study can be found in reference 22.



- Candidate configuration
- Design requirement
  - Obtain two pairs of closely spaced vibration frequencies

FIGURE 4

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[*] Carried out and communicated to the author by Dr. Michael F. Card of the NASA Langley Research Center.

COFS-II MODEL FOR FREQUENCY SPACING STUDY

The simple model of a COFS-II configuration shown on the right in figure 5 was used in this study. The model which is based on the geometry derived from reference 23 is modeled as an equivalent beam with 17 joints. More detail on the model can be found in reference 22. The properties of the mast are fixed except for the length L_1. In the beam segment from the top of the mast to the tip of the boom, none of the properties are fixed.

The six design variables are shown below: the mast length (L_1), the boom length (L_2), the boom cross-sectional area (A), the two boom area moments of inertia (I_{yy} and I_{zz}), and the concentrated mass (M) at the tip of the beam. Since the structure is to be deployable to an arbitrary length in increments of two-bay lengths and must fold inside a canister on the Shuttle, the mast length L_1 is allowed to vary between 40 and 60 meters and the boom length L_2 between 1 and 25 meters. The tip mass, M, representative of an attachment such as an antenna is allowed to vary between 10 and 30 kg. The range of values for A, I_{yy}, and I_{zz} are chosen to prevent mode switching (i.e. want to ensure f_3 is paired with f_4 and f_5 is paired with f_6).

Design variables

L_1 - mast length

L_2 - boom length

A - boom cross-sectional area

I_{yy}, I_{zz} } Boom moments of inertia

M - mass attached to end of boom

Model

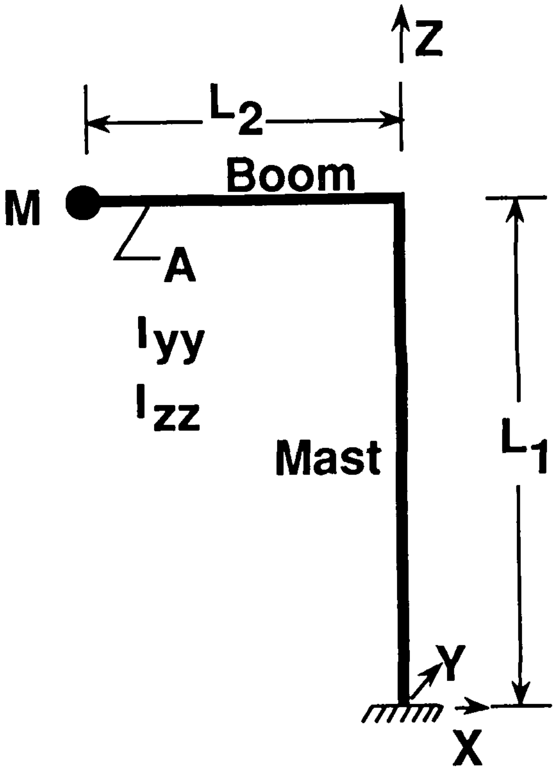

FIGURE 5

OPTIMIZATION FORMULATIONS FOR COFS-II FREQUENCY SPACING STUDY

Two optimization formulations (fig. 6) were tried for the COFS-II study. Since the aim of the study was to develop the methodology for closely-spacing two pairs of frequencies, the optimization problem was first formulated in terms of a frequency spacing objective function (Formulation 1). The objective function was defined so that minimizing the objective function would cause the close-spacing of the frequency pairs. The only constraints on the problem were upper and lower bounds on the design variables denoted by ϕ_{i_L} and ϕ_{i_U}, respectively. The second formulation (Formulation 2) was a more conventional structural optimization formulation in which mass was minimized. The design requirements include two pairs of adjacent frequencies to be closely spaced - i.e., f_3 and f_4 be within a specified arbitrarily small ε_1 while f_5 and f_6 be within a specified arbitrarily small ε_2. These latter conditions are modeled as constraints in the optimization along with upper and lower bounds on the design variables denoted by ϕ_{i_L} and ϕ_{i_U}, respectively. For both formulations the design variables are L_1, L_2, A, I_{YY}, I_{ZZ}, and M (fig. 5).

	Formulation 1	**Formulation 2**
Objective function	$\left[\left(\frac{f_6-f_5}{f_6}\right)^2+\left(\frac{f_4-f_3}{f_4}\right)^2\right]^{1/2}$	**Mass**
Constraints	$\varphi_{i_L} \le \varphi_i \le \varphi_{i_u}$	$\varphi_{i_L} \le \varphi_i \le \varphi_{i_u}$ $\left\lvert\frac{f_4-f_3}{f_4}\right\rvert \le \varepsilon_1$ $\left\lvert\frac{f_6-f_5}{f_6}\right\rvert \le \varepsilon_2$
Design variables (φ_i)	$L_1, L_2, A, I_{yy}, I_{zz}, M$	

FIGURE 6

COFS-II OPTIMIZATION RESULTS

Results of the optimization procedure using Formulation 2 are shown in figure 7. Plots of vibrational frequency as a function of design cycle are shown on the left. A design cycle is a finite element analysis followed by an optimization step. As shown in the figure the first pair of frequencies (f_3 and f_4) are closely spaced after 5 design cycles. After about 16 design cycles, both pairs of frequencies are closely spaced. A detailed discussion of why the optimization procedure is able to closely space the first pair of frequencies (f_3 and f_4) so quickly but requires 11 more cycles to closely space the second pair of frequencies (f_5 and f_6) can be found in reference 22. A plot of the mass as a function of design cycle is shown on the right. The optimization procedure obtains a design which is able to closely-space two pairs of adjacent frequencies and provides some reduction in total mass (approximately 11 kg). Results for Formulation 1 are not shown since this formulation was not successful. The reasons for this will be discussed in figure 8.

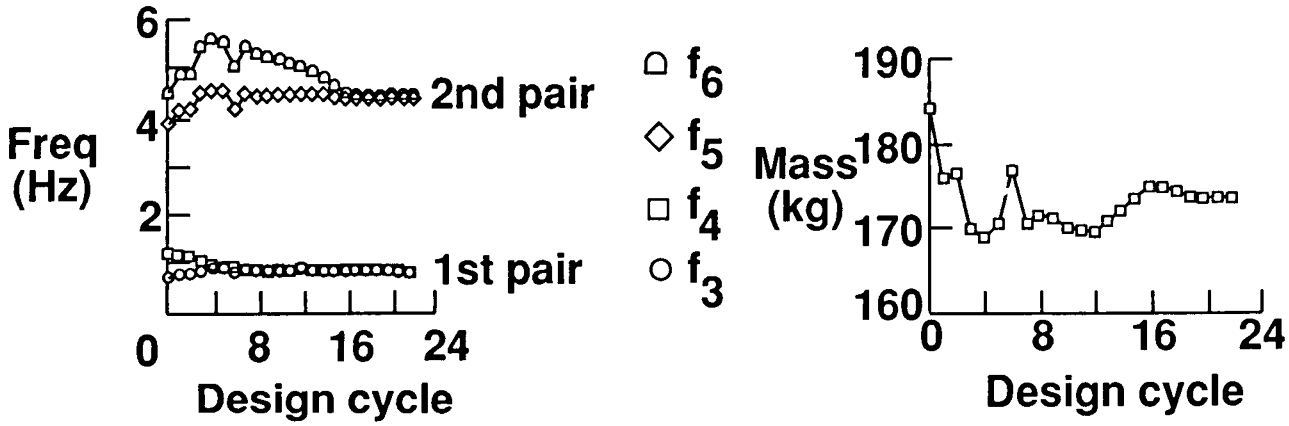

FIGURE 7

OBSERVATIONS FROM THE COFS-II FREQUENCY SPACING STUDY

Of the two optimization formulations tried for the COFS-II frequency spacing study only Formulation 2 was successful. With Formulation 1 the optimizer initially had some success in closely spacing one pair of frequencies (f_3 and f_4) for the first several cycles. When the optimizer tried to closely space the second pair of frequencies (f_5 and f_6), the first pair separated. At first it was thought it might not be possible to closely space f_5 and f_6 so two separate optimization problems were tried - one where one pair of frequencies (f_3 and f_4) were to be paired and one where the frequencies (f_5 and f_6) were to be paired. It was found that both pairs could be closely spaced separately. As a result, the second formulation (Formulation 2) was developed in which frequency spacing conditions were formulated as constraints. This formulation was successful. In retrospect one difficulty with Formulation 1 may have been due to the use of CONMIN with no constraints (other than side constraints on the design variables). Since CONMIN uses the method of usable feasible directions, it tends to follow active constraints in its search for an optimum. However by examining the plots (fig. 7) for Formulation 2, it appears that a stronger reason why Formulation 1 did not work was that the pairings were conflicting. The optimizer could not adjust the spacing of one pair without hurting the spacing of the second pair. However, when the pairings were used as constraints, the optimizer increased the spacing between the second pair of frequencies (f_5 and f_6) to decrease the spacing between the first pair (f_3 and f_4) and then finally decreased the spacing between the second pair later (around cycle 16) in the optimization process.

- Formulation 1
  - No convergence
  - Only one pair of frequencies could be closely spaced at a time
    - CONMIN performs best for constrained problems
    - Conflicting goals in objective function
- Formulation 2
  - Converged
  - Two pairs of frequencies closely spaced
  - Observation of convergence behavior revealed reason for poor convergence of Formulation 1

FIGURE 8

COFS-I BUCKLING DEFICIENCY STUDY

The next study involved the COFS-I flight mast shown fully deployed from the Space Shuttle in figure 9. The mast is approximately 60 meters long and consists of 54 bays of single-laced latticed beams with unequal area longerons (two "weak" longerons and one "strong" longeron). The "strong" longeron is located on the centerline of the Shuttle. The longerons have different cross-sectional areas to promote the coupling between modes. Further details of the COFS-I flight mast can be found in references 15, 16, and 19.

The mast was originally designed using parametric studies to have one pair of closely spaced frequencies (the first torsional and the second bending frequencies). It was subsequently determined that there were some deficiencies with the original design. In particular, the diagonal members of the original COFS-I design might buckle during deployment. There was also a concern that individual member frequencies might interact with global frequencies of the mast (i.e. be in the bandwidth which was to be tested in the flight experiment). An in-house redesign team was formed to address these issues. As part of this effort, an optimization procedure based on the previous COFS-II study was formulated and applied using a detailed model of the original COFS-I configuration to determine if it was possible to meet the additional design requirements and maintain the close-spacing of the frequencies. The design requirements, shown below, are that the first natural frequency of the diagonal be greater than 15 Hz, the first torsional and second bending frequencies be within one percent, the first natural frequency of the mast be greater than 0.18 Hz, minimum gage conditions (e.g. diagonal wall thickness be greater than 0.56mm), and the condition that the "weak" and "strong" longerons remain the same. For this study the mast was analyzed at its fully deployed position. It was felt that addressing the individual member frequency concern would also help alleviate the buckling during deployment concern.

- **Issues - original design deficient**
  - **Potential buckling of diagonal during deployment**
  - **Interaction of individual member frequencies with global frequencies**
- **Design requirements**
  - **1st natural frequency of diagonal ≥ 15 Hz (local frequency and buckling requirements)**
  - **1st torsion and 2nd bending frequencies within 1%**
  - **1st natural frequency of mast ≥ 0.18 Hz**
  - **Minimum gage, e.g. diagonal wall thickness ≥ 0.56mm**
  - **"Weak"/"strong" longerons**

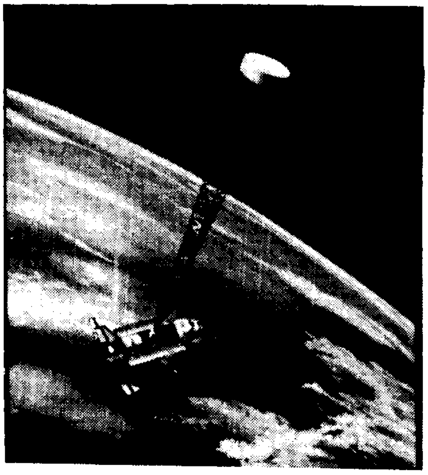

FIGURE 9

COFS-I MODEL FOR BUCKLING DEFICIENCY STUDY

A finite element model of the entire COFS-I mast and Shuttle consisting of 360 joints is used in this study. The Shuttle is modeled as a stick model with very stiff beam elements. The battens, longerons, and diagonals of the mast are modeled by tubes which have bending, torsional, and axial stiffnesses. The model includes lumped masses to represent hinges, deployer retractor assembly, sensor and actuator platforms, etc. Further details of the finite element model can be found in reference 19.

Shown on the left of figure 10 is a typical 2-bay segment of the mast. In order to have minimal impact on the original design, a limited number of quantities are allowed to vary. The number of bays, all lengths of individual members (battens, longerons, and diagonals), and all physical properties of the battens are held constant. The outer radii of the longerons are also held constant to permit the mast to fold into a canister in the Shuttle without redesigning the hinges. The inner radii (R_S and R_W) of the longerons and the inner and outer radii of the diagonals (R_D and R_o, respectively) are allowed to vary in order to meet the design requirements discussed above. The four design variables are shown on the right.

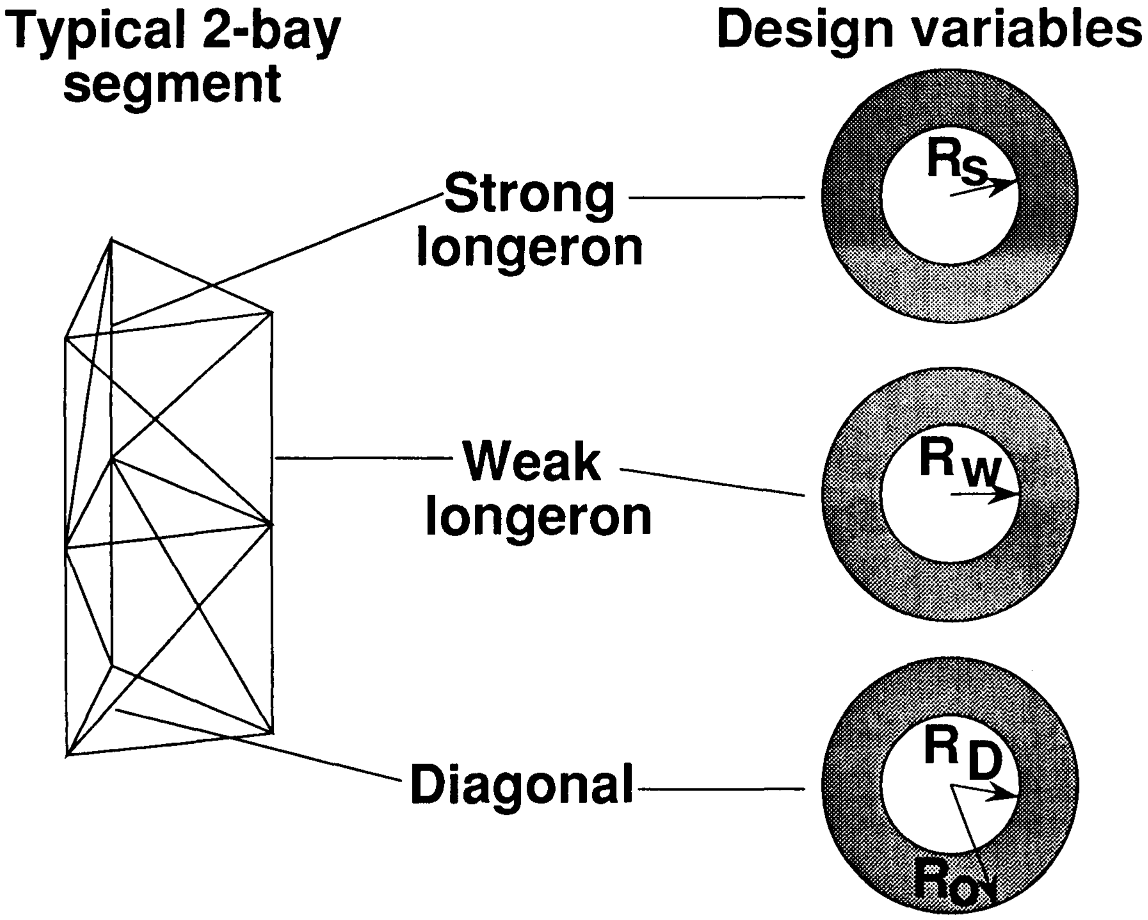

FIGURE 10

OPTIMIZATION FORMULATIONS FOR COFS-I BUCKLING DEFICIENCY STUDY

The two optimization formulations used for the COFS-I buckling deficiency study are shown in figure 11. The major difference between the two formulations was in the choice of objective function. In Formulation 1, the objective function was the total mass with frequency spacing used as a constraint. In Formulation 2 the objective function was a measure of the spacing between the first torsional and second bending frequencies denoted by f_T and f_B, respectively. No limitations on the mass were included. The design variables (R_S, R_W, R_D, and R_O) and the remaining constraints were the same for both formulations. The first constraint is that the first natural frequency (f_D) of the diagonal be greater than 15 Hz. The diagonal frequency is calculated from a simple formula based on assumptions of simply-supported ends with the mass of the hinge concentrated at the center of the diagonal (ref. 24). This requirement is a stiffness constraint to ensure that individual member frequencies of the diagonals are outside the mast frequency range in which frequencies are to be closely spaced (to preclude interaction of member frequency upon the global frequency). Although individual member frequencies of the longerons and battens are also of concern, it is felt that individual member frequencies of the diagonals are most likely to be in the mast frequency range due to their length and the large mass of the hinge. The next requirement is that the first natural frequency (f_1) of the mast be greater than 0.18 Hz. This requirement assures that the frequencies of the mast do not couple with those of the Shuttle control system. Another requirement is that the inner radius R_W of the weak longeron be at least 0.254mm larger than the inner radius R_S of the strong longeron (this is the "weak"/"strong" longeron design requirement shown on the previous figure). The last requirement is a minimum gage requirement on the wall thickness (Δt) of the diagonal members (the minimum wall thickness must be greater than 0.56mm). In addition side constraints (lower and upper limits denoted by ϕ_{i_L} and ϕ_{i_U}, respectively) were imposed on the design variables.

	Formulation 1	**Formulation 2**
Objective function	**Mass**	$\left[\left(\frac{f_T - f_B}{f_T}\right)^2\right]^{1/2}$
Design variables (φ_i)	R_S, R_W, R_D, R_O	R_S, R_W, R_D, R_O
Constraints	$\left\lvert\frac{f_T - f_B}{f_T}\right\rvert \leq 0.01$	
	$f_D \geq 15$ Hz	$f_D \geq 15$ Hz
	$f_1 \geq 0.18$ Hz	$f_1 \geq 0.18$ Hz
	min. gage	min. gage
	$(R_W - R_S \geq \Delta)$	$(R_W - R_S \geq \Delta)$
	$\Delta t_D \geq 0.56$mm	$\Delta t_D \geq 0.56$mm
	$\varphi_{i_L} \leq \varphi_i \leq \varphi_{i_u}$	$\varphi_{i_L} \leq \varphi_i \leq \varphi_{i_u}$

FIGURE 11

OPTIMIZATION RESULTS FROM COFS-I BUCKLING DEFICIENCY STUDY

Results for the COFS-I buckling deficiency optimization study using Formulation 1 are given in figure 12. Plots which show convergence of the COFS-I design give the designer insight into the design process by allowing him to see trade-offs between design requirements. The optimization procedure begins with four satisfied design requirements (two of which are active). As shown on the upper left, initially the frequencies f_B (second bending) and f_T (first torsional) are closely spaced and the Shuttle requirement on the first natural frequency f_1 of the mast is active (f_1=0.188Hz). As seen in the upper right figure, the requirement on the weak and strong longerons (R_W-R_S) and the diagonal wall thickness (R_O-R_D) are satisfied with the latter requirement being active. However, from the lower left figure, initially the diagonal frequency (f_D=11.5 Hz) is lower than the required value of 15 Hz. As stated earlier, the diagonal frequency requirement was not considered in the original design. As the optimization process proceeds, the values of the design variables are changed until the diagonal frequency requirement is satisfied (lower left). The two frequencies (f_B and f_T, upper left) are not as close as they were initially since the diagonal frequency works against this requirement. Specifically, when the diagonal frequency f_D is increased by an increase in stiffness, the first torsional frequency f_T is also increased. The "dips" in the diagonal frequency and the frequency pairs at cycles 9, 13, and 20 are partly due to the optimizer which attempts to satisfy all constraints even at the expense of increasing the objective function and partly due to the linearization of the problem. The optimizer concentrates on satisfying the diagonal frequency constraint until cycle 8, when it tries to satisfy the frequency spacing requirement. The optimizer chooses values for the four radii which closely space the frequencies (see cycle 9, upper left), but those choices lower the diagonal frequency (cycle 9, lower left). Now the optimizer tries to satisfy this diagonal frequency constraint which, as mentioned previously, works against the frequency spacing requirement (see upper left, cycles 10-12). This same process occurs again at cycles 13 and 20. The spacing of the two frequencies (f_B and f_T) cannot be made closer than 0.18 Hz. The "dips" are also due to the linearization of the problem. During the optimization process, "mode switching" occurs at cycles 9, 13, and 20. For example, if at the beginning of the cycle, the second bending mode is associated with f_{10} and the first torsional mode is associated with f_{11}, changes in the radii can cause the second bending mode to be associated with f_9 and the torsional mode with f_{11}. However, the optimizer is choosing values for the design variables based on derivative information at the start of the cycle (i.e. which mode is torsional and which mode is second bending). This is rectified when a full analysis is performed. The design process is also being limited by the minimum gage requirements - namely, R_D and R_W are at their upper and lower bounds, respectively. The inner radius, R_W, is within 0.25 mm of minimum gage (limited by the fourth design requirement upper right). A plot of the objective function (mass of the Mast) as a function of design cycle is shown on the lower right of figure 12. The optimization procedure obtains a design for the mast which better satisfies the design requirements at the expense of an additional 40 kg of mass. This increase in mass from the original design is mainly due to the diagonal frequency requirement.

OPTIMIZATION RESULTS FROM COFS-I BUCKLING STUDY (Formulation 1)

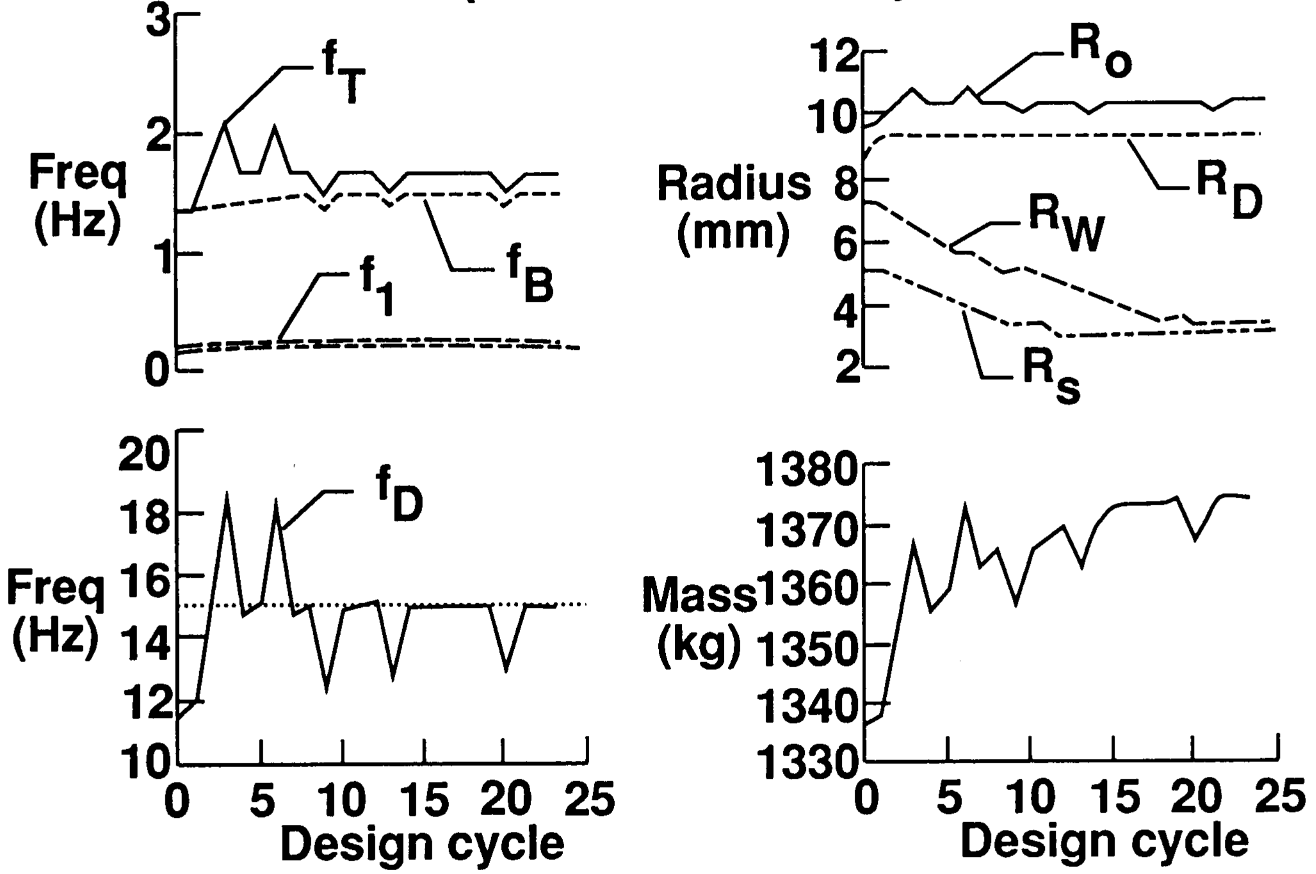

FIGURE 12

OBSERVATIONS FROM COFS-I BUCKLING DEFICIENCY STUDY

As shown in figure 13, in the previous study (COFS-II), two formulations were tried. Unlike the previous study, both formulations were successful and converged to the same design. From both studies, it is concluded that no feasible design exists which can be obtained by simply varying longeron radii and diagonal tube thickness within the prescribed limits. Therefore, there is a need for more design freedom in the optimization procedure in order to achieve a fully satisfactory design.

- Two formulations used for frequency spacing
  - Constraint-based (successful)
  - Objective function-based (successful)
- Formulations gave identical results
- Results showed need for more design freedom

FIGURE 13

COFS-I STRUCTURAL REDESIGN ACTIVITY

In May 1987 a problem (fig. 14) arose during the final experiment definition phase of the COFS-I project before the system requirement review. The project faced severe cost overruns and possible failure to meet schedule deadlines. In addition there were concerns whether the mast would meet some design requirements. An activity at Langley addressed these concerns. This section of the paper will describe the role of optimization in that activity.

In order to meet the proposed design and science requirements, the mast had been designed with a high modulus material (P75 graphite) in the longerons. This material had never been flight tested and there was concern for its performance. If this high modulus material (P75) could be replaced by a lower modulus material (HMS4 graphite) which had been flight qualified, flown, and could still meet all the design and science requirements (close-spacing of two adjacent frequencies), then there could be a cost savings. If the science requirements could not be met using the 54-bay length with the lower modulus material, the question was how short would the mast have to be to use the lower modulus material. These issues had to be addressed and answered in a very short time (originally approximately six weeks). Finally, there was to be minimal impact on the existing design. For example, no hinge or individual length changes were permitted. The deployment mechanism constrained length changes to 2-bay increments.

- Issues - cost savings associated with material choice
  - Candidate material (P75) has desirable characteristics
    - High modulus
    - Resulting design meets science requirements
  - Alternate material (HMS4) has lower modulus but
    - flight qualified
    - flight experience
- Could HMS4 be used?
- Would mast need to be shortened to permit HMS4 to be used and still meet science requirements?
- Short time frame for decisions
- Minimal impact on existing design
  - no hinge changes
  - no individual length changes
  - no outer diameter changes

FIGURE 14

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APPROACH

To address the issues described in figure 14 and meet schedule deadlines, the optimization procedures discussed previously were extended to included redesigning the mast to see if the lower modulus material could be used for the longerons. As shown in figure 15, the approach to the problem was first to identify the design requirements used to design the mast originally and then incorporate as many of these requirements as possible into an optimization procedure. Since there was to be a minimal impact on the existing design, only the inner radii of the longerons and diagonals and modulus of the longerons were allowed to vary. The optimizer would determine the modulus and wall thicknesses of the longerons and the wall thickness of the diagonal. The radius of the diagonal elements was allowed to change since from the previous COFS-I study it was known that the diagonal radius would have to change in order to satisfy the individual member frequency requirement. Once the optimum modulus and wall thicknesses of the longerons and diagonals were found, the closest ply layup would then be determined manually . The ease of manufacturing would also be verified. If this design looked "good" from the ease of a manufacturing point of view, this design would be offered as a possible replacement for the COFS-I mast design. If the design looked "bad" from the ease of manufacturing point of view, then a parameter such as wall thickness (possibly a new design variable) would be added and the optimization procedure would be repeated. There was also the possibility that new design requirements could be imposed.

- Extend optimization procedures developed in two previous studies
- Identify design requirements used for existing design
- Incorporate as many requirements as possible in extremely short time
- Address material issue:
  - Use optimization procedure to determine modulus and wall thickness
  - Manually determine ply layup
  - Verify manufacturability

FIGURE 15

DESIGN REQUIREMENTS

The design requirements which were used in the COFS-I structural redesign activity are shown in figure 16. On the left are the flight mast requirements which the design must meet. The first requirement is that the first natural frequency (f_1) of the mast be between 0.15 Hz and 0.2 Hz. This requirement assures that the frequencies of the mast avoid those of the Shuttle control system. The second requirement is that the first torsional and the higher of the second bending frequencies be approximately equal for a beam three bays shorter than its fully deployed length. The third requirement is that the first natural frequency (f_D) of the diagonal be greater than 18 Hz. This requirement was to assure that the diagonal frequency would be above the bandwidth which would be tested in the flight experiment. The fourth requirement is that the fundamental bending frequencies about the two principal axes be "distinctly different". In addition the mast must be able to withstand a tip deflection of 0.2m and a tip rotation of 2 degrees when fully deployed.

Shown on the right is how the mast design requirements were interpreted and implemented in the optimization procedure. There were some differences between these requirements and those used in the optimization procedure. Some of these changes were due to a better interpretation of the design requirements and some were due to the imposition of new requirements during the redesign activity. To save time and analysis effort, all design requirements were implemented at the fully deployed length so that only one finite element model would be required. The first difference shows up in the bounds on the first natural frequency. The same lower bound of 0.18 Hz used in the previous studies was used here, but the upper bound was changed from 0.2 Hz to 1 Hz when it was found that using 0.2 Hz as the upper bound prevented the optimizer from finding a feasible design. Later, it was determined that there was no reason why an upper bound of 1.0 Hz should not be used. In most cases the optimum designs gave a value of approximately 0.28 Hz for the first frequency. The tip rotation and tip deflection requirements were replaced by constraints on the Euler buckling loads in individual members. The critical Euler buckling loads $P_{S,cr}$, $P_{W,cr}$, and $P_{D,cr}$ for a strong longeron, weak longeron, and diagonal member respectively had to be greater than corresponding loads in the member denoted by P_S, P_W, and P_D (determination of these loads proved to be a challenge and will be discussed shortly in the observations section). A weight restriction on the mast was also added to the design requirements. The mast must fit inside a canister on a platform in the Shuttle. There were restrictions on how much weight this platform could hold due to launch and landing loads. The weight requirement was expressed in terms of the tubing weight (longerons, diagonals, and battens). Minimum gage wall thicknesses were also imposed.

DESIGN REQUIREMENTS

Mast's Requirements	Optimization Implementation
1st natural frequency of mast greater than 0.15 Hz and less than 0.2 Hz	$0.18 \le f_1 \le 1.0$ Hz
1st torsion and higher one of 2nd bending frequencies be approximately equal for beam 3 bays shorter than fully deployed length	$\left\lvert \frac{f_T - f_B}{f_T} \right\rvert \le \varepsilon$
1st natural frequency of diagonal greater than 18 Hz	$f_D \ge 18$ Hz
Fundamental bending frequencies about the 2 principal axes be "distinctly different"	"Weak"/"strong" longeron requirement, $R_W - R_S \ge \Delta$
Fully deployed mast withstand tip deflection of 0.2 m and tip rotation of 2 degrees	$P_{S,cr} \ge P_S$, $P_{W,cr} \ge P_W$, $P_{D,cr} \ge P_D$
	Weight of tubing $\le \overline{W}$ Minimum gage wall thickness

FIGURE 16

COFS-I STRUCTURAL REDESIGN ACTIVITY OPTIMIZATION FORMULATIONS

The two optimization formulations developed for the COFS-I structural redesign activity are shown below in figure 17. Both formulations have the frequency spacing as the objective function and the same set of constraints. They differ in the number of design variables. As noted below the pairs of frequencies to be closely-spaced and some of the constraint limits are expressed in generic terms (e.g. f_A, f_B, P_S, P_W, P_D, $\overline{W}$, and $\overline{\Delta t_D}$). No single optimization formulation can be shown as in the previous studies since design requirements shown on the previous page and even design variables were continually augmented and clarified throughout the study. Some of the changes were due to a better interpretation of the mast design requirements. While other changes were due to the addition of new requirements which should have been included, still other changes involved insights which came from some of the results of the optimization procedure.

Shown below are two of the formulations used. Formulation 1 addressed the issues discussed in figure 14. During the study several "what if" questions arose. For example, instead of trying to closely space the first torsional and second bending frequencies, could the third torsional and second bending be closely spaced. Another question was could the diagonal frequency be even higher than 18 Hz. This led to several studies where the diagonal frequency lower limit was 20, 25 and even 30 Hz. In addition, from the results of Formulation 1 (four design variables), the question was asked what if the material in the diagonal were changed to the same material (HMS4 graphite) as the longerons, could the 54 bay length be used for the mast, and if, not what length could be used and still meet all the design requirements. This led to Formulation 2 (five design variables) shown on the right of figure 17. In addition, the minimum diagonal wall thickness was adjusted due to questions about the ease of manufacturing (handling qualities) of tubes with ply layups corresponding to the optimum wall thickness and modulus determined by the optimizer. The tubing weight limit $\overline{W}$ was a function of the mast length.

	Formulation 1	**Formulation 2**
Objective function	$\left[\left(\frac{f_A - f_B}{f_A}\right)^2\right]^{1/2}$	$\left[\left(\frac{f_A - f_B}{f_A}\right)^2\right]^{1/2}$
Design variables (φ_i)	R_S, R_W, R_D, E_L	R_S, R_W, R_D, E_L, E_D

Constraints
(Both Formulations)

$$0.18 \text{ Hz} \leq f_1 \leq 1.0 \text{ Hz}$$
$$f_D \geq 18 \text{ Hz}$$
$$R_W - R_S \geq \Delta$$
$$P_{s,cr} \geq P_s$$
$$P_{W,cr} \geq P_W$$
$$P_{D,cr} \geq P_D$$
$$W \leq \overline{W}$$
$$\Delta t_D \geq \overline{\Delta t_D}$$
$$\varphi_{i_L} \leq \varphi_i \leq \varphi_{i_U}$$

FIGURE 17

SUMMARY OF CASES STUDIED

As mentioned on figure 17 many different cases were optimized during the COFS-I structural redesign activity. Figure 18 presents a summary of the cases studied during the redesign activity. The cases optimized included different material for the longerons and diagonals, different frequencies to be closely spaced (first torsional and second bending frequencies or third torsional and second bending frequencies), various minimum values for the diagonal frequencies (18, 20, 25, and 30 Hz) and different minimum diagonal wall thicknesses (20 mils, 30 mils, and 40 mils). In addition, the cases mentioned above were optimized for different mast lengths, i.e. number of bays (42, 44, 46, 48, 50, 52 and 54 bays).

- **Different materials (P75, HMS4)**
- **Different frequencies to be paired**
- **Different diagonal frequency lower limits**
- **Different wall thickness limits**
- **Different Mast lengths**

FIGURE 18

RESULTS OF COFS-I STRUCTURAL REDESIGN ACTIVITY

During the redesign activity over 60 optimum designs were obtained. Nine "official redesigns" were obtained. By "official" it is meant that these designs warranted further analyses to see if they met additional requirements such as ease of manufacturing not included in the optimization procedure. These "official" redesigns were for HMS4 graphite. The optimization procedure was formulated, implemented, and results obtained in less than four months (figure 19).

- Total number of optimized designs obtained - 60
- Nine candidate redesigns produced
  - All used HMS4
  - All met design requirements
- Accomplished in less than four months

FIGURE 19

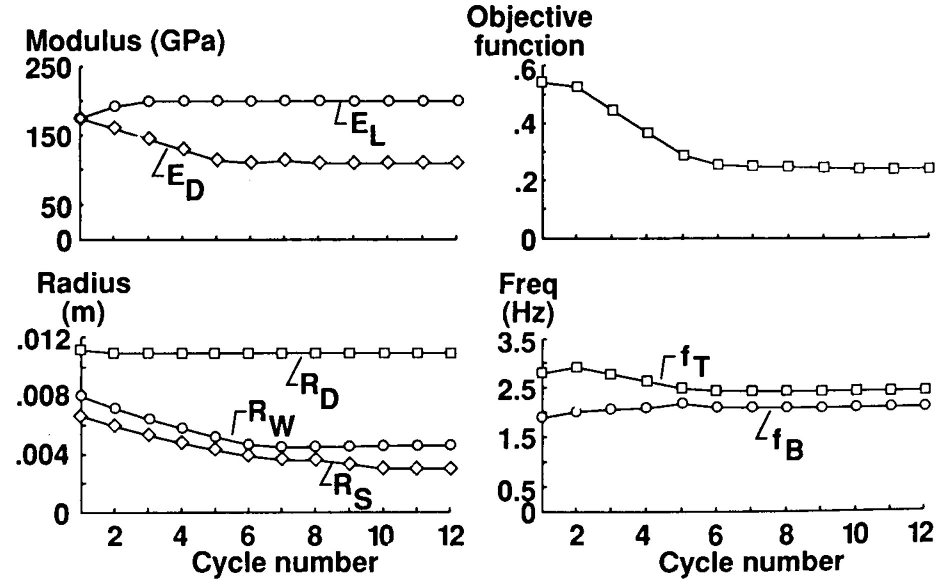
TYPICAL OPTIMIZATION RESULTS FOR COFS-I STRUCTURAL REDESIGN ACTIVITY

Over 60 optimum designs were obtained during the COFS-I structural redesign activity. Figure 20 presents optimization results for one of the nine "official" redesign candidates. This design is for a mast with 42 bays, HMS4 material in the longerons and diagonals, a maximum diagonal wall thickness of 40 mils, and a tubing weight limit of 125 kgs. The first torsional f_T and second bending f_B frequencies were to be closely spaced. The diagonal frequency f_D had to be greater than 18 Hz. The limit loads in the longerons and diagonals were 16000N for P_S and P_W and 1955N for P_D.

Design variable and objective function histories are shown in the upper half of figure 20. Histories of the two frequencies (f_T and f_B) comprising the objective function are also given. Design requirement histories are shown in the lower half of the figure. To increase the second bending frequency f_B, the optimizer increased the longeron modulus E_L to its upper limit and decreased the inner radii of the longerons (R_W and R_S). To lower the first torsional frequency f_T, the optimizer decreased the diagonal modulus E_D to its optimum value. R_D and R_S reached their respective lower limits.

The effect of these design variable changes on the design requirements is shown in the lower portion of figure 20. The diagonal frequency f_D and the first natural frequency f_1 met the requirements throughout the design process (with initial values of 22.5 Hz and 0.23 Hz and final values of 19.4 Hz and 0.27 Hz, respectively). The buckling loads in the strong longeron ($P_{S,cr}$) and the diagonal ($P_{D,cr}$) were adequate throughout the design process. The strong longeron buckling $P_{S,cr}$ increased from an initial value of 16258N to a final value of 20820N and both its modulus (E_L) and wall thickness increased. Since the modulus was at its upper limit after three cycles, the optimizer increased the wall thickness of the longeron (i.e., by decreasing the inner radius R_S) to raise the second bending frequency f_B. Initially with the HMS4 material, the constraint on the buckling load in the weak longeron was violated ($P_{W,cr}$ = 13906N). The optimizer satisfied this constraint by increasing the wall thickness of the weak longeron (i.e. decreasing the inner radius R_W). In the final design, the buckling load requirements for both the weak and strong longerons ($P_{S,cr}$ = 20820N and $P_{W,cr}$ = 20305N, respectively) were well satisfied. However, the buckling load in the diagonal (denoted by $P_{D,cr}$) was at its limiting value of 1955N. The optimizer increased the tubing weight W from an initial value of 92.5 kg to its upper limit of 125 kg to satisfy the buckling load constraints and increase the second bending frequency f_B. In the final design the diagonal wall thickness denoted by Δt_D (lower right) was at its minimum value. The "weak"/"strong" longeron requirement (R_W-R_S) kept R_W from reaching its lower limit.

TYPICAL OPTIMIZATION RESULTS FOR COFS-I STRUCTURAL REDESIGN ACTIVITY
42 Bay Mast



TYPICAL OPTIMIZATION RESULTS FOR COFS-I STRUCTURAL REDESIGN ACTIVITY, CONCLUDED
42 Bay Mast

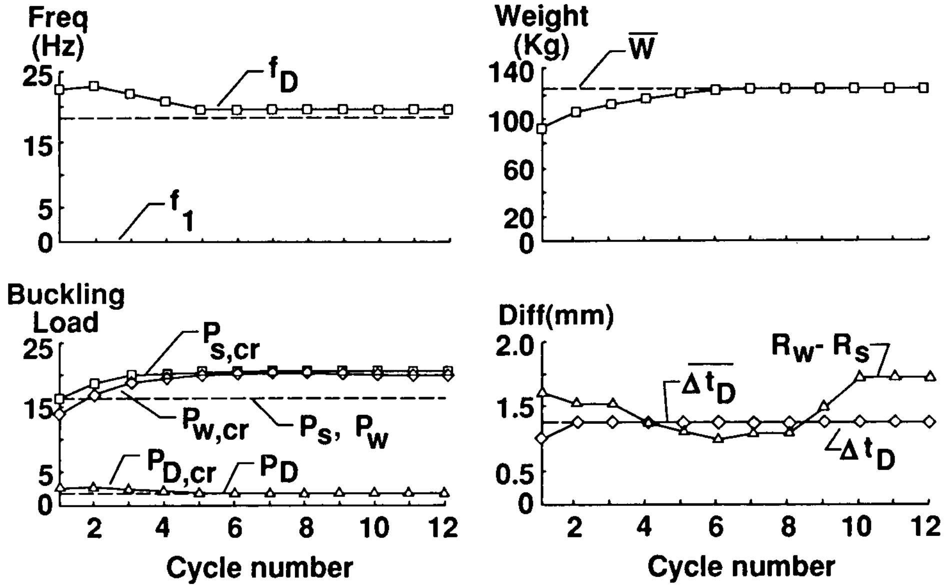


FIGURE 20

OBSERVATIONS FROM COFS-I STRUCTURAL REDESIGN ACTIVITY

As shown in figure 21, the use of optimization techniques can be extremely helpful when applied to an actual design activity such as the one just described. Once the procedure is developed, the designer is able to look at different options and answer "what if" questions he may not have time to answer by doing parametric studies. Since convergence is rapid (usually in less than 12 cycles), the optimization procedure allows the designer to look at many different options in a very short time. The designer is offered many avenues he may not have in a normal redesign activity when faced with time limitations and confined to using only parametric studies. Even infeasible designs can be important since they give the designer options he could have if willing to relax some of the design requirements. For example, if he were willing to accept a lower diagonal frequency than originally specified, he might obtain a candidate design with a longer mast.

Needless to say, optimization procedures are not a substitute for engineering judgment. The designer must be able to interpret and incorporate design requirements into the procedure. Sometimes this is not an easy task. For example, the buckling load requirement (the Euler buckling load greater than the limit load in the member) proved to be a troublesome constraint. Initially, a simplifying assumption was made to meet schedule deadlines. The limit load was defined as a safety factor (2.8) times the working load in the member. This working load was determined by applying the tip rotation and tip deflection requirement (fig. 17). Assuming that the limit loads varied in the analysis but were constant in the derivative calculations made it easy to obtain the derivatives of the constraints, but the constraint functions determined by the linear Taylor series approximations were inaccurate. Allowing these limit loads to vary during the derivative calculations made the calculations more costly in computer time and convergence, but the approximate constraint functions were more accurate. The limit loads were nevertheless very sensitive to changes in the design variables and the optimization procedure had trouble converging to a feasible design. The procedure appeared to be converging very slowly and there was no time to complete the convergence since deadlines were approaching. At the same time, communication with the contractor resulted in a better interpretation of how to obtain these limit loads. Since the hinges were already designed to withstand given loads, it was decided to use these same loads as limit loads for the longerons and diagonal. After this, the optimization procedure converged rapidly (less than 12 cycles).

A final observation is that optimization practitioners must be aware of the ease of manufacturing designs. Consideration of handling qualities led to an increase in the minimum wall thickness for the diagonals. Questions about the wall thickness of the longerons being too thick (could graphite tubes with very small inner diameters be manufactured?) led to changes in lower bounds on the design variables (inner radii).

OBSERVATIONS FROM COFS-I STRUCTURAL REDESIGN ACTIVITY

- Optimization can be powerful tool for practical engineering decisions
- Designer can look at different alternatives ("what if" questions)
- Designer can quickly determine effect of different options on design
- Infeasible design also important - give designer options if willing to relax a design requirement

BUT

- Not a substitute for engineering judgment - examples
  - Buckling load constraint determinations
  - Manufacturing considerations

FIGURE 21

SUMMARY

The paper described experiences gained in optimizing Control of Flexible Structures (COFS) configurations. Optimization procedures were developed to systematically provide closely spaced vibration frequencies. The optimization procedures combined a general-purpose finite-element program for eigenvalue and sensitivity analyses with formal mathematical programming techniques. The formal mathematical programming technique combined a general-purpose optimization program and approximate analyses.

Results were presented for three studies. The first study used a simple model of a typical COFS-II configuration to obtain a design with two pairs of closely spaced frequencies. Two formulations were developed: an objective function-based formulation; and a constraint-based formulation for the frequency spacing. It was found that conflicting goals were handled better by a constraint-based formulation. The second study used a detailed model of the COFS-I configuration. The structure was to be designed to have one pair of closely spaced frequencies while satisfying requirements on local member frequencies and manufacturing tolerances. Two formulations were again developed. Both the constraint-based and the objective function-based formulations performed reasonably well and converged to the same results. However, no feasible design solution existed which satisfied all the design requirements for the choices of design variables and the upper and lower design variable values used. It was concluded that more design freedom was needed to achieve a fully satisfactory design. The third study was part of a redesign activity in which a detailed model was used and actual design requirements were incorporated. The use of optimization in this redesign activity allowed the project engineers to investigate numerous options (such as number of bays, material, minimum wall thickness, minimum diagonal wall thicknesses) over a relatively short period of time. The procedure provided data (60 designs in a four month period) for judgments on the effects of different options on the design. Finally the optimization results permitted examination of various alternatives and answers to many "what if" questions in a relatively short time. (See figure 22.)

- **Related experiences in optimizing COFS structures**
- **Many "what if" questions were answered**
- **Proper problem formulation-important**
  - **Objective function selection**
  - **Design variable selection**
  - **Conflicting goals work best as constraints**
- **Optimization found to be powerful tool in engineering design process**

FIGURE 22

C-6

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