https://ntrs.nasa.gov/search.jsp?R=19890015858 2020-03-20T01:48:06+00:00Z

N89-25229

AN IMPROVED ALGORITHM FOR OPTIMUM STRUCTURAL DESIGN WITH MULTIPLE FREQUENCY CONSTRAINTS

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SUMMARY

This paper presents an optimality criterion (OC) method for minimum-weight design of structures having multiple constraints on natural frequencies. In this work a new resizing strategy is developed based on "relaxation" techniques. A computationally adaptive control parameter is used in conjunction with existing OC recursive formulae to promote convergence of optimum structural designs. Some considerations regarding the coupling of the modified Aitken accelerator with the OC method are discussed. Improved and rapidly converged minimum-weight designs are obtained when using an under-relaxed recursive scheme combined with the modified Aitken accelerator.

INTRODUCTION

In recently published literature regarding structural optimization with multiple frequency constraints [1,2], the algorithms were applied to truss systems, taking advantage of their special characteristics (i.e., single design variable per element, structural matrices linearly proportional to the design variables, constant stress elements, etc.). In search for optimal values of design parameters in minimum weight design the iterative approach based on alternately satisfying the constraints (scaling) and applying an "optimality criterion" (resizing) may give oscillatory results which might not converge; or they may converge to local extrema at the expense of an increased number of iterations.

The resizing formulae used in [1,2] employed an exponential control parameter as the step size. The control parameter was reduced to stabilize the iterative design cycle and to assure convergence. Basically, the control parameter was kept constant through all the iterations, as the structural weight was continually reduced; or if a sudden rise in the weight was observed, the iterative design cycle was momentarily stalled and the control parameter was reduced until a decrease in the weight was obtained. For various single and multiple frequency constraint conditions, optimum designs were presented. Although, the final designs being the real optimum were questionable.

In this work a new resizing strategy is developed based on "relaxation" techniques. A computationally adaptive control parameter is used in conjunction with OC recursive formulae currently used to obtain minimum weight design of truss system [1-3]. The new control parameter is adjusted by monitoring the local histories of scaled weights calculated in the iterative design cycle. As the step size is reduced, the rate of convergence is reduced. Hence, the convergence rate is increased using an acceleration technique. The modified Aitken accelerator [4,5] is implemented to extrapolate values of structural weight from the local history of the design cycle to accelerate convergence towards an optimal design. Structural element sizes and natural frequencies are presented for optimally designed truss systems under various frequency constraint conditions. Design cycle histories of structural weights and control parameters are charted to compare the performance of different recursive strategies to modify the design variables and to estimate the Lagrange multipliers.

FREQUENCY ANALYSIS

The square of the j^{th} natural frequency for the case of undamped vibration of a discretized structure can be written as

$$\omega_j^2 = \{q_j\}^T[K]\{q_j\} \tag{1}$$

where [K] is the stiffness matrix, and $\{q_j\}$ is the j^{th} vibration mode normalized with respect to the total mass, $[M]=[M_s+M_c]$, consisting of structural and nonstructural mass. The gradient of the natural frequency with respect to the design variables x_i (member cross-sectional areas) is obtained by differentiating Eq. (1). The result is

$$(\omega_j^2)_{,x_i} = (1/x_i)\left[\{q_j\}_i^T[k_i]\{q_j\}_i - \omega_j^2\{q_j\}_i^T[m_i]\{q_j\}_i\right] \tag{2}$$

where $\{q_j\}_i$, $[k_i]$, and $[m_i]$ denote components of the structural matrices associated with the i^{th} element x_i, and $(\ )_,$ represents a partial differentiation.

OPTIMIZATION PROCEDURE

The optimization problem is defined as

minimize the structural weight

$$W(x_i) = \rho_i\ l_i\ x_i \qquad (i=1,2,\ldots,n) \tag{3}$$

subject to m constraints

$$\begin{aligned} g_j(x_i) &= \omega_j - \omega_j^* = 0 \qquad (j=1,2,\ldots,k) \\ g_j(x_i) &= \omega_j - \omega_j^* < 0 \qquad (j=k+1,\ldots,m) \end{aligned} \tag{4}$$

where ρ_i is the mass density, x_i is the design variable, and l_i is the length of the element. In Eq. (4) ω_j and ω_j^* are the actual and the desired values of the frequency constraints. In addition, minimum limits are prescribed on the design variables: $x_i > x_i^l$.

Using Eqs. (3) and (4), the Lagrangian function, L, can be written as

$$L(x_i, \lambda) = \rho_i\ l_i\ x_i - \lambda_j\ (\omega_j - \omega_j^*) \qquad (i=1,2,\ldots,n) \quad (j=1,2,\ldots,m) \tag{5}$$

where λ_j are the Lagrange multipliers.

Differentiating Eq. (5) with respect to the design variables and setting the resulting equations to zero, the optimality criterion can be written as:

$$e_{ij} \lambda_j = 1 \qquad (i=1,2,...,n \; ; \; j=1,2,...,m) \tag{6}$$

where

$$e_{ij} = \frac{(\omega_j)_{,xi}}{(W)_{,xi}} \tag{7}$$

where the Lagrangian energy density, e_{ij}, represents the ratio of the gradient for the natural frequency constraint (Eq. 2) to the gradient of the objective function, given as $(\rho_i l_i)$.

Using Eq. (6), one can write recursive relations to modify the design variables. Recursive relations to estimate the Lagrange multipliers can be written by assuming that all the constraints in Eq. (4) are equality constraints [3]. In either case these recursive relations can be written in an exponential or a linearized form. In the exponential recursive relations the design variables (or Lagrange multipliers) are modified by multiplying them by a quantity which is equal to unity at the optimum, and in the linear recursive relations the design variables (Lagrange multipliers) are modified by adding a quantity which is equal to zero at the optimum. Note that the linear recursive relation for the Lagrange multipliers is an approximation to a set of linear equations that can be used to determine the Lagrange multipliers [1,3]. Nonetheless, it is possible to promote the convergence of these relations by incorporating a simple technique known as relaxation. Such a modification is used in this work:

To modify the design variables:

$$x_i^{k+1} = x_i^k + s\,[(e_{ij}\,\lambda_j)_k^{(1/r)} - 1]\; x_i^k \tag{8}$$

$$x_i^{k+1} = x_i^k + s\,[\{1 + (1/r)\,(e_{ij}\,\lambda_j - 1)\}_k - 1]\; x_i^k \tag{9}$$

To estimate the Lagrange multipliers:

$$\lambda_j^{k+1} = \lambda_j^k + s\,[(\lambda_j \,/\, \lambda_j^*)_k^{(1/b)} - 1]\;\lambda_j^k \qquad ; \quad (b=1/r) \tag{10}$$

$$\lambda_j^{k+1} = \lambda_j^k + s\,[\{(b+1)/b\} - (1/b)(\lambda_j \,/\, \lambda_j^*)_k^{(1/b)} - 1]\;\lambda_j^k \tag{11}$$

where the superscripts k and k+1 denote iteration numbers. The quantity (1/r), is the step size used in the algorithms reported in [1,2]. In the present algorithm this step size is immobilized by setting it to a constant value, 1/r=0.5. Alternatively, a more adaptive control parameter s, is utilized. At the beginning of the design cycle the control parameter is set to unity. Henceforth, the value of s is adjusted by monitoring the local histories of structural weights calculated in the iterative design cycle. For structural weight histories exhibiting an oscillatory pattern of convergence, an optimal value of s is chosen using the following algorithm: If $W^{k+1} > (1/2)[W^k - W^{k-1}]$ or $W^{k+1} > W^k > W^{k-1}$ and the current value of s is above a specified minimum value, then s is reduced to s/2. (For structural weight histories displaying a pattern of convergence other than oscillatory, the algorithm can be appropriately refined. At the optimum the optimality criterion (Eq. 6) and the constraints (Eq. 4) are satisfied. Hence, Eqs. (8-9) converge to $x_i^{k+1} = x_i^k$ and Eqs. (10-11) converge to $\lambda_j^{k+1} = \lambda_j^k$.

The j^{th} Lagrange multiplier for the j^{th} frequency constraint also can be approximated by a simple expression derived from a single constraint condition [2,6]

$$\lambda_j = W / \omega_j^2 \, m_1 \tag{12}$$

where

$$m_1 = \frac{q_j^T [M_s] q_j}{q_j^T [M] q_j} \tag{13}$$

Equation (12) is used as initial values in the recursive Eqs. (10-11).

After the structural members are modified using Eqs. (8) or (9), they are uniformly scaled by a factor f_j corresponding to the j^{th} frequency constraint. The relationship between the unscaled design x_i and the scaled design x_i^s is given by

$$x_i^s = f_j x_i \tag{14}$$

The scale factor f_j is computed as follows [2,6]:

$$f_j = \frac{m_1 R_j^2}{1 - R_j^2 m_2} \quad , \quad R_j^2 m_2 < 1$$

$$f_j = R_j^2 \quad , \quad \text{otherwise} \tag{15}$$

where

$$m_2 = \frac{q_j^T [M_c] q_j}{q_j^T [M] q_j} \tag{16}$$

and R_j^2 represents the frequency target ratio given by

$$R_j^2 = \omega_j^{2*} / \omega_j^2 \tag{17}$$

In search for optimal values of design parameters in minimum weight design the iterative approach based on alternately satisfying the constraints (Eqs. 14-17) and applying the optimality criterion (Eqs. 6) may give oscillatory results which might not converge; or they may converge to local extrema at the expense of an increased number of iterations. The control parameter s, adopted in Eqs. (8-11) controls the step size of the recursive relations and stabilizes the convergence of the iterative design cycle. A drawback is that the convergence rate is slowed as the control parameter is reduced. This is primarily due to the fact that a smaller value of s reduces gains towards meeting the optimality condition. Hence, the convergence rate of the iterative process is improved with two-fold objectives in mind: (1) maintaining as large of a value for s as possible during the design cycle, and (2) extrapolating structural information from the local history of the design cycle to accelerate the convergence rate.

MODIFIED AITKEN ACCELERATOR

The convergence rate of the iterative process can be enhanced by using an accelerator. An appropriate one has been proposed by Boyle and Jennings [4,5]. The Aitken accelerator is a numerical technique whereby three consecutive results of an iterative process are extrapolated to obtain improved results on the assumption that the error curve of the iterative process decays exponentionally. The adaptability of Aitken's accelerator for the computer, however, is unpredictable given the possibility of a singular denominator in the predictor algorithm. Nonetheless, a modified Aitken accelerator was developed by Jennings [5] for general multivariable iterative problems. The predictor algorithm for the modified Aitken accelerator requires only one division as opposed to one for each variable and allows the divisor to be chosen to avoid the possibility of a zero value.

In this work, local histories of structural weight are monitored for convergence patterns which are not monotonic. If the structural weight histories before the current design exhibit an oscillatory pattern of convergence and show a mark increase in value, then continued computations with the current design are bypassed while an improved design (i.e., one that will result in a reduced scaled weight) is obtained using the modified Aitken accelerator.

Let x_i^{k-3}, x_i^{k-2}, and x_i^{k-1} be design variables obtained from three consecutive iterations of the design cycle and let x_i^k be the desired variables for a current design. By letting

$$d_1 = x_i^{k-2} - x_i^{k-3} \qquad d_2 = x_i^{k-1} - x_i^{k-2} \qquad (18)$$

the adopted procedure [5] for finding an improved (accelerated) design x^a, may be written as

$$x_i^a = x_i^{k-1} + Sd_2 \qquad (19)$$

where S, is defined as the acceleration factor

$$S = (d_2 - d_1)^T(-d_2)[(d_2 - d_1)^T(d_2 - d_1)]^{-1} \qquad (20)$$

In general, x_i^a satisfies neither the optimality condition (Eq. 6) nor the frequency constraints. Nonetheless, the design cycle is continued after an acceleration by applying the optimality condition with the improved design x_i^a:

$$x_i^k = x_i^a + s\,[(e_{ij}\,\lambda_j)_a^{(1/r)} - 1]\,x_i^a \quad (21)$$

$$x_i^k = x_i^a + s\,[\{1 + (1/r)\,(e_{ij}\,\lambda_j - 1)\}_a - 1]\,x_i^a \quad (22)$$

$$\lambda_j^k = \lambda_j^a + s\,[(\omega_j\,/\,\omega_j^*)_a^{(1/b)} - 1]\,\lambda_j^a \quad ; \quad (b=1/r) \quad (23)$$

$$\lambda_j^k = \lambda_j^a + s\,[\{(b+1)/b\} - (1/b)(\omega_j\,/\,\omega_j^*)_a^{(1/b)} - 1]\,\lambda_j^a \quad (24)$$

The k^{th} design is then scaled to satisfy the constraints using Eq. (14). (Note that the control parameter s is appropriately adjusted as previously outlined.)

OPTIMIZATION ALGORITHM

The main steps of the present optimization algorithm are

(1) Assign uniform sizes to all elements (set s=1, k=1).
(2) Perform frequency analysis (Eqs. 1-2)
(3) Scale the design until frequency constraints are obtained within the required accuracy (Eqs. 14-17).
(4) Calculate the scaled weight of the structure using (Eq. 3) and the scaled design variables (Eq. 14).
(5) For iteration k=4 or greater, check for oscillatory convergence pattern of scaled weights in the last three consecutive k-1, k-2, k-3 iterations. If this is the case, then apply the modified Aitken accelerator to obtain an improved design (Eq. 19) and modify the design variables using Eqs. (20 or 21 and 12, 22 or 23). Else continue to step (6).
(6) Determine the Lagrange multipliers (Eqs. 10,11 or 12).
(7) Modify the design variables (Eqs. 8 or 9).
(8) Repeat steps 2, 3 and 4.
(9) For iteration k=3 and greater: If $W^{k+1} > (1/2)\,[W^k - W^{k-1}]$ or $W^{k+1} > W^k > W^{k-1}$ and s > specified minimum value, then s is equal to s/2 and go to step 7; Else go to step 5.
(10) Steps 5-9 represent one iteration in the design cycle history.
(11) Repeat until difference in weight is less than specified tolerance.

RESULTS AND DISCUSSIONS

The effectiveness of the above algorithm was demonstrated by designing a 10 member truss (Figure 1), a classical problem in the structural optimization literature [1-3]. The elastic modulus was 10^7 psi, and the weight density was 0.1 psi. A nonstructural mass of 2.588 lb-sec^2/in was added to the four free nodes. All the member cross-sectional areas were given uniform sizes for an initial design. During the design cycle history, a lower limit value of 0.1 in^2 was imposed on member sizes. The natural frequencies and mode shapes were computed using a Jacobi method.

Table 1 presents initial and final frequencies and structural weight at the optimum design for various frequency constraint conditions. Table 2 gives the optimum member sizes. As indicated in Tables 1 and 2, Eq. 8 was used to modify designs and Eq. 12 was used to estimate the Lagrange multipliers. Reference [2] presents optimization studies using similar formulae for the same ten member truss subjected to the same constraint conditions. In Tables 1 and 2 these results are shown in parentheses for comparison with those obtained in the present analysis.

The ten member truss was designed with both single and multiple frequency constraint conditions (Tables 1 and 2). The first set of results (Case 1) was obtained with a single constraint on the second frequency (ω_2=10.0Hz). The next three sets of results involved multiple frequency constraints: (Case 2) ω_1=7.0Hz, ω_2>15.0Hz; (Case 3) ω_1=7.0Hz, ω_2>15.0Hz, ω_3>20.0Hz; (Case 4), ω_1>3.5Hz, ω_2>10.0Hz, ω_3>14.0Hz.

At the initial design the structural weight was 4000 lbs. Furthermore, the first eight frequencies were on the average approximately 4.26 percent higher than those of reference [2]. Resulting design weights obtained by the present analysis were significantly lower than those obtained in [2]. For example, Cases 1 and 4 showed mark improvements in structural weight with approximately 15.7 and 15.88 percent decreases, respectively. Additionally, the first eight frequencies calculated for Cases 1 and 4 were decreased by an average of approximately 11.13 and 12.63 percent, respectively. For Case 4 the constraint on the first and second frequencies was met to within 6 and 3.5 percent, respectively. The third frequency constraint in Case 4 was completely satisfied, as well as the second frequency constraint in Case 1. In Case 2 a 3.01 percent decrease from the weight reported in [2] was calculated, while a 9.78 percent decrease in weight was obtained for Case 3. For the calculated frequencies in Cases 2 and 3, there was less than 1 average percent change from those reported in [2].

Reference [1] presents optimization studies of a thirty-eight member truss (Figure 2) with multiple frequency limits. The elastic modulus and weight density of the material were 10^7 psi and 0.1 lb/in^3, respectively. At nodes 8 and 14 a nonstructural mass of 0.5 lb-sec^2/in was included. Lower limit on the design variables was 0.005 in^2.

Tables 3 and 4 show design cycle histories of structural weight and frequencies of a 38 member truss when a specified band between the square of the first and second frequency is increased: [(Case i) ω_1^2=2500rad^2/sec^2, ω_2^2>2500rad^2/sec^2; (Case ii) ω_1^2=2500rad^2/sec^2, ω_2^2>3000rad^2/sec^2, respectively.] The first set of results were obtained by the present analysis using Eq. (8) to modify the design variables and Eq. (12) to estimate the Lagrange multipliers. The asterisk associated with an iteration indicates the use of the modified Aitken's accelerator (Eq. 19) and the corresponding Eqs. (21-24). The second set of results was taken from reference [1]. Khot [1] reported real optimum designs because the minimum weights obtained were equal to the dual weight of the structure, which was the difference between the total weight and the weight of passive elements (such as elements at minimum gage).

From Tables 3 and 4 it is seen that with the relative areas of all members equal to unity the initial scaled weight of the structure was 27.74 compared to 52.30 obtained in [1]. The present authors could not reach any justifiable conclusions for the difference. However, it is seen that the weights obtained in this work quickly converged to a 8.61 percent lower weight for Case (i) (Table 3). Comparing the results for Case (ii) (Table 4), it is seen that the present algorithm calculated a 7.04 percent decrease in weight.

The example ten bar truss was redesigned with the constraints of Case 4. The two recursive relations used to modify the design variables were (1) the exponential relation (Eq. 8); (2) the linear relation (Eq. 9). The Lagrange multipliers for the above two cases were determined by using (1) the approximate relation [2] (Eq. 12); (2) the exponential relation (Eq. 10); (3) the linear relation (Eq. 11).

The design cycle history of structural weights using combinations of the above recursive formulae (Cases A-F) is given in Table 5. This table also contains CPU time (sec) using double precision arithmetic on a 32-bit IBM machine. Table 6 gives design cycle histories of the control parameter s, used in Cases A-F. (Note that the control parameter is not used in Eq. (12) of Cases A-B). The iteration history for the six cases is shown in Figure 3. At this time it is premature to draw general conclusions at this time on which case performs the best in a wide variety of design situations. Although all the cases appear to illustrate an average degree of convergence, a value of s near unity is preferred in Eq. (8 or 9) because this ensures a larger contribution in satisfying the optimality condition. This inevitably leads to a more rapid convergence to a lower weight. Hence, Case B appears to perform the best for the example problem.

The example thirty-eight bar truss was redesignedwith the constraints of Cases (i) and (ii). The design cycle history of structural weights using the recursive Cases A-F is given in Tables 7 and 8. The iteration history for the six cases is shown in Figures 4 and 5. In Figure 4, all the recursive cases appear to converge with Case A producing the lowest weight. The curve for Case B displays the most stable convergence. As the frequency band is increased in Figure 5, all the cases appear to converge to the same weight, but the path of convergence is more dispersed.

CONCLUSIONS

In this paper, minimum weight designs of truss systems with multiple frequency constraints were obtained using OC methods with a new resizing strategy based on relaxation techniques. A computationally adaptive control parameter was used in conjunction with available OC recursive formulae. To increase the overall rate of convergence, the modified Aitken accelerator was employed during the design cycle. Several recursive schemes to modify the design variables and to estimate the Lagrange multipliers have been compared. It is premature to generally state which scheme was superior for frequency constraint design problems until more case studies are complete. Minimum weight designs were obtained for various frequency constraint conditions, even though their design may be undesirable due to other practical considerations. Practical extensions of this work call for including displacement constraints.

ACKNOWLEDGMENTS

This research effort has been supported by the Air Force Office of Scientific Research at Air Force Wright Aeronautical Laboratories, WPAFB, Ohio. The authors gratefully acknowledge the helpful assistance given by Dr. R.S. Sandhu, Professor, Department of Civil Engineering at The Ohio State University.

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Table 1 Ten Bar Truss

*Initial and Final Frequencies (Hz) in Different Constraint Conditions**

Frequency No.	Initial Design	$\omega_2 = 10.0$	$\omega_1 = 7.0$ $\omega_2 \geqslant 15.0$	$\omega_1 = 7.0$ $\omega_2 \geqslant 15.0$ $\omega_3 \geqslant 20.0$	$\omega_1 \geqslant 3.5$ $\omega_2 \geqslant 10.0$ $\omega_3 \geqslant 14.0$
1	9.18 (8.96)	3.04 (3.26)	7.00 (7.00)	7.00 (7.00)	3.71 (4.40)
2	27.31 (27.08)	10.00 (10.00)	15.45 (15.58)	16.30 (15.61)	10.35 (12.14)
3	29.79 (27.45)	10.00 (10.19)	17.36 (16.93)	20.15 (20.17)	14.00 (14.00)
4	53.87 (51.25)	11.44 (16.01)	18.83 (18.75)	20.24 (20.77)	14.33 (17.89)
5	61.06 (58.00)	12.86 (18.08)	28.36 (29.13)	29.08 (28.76)	16.84 (19.58)
6	68.35 (64.73)	17.34 (22.96)	29.71 (30.30)	29.88 (29.76)	19.52 (22.96)
7	69.95 (66.87)	26.01 (25.21)	47.70 (46.93)	48.52 (53.88)	30.33 (34.01)
8	82.11 (80.85)	26.81 (27.25)	50.31 (49.67)	51.41 (56.03)	31.84 (35.72)
Weight (lb)	4000.0 (4000.0)	256.7 (304.5)	1137.3 (1172.6)	1180.4 (1308.4)	411.5 (489.17)

Notes :

* present analysis using exponential resizing and approximate Lagrange multiplier formulae.

( ) via reference [2].

Table 2 Ten Bar Truss
*Optimum Design Variables (in^2) in Different Constraint Conditions**

Element No.	$\omega_1 = 10.0$	$\omega_1 = 7.0$ $\omega_2 \geqslant 15.0$	$\omega_1 = 7.0$ $\omega_2 \geqslant 15.0$ $\omega_3 \geqslant 20.0$	$\omega_1 \geqslant 3.5$ $\omega_2 \geqslant 10.0$ $\omega_3 \geqslant 14.0$
1	0.887 (0.910)	5.769 (5.511)	5.254 (5.672)	1.021 (2.306)
2	0.889 (0.821)	1.944 (1.937)	2.446 (3.823)	1.211 (1.304)
3	0.887 (0.910)	5.769 (5.511)	5.254 (5.672)	1.021 (2.306)
4	0.889 (0.821)	1.944 (1.937)	2.446 (3.823)	1.211 (1.304)
5	0.360 (0.768)	0.125 (0.207)	0.125 (0.646)	0.213 (0.639)
6	0.208 (0.570)	0.448 (0.414)	0.720 (0.321)	0.343 (0.557)
7	0.788 (0.712)	3.302 (3.616)	3.739 (4.191)	1.661 (1.029)
8	0.788 (0.712)	3.302 (3.616)	3.739 (4.191)	1.661 (1.029)
9	0.276 (0.581)	2.211 (2.414)	2.109 (1.604)	0.605 (0.800)
10	0.276 (0.581)	2.211 (2.414)	2.109 (1.604)	0.605 (0.800)
Weight (lb)	256.7 (304.5)	1137.3 (1172.6)	1180.4 (1308.4)	411.5 (489.17)

Notes :
* present analysis using exponential resizing and approximate Lagrange multiplier formulae
( ) via reference [2].

Table 3 Thirty-Eight Bar Truss
Design Cycle History for $\omega_1^2 = 2500$ and $\omega_2^2 \geqslant 2500$ $(rad/sec)^2$

Iter. No. #	ω_1^2	ω_2^2	Weight (lbs)	Iter. No. +	ω_1^2	ω_2^2	Weight (lbs)
1	2500	3667	27.74	1	2500	8560	52.30
2	2500	2907	25.74	2	2500	6451	39.41
3	2500	3111	30.65	3	2500	5061	33.29
4	2500	4497	* 28.56	4	2500	4326	31.01
5	2500	3435	26.29	5	2500	4033	29.83
6	2500	2759	25.26	6	2500	3706	28.94
7	2500	2507	24.93	7	2500	3314	28.26
				8	2500	2880	27.72
				9	2500	2537	27.36
				10	2500	2501	27.29
				11	2500	2500	27.28

Notes :
\# present analysis using exponential resizing and approximate Lagrange multiplier
\+ via. reference [1]
* with acceleration

Table 4 Thirty-Eight Bar Truss
Design Cycle History for $\omega_1^2 = 2500$ and $\omega_2^2 \geq 3000$ $(rad/sec)^2$

Iter. # No.	ω_1^2	ω_2^2	Weight (lbs)	Iter. + No.	ω_1^2	ω_2^2	Weight (lbs)
1	2500	3667	27.74	1	2500	8560	52.30
2	2578	3000	26.58	2	2500	6451	39.41
3	2500	3789	26.89	3	2500	5061	33.29
4	2500	3123	* 25.77	4	2500	4326	31.01
				5	2500	4033	29.83
				6	2500	3706	28.94
				7	2500	3314	28.26
				8	2500	3016	27.80
				9	2500	2998	27.72
				10	2500	2999	27.72
				11	2500	3000	27.72

Notes :
- # present analysis using exponential resizing and approximate Lagrange multiplier
- + via. reference [1]
- * with acceleration

Table 5 Ten Bar Truss

Design Cycle History of Structural Weight
Using Various OC Recursive Formulae
$\omega_1 \geq 3.5$, $\omega_2 \geq 10$ & $\omega_3 \geq 14$ *Hz.*

Iter. #	A	B	C	D	E	F
1	778.192	469.222	778.192	469.222	778.192	469.222
2	646.754	502.846	647.753	533.498	647.753	533.498
3	750.374	606.414	744.672	674.662	744.672	672.496
4	* 445.400	436.027	* 439.545	628.077	* 439.545	618.992
5	467.874	463.789	528.799	462.462	511.869	457.601
6	* 438.693	* 442.361	* 407.225	484.560	* 409.063	458.850
7	436.855	422.566	448.569	491.267	436.235	* 455.656
8	425.245	404.818	* 430.650	* 480.714	* 430.713	454.152
9	422.174		427.603	472.686	418.765	450.901
10	421.555		414.426	465.015	406.346	447.713
11	416.449		418.468	457.683		444.588
12	411.556		* 415.834	450.670		441.523
13			414.329			438.518
14						435.571
15						432.681
16						429.847
17						427.068
c.p.u. (sec.)	(3.16)	(1.86)	(3.33)	(3.17)	(2.28)	(3.99)

Notes :
- A exponential resizing/approximate Lagrange multipier formulae
- B linear resizing/approximate Lagrange multiplier formulae
- C exponential resizing/exponential Lagrange multiplier formulae
- D linear resizing/exponential Lagrange multiplier formulae
- E exponential resizing/linear Lagrange multiplier formulae
- F linear resizing/linear Lagrange multiplier formulae
- * with acceleration

Table 6 Ten Bar Truss

Design Cycle History of Control Parameter β
Using Various OC Recursive Formulae
$\omega_1 \geqslant 3.5$, $\omega_2 \geqslant 10$ & $\omega_3 \geqslant 14$ *Hz.*

Iter. #	A	B	C	D	E	F
1	1.	1.	1.	1.	1.	1.
2	1.	1.	1.	1.	1.	1.
3	1.	1.	1.	1.	1.	1.
4	0.5	1.	0.5	1.	0.5	1.
5	0.5	1.	0.5	1.	0.5	1.
6	0.5	0.5	0.5	0.125	0.5	1.
7	0.25	0.5	0.125	0.0625	0.125	0.0625
8	0.125	0.5	0.125	0.0625	0.125	0.0313
9	0.0625		0.125	0.0625	0.125	0.0313
10	0.0625		0.125	0.0625	0.125	0.0313
11	0.0625		0.0156	0.0625		0.0313
12	0.0625		0.0156	0.0625		0.0313
13			0.0156			0.0313
14						0.0313
15						0.0313
16						0.0313
17						0.0313

Notes :
- A exponential resizing/approximate Lagrange multiplier formulae
- B linear resizing/approximate Lagrange multiplier formulae
- C exponential resizing/exponential Lagrange multiplier formulae
- D linear resizing/exponential Lagrange multiplier formulae
- E exponential resizing/linear Lagrange multiplier formulae
- F linear resizing/linear Lagrange multiplier formulae
- * with acceleration

Table 7 Thirty-Eight Bar Truss

Design Cycle History of Structural Weight
Using Various OC Recursive Formulae
$\omega_1^2 = 2500$ & $\omega_2^2 \geqslant 2500$ $(rad/sec.)^2$

Iter. #	A	B	C	D	E	F
1	27.742	30.186	25.741	26.965	25.741	26.965
2	25.741	26.965	30.653	25.293	30.653	25.293
3	30.653	25.379	215.117	29.667	215.117	29.322
4	* 28.564	25.613	34.820	* 27.886	34.820	* 27.636
5	26.288	* 25.438	31.565	26.434	31.565	26.264
6	* 25.259	25.297	26.756	25.500	26.756	25.376
7	24.928		25.119	25.153	25.119	25.044

Notes :
- A exponential resizing/approximate Lagrange multipier formulae
- B linear resizing/approximate Lagrange multiplier formulae
- C exponential resizing/exponential Lagrange multiplier formulae
- D linear resizing/exponential Lagrange multiplier formulae
- E exponential resizing/linear Lagrange multiplier formulae
- F linear resizing/linear Lagrange multiplier formulae
- * with acceleration

Table 8 Thirty-Eight Bar Truss
Design Cycle History of Structural Weight
Using Various OC Recursive Formulae
$\omega_1^2 = 2500$ & $\omega_2^2 \geq 3000$ $(rad/sec.)^2$

Iter. #	A	B	C	D	E	F
1	27.742	30.186	26.888	26.965	26.888	26.965
2	26.577	26.965	29.175	32.344	29.278	32.344
3	26.888	28.090	37.388	32.410	211.307	32.518
4	* 25.776	* 27.450	27.629	26.922	72.092	26.964
5		26.287	26.085	25.920	* 31.820	
6		26.154			* 27.844	
7		25.987			26.752	
8					25.976	

Notes :
A exponential resizing/approximate Lagrange multipier formulae
B linear resizing/approximate Lagrange multiplier formulae
C exponential resizing/exponential Lagrange multiplier formulae
D linear resizing/exponential Lagrange multiplier formulae
E exponential resizing/linear Lagrange multiplier formulae
F linear resizing/linear Lagrange multiplier formulae
* with acceleration

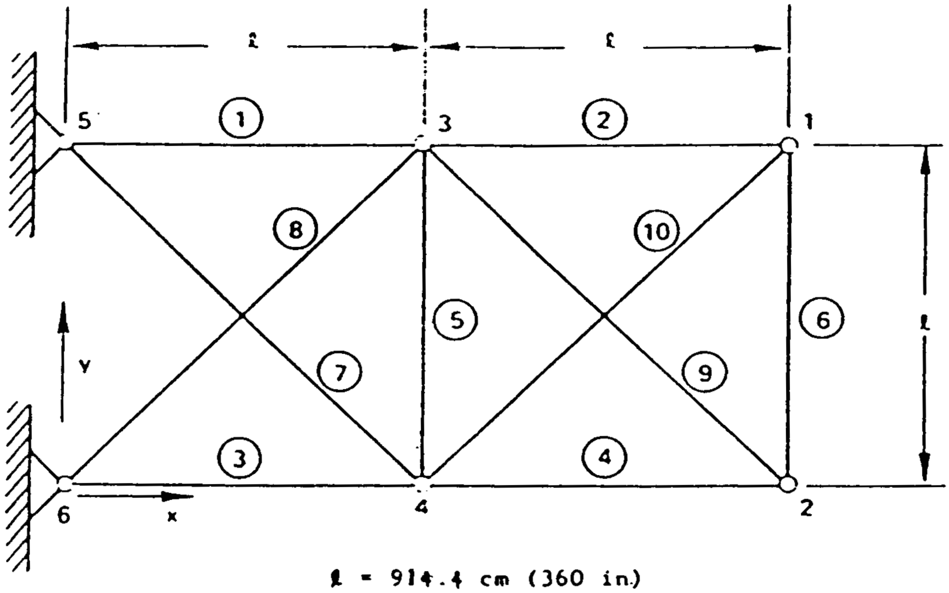


Figure 1 Ten bar truss

Element	Connecting Nodes	Element	Connecting Nodes	Element	Connecting Nodes
1	1-2	14	7-9	27	13-16
2	1-3	15	7-10	28	14-15
3	1-4	16	8-9	29	14-16
4	2-3	17	8-10	30	15-17
5	2-4	18	9-11	31	15-18
6	3-5	19	9-12	32	16-17
7	3-6	20	10-11	33	16-18
8	4-5	21	10-12	34	17-19
9	4-6	22	11-13	35	17-20
10	5-7	23	11-14	36	18-19
11	5-8	24	12-13	37	18-20
12	6-7	25	12-14	38	19-20
13	6-8	26	13-15		

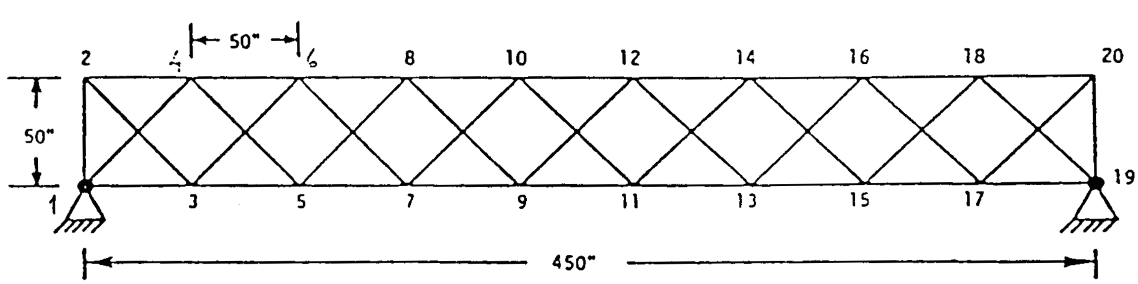


Figure 2 Thirty-eight bar truss

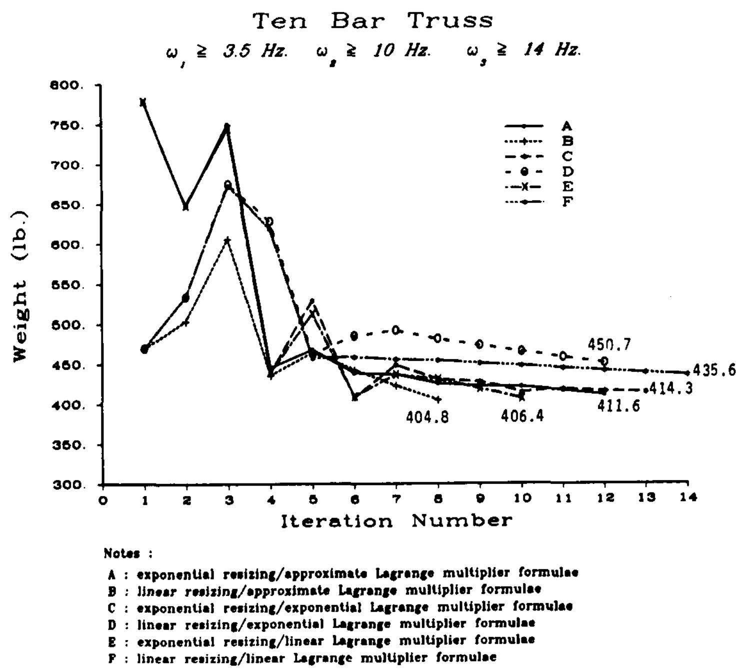


Figure 3 Iteration history for ten bar truss with multiple frequency constraints

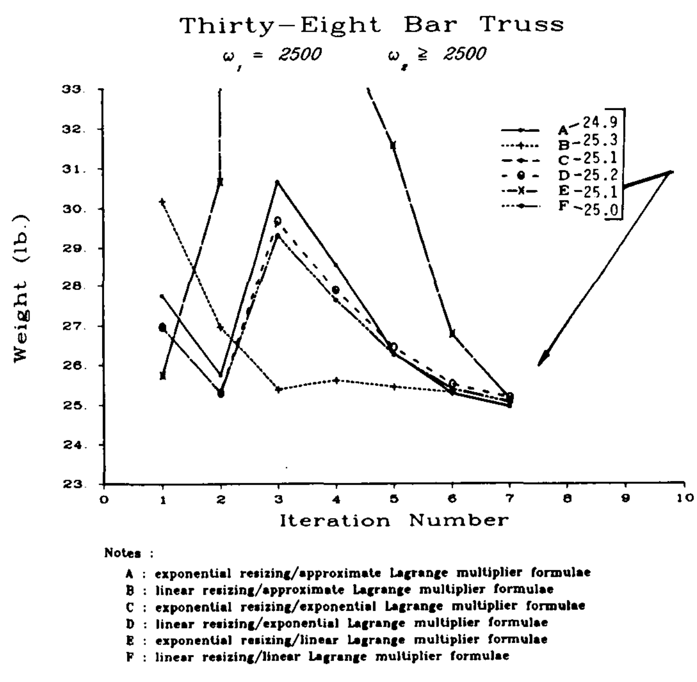
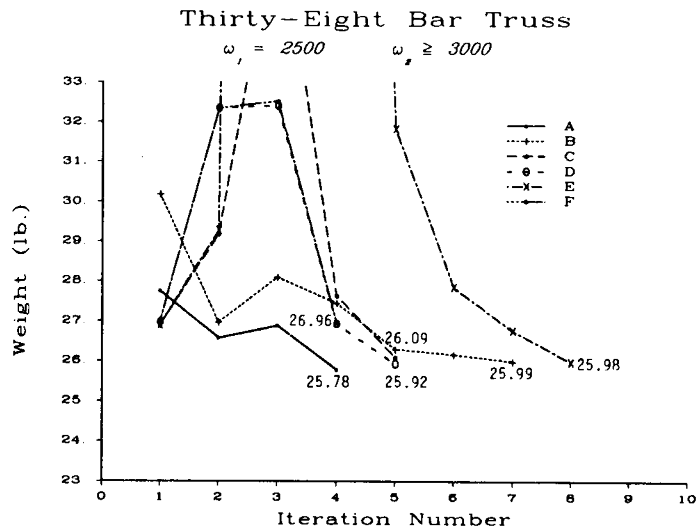


Figure 4 Iteration history for thirty-eight bar truss with multiple frequency constraints



Notes :

A : exponential resizing/approximate Lagrange multiplier formulae
B : linear resizing/approximate Lagrange multiplier formulae
C : exponential resizing/exponential Lagrange multiplier formulae
D : linear resizing/exponential Lagrange multiplier formulae
E : exponential resizing/linear Lagrange multiplier formulae
F : linear resizing/linear Lagrange multiplier formulae

Figure 5 Iteration history for thirty-eight bar truss with multiple frequency constraints