Fermi National Accelerator Laboratory

Pair Production of Helicity-Flipped Neutrinos in Supernovae

Armando Pérez

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Box 500, Batavia, Il. 60510-0500 USA

and

Departamento de Física Teórica
Universidad de Valencia
46100 Burjassot (Valencia) Spain

and

Raj Gandhi

NIKHEF
P.O. Box 41882
1009DB Amsterdam
The Netherlands.

Abstract

We calculate the emissivity for the pair production of helicity-flipped neutrinos, in a way that can be used in supernova calculations. We also present some simple estimates which show that such process can act as an efficient energy-loss mechanism in the shocked supernova core, and we use this fact to extract neutrino mass limits from $SN1987A$ neutrino observations.

1 Introduction

The observation of neutrinos from SN1987A [1,2], in fair agreement with predictions from supernova models, has been used by several authors to bound the properties and interactions of various exotic and non- exotic particles [3,4,5,6,7,8]. For those particles which act as an efficient energy loss mechanism for the supernova, the simplest constraint on their masses and couplings can be derived from the fact that the total energy carried away by them cannot be greater than the available energy of the star. The detected neutrino flux has substantiated several features of supernova theory which are now accepted as being standard [9,10]. First, the total emitted energy is $2 - 4 \times 10^{53}$ ergs . Secondly, it is emitted in the form of neutrino- antineutrino pairs of all species (with roughly equal amounts carried by each) formed inside the core via the Z^0 exchange [1] process $e^+ + e^- \rightarrow \nu + \bar{\nu}$. Finally, these neutrinos are trapped in the core and undergo slow thermal diffusion for several seconds until they reach the neutrinosphere, where they are released in large quantities.

If neutrinos are massive Dirac particles, then it is possible to produce neutrino- antineutrino pairs via the above process such that one of them is non-interacting, i.e either a positive helicity neutrino or a negative helicity antineutrino, which does not undergo diffussion but leaves the core much faster than its trapped partner[2]. As will be shown, this process can lead to significant energy loss on a much shorter timescale from the shocked core, depending on how massive the neutrinos are. Our main aim in this paper is to provide suitable expressions for the emissivity of this process in a form which can be easily incorporated into realistic supernova models to evaluate the energy lost in the form of these flipped neutrinos. To illustrate that the process can have significant consequences, we also derive, using general considerations, approximate expressions for these energy

---

[1]Electron type neutrinos are also produced via the W exchange channel.

[2]We confine our discussion to the Standard Model, [11,12,13] minimally extended to include Dirac masses for the neutrinos. A right-handed Majorana neutrino interacts in a manner similar (but not identical) to a right-handed Dirac anti-neutrino and hence cannot provide an avenue for rapid cooling.

losses. The emissivity of the process is proportional to the mass squared of the neutrino and the seventh power of the core temprature. We show that, using these expressions , it is possible to reliably exclude neutrino masses in the range $1 MeV - 100 MeV$. The above range of $1 - 100$ MeV is obtained using low core temperatures [21], and hence is on the pessimistic side. If higher temperatures are used, as is the case in some supernova models [22] then it is possible to exclude all neutrino masses between 100 KeV - 100 MeV using the considerations discussed below. As we will discuss, these mass limits would be improved by performing full supernova calculations.

This paper is organized as follows. In Section 2, we analyze the production amplitudes for helicity- flipped neutrinos. In Section 3, we calculate the emissivities associated to this processes, in a way that can be used in supernova calculations, and we make some estimations about mass limits. [3]. We end by summarizing our most relevant conclusions.

2   Pair production of helicity-flipped neutrinos

In this section we obtain the cross-section for the Z^0 exchange process

$$e^+(p_1) + e^-(p_2) \rightarrow \bar{\nu}_i(k_1, \lambda_1) + \nu_i(k_2, \lambda_2) \tag{2.1}$$

Here the k_i and p_i are the particle momenta and λ is the neutrino helicity. We recall that within the context of the standard theory, a neutrino interaction eigenstate (ν_L or ν_R) is a superposition of helicity (λ) eigenstates $\nu_\pm$, where $\lambda = \vec{\sigma}.\vec{p} = \pm 1$. For a relativistic particle, this translates into the statement that a ν_L is predominantly in the $\lambda = -1$ state and a ν_R is predominantly in the $\lambda = +1$ state, with small admixtures of the opposite helicity, of order m/E_ν. In particular, we are interested in the case where the final state neutrino has positive helicity, and is 'almost' non-interacting.

---

[3]Current experimental bounds on neutrino masses from particle accelerators are $m_{\nu_e} \leq 18 eV$, $m_{\nu_\mu} \leq 0.25 MeV$, $m_{\nu_\tau} \leq 35 MeV$ [14]. More stringent bounds on the masses of stable neutrinos have been derived from big bang cosmology [15,16,17]. The neutrinos we consider here would reasonably be expected to decay via mixing and other modes. Bounds from cosmology and astrophysics on the masses and lifetimes of unstable neutrinos have been derived in [18,19,20].

In what follows below, the subscipts 1 and 2 denote antiparticle and particle respectively. The amplitude for (2.1) is given by:

$$\mathcal{A} = \frac{G_F}{\sqrt{2}}\bar{u}(k_2,\lambda_2)\gamma^\mu(1-\gamma^5)v(k_1,\lambda_1)\cdot\bar{v}(p_1)\gamma_\mu(C_V-C_A\gamma^5)u(p_2) \tag{2.2}$$

The u and v are the usual Dirac spinors, and use has been made of the fact that, when the processes (2.1) occur in the core of the collapsed star, the center of mass energies are at most 1 GeV , hence the amplitude may be written in its effective four-fermion form. The electron and positron helicity indexes have been supressed since they will be averaged over. We then write

$$\frac{1}{2}\cdot\frac{1}{2}|\mathcal{A}|^2 = \frac{G_F^2}{8}N^{\mu\nu}E_{\mu\nu} \tag{2.3}$$

(where the spin averaging factors have been explicitly shown) with

$$N^{\mu\nu} = \frac{1}{4}Tr[(\not{k}_2+m)(1+\gamma^5\not{s}_2)\gamma^\mu(1-\gamma^5)(\not{k}_1-m)(1+\gamma^5\not{s}_1)\gamma^\nu(1-\gamma^5)] \tag{2.4}$$

and

$$E^{\mu\nu} = Tr[(\not{p}_2+M)\gamma^\mu(C_V-C_A\gamma^5)(\not{p}_1-M)\gamma^\nu(C_V-C_A\gamma^5)] \tag{2.5}$$

Here s_1 and s_2 are the spin four-vectors associated with the anti-neutrino and neutrino respectively, while m and M are the neutrino and electron masses. These spin vectors satisfy the Lorentz invariant conditions

$$s_i\cdot s_i = -1;\quad s_i\cdot k_i = 0; \tag{2.6}$$

and for a relativistic neutrino the additional constraint

$$\vec{s}_i\|\lambda_i\vec{k}_i \text{ for } i = 1,2 \tag{2.7}$$

holds, where

$$k^\mu = (E_\nu, |\vec{k}|\hat{\vec{k}}) \tag{2.8}$$

3

with $\hat{\vec{k}}$ being a unit vector along the three-momentum of the neutrino .

We now introduce two four-vectors associated with the neutrino pair:

$$K_1^\mu = k_1^\mu + ms_1^\mu \; ; K_2^\mu = k_2^\mu - ms_2^\mu; \qquad (2.9)$$

In conjunction with the properties in (2.6) and (2.7), these will allow us to write the amplitude squared for the process under consideration in a compact and physically revealing form. As a first step towards this, we note that the spin vector may be expressed as

$$s^\mu = \lambda m^{-1}(|\vec{k}|, E_\nu \hat{\vec{k}}) \qquad (2.10)$$

Using this and (2.9), we see that one may write

$$K_1 = \eta_1(1, \hat{\vec{k}}_1); \; K_2 = \eta_2(1, -\hat{\vec{k}}_2); \qquad (2.11)$$

with

$$\eta_1 = E_\nu + (E_\nu^2 - m^2)^{1/2} \; ; \eta_2 = E_\nu - (E_\nu^2 - m^2)^{1/2} \qquad (2.12)$$

Note that for $m << E_\nu$ we have:

$$\eta_1 \approx 2E_\nu \; ; \eta_2 \approx \frac{m^2}{2E_\nu} \qquad (2.13)$$

We can now evaluate the traces and the contraction $N^{\mu\nu} E_{\mu\nu}$ in a straightforward way to obtain:

$$
\begin{aligned}
N^{\mu\nu} E_{\mu\nu} = \; & 16(C_V + C_A)^2(p_1 \cdot K_1)(p_2 \cdot K_2) + \\
& 16(C_V - C_A)^2(p_1 \cdot K_2)(p_2 \cdot K_1) + \\
& 16(C_V^2 - C_A^2)M^2(K_1 \cdot K_2)
\end{aligned}
\qquad (2.14)
$$

Here C_V, C_A are the usual weak vertex factors. From this expression and equations (2.11) and (2.13) above one sees that the amplitude vanishes for massless neutrinos , as it should . Further, the expression (2.14) is akin to the usual weak pair production

4

amplitude with the replacement $K_i \rightarrow k_i$. Finally, the flip and non-flip cross-sections are related by more than just a simple factor of $m^2/4E_\nu^2$, since the η_i carry a sign affixed to the three-momentum, which in general depends on the nature of the final state (i.e whether it is a particle or anti-particle) and its helicity eigenvalue [5] .

In the next section we proceed to evaluate the emissivity for this process using (2.14).

3   Calculation of emissivity

We now proceed to calculate the emissivity associated with the pair production of helicity-flipped neutrinos. This is given by

$$Q_m = 4 \int \frac{d^3 \vec{p_1}}{(2\pi)^3} \frac{d^3 \vec{p_2}}{(2\pi)^3} f_{e^-}(E_{e^-}) f_{e^+}(E_{e^+}) \, \epsilon(\vec{p_1}, \vec{p_2}) \tag{3.1}$$

In the last equation Q_m is the emissivity, $\vec{p_1}$ ($\vec{p_2}$) are the e^- (e^+) momenta, E_{e^-} and E_{e^+} are their energies, and f_{e^-} (f_{e^+}) are their respective equilibrium Fermi-Dirac distribution functions, which are assumed to be

$$f_{e^-}(E_{e^-}) = \frac{1}{1 + \exp(E_{e^-} - \mu_{e^-})/T} \tag{3.2}$$

$$f_{e^+}(E_{e^+}) = \frac{1}{1 + \exp(E_{e^+} + \mu_{e^-})/T} \tag{3.3}$$

Here, μ_{e^-} is the chemical potential for the electrons and T is the temperature (we take $K_B = 1$ for the Boltzmann constant)

In eq. (3.1), $\epsilon(\vec{p_1}, \vec{p_2})$ stands for

$$\epsilon(\vec{p_1}, \vec{p_2}) = \int \frac{d^3 \vec{k_1}}{2k_1^0} \frac{d^3 \vec{k_2}}{2k_2^0} \, k_2^0 \, v \, d\sigma(p_1 p_2 \rightarrow k_1 k_2) \tag{3.4}$$

where v is the relative $e^+ - e^-$ velocity, and $d\sigma(p_1 p_2 \rightarrow k_1 k_2)$ is the differential cross section for the process described in (2.1) (later on, we will also include the process $e^+ e^- \rightarrow$

$\nu_L \bar{\nu}_L$, the cross section being the same that for $e^+e^- \to \nu_R \bar{\nu}_R$). Since the typical energies which are involved are in the order of magnitude of $\sim 100 MeV$, electrons will be treated as relativistic. Therefore, we have

$$Q_m = \frac{1}{2\pi^4} T^7 \int_0^\infty dx_1 dx_2 x_1^2 x_2^2 f_{e^-}(x_1) f_{e^+}(x_2) \int_{-1}^1 d(\cos\theta) \left[\frac{\epsilon(\vec{p}_1, \vec{p}_2)}{T} \right] \quad (3.5)$$

Where $x_1 = E_{e^-}/T$, $x_2 = E_{e^+}/T$ and θ is the angle formed by $\vec{p}_1$ and $\vec{p}_2$.

We have performed the integrals in (3.4). After a lenghty (although strightforward) calculation, we obtain

$$\epsilon(\vec{p}_1, \vec{p}_2) = \frac{G_F^2 m^2}{32\pi} \Omega(\vec{p}_1, \vec{p}_2) \quad (3.6)$$

The function $\Omega(\vec{p}_1, \vec{p}_2)$ is given by

$$\Omega(\vec{p}_1, \vec{p}_2) = E \left\{ (C_V^2 + C_A^2) \left[C_1(\alpha) + C_2(\alpha) \frac{E_{e^+} E_{e^-}}{E^2} (1 - \cos\theta)^2 + C_3(\alpha)(1 - \cos\theta) \right] + \right.$$

$$\left. 2 C_V C_A C_4(\alpha) \frac{E_{e^-} - E_{e^+}}{E} (1 - \cos\theta) \right\} \quad (3.7)$$

where $E = E_{e^-} + E_{e^+}$, $\alpha = |\vec{p}_1 + \vec{p}_2| / E$ and

$$C_1(\alpha) = -\frac{[(\alpha^6 - \alpha^4 - 4\alpha^3 + 3\alpha^2 + 4\alpha - 3) f_l(\alpha) - 2\alpha^5 + 8\alpha^4 - 4\alpha^3 - 8\alpha^2 + 6\alpha]}{2\alpha^5} \quad (3.8)$$

$$C_2(\alpha) = \frac{[(3\alpha^4 - 4\alpha^3 + 2\alpha^2 - 4\alpha + 3) f_l(\alpha) - 6\alpha^3 + 8\alpha^2 - 6\alpha]}{2\alpha^5} \quad (3.9)$$

$$C_3(\alpha) = \frac{[(\alpha^6 - 3\alpha^4 + \alpha^2 + 4\alpha - 3) f_l(\alpha) + 2\alpha^5 + 4\alpha^4 - 8\alpha^2 + 6\alpha]}{2\alpha^5} \quad (3.10)$$

$$C_4(\alpha) = \frac{[(3\alpha^4 - 4\alpha^3 + 2\alpha^2 - 4\alpha + 3) f_l(\alpha) - 4\alpha^5 - 6\alpha^3 + 8\alpha^2 - 6\alpha]}{5} \quad (3.11)$$

6

In the last equations, $f_l(\alpha) = \log(\frac{1+\alpha}{1-\alpha})$. The integral in $\cos\theta$ can be performed numerically, for different values of E_{e^-} and E_{e^+}. We realized that, for our purposes, the result of this integral can be approximated by the simple expression

$$\int_{-1}^{1} d(\cos\theta) \left[\frac{\Omega(\vec{p}_1, \vec{p}_2)}{T}\right] = \lambda_1(C_V^2 + C_A^2)(x_1 + x_2) - \lambda_2 C_V C_A(x_1 - x_2) \qquad (3.12)$$

Here, λ_1 and λ_2 are two numerical constants to be determined later. By inserting (3.6) and (3.12) in eq. (3.5) one can get a final expression in terms of the relativistic Fermi functions $F_n(\eta)$, defined as

$$F_n(\eta) = \frac{1}{n!} \int_0^{\infty} \frac{x^n dx}{1 + \exp(x - \eta)} \qquad (3.13)$$

here, η is the electron degenerancy parameter $(\eta = \mu_{e^-}/T)$. We compared the resulting expression with the exact numerical integral, as given in (3.5). We then found that $\lambda_1 \simeq 2.5$ and $\lambda_2 \simeq 8.0$ is a good choice for $\eta < 10$, whereas $\lambda_1 \simeq 3.5$ and $\lambda_2 \simeq 9.0$ seems more appropiate when $\eta > 10$. In that way, we obtain the final expressions

$$Q_m = \frac{3G_F^2}{8\pi^5} m^2 T^7 H(\eta) \qquad (3.14)$$

with the following approximations for $H(\eta)$:

$$H(\eta) = 2.5(C_V^2 + C_A^2)A^+(\eta) - 8C_V C_A A^-(\eta) \qquad (3.15)$$

$(\eta < 10)$

$$H(\eta) = 3.5(C_V^2 + C_A^2)A^+(\eta) - 9C_V C_A A^-(\eta) \qquad (3.16)$$

$(\eta > 10)$

where

$$A^{\pm}(\eta) = F_3(\eta)F_2(-\eta) \pm F_2(\eta)F_3(-\eta) \qquad (3.17)$$

In eq. (3.14) we introduced a factor of 2 to account for the combined emissivity of the two processes $e^+e^- \to \nu_L\bar{\nu}_L$ and $e^+e^- \to \nu_R\bar{\nu}_R$

Substituing for the values of the constants, one obtains

$$\frac{Q_m}{erg.cm^{-3}s^{-1}} = 5.281 \times 10^{29}(m/1MeV)^2(T/10MeV)^7 H(\eta) \qquad (3.18)$$

In Fig. 1 we show the two approximations given in (3.15) and (3.16) (dashed line and dot-dashed line, respectively) for $H(\eta)$. We also have plotted the values that result from a direct numerical integration of (3.5). The error in using this approximate expressions is only a few percent.

As one can see from eq. (3.14-17) the emissivity is highly dependent on the temperature and electron degenerancy in the supernova core. Therefore, in order to get an appropiate neutrino mass limit, one should incorporate our expressions in a realistic supernova calculation. However, in order to motivate such a calculation, we present some simple estimates. The highest temperatures in a supernova collapse are reached in the shocked, outer core, during the first few seconds of the neutron star cooling. In this region, the temperature amounts to several tens of MeV, and $\eta \approx 0$ (see ref [21]. For this model, the peak temperature is higher than $35MeV$).

In Fig. 2, we show our estimates for the total luminosity of the shocked core in the form of helicity-flipped neutrinos, for various temperatures (we assumed a characteristic core radius $R = 10\ Km$). As can be seen, for modest temperatures $T = 35MeV$ [21] one can exclude neutrino masses larger than about $1MeV$, using arguments identical to those in ref. 5. However, for supernova models with higher temperatures $T \approx 70MeV$ [22,23], the corresponding mass limit is about $100KeV$ (we are convinced that this limits can be improved by performing realistic calculations, since the neutrino luminosity would

be integrated over a few seconds).

We now address the question of whether very massive neutrinos can be ruled out from the above considerations. For these neutrinos, the re-flipping process into standard neutrinos can proceed via scattering with targets such as electrons, neutrons and protons in the core [5]. In fact, for neutrino masses larger than a few MeV, the mean free path associated with re-flipping becomes comparable to the core radius, and hence one could claim that flipped neutrinos will be trapped, rather than freely escaping. However, they will be trapped only temporarily, because neutrinos will continue to be flipped. The important criterion is when transport is more effectively done by flipped neutrinos rather than unflipped ones. Hence, the relevant quantity is the difussion time-scale associated with re-flipping

$$t_{diff} = \frac{3R^2 n \sigma_{flip}}{\pi^2 c} \tag{3.19}$$

with n being the number density of the targets, and σ_{flip} the cross section for re-flipping (which is proportional to the neutrino square mass). If t_{diff} is of the order of 1 sec. or so, helicity-flipped neutrinos become effectively trapped, and no longer act as an energy loss source. For standard values in (3.19), this only happens when the neutrino mass is $m > 100 MeV$ [5]. Therefore, neutrinos which are more massive than this limit can not be excluded by helicity-flipping processes (however we note that such high masses for μ and τ neutrinos are anyways ruled out by accelerator limits and hence reflipping is not a relevant concern here).

4   Conclusions

We have calculated accurate expressions for the emissivity due to pair production of helicity-flipped neutrinos with a Dirac mass. We showed that this process can act as an efficient energy-loss mechanism in the shocked core of a supernova. Therefore, this can

be used to put limits on the neutrino mass, by using the data of the detected neutrinos from $SN1987A$.

Because of the high temperature-dependence of the corresponding emissivity, these mass limits are better in the case of supernova models with large core temperatures [22,23]. If these models are reliable, all Dirac neutrino masses in the range $100KeV$-$100MeV$ can be ruled out by our simple estimates. A full supernova calculation should certanly lower the $100KeV$ limit (probably within a factor of 3 or so), since the neutrino luminosity would be integrated over several seconds. In this way, mass limits which are comparable to the ones obtained from scattering helicity- flipping processes [5,7] can be reproduced by an independent mechanism.

If one addopts models with much lower core temperatures [21], the corresponding (pessimistic) mass limit is higher in about a factor of 10. However, even in this case, the extreme sensitivity of the supernova explosion to neutrino properties (specially for the so-called delayed explosion scenario [22,24]) could lead to a much more stringent limit.

References

[1] K. Hirata et al. , *Phys. Rev. Lett.* , **58** , (1987) , 1490.

[2] R. Bionta et al. , *Phys. Rev. Lett.* ,**58** , (1987) , 1494.

[3] R. Barbieri and R.N. Mohapatra, *Phys. Rev. Lett.*, **61** 1988,27 .

[4] J. Ellis and K. Olive, *Phys. Lett. B*, 193, (1987), 525

[5] K. Gaemers , R. Gandhi and J. Lattimer, NIKHEF prerprint 88/18, to appear in *Phys. Rev. D.*

[6] J. Lattimer and J. Cooperstein , *Phys. Rev. Lett.*, **61** (1988) 23.

[7] G. Raffelt and D. Seckel, *Phys. Rev. Lett.*, **60** , (1988) ,1793.

[8] M. Turner, *Phys. Rev. Lett.*, **60** , (1988) ,1797.

[9] J.M. Lattimer, *Nucl. Phys.* ,**A478**, (1988), 199.

[10] D. Schramm in *Proc of the La Thuile Conference*, February 1988.

[11] S. Glashow , *Nucl. Phys.*, **22**, (1961), 579.

[12] S. Weinberg , *Phys. Rev. Lett.*, **19** ,(1967) 1264.

[13] A. Salam in *Elementary Particle Physics, Nobel Symp.* 8, ed. N. Svartholm, (Almqvist and Wilsell, Stockholm, 1968).

[14] Review of Particle Properties, *Phys. Lett.*, **B204** , (1988).

[15] R. Cowsik and J. McClelland, *Phys. Rev. Lett*,29, (1972),669.

[16] B. Lee and S. Weinberg, *Phys. Rev. Lett.*, **39**, (1977),165.

[17] J. Gunn, B. Lee, I. Lerche, D. Schramm and G. Steigman, *Ap. J.*, **223**, (1978),1015.

[18] D. Dicus, E. Kolb, V. Teplitz, *Ap. J.*, **221**,(1978),327.

[19] D. Dicus, E. Kolb, V. Teplitz, *Phys. Rev Lett*, **39**, (1977),169.

[20] R. Cowsik, *Phys. Rev. Lett*, **39**, (1977),784.

[21] A. Burrows and J.M. Lattimer, *Ap. J.* **307**, (1986)178

[22] J. Wilson in *Numerical Astrophysics*, ed J. Centrella et al, (Jones and Bartlett, Boston, 1985).

[23] R. Mayle and J. Wilson, submitted to *Ap. J.* (**1987**).

[24] H.A.Bethe and J.R. Wilson, *Ap. J.* **295**, (1985)14

Figure Caption

Fig. 1.- The function $H(\eta)$ (solid line), as given by a numerical integration of (3.5). Also shown, the two approximations corresponding to eq. (3.15) (dashed line) and (3.16) (dot-dashed line).

Fig. 2.- Total emissivity for helicity-flipped neutrinos in the core, as a function of the neutrino mass, for three different core temperatures. A typical core radius $R = 10 Km$ has been assumed.
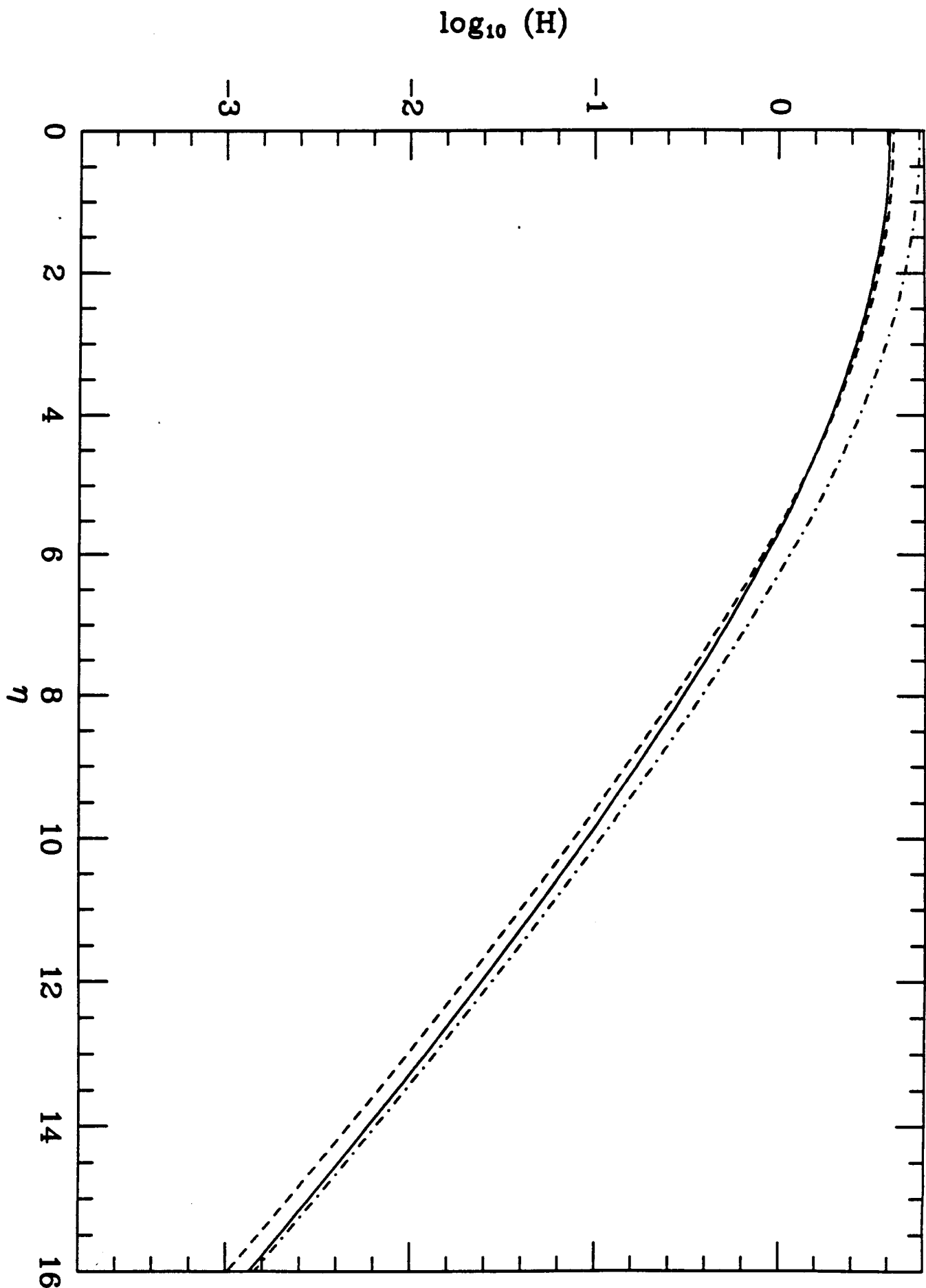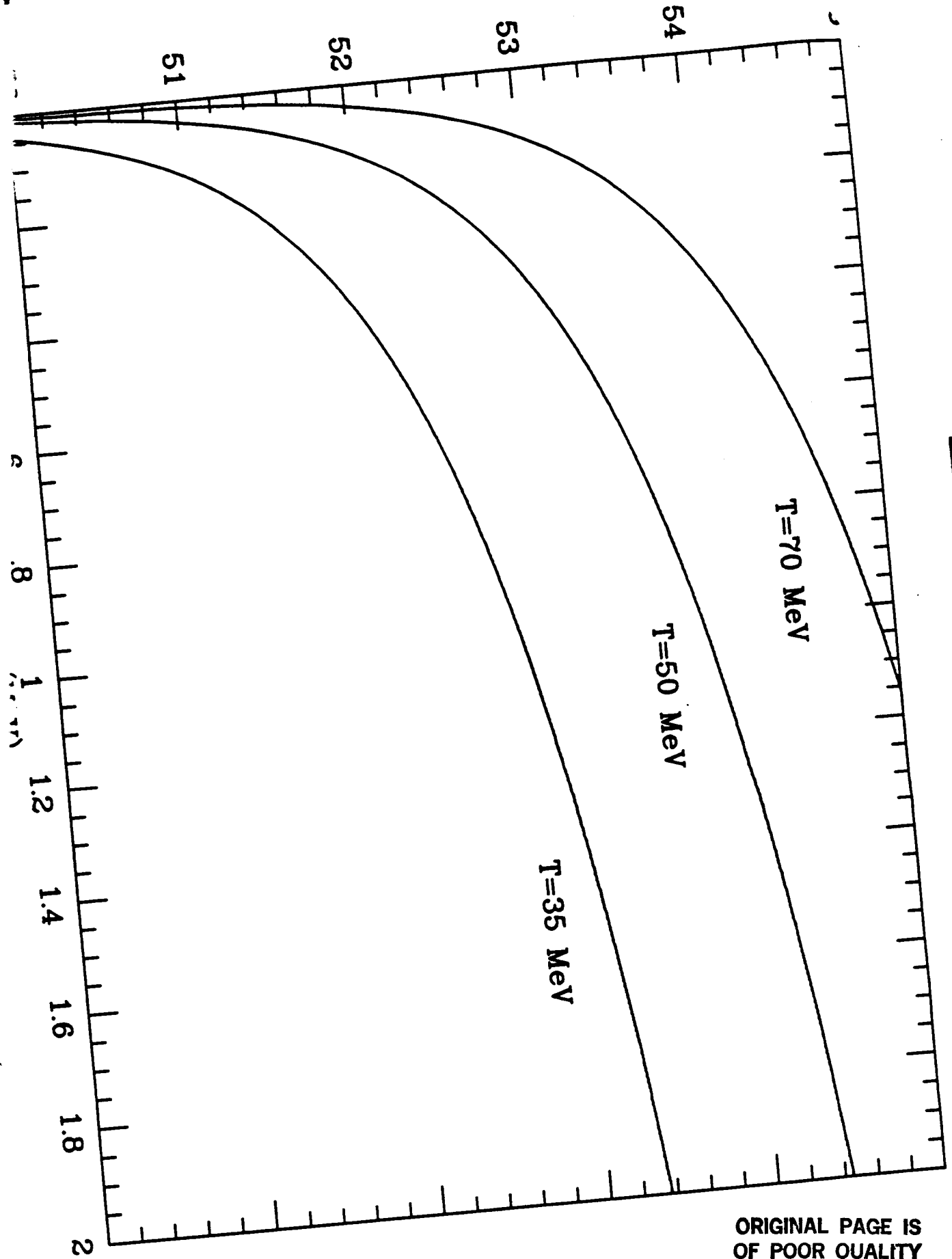
FIGURE 1

$\log_{10} (L_\nu /\text{erg. s}^{-1})$

T=70 MeV

T=50 MeV

T=35 MeV